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University of Southampton
Southampton, SO9 5NH
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The International Commission on Mathematical Instruction

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ICME 7 QUEBEC 1992

SEVENTH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

The 7th International Congress on Mathematical Education (ICME-7) will be held in the city of Québec (Canada) from August 16 to 23, 1992. It will be the seventh in a series of congresses of the International Commission on Mathematical Instruction (ICMI), following those of Lyons (1969), Exeter (1972), Karlsruhe (1976), Berkeley (1980), Adelaide (1984) and Budapest (1988).

In an effort to meet the diverse needs and interests of the 3000-3500 expected participants, the programme will cover all of the major areas of mathematics education at the elementary, secondary and post-secondary levels. Activities will include lectures, working groups, topic groups, workshops, short communications, posters, project presentations, and films, as well as exhibitions of textbooks, software, and other types of materials. Here are a few examples of themes that will be discussed during the congress:

Improving students' attitudes and motivation
Mathematics for early school leavers
Innovative assessment of students in mathematics
Students' misconceptions and inconsistencies of thought
The impact of calculators on the elementary school curriculum
The role of geometry in general education
Probability and statistics for the future citizen
Modelling activities in the classroom
Students' difficulties in calculus
Undergraduate mathematics for different groups of students
Pre-service and in-service teacher education
Methodologies for research in mathematics education

Founded in 1608 by Samuel de Champlain, the city of Québec, capital of the Canadian province of the same name, is the cradle of French civilization in North America. Because of its unique charm, its historical past and its exceptional location dominating the majestic Saint-Laurent river, Québec is a privileged place which attracts tourists from all over the world. The ICME-7 Congress will take place on the campus of Université Laval, which offers facilities and services making it a most convenient place to hold such an international event.

The Second Announcement of the congress will be published in 1991 and will contain detailed information about the programme, as well as forms for registration, accommodation, and submission of short communications or posters. In order to receive it, please write to: **Congrès ICME-7 Congress, Université Laval, Québec QC, Canada, G1K 7P4,** or communicate with the secretariat of the congress by phone: (418) 656-7592, or by fax: (418) 656-2000, or by electronic mail: ICME-7@LAVALVM1.BITNET

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An ICMI Study on:

ASSESSMENT IN MATHEMATICS EDUCATION AND ITS EFFECTS

Discussion Document

The International Program Committee appointed by the International Commission on Mathematics Instruction (ICMI) announces a study on:

"Assessment in Mathematics Education and its Effects"

The study consists of two components, a **conference**, to be held in Spain, 11-16 April, 1991, and a **volume**, to be published in the ICMI Studies series based on (the contributions to and products of the outcomes of) the conference. This discussion document: (1) provides background on the study, and outlines its aims, scope, and issues; (2) announces a call for papers; and (3) provides preliminary practical and organizational information to potential contributors to the study.

BACKGROUND AND OUTLINE OF THE STUDY

Why an International Study on Assessment?

Each ICMI study is conducted to contribute to the understanding and tackling of a specific topic or problem area which is of current importance to mathematics education in countries in different parts of the world.

Why is **assessment** of current importance to mathematics education in an international context?

The Program Committee believes that in many countries difficult questions, serious problems, and major challenges exist with respect to assessment in mathematics education. The committee also believes that many current assessment practices are counterproductive, the products of (and supports for) outmoded educational traditions which fail to meet today's views about mathematical and societal needs.

These challenges would <u>not</u> exist:

- if society were not experiencing rapid and substantial changes, and if these changes were not in turn bringing about changes in mathematics instruction in ways that call for new approaches to assessment;

- if the roles, functions, and effects of contemporary methods of assessment were completely clear and well understood;

- if there were no divergent aims or conflicting interests and no unintended, undesired, or sometimes even dangerous side- effects involved in the current methods of assessment;

- if we had devised and implemented methods of assessing, in a valid and reliable way, the essential knowledge, insights, abilities, and skills related to mathematics and its place in the world; and

- if current assessment procedures could provide genuine assistance to (a) the individual learner in monitoring and improving his or her acquisition of mathematical insight and power, (b) the individual teacher in monitoring and improving his or her teaching, guidance, supervision, and (c) textbook authors, curriculum planners and authorities, in adequately shaping the framework of mathematics instruction.

Unfortunately, the Program Committee is convinced that most of these conditions are problematical in schools, colleges and universities throughout the world today. Failure to confront these issues raises important and significant questions which must now be addressed.

In brief: if, in mathematics, it were easy to employ effective, harmonious, assessment procedures free from serious internal or external problems, an ICMI study on assessment would not be relevant. Nor would it be relevant if the difficulties were present only in a few isolated nations, for then an international perspective would not be justified.

It has become clear, however, that serious questions about mathematics assessment are being raised in countries throughout the world. Although we recognize that the purposes, roles, functions, and practices of assessment may be viewed very differently in different educational systems and in different societies, there is no doubt that the crucial questions, problems, and challenges now being addressed are very similar worldwide. Thus, the 1991 conference and volume should be of common interest to researchers and practitioners of mathematics education all over the world. What faces mathematics educators are matters and issues about assessment of a fundamental rather than of an incidental nature. Thus, the goals of the study are: to present and examine current assessment practices in many nations; and to identify examples, practices, and ideas that will enable assessment to become universally a positive influence on instruction by contributing to link together the purposes, implementation and outcomes of any mathematical program.

The Function of Assessment

In every educational system, the purpose of assessment in mathematics education is to provide information, gathered in a specific manner, for a specific constituency, about the mathematical performance of a student or a group of student. The objective of gathering this information is to assist "someone" to make decisions about students, teachers, or programs. The information sought may be relative (i.e., it may compare one outcome with a larger sample of outcomes), or absolute (i.e., it may determine the quality of the particular outcome). Finally, the information may be reported in either a qualitative or a quantitative form.

What varies in different assessment approaches is:

(1) the kinds and forms (relative, absolute; qualitative, quantitative) of the information to be collected;

(2) the way of gathering such information;

(3) the profesional position of those who gather the information;

(4) the "someone" receiving and using the information;

(5) the types of outcome performances;

(6) the unit of aggregation of the information (individual, group, class, cohort, nation);

(7) the types of decisions or actions that might be taken as a result of the information; and

(8) the students, teachers or programs that are the objects of the decisions or actions to be taken.

The possible variations in each of those components resulting from combining the diverse approaches encountered in actual practice is, of course, tremendous. Yet, we are dealing only with variations on a theme. And it should not be forgotten that the salience of certain elements tends to set the entire character of the assessment approach adopted.

Traditionally, the purpose of student assessment in mathematics education has been to determine whether a given individual should be granted access to certain privileges. A typical privilege is *either* a "license" to practice a profession or vocation based on mathematics - for instance, as a surveyor, an actuary, or a teacher of mathematics in a school or university - or, more importantly, a "ticket" to further education in the next grade, in a new subject, in a new institution, for which achievements in mathematics have been made part of the entrance requirement. An examination results in a verdict: passed or failed, admit or deny admission. It is typical of this conception of assessment that an evaluation is not being made of the assessment procedures themselves, nor of the curriculum, the institution, the textbooks, the instruction, the teachers, the assessors, and so forth. Only the individual student, viewed as an *object*, is being judged, and decisions or actions concerning the student are the focus of attention.

This function of assessment as a tool for the selection or placement of people continues to be strong, even dominant, in most educational systems throughout the world. However, a broader conception of assessment has begun to emerge in recent decades. Information about student performance is now being used to judge other components.

Information about the outcomes of mathematics instruction is now being sought for the general purpose of informing and guiding teaching practice, teacher performance, and curriculum development. Given this development, an important question then is: *Does mathematics instruction function satisfactorily in relation to different groups of learners?*

If not, where are the problems? With the students? With their family backgrounds? With the teacher(s)? With the institutional environment? With the textbooks? With the curriculum or program? So, we are led to an ensuing four-fold question: What instructional procedures, under what conditions, and for what students, are effective in achieving what types of learning in mathematics?

If the answer is yes, are further improvements possible? The "someone" who is, or should be, interested in knowing the answers to these questions varies, ranging from the individual student to local or national curriculum authorities and administrative and government agencies. The system which attempts to obtain the answers varies in a similar manner. The decisions or actions to be taken as a result of the information gathered are not limited to addressing the individual student. They may address any element in the spectrum of components which constitute mathematics instruction.

First of all, the learner is no longer considered as merely an object of assessment, but as an **autonomous individual** too, as an individual with his or her own dynamic to grow and develop, a person who has the right to demand a certain quality from the mathematics instruction he or she is receiving. In modern society, mathematical competence is essential not only to obtaining access to careers (whether educational or vocational/professional), but it is also crucial in exercising active, responsible citizenship. The learner may want to insist that mathematics instruction should provide such competence as well. When considering the individual student as an independent person, there is a growing trend in assessment to respect the integrity of what the student knows, how the student knows, and why he or she seeks to know. Mathematical knowledge is being increasingly viewed as a working force in the life and being of the student and has to be assessed as such.

The outcomes of mathematics instruction are now regarded as having much wider implications than before. More emphasis has been put on students' ability to actively and creatively deal with mathematical ideas, concepts, topics, problems, and issues within mathematics itself as well as in extra-mathematical contexts. Problem posing and problem solving, modeling and applications, open-ended situations, investigations, scientific debate, and so forth, have been introduced in mathematics instruction in many areas and at many levels. Briefly, it has become important to give students as much opportunity as possible to engage in the same *kind* of activities and processes, although not at the same level, as mathematical professionals in a broad meaning of that term: thinking and acting mathematically. So, mathematics instruction is being given many more dimensions than before.

When the outcomes of mathematics instruction are perceived in a wider way, the information about them also must be perceived in this wider way. The crucial question here is: To what extent do we possess the means to obtain valid, authentic information about student performance? Although in recent years much work has been done to acquire more valid information about the outcomes of mathematics instruction, the methods of gathering such information lag far behind the need for it. This is due not only to inertia in educational systems but also to insufficient ingenuity, research, and development in the field, and to insufficient resources for creating new methods of assessment.

Finally, we should not forget that any system of assessment strongly influences, for better and for worse, the educational system in which it is embedded. The way mathematics instruction functions, as well as the entire spirit in which it takes place, is strongly influenced by assessment methods. Assessment is not just a separate appendix to mathematics instruction; it is one of its crucial components.

Scope of the Study

The changing perception of assessment outlined above encompasses, in principle, each agent and component of mathematics instruction as an actual, and potential, object of assessment. In this context, the terms *assessment* and *evaluation* are often used interchangeably without a clear difference in meaning being made between them. The Program Committee suggests that *assessment* be used to refer to the outcome of mathematics instruction as reflected in the performance of **students** as individuals or in groups, whereas *evaluation* should deal chiefly with the use of such performance information to make judgements about instructional **programs**, **curricula**, and appraisal of teachers. In order to obtain a clear focus of the present study, and in order to limit it to a tractable size, the dominant emphasis will be on **assessment** as just defined, but within the wide conception of outcomes outlined in the previous section. This limitation does not imply that issues related to evaluation are to be left out of consideration in the study. However, the evaluation of programs, curricula, and teachers will primarily be addressed as they are reflected in the assessment of students' performance. To avoid possible misunderstandings of this: it is the firm view of the Program Committee that programs, curricula, and teachers should not be judged solely on the basis of student performances.

Problems and Issues

The problems and issues to be addressed in the study conference concentrate on the various purposes, roles, and functions of assessment of students' mathematical performance. In particular, to contrast practices among countries, the following fundamental questions about assessment procedures and their uses need to be addressed:

* What are the significant historical developments in the philosophy and evolution of assessment and evaluation?

* For what purposes is informtion about students' mathematical performances being gathered? (To help teachers make instructional decisions? To assist students in monitoring and controlling their own learning process? To select or place students? To evaluate the effects of new programs?)

* What are the units on which information is being aggregated - the individual student, group, class, teacher, institution, program, system?

* For what kinds of mathematical tasks are students' performances being assessed (short technical exercises, long tasks, extended problems, portfolios, project reports)? And what kind of information is being gathered (examination of written items, of oral responses or responses to oral questions; observation of performance)?

* Who gathers the information?

* How is information gathered, coded, and recorded?

* How is the coded data aggregated and analyzed?

* What kinds of decisions or actions are taken on the basis of the information gathered?

* Are new procedures being developed/tried out?

* Are there conflicting views or interests between different segments of the educational system in relation to assessment and evaluation (e.g., between government authorities and mathematics educators)?

* What are the important differences in the assessment practices of different countries?

* How *useful* are international performance comparisons?

While the questions above serve primarily descriptive purposes, the following questions focus on the analysis of different modes of assessment with particular regard to their influences and effects:

* What are the theoretical and empirical foundations of current assessment procedures, and to what extent are these procedures valid, reliable, efficient?

* What are the known influences of external assessment practices on mathematics instruction?

* Are there examples of assessment practices which are known to influence instruction positively? What aspects should be maintained and encouraged?

* Are there examples of assessment practices which negatively influence instruction; for example, by focusing instruction on assessment and tests rather than on more general goals?

* How do different assessment modes influence the social environment in the classroom?

* In what respects are teachers good or bad judges of student performance? And to what extent can they be trained to be good judges?

* How does the teacher's assessment role conflict with his/her supportive role?

* In many countries, university professors are considered professionally capable of assessing students justly, validly, and accurately, whereas school teachers are not; thus, external examinations are judged necessary. Does this make any sense?

* It is widely recognized that most current assessment practices deal mainly with independent facts and skills. Many of these practices have a high degree of reliability but a low degree of validity. To what extent is it possible to devise assessment modes which are both valid and reliable? How well do we assess authentic abilities such as the capacity for scientific debate, problem formulation, and problem solving, modelling, application, etc.?

* What assessment modes are suitable in relation to different types of tasks, such as short technical exercises, long tasks, extended problems, project work, etc.?

* How can assessment be embedded harmoniously into instructional practices as an instrument to serve the needs of both teachers and students in everyday instruction

- performing tasks in natural situations and in contexts psychologically close to the learner, not in isolation; respecting the cultural setting of mathematics; making use of a variety of ways of accomplishing specific tasks?

* What are the main obstacles to devising and implementing innovations in assessment, and what can be done to overcome these obstacles?

Structure of the Study

To accommodate a thorough treatment of the issues listed above, the study will contain four sections.

1. A **descriptive** section:

In this section, the most important conceptions and modes of assessment practiced in different countries or educational systems will be identified and described. Emphasis will be on *archetypes* rather than on peculiarities.

2. An **analytic** section:

This section will establish a framework for analyzing goals, functions, effects, consequences, limitations, possibilities, difficulties, and problems, related to the assessment of students' mathematical capabilities. By means of this framework, specific analyses regarding the issues listed above will be carried out. Furthermore, important empirical or theoretical research contributions to the field will be presented.

3. A selection on the presentation and discussion of **innovative/experimental** cases: In many places around the world, very interesting innovative/experimental work on new modes of assessment and evaluation has been or is being done. This section will present and discuss a number of the most interesting examples.

4. A **statement** section:

This section will present a formulation of policy statements and recommendations if appropriate. At the least, it will serve to identify important open issues as well as objectives for future research and development.

CALL FOR PAPERS

Given this background, the International Program Committee invites individuals and groups to propose or submit contributions to the study for consideration by the Committee. Contributions should be related to the problems and issues identified in the present document and fit into at least one of the four sections of the study just described. Participation in the conference is *only at the invitation* of the Program Committee, but those who submit a contribution are encouraged to apply for an invitation.

In addition to calling for papers, the Program Committee will solicit contributors to address specific questions, to present research contributions, and to share examples of innovative work at the conference. We invite suggestions on topics and names of potential contributors. Also, comments regarding the structure of the conference will be welcome.

The Program Committee will meet in November 1990 to make major decisions about the conference program. For matters regarding the program, please contact:
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PRACTICAL INFORMATION

The study conference will take place in:

Calonge (Costa Brava), SPAIN, April 11-16, 1991.

The number of participants will be limited to about 75. The local organization of the conference will be taken care of by the *Federación Española de Sociedades de Profesores de Matemàticas* and the local organizer will be:
Professor Claudi Alsina
Secció Matemàtiques, ETSAB
Universitat Politècnica de Catalunya
Diagonal 649
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The International Program Committee consists of:

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David Robitaille, University of British Columbia, Canada

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Past ICMI Studies

Readers are reminded that

Mathematics and Cognition, the research synthesis produced by the Psychology of Mathematics Education group, has now been published by the Cambridge University Press.

Hardcover	ISBN	0521	366089
Paperback	ISBN	0521	367875

The Popularization of Mathematics, the report of the 1989 Leeds symposium, should appear later this year.

We reprint on following pages a report from the Leeds Working Group on **'Games and Puzzles'**, which we hope will interest readers and which could not be reprinted in its entirety in the Leeds proceedings.

GAMES AND PUZZLES

The popularisation of mathematics begins, or fails to begin, at the start of, or even before, formal schooling. Thus, much of the following is directly applicable to the mathematics classroom. However, games and puzzles can be used both outside and after formal schooling to develop, improve and enhance the image of mathematics held by the public. In what follows, non-classroom settings for mathematical games and puzzles will be referred to as workshops.

First, however, it is necessary to address the question of what is meant by games and puzzles in the context of popularising mathematics. The following working definitions are suggested.

A game is a group activity, engaged in for pleasure, with a structure which permits a mathematical interpretation.

A puzzle is a game for one person or a group of people acting collaboratively rather than competitively. Perhaps we can describe puzzles as "one-mind" games.

WHAT MATHEMATICS CAN BE DERIVED FROM GAMES?

(a) The Growth of Mathematics

Historically mathematics possesses both artistic and game-like components, which are so significant that any field of mathematical work which does not attain a certain level of aesthetic satisfaction "remains unstable, reaching for a more polished expression that might convey a unitary, harmonious, pleasurable, amusing vision, in the same way as an unfinished symphony or poem stretches out in the mind of the author for the most beautiful possible form". (Guzman, 1989.)

Mathematicians who can be seen to have been motivated by a game-like spirit include Fibonacci, Cardano, Fermat, Pascal, Liebniz, Euler, D. Bernoulli, Gauss, Hamilton and von Newmann. Similarly many branches of mathematics have, by their very nature, suggested an abundance of games and puzzles. Besides arithmetic, geometry, number theory which are the traditional sources of recreations, more modern branches of mathematics such as topology, combinatorial geometry, graph theory, logic, probability theory can be drpawn on. In all these older and younger fields there are open problems of an amusing and attractive nature, which, like Fermat's last theorem, are easy to state and difficult to solve, and which await the creation of new thinking processes that can throw some light on their solution.

Again, quoting Guzman (1990) "Mathematics is a great and sophisticated game ... The attempts to popularize mathematics through its applications, its history, the biography of the most interesting mathematicians, through the relationship with philosophy or other aspects of the human mind can serve very well to let mathematics be known by many persons. But possibly no other method can convey what is the right spirit of doing mathematics than a well chosen game."

The above paragraphs consider the role of games in relation to the growth and popularity of mathematics as a subject. The role of games and puzzles in the mathematical development of the individual, is of parallel significance.

(b) Games and the Mathematics Learner

Many adults testify to having developed facility at arithmetic through games played in childhood. In western cultures these include such recreations as card games, dominoes and even darts. Perhaps even more powerful as "arithmetic tutors" are the many variations of the game mancala which is widely played in African, Asian, Caribbean, and South American countries.

In recent years, and in many parts of the world, computer games seem to have replaced, and possibly even improved on, the teaching of geometry in developing spatial sense/awareness.

Sometimes, the mathematical component of a game can be recognised and built on immediately. Sometimes, however, playing games early in life can lead to much later recognition of the mathematics involved. This is well illustrated in the following account (Walter, 1986).

> "People often ask me how I decided to study mathematics and I have rarely had a chance to tell the whole story. Actually, I am not sure I know the whole story. What I think influenced me may not, in fact, be a factor.
>
> My father was in the bead business and I know I had boxes full of beads to play with. I must have done much threading, counting and pattern making. Could my interest in symmetry stem from trying to make "balanced" necklaces? I also had sets of parquet blocks and I loved to make the designs provided on sheets. Is that how I got my intuitive feelings for geometry? ... Another favourite toy - wooden pieces of various creatures cut into thirds so that one could mix and match them. (No I was not precocious and did not work out how many different combinations were possible!) ... One other toy should be mentioned because thereby hangs a tale. We are always so ready to test students the minute that have learned something - but I realised, only perhaps 40 years later, that I did learn something from this toy.

It was a box on which one could place various sheets of paper with questions and answers on them. By using two metal wires the bulbs lit up if the correct answer was chosen for the question. Well I soon found out that the relative positions of questions and answers was the same for each sheet - and I felt a bit jipped. Only many years later did I realise that I was dealing with a very simple version of isomorphism."

Games have another important function for the learner as is pointed out in Ainley (1988), who discusses the importance of games which

"not only enable children to learn mathematics, but also provide rare opportunities for children to do real mathematics in the classroom.

Real mathematics here means mathematics which is important and meaningful to children, and doing what real mathematicians do, using mathematical processes and thinking in a mathematical way. These two meanings are related since if children are thinking mathematically then the mathematics they tackle is more likely to become important to them personally. Equally, if children engage in problems and activities which are important for them, they are more likely to see the value of mathematical processes."

Ainley goes on to discuss the similarities and differences between learning to read and write and leaning mathematics. She points out that once a child can read and write, even to quite a limited extent, they can use these skills for their own purposes, but that occasions for using mathematics learning can be less obvious. She goes on to claim that:

"Mathematical games are one way of providing the equivalent of children's books and comics; within a game there is a context for using some mathematics that you have learned, and that context is real for children because they engage with it and the outcome matters to them."

WHAT ARE THE CHARACTERISTICS OF A GOOD (MATHEMATICAL) GAME?

As the above discussion may suggest, there are a variety of games which are good in specific situations, some which are good in many situations, but it is not to be expected that any game will be successful in all situations. What may be crucial is that an appropriate game or puzzle is encountered by a particular learner. Also important are both the context in which the game is set and who the fellow players are, with different populations group responding differently to different games. Nevertheless, there are general points to be made.

In general, games are closed systems governed by sets of rules. Games with relatively simple rules are very valuable in that they can be learned quickly and played independently of an external instructor or judge. Without an external arbitrator, players may need to justify their moves to each other and thus begin to develop the power to express arguments clearly and unambiguously, an important aspect of mathematics.

On the other hand, the rules which effect the closure are arbitrary, and may be varied or extended, sometimes spontaneously by the players themselves, sometimes at the suggestion or with the support of a teacher or workshop leader.

Games also involve a target and a set of activities which contribute to that target. In a good game, both target and activities will motivate the players to keep trying.

So far it has been assumed that games are competitive, and puzzles co-operative in their nature. Sometimes the competitive nature of games is helpful in enhancing motivation, especially when the blend of strategy and luck is such as to equalise chances of winning between players of differing sophistication. Sometimes, however, if the structure of the game is sufficiently engaging, players will unite to find the "best move" in a set of circumstances, whichever player is likely to benefit.

A game of pure chance is unlikely to be mathematically enlightening, though sometimes players may become interested in the probability of winning.

Similarly a game of pure strategy, where that strategy can be fully appreciated and utilised by an unsophisticated player, as for example in "Noughts and Crosses" may become dull if played repetitively. However, if the object of the exercise moves from simply playing to win, on to describing how to win, and even to a classification of simple strategy games, then mathematical interest is again invoked.

IS IT POSSIBLE TO FIND A GAME FOR EVERY ASPECT OF MATHEMATICS?

Whether we believe that mathematics is invention or discovery, it is the case that all known aspects of mathematics were once problems or more formally research questions. Mathematicians are frequently heard to say "I played with the idea", hence, it can be claimed with some justification that all mathematics was once a puzzle or game.

However, it is not practical to introduce all mathematical ideas in the form of the original question addressed. Some mathematics comes in a form where the original "game" is recognisable, while other aspects may need considerable ingenuity to present them through more open game-like approaches. Indeed, sometimes exposition may be the appropriate way to share a mathematical idea whether we are teaching or popularising. All learners benefit from a variety of ways of working.

Although instances have been quoted of the value of games in providing a context for the consolidation of mathematical content, perhaps they are most valuable in introducing or supporting the development of the process of doing mathematics. Particular games may also model aspects of the nature of mathematics itself.

For example, a game like chess provides experience of an axiomatic system. The first step in learning chess is to familiarise oneself with the objects (or elements) and rules (or axioms). By playing, specific problems will be encountered. With well matched opponents the difficulty of problems recognised and tackled will increase in complexity with time and experience.

A recent article (Times Educational Supplement, 8th September, 1989) describes a successful way of teaching and learning chess which suggests interesting parallels for mathematics learning.

> "First of all, we just play with pawns on the board, so that children can get to know the moves this particular piece can make. Even the pawn game can be quite challenging, with the winner being the first person to reach the other end and turn that pawn into a queen. So after they've mastered the pawns, that's the next piece we introduce, followed by all the other pieces until they are finally playing with the full set."

This is followed up by exercises like the Knight's Driving Test, in which the knight is challenged to take all eight pawns in just 18 moves.

> "... with all these mini-games and exercises, children can progress at their own pace and only tackle the complexity of full chess when they're ready. In the meantime, they aren't bored and they aren't discouraged."

Some computer games can offer a complementary situation to players. In these, pressing specific keys has well-defined but undisclosed effects. The players discover the rules through a mixture of trialling, conjecturing and testing hypotheses.

SOURCES AND CONTEXTS

Sometimes ready made games exist which have proven value in enhancing some aspect of mathematical thinking. Many of these can be described as classical or traditional. Sometimes these are specific to a particular culture, and sometimes they are widespread, though different may have emerged in different settings. Two examples which have already been mentioned are mancala and chess. There are also many more modern games and puzzles which catch the imagination. These include variations on the game of NIM, and the popular puzzle 'Rubik's cube'.

Alteratively, games may be adapted or invented by teachers or popularisers, in order to introduce or embody a mathematical concept or principle, or to practice a technique, but it is important that such inventions should encourage the enjoyment associated with games playing in everyday life. An example of such an adaptation is the "FROGS" puzzle described by Hatch and Shiu (1990).

Mathematical games and puzzles can be presented in a group setting when there is a community of volunteers as in children's workshops, parents' evenings or a TV show audience.

They can be offered less formally through TV and radio programmes, crackers, beer mats, fortune cookies, matchboxes and columns in newspapers and journals ranging from in-flight magazines to Martin Gardner-type columns in the Scientific American.

A number of references are listed at the end of this article. Some of these are source books offering a comprehensive list of descriptions or presentations of existing games and puzzles. Others address particular aspects of the relationship between mathematics and games.

HOW DO I RESPOND TO SOMEONE WHO SAYS "THIS IS NOT REAL MATHEMATICS"?

As in the opening section 'What mathematics can be derived from games?' there are two perspectives on this question.

(a) The Learner's Perspective

When a school child says "This is not real maths", this is likely to mean that the activity is not recognisable as anything in the mathematics textbook or examination. From a parent the comment may mean that "this is not like the mathematics I did at school".

As indicated in the Ainley article, with young children absorption in the game may in itself invest the activity with reality. However, as children grow older the opinions of both parents and peers may prevail over those of teachers. It may be particularly difficult to introduce games and puzzles to older children who have only experienced formal and didactic presentations of mathematics.

Teachers and workshop leaders then have a responsibility to help learners to see links and relationships between particular games and puzzles and school mathematics. It may be desirable to create a continuum of experience, a cycle of "work" and "play" which includes providing a chance to reflect on what has been done both in terms of content and of process.

There is a danger that stopping short of mathematics popularises the image of mathematics rather than mathematics itself. In general often the best use of games and puzzles is to popularise not to trivialise, to encourage persistence with mathematical ideas, to encourage participants to look for extensions beyond the immediate activity. However, it should be recalled that sometimes the "learning" will not be recognised for many years, and that such learning is also significant. Also there are occasions when successfully playing the game or solving the puzzle is worthwhile in itself, and space should be made for these occasions.

Again, it is worth emphasising the importance of offering appropriate games. To alleviate negative and fearful views of mathematics held by pupils, it may be helpful to offer something which does not look like classroom mathematics, whereas to broaden and develop a narrow, content-focused stereotyped view of mathematics it is better to offer something in which the mathematics is overt, but which presents a game-like challenge.

(b) The Mathematician's Perspective

A mathematician who claims that an activity is not real mathematics may mean that the mathematics involved is not sufficiently significant, though some games and puzzles *are* accepted as being mathematics, e.g. NIM, Conway's work, the construction of magic squares, etc.. Such objections may be overcome if the mathematician is persuaded to observe or better still participate in a workshop and discover at first hand how the mathematically unsophisticated may be drawn into quite profound thought from an apparently simple starting point.

They may also accept the reminder that in pure mathematics we often do not know what the consequences of solving a particular problem will be. Examples include the Konigsberg bridge problem which led to the study of networks, and investigations of the 4-colour problem which led to new methods of proof.

They might further consider those great mathematicians who were themselves fond of games, and ask if simple problems, puzzles and games are a means of cutting one's mathematical teeth.

Finally, it is worth suggesting that the most important thing we can learn from games is to play mathematics in the spirit of a game.

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Leeds, September 1989

Miguel de Guzman
Gillian Hatch
Pat Hiddleston
Christine Hopkins
Marion Walter
David Wheeler
Claudia Zaslavsky

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INSTITUTIONS FOR MATHEMATICS EDUCATION

5. MATHEMATICAL SCIENCES TRUST SOCIETY - INDIA

This society has been formed to serve education and research in all mathematical sciences i.e. in pure and applied mathematics, operations research, mathematical statistics, mathematical economics, mathematical social sciences, mathematical biology, computer sciences etc. It has a library of its own and has a publication programme. It has so far published twenty volumes of the Bulletin of Mathematical Association of India, 8 volumes of 'Fascinating World of Mathematical Sciences' containing expository articles on Nature, Applications, Education, History and Biography of Mathematical Sciences by Professor J.N. Kapur, 2 volumes of Eminent Indian Mathematicians of the 20th Century, a volume of the Proceedings of an International Conference on Mathematics Education held in India, and a volume on Excellence in Teaching.

MATHEMATICAL ASSOCIATION OF INDIA

This was established in 1969 for developing mathematics education at undergraduate and postgraduate levels and to promote applications of mathematics. It has published Bulletin Mathematical Association of India regularly since then. This contains expository and educational articles on mathematics. The Delhi chapter of MA has organised twelve Dates with Mathematics on 13th April every year in which three lectures in memory of eminent mathematicians are organised. Distinguished Service Award to a mathematician is given every year.

MATHEMATICS EDUCATION JOURNAL

Mathematical Education is a quarterly journal for teachers and students of undergraduate and postgraduate classes. It is sponsored by the University Grants Commission of India and is edited by Professor J.N.Kapur, who is the ICMI National Representative of India, in collaboration with Professors M.K. Singal, D.K. Sinha, P.C. Vaidya and S.P. Arya. It is published by Macmillans Limited 22 Patalaos Road, Madras 600026. Its subscription is Rs 80 per annum. Five volumes have already been published.

The journal contains articles on Mathematics Education at all levels, expository articles on the history, Culture, excitement and relevance of mathematics, on nature of mathematics, on applications of mathematics, in physical/biological, social and management and engineering sciences.

Its special features include a problem section, book reviews section, news and notes section, classroom notes, graphs of curves etc. The journal also includes interviews with eminent mathematicians. It also gives a coverage of mathematical events in India and abroad. It includes special articles on role of computers and calculators in mathematics education and research, on mathematics in industry and on curriculum innovations.

Further information about the Journal, and the activities of the Trust and the Association can be obtained from:

Professor J.N. Kapur, Mathematical Sciences Trust Society
C-766 New Friends Colony, New Delhi 110065, India.

JOURNALS ON MATHEMATICS EDUCATION

No. 7 Journal für Mathematik-Didaktik (JMD)

Aims

The Journal für Mathematik-Didaktik (JMD) is an international scientific journal in which appear original contributions in German, English or French from the whole field of research and development in mathematics education (didactics of mathematics). It is the aim of the JMD to contribute to the further development of mathematics education as a scientific discipline, especially to the establishing and guaranteeing of standards of excellence in the didactics of mathematics. The JMD is open with regard to contents (also and in particular towards related disciplines) as well as methods. In any case, contributions have to essentially bear upon the leaning and teaching of mathematics.

The readers of the JMD are expected to be interested in scientific research and development in mathematics education. So, in the first place, readers are researchers or teacher educators. But many teachers also belong to the readership of the JMD.

It is indispensable for articles published in the JMD to be of high quality as a contribution to research. This is to be guaranteed by certain criteria which articles are to fulfil, such as relevance to the learning and teaching of mathematics, originality, inventiveness, reference to the "state-of-the-art", suitability and correctness of contents and methods, or stringency and consistency of the argumentation.

Organization

JMD was launched in 1980. Each volume consists of four issues. At present, volume 11 (1990) is being published. The price for one volume is DM 48. For members of the "Gesellschaft für Didaktik der Mathematik", the membership fee includes a subscription to the JMD.

The responsibility for the planning and further development of the JMD is in the hands of three editors, elected on a temporary basis. The editors decide on acceptance or rejection of submitted articles, supported as a rule by two additional referees. The editors also receive support from an advisory board consisting of twelve scientists.

Further information can be obtained from the publisher: Schöningh-Verlag Paderborn (FR Germany), Postfach 2540, D-4790 Paderborn, FR Germany.

Contents

The JMD contains articles on all topics and at all levels of the learning and teaching of mathematics. If the articles are roughly divided into categories (some of which, of course, overlap), then each of the following categories contains about one fourth of the articles published so far:

- Basic questions on the learning and teaching of mathematics (such as aims, learning theories, assessment or teaching conceptions),
- Curriculum development in mathematics (including analyses of mathematical subject matter),
- Empirical investigations into the learning and teaching of mathematics,
- Basic questions of mathematics education as a discipline.

The spectrum of topics can be seen in the following **examples** of articles taken from the existing volumes.

- A. Kirsch: On the training in mathematics of future teachers - with respect to the practice of geometry teaching (1980).
- E. Wittmann: Relations between operative "programmes" in mathematics, psychology and mathematics education (1981).
- S. Schmidt/W. Weiser: Numbers and comprehension of numbers of preschoolers: Counting and the cardinal aspect of natural numbers (1982).
- G. Herden et al.: An investigation on the discussion of problems in handling the concept of convergence (1983).
- W. Dörfler: Pocket calculators and mathematical reflection (1984).
- E. Cohors-Fresenborg: Different representations of algorithmic concepts (1985).
- G. Brousseau: Trends in research on mathematics education in France (1986).
- P. Bender: Critique of the Logo philosophy (1987).
- K. Hasemann: Models of cognitive science and mathematical learning processes (1988).
- E. Puchalska/A. Zemadeni: A structural categorization of verbal problems with missing, surplus or contradictory data (1988).
- H.-J. Vollrath: Functional thinking (1989).

- W. Peschek: Abstraction and generalizing in the mathematical learning process (1989).

Most of the articles in the JMD are written in German. In order to be accessible to everyone, each article commences with a summary in English.

Besides "regular" research articles, the JMD also contains "contributions for discussion" which either refer to published articles or to problems of immediate interest in mathematics education. Finally, the JMD brings regular information on recent doctoral dissertations or habilitations in the field of didactics of mathematics.

Werner Blum, Kassel, on behalf of the editorial board of the JMD

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