ICMI

Bulletin

of the International Commission on Mathematical Instruction

No. 21

December 1986

Secretariat
Centre for Mathematics Education
University of Southampton
Southampton, SO9 5NH
England

The International Commission on Mathematical Instruction

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DECEMBER 1986

Editors: Keith Hirst and Geoffrey Howson
Centre for Mathematics Education
University of Southampton
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INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION

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A Letter from the President

A new Executive Committee of ICMI was elected by the General Assembly of the International Mathematical Union in Oakland, for the period 1987-1990. The continuity is insured: A.G. Howson is reelected as secretary and myself as president. Moreover, O. Lehto, as secretary of IMU, remains in the E.C. of ICMI ex officio. Apart from these fixed points the whole E.C. of ICMI is renewed. The Vice-Presidents, Lee Peng Yee and E. Lluis, are from Singapore and Mexico - two faces of the Third World -. H. Fujita, J. Kilpatrick and M. Niss represent different trends in mathematics and mathematical education, and come from very different countries - Japan, USA, and Denmark -. J. van Lint, from the Netherlands, is the new IMU representative in ICSU-CTS (the Committee on the Teaching of Science in the International Council of Scientific Unions), with which we cooperate closely. Last but not least, L. Faddeev, from Leningrad, a world renowned physicist and mathematician, is member ex officio of the E.C. of ICMI as president of IMU.

The Executive Committee whose term now ends has been impressively active and efficient, though the financial means of ICMI did not allow many formal meetings. In particular, it decided and organized the series of studies on different topics of current interest now known as ICMI studies. It discussed a number of important questions - such as the International Congresses on Mathematics Education. It consisted of prominent personalities in mathematics and mathematical education, with interests in many other fields of human activities. Let me mention the particular attention and help we received from IMU, through O. Lehto and J. Moser. All members of the E.C. had been involved in mathematical education for many years: the vice-presidents, B.Christiansen and Z. Semadeni, as well as B.F. Nebres, M.F. Newman, H.O. Pollak - all are famous names and highly-regarded people.

All have given much to ICMI and we hope that we shall still be able to call upon their experience and help in the coming years. In particular, we note how Bent Christiansen has served for three terms as a Vice-President of ICMI, and has during that time made many outstanding contributions, for example, to the Nairobi Seminar on language and to the production of Volume IV of UNESCO's <u>New Trends in Mathematics Teaching.</u>

Let me add a few words on a man whom all mathematicians and educators consider with respect, admiration, and warm affection. He is one of the founders of modern topology, and the methods he introduced ("coverings", "extension") are of constant use in analysis and other parts in mathematics. He dedicated himself for years to mathematical education, with a very personal and deep insight on social implications. He was president of ICMI in the period 1979-1982, and stayed in the Executive Committee as Past President in 1983-1986. He remains with us as a great example, a friend, and adviser. Long live Hassler Whitney!

ICMI consists of its Executive Committee and of representatives of organizations which are members of IMU -usually, national organizations, therefore, national representatives -, together with representatives of countries which are not represented in IMU, and so extend ICMI's range. There is only <u>one</u> ICMI General Assembly every four years, at the occasion of ICME. But all members of ICMI are invited to take part in ICMI's life, and to develop actions of national subcommissions of ICMI whenever this is possible. The Executive Committee would appreciate suggestions or proposals coming from all parts of the world. Mathematical teaching and learning is a worldwide concern more than ever before. ICMI helps international cooperation in this matter. International cooperation in and for a peaceful world is my best wish at the occasion of the New Year.

Jean-Pierre Kahane

ICME 6

Preparations for ICME 6 (July 27 - August 3, 1988) are advancing. The International Program Committee met again in Budapest in July.

Invitations to give plenary lectures have been accepted by:

László Lovász (Budapest) (Trends in mathematics with special emphasis on algorithmic ideas and their relevance to the learning and teaching of mathematics)

Bienvenido Nebres (Manila) (Trends in School mathematics with particular reference to developing countries)

Gérard Vernaud (Paris) (Cognitive psychology relevant to the learning and teaching of mathematics)

A sub-plenary lecture will be given by

Andrei Ershov (Novosibirsk) (A new school course linking mathematics and computer science)

Below we reprint two papers issued by the IPC. One relates to the procedure for requesting 'Project' presentations, the other explains the philosophy behind the Theme and Action groups.

Readers are reminded that requests for the second announcement should be made to ICME-6, Janos Bolyai Mathematical Society, Budapest, POB 240, H-1368, Hungary.

POLICY ON PROJECTS FOR

ICME 6 / 1988 BUDAPEST

The International Program Committee has formulated the following guidelines and procedures for those projects wishing to make a presentation at ICME 6,

1. Projects are asked to submit a full (1-2 pages of A4) description indicating:

   (a) The project's relevance to the scientific work of the congress (e.g. to which action and theme groups its work is relevant);

   (b) the project's activities and aims including, for example its contributions to research and development, the form in which its materials (if any) (will) appear (e.g. textual material, software, films,...), its involvement in teacher training and procedures for student assessment, etc.

2. A project presentation will take one of the following three forms:

   (a) one short oral communication which might possibly be accompanied by a temporary (i.e. ~ 2 hour) display and exhibition

   (b) one or two poster presentations;

   (c) one larger-scale exhibition in either individual (or perhaps shared) rooms and/or corridors which will be open throughout the congress (except on the opening, closing and excursion days).

   (Note: the corridor exhibits should be secure.)

3. In the case of projects with standing exhibitions:

   (a) It is not intended to allocate any time for oral presentations. Individuals associated with such projects will be expected to make contributions to the congress' scientific work, particularly Action, Theme and Topic groups. (It should not, however, be assumed that Chief Organisers will automatically set aside time for presentations from such individuals.)

(b) A project will be expected to have personnel on duty at its standing exhibition at all times when the congress exhibition is formally open in order to explain the project's work to visitors and to answer their questions.

(c) It is expected that a nominal charge will have to be made (based on the number of square metres of corridor or room space used) to all projects allocated a standing exhibition.

4. Projects should indicate clearly which type of presentation they wish to make. Projects applying to have a standing exhibition are asked to supply the following additional information:

   (a) the amount of room/corridor space they would like to hire,

   (b) the apparatus/display boards, micros, videos/ they would wish to use <u>and</u> also that which they themselves would bring.

   Information on hiring possibilities and charges will be supplied later.

5. On the basis of the applications received, the IPC will determine which projects will be invited to give short oral communications, to make poster presentations, or to mount standing exhibitions. In the case of the last named, projects will also be informed of the amount and nature of space allocated.

6. Decisions will be made by December, 1987 and conveyed to projects as soon as possible. For that reason it is essential that all requests for project presentations, accompanied by the requested data, should be submitted to the Congress Organisers not later than 31 July, 1987.

BACKGROUND ON ACTION AND THEME GROUPS

Action and Theme groups are seen as a way of involving all participants of ICME-6 as active members of groups considering two aspects of their professional work. This involvement is intended to be four 90-minute sessions in each group. Thus it represents, in total, about a fourth of the formal program for each participant.

In Action groups the participants will consider their work from the point of view of the age group of the students they are principally interested in. The division into seven such groups is described in the first announcement. This division is essentially the same as for ICME-5. In Theme groups the participants will consider their work from a thematic point of view. The division into seven such groups is described in the first announcement. This division while having significant overlap with that used in ICME-5 has some novel features for ICME-6.

The emphasis in these groups is an active involvement in the work of the Congress by all participants. Thus the program for each group should include significant time for discussion in groups of reasonable size. ICME-5 demonstrated that such groups are essential if participants from different countries with different institutional backgrounds are to be able to assimilate the ideas, proposals and experiences of others and to respond to them in terms of their own experience. In the learning and teaching of mathematics the cultural environment of a participant may mean that ideas and activities that prove useful or even wonderful elsewhere do not necessary appeal. Thus new ideas or experiences in mathematics education need discussion as well as presentation before a proper appreciation of them can be formed. Variants are likely to emerge in the process of discussion which give added scope to the original stimulus.

Thus the panels must choose a series of subthemes whose breadth is such that a participant interested in their Action- or Theme group should be able to choose one which is of direct interest. The panels' choice should not omit such an area which would be preferred by a significant proportion of its clients.

The panel must then agree on a structure for the work of the group. This should provide for each group a serious starting point and emphasize that the level of discussion (and that of the presented papers) will assume some experience and knowledge of the area. To this end each panel is asked to prepare and distribute about ten pages of written material which describe the structure of the group and provides a description of the background. To further stimulate discussion, it may be desirable to select some people to present brief position papers with the aim of raising issues. These papers should not suppress discussion by attempting to present something for everyone.

The local organisers will give figures to the panels on intended participants expressing interest in the work of the given groups.

Panels may (in certain cases should) cooperate on some subthemes. Panels will also be required to prepared a brief statement about 100 words for the 2nd announcement.

THE EDUCATION OF MATHEMATICS TEACHERS

The Conference was held during the International Congress of Mathematicians (ICM) in Helsinki 1978 as a symposium of the International Commission on Mathematical Instruction (ICM), in cooperation with UNESCO and the Institute for the Didactics of Mathematics (IDM). The 280 pages proceedings include contributions by: U. D'Ambrosio, T.J. Cooney, T.J. Fletcher, H.B. Griffiths, M.G. Kantowsky, A.Z. Krygowska, W. Kuyk, J.A. Marasigan, H. Mehrtens, M. Otte, A. Revuz, H.G. Steiner, S. Touré, D. Wheeler, H. Wussing, the Teaching Committee of the British Mathematical Assosciation. The book is available (for DM 9.50 ≈ 5 US$ plus postage) from: IDM, University of Bielefeld, P.B. 8640, D-48 Bielefeld, F.R. Germany.

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Newsletter — Professor Charles V. Jones, Department of Editor Mathematical Sciences, Ball State University, Muncie, Indiana 47306, U.S.A.

ICMI STUDIES

Readers are warned that only a very few copies of the Supporting Papers to the Computer Study remain unsold. Any remaining copies can be obtained from IREM, 10 rue du Général Zimmer, 67084, Strasbourg, France. The price is 100 French Fancs (including postage). These papers will not be reprinted.

The Proceedings of the Strasbourg meeting, The Influence of Computers and Informatics on Mathematics and its Teaching, are still available from the Cambridge University Press (Hard covers ISBN 0 521 324015, Paperback 0 521 31189 6).

The report School Mathematics in the 1990s, which resulted from the Kuwait Seminar, is being published by the Cambridge University Press in December, 1986 (Hard covers 0 521 33333 4, Paperback 0521 33614 7).

A seminar on 'Mathematics as a Service Subject' is to be held in April, 1987 at Udine. The discussion document has appeared in, for example, L'Enseignement Mathématique (32 (1986), 159-172), La Gazette des Mathématiciens, The Bulletin of the Institute of Mathematics and its applications, (May/June 1986) and Notiziario della Unione Matematica Italiana. (August/September 1986)

FORTHCOMING INTERNATIONAL MEETINGS

6–10 April, 1987.	'Mathematics as a Service Subject' International Centre for the Mechanical Sciences, Udine, Italy.
13–18 April, 1987.	'Renovation de l'Enseignement des Mathématiques et Informatique dans les Pays en Developpement' Yamoussoukro, Côte d'Ivoire, (Société Mathématique de Côte d'Ivoire, 08 B.P. 2030, Abidjan 08, Côte d'Ivoire)
17–19 May, 1987.	'Informatics and the Teaching of Mathematics' Sofia, Bulgaria, (Peter Bollerslev, Røjlevangen 40, DK2630 Taastrup, Denmark)
1–3 June, 1987.	Fourth S.E. Asian Conference on Mathematical Education, Singapore. (Dr. Ong Sit Tui, c/o Mathematics and Computer Studies Department, Institute of Education, 469 Bukit Timah Road, Singapore 1025.)
12-17 July, 1987	TME "Investigating and Bridging the Teaching-Learning Gap", Antwerp, (Belgium), (Professor A. Vermandel, Department of Teacher Education, TME-Conference Secretariat, Universiteitsplein 1, B-2510 Antwerp, Belgium.)
12-16 July, 1987	Seventh Inter-American Conference on Mathematics Education Santo Dominigo, D.R.

8-11 September, 1987	3rd International Conference on the Teaching of Mathematical Modelling and Application, Kassel (FRG) (Professor Dr. W. Blum, Universität Kassel (GHS), Fachbereich Mathematik, Heinrich-Plett-Str. 40, D-3500 Kassel, F.R. Germany
27 July-3 August 1988	ICME 6 Technical University, Budapest, Hungary.

A NEW JOURNAL

A new Spanish-medium journal on mathematics education, Elementos de Matemática, has recently appeared in Argentina. It is specially designed for mathematics teachers in secondary schools. It is supported by CAECE University and will be used by the Argentine Commission on Mathematical Instruction to publicise its activities.

Further details can be obtained from

Elementos de Matemática, CAECE, Avda. de Mayo 1396, 5° piso, Código Postal 1085, Buenos Aires, Argentina.

NEW MEMBERS OF ICMI

Two new members are welcomed to ICMI, the Chinese Mathematical Society representing the People's Republic of China, and Kuwait. To celebrate the new representation of People's Republic of China in ICMI we reprint below the text of a talk given by Professor Ding Er-Sheng at the ICMI sessions at last August's International Congress of Mathematicians, Berkeley, USA.

A BRIEF INTRODUCTION TO MATHEMATICAL EDUCATION IN THE PEOPLE'S REPUBLIC OF CHINA

Before I go into the main part of my article, I think I had better give some background concerning the educational system of China in general.

1. Educational system in China

Type of School	Duration	Age-range
primary school	5 or 6 years	6/7 - 12
junior middle school	3 years	12 - 15
senior middle school	3 years	15 - 18
college or university	4 years	18 - 22
postgraduate school	3 years	22 - 25

There are 832309 primary schools: 46% of them are 6-year primary schools and most of them are in cities.

We are now popularizing 9-year compulsory education, i.e. popularizing junior middle school.

II. The numbers of primary and middle schools, students and teachers.

	numbers of schools	numbers of students	numbers of teachers
middle schools	104946	50932900	2967600
primary schools	832309	133701800	5376800

III. Curricula of general middle schools

grades	Junior Middle Schools			Senior Middle Schools			Total
	1	2	3	1	2	3	
	teachings hours per week						
Subjects politics	2	2	2	2	2	2	384
Chinese	6	6	6	5	4	4	1000
mathematics	5	6	6	5	5	5	1026
foreign language	5	5	5	5	5	4	932
physics		2	3	4	3	4	500
chemistry			3	3	3	3	372
history	3	2		3			266
geography	3	2			2		234
biology	2	2				2	192
physiology & health			2				64
physical education	2	2	2	2	2	2	384
music	1	1	1				100
art	1	1	1				100
TOTAL	30	31	31	29	26	26	5554
Optional subjects					4	4	240

As you may know, nationally-unified curricula, syllabi and textbooks approved by the National Commission of Education are adopted in China. The Textbooks of Mathematics for Primary and Middle Schools are based on such guidelines as enriching advanced scientific knowledge, reinforcing the teaching of basic knowledge and fostering the student's abilities, thus laying a solid foundation in mathematics for primary and middle school students in order to meet the needs of training talented people for their future work in China's socialist modernization. The purpose of teaching mathematics in middle school is to enable the students to gain a good knowledge of the basic mathematics necessary for socialist construction and the study of modern science and technology, to have the ability to perform quick and correct operations, to think logically and to use spatial imagination in order to gradually develop the ability to analyse and solve problems.

According to these teaching aims, proceeding from the needs of realizing the four modernizations of our country and taking into consideration what the students can absorb, we have carefully selected the traditional contents of school mathematics, have cut out the not particularly useful content, such as over-elaborate operations of addition, subtraction, multiplication and division in arithmetic, complicated identity transformations in algebra and trigonometry, and difficult proofs in geometry, etc. Then we have added in primary school some preliminary knowledge of algebra and geometry, and in middle school some preliminary knowledge of differential and integral calculus, statistics, probability and the algebra of logic. Finally we have also infiltrated some new ideas in modern mathematics into the teaching materials, such as ideas of sets and correspondence and their symbols. But we have not used the theory of sets as a basis to explain the teaching materials, nor have we widely used the language and symbols of sets, because at this moment it is not necessary and is very difficult to achieve good results in teaching.

These textbooks of mathematics were published in 1978, and from that time they have been used throughout the country. During this period there have been some small revisions, the biggest one being that from 1983. Unified textbooks are divided into two sets. One of them is basic and the other is higher. The parts for junior middle school are the same. The parts for senior middle school differ only by cutting out calculus from the higher texts.

The main contents of mathematics for junior middle schools include:

I. <u>Algebra</u>

1. Rational numbers and real numbers, and their operations.

2. Polynomials, addition, subtraction, multiplication and division of polynomials, factorization.

3. First-degree and quadratic equations in one variable. Systems of first degree and second degree equations in two variables. Simple higher-degree equations.

4. First-degree and quadratic inequalities in one variable.

5. Fractions and fractional equations, radicals.

6. Functions and graphs: direct variation and inverse variation, linear and quadratic functions.

7. Exponents and common logarithms.

8. Elementary statistics: mean, mean-squared deviation, standard deviation, frequency distribution.

II. Geometry

1. Straight lines, intersecting lines and parallel lines.

2. Triangles.

3. Quadrilaterals.

4. Area, Pythagoras' theorem.

5. Similar figures.

6. Circles.

7. Descriptive Geometry: principal views.

The main contents of mathematics for senior middle schools include:

I. Algebra

1. Sets, correspondence, function, power functions, exponential functions, logarithmic functions, trigonometric functions, inverse trigonometric functions and simple trigonometric equations.

2. Sequences, limits, mathematical induction.

3. Determinants and systems of linear equations, inequalities.

4. Complex numbers: the concept and operations of complex numbers.

5. Permutations, combinations and the binomial theorem.

6. Probability: equally likely events, mutually exclusive events, independent events, composite experiments.

II. Solid Geometry.

1. The basic properties of planes, relation of position between lines and planes.

2. Surface areas and volumes of prisms, pyramids, cylinders, cones, frustums and spheres.

III. Plane Analytic Geometry

1. Lines.

2. Quadratic curves, circle, ellipse, parabola, hyperbola, translation and rotation of axes.

3. Polar coordinates and parametric equations.

IV. Preliminary knowledge of differential and integral calculus

1. The derivative and the differential, and their applications.

2. The indefinite integral and its applications.

The senior middle school with basic requirements does not cover section IV.

The nationally-unified textbooks of mathematics successfully meet the needs of general schools. Through learning these textbooks, students can gain systematic basic knowledge of mathematics and basic skills. But with one set of textbooks, even though it's excellent, it is difficult to fit the requirements of all schools throughout the country. On the other hand, our socialist modernization increased the requirements of mathematical education and mathematics education itself has made great progress. For these reasons the mathematics education must be changed. We are now in a period of reforming mathematics education. We are revising the mathematics curriculum, adjusting the contents and improving the teaching methods. Recently, there are more and more experiments in both areas (teaching materials and teaching methods).

I. Firstly, I would like to indicate the general guidelines of such reforms.

The important common guideline is that, "education should face the modernization, face the world, and face the future". Through wide practice and deep research the following principles have been formulated:

(1) The aims of teaching should change from teaching knowledge only, to also developing abilities, especially fostering the student's fine qualities and attitudes.

(2) The contents of teaching should change from teaching classical book knowledge to clarifying fundamental principles and laws, structures of learnt knowledge, to understanding new basic ideas and methods, to popularizing computer education.

(3) The style of teaching should change from giving priority to lectures by teachers to developing initiative and the subjective role of students under the guidance of teachers.

(4) The methods of teaching should change from "forced-feeding methods of teaching" to "heuristic methods", from "rote" to "understanding".

(5) The form of teaching should change from class-based instruction to "second classroom" activities, i.e. group activities. So that students may lay a solid foundation in "first classroom" and develop interest and speciality in "second classroom".

(6) The teaching aids should change from "white chalk and blackboard only" to the widespread use of modern teaching aids.

II. Secondly, I would like to briefly introduce some experiments on reform in contents of teaching.

Recently, there are more and more experiments on reform in contents carried out in my country. Among them we have 8 experiments with wide scope and very good results. I am now going to present a survey and analysis of these experiments, especially on working out "Experimental Textbook of School Mathematics". I am able to describe this undertaking in some detail because I've been a participant ever since 1978, the beginning of this project.

<u>The guidelines and structure of "Experimental Textbook".</u>

In order to meet the requirement of the modernization of mathematical education we try to

- make the contents of the text fundamental, simple and practical

- come back to the essentials of mathematics and avoid adhering to the abstract form.

- follow the logical train of thought and explain fundamental concepts in simple terms.

"To make the contents fundamental, simple and practical" is the principle of selecting materials. It just reflects the relationship between theory and practice. As we know, the more general, simple and fundamental the material is, the more applications it has. So first of all we should clarify the priorities of fundamental mathematics. In our opinion, the main contents of algebra may consist of the following four topics:

(1) The system of numbers: the laws of operations of numbers are most basic and commonly used.

(2) Solving algebra equations: to solve the algebraic equations of lower degree we need only the common properties of the system of numbers, and the techniques of elimination and completing the square. But to solve equations of higher degree we have to have the completeness of the system of real numbers. However, if we consider solving equations from the viewpoint of function, the intermediate value theorem and Sturm's theorem are necessary.

(3) Operations with polynomials, such as addition, subtraction, multiplication and division, as well as synthetic division. The remainder theorem and the division algorithm.

(4) Indeterminate coefficient methods.

The content of geometry in school should be the fundamental concepts and major properties, such as the theorems of parallelism, theorems on similar triangles, the Pythagorean theorem, and the substance of vector and analytic geometry. Mathematical analysis is a sort of mathematics of variables. Besides the concepts of function, limit, continuity and so forth, the rate of change plays an important role in mathematical analysis. The most important technique is approximation. Moreover, the theory of probability and statistics should also be given attention in this text.

"To come back to the essentials of mathematics and to avoid adhering to the abstract form": The essentials of any theory are easier to understand and more active in use than the abstract form. For example, the laws of operations of numbers, which are the most basic and important in elementary algebra, are so simple and general that they can be easily understood and are commonly used everywhere. Naturally by formalization, these laws of operations must lead to the theory and operations of polynomials. But traditionally the algebra texts usually begin with the formal theory of polynomials. It makes the students feel that it is difficult to understand, so that it does not arouse their interest. In the "Experimental Textbook", to begin with, we tell the students that it is the cardinal principle to solve practical problems by applying flexibly the laws of operations of numbers. Then it is natural for the student to feel that there must be a theory of polynomials. At that moment, the chapter on polynomials comes in the text. In this case the student will easily understand the essentials of polynomials and will know where the concept of polynomial comes from.

The above is only one of the examples to show how to follow the principle "come back to the essentials of mathematics and avoid adhering to the abstract form". There are many such examples in the "Experimental Textbook".

"To follow the logical train of thought and explain fundamental concepts in simple terms" requires dealing with the contents naturally, following the process of development and the law of cognition. And at the same time we should pay attention not only to reasoning but also to developing the ability of students to master the principles and skills of mathematics.

For this purpose, at the heart of the matter, we should deal carefully with the following four transition points in school mathematics:

(1) The first is that from arithmetic to algebra. Our experiment shows that it is relatively easy to go through this turning point, if we concentrate ourselves on the role of common properties of system of numbers.

(2) The second is that from intuitive geometry to Euclidean geometry, i.e. from experimental geometry to deductive geometry. Through studying experimental geometry the fundamental concepts and properties of geometry must be established and will form a system of "common properties of space". Then by introducing the terms of "set" and "logic" into courses, experimental geometry will be transformed into Euclidean deductive geometry.

(3) The third will be the one from Euclidean elementary geometry to analytic geometry, namely to the algebraization of geometry. In order to algebraize geometry we should at first algebraize the most basic geometrical quantity, i.e. the displacement consisting of direction and distance. This will lead to the concept and properties of vectors. In this case the parallelogram law will be translated into the commutative law of vector addition, the theorem of similar triangles into the distributive law of vector multiplication, the Pythagorean theorem, in the broad sense, into the distributive law of the inner product of vectors. In one word, the "common properties of space" will be translated into a set of useful properties of vector operations. In other words, the laws of vector operations are the algebraized geometrical axioms. So the key to dealing with third transition point is to take great pains to analyse the above translations.

(4) The fourth is that from mathematics of constants to the mathematics of variables. With this transition there is a great leap not only in concepts but also in approaches. It is necessary to do this preparation as early as possible for overcoming the difficulties in this. For instance, the elementary concepts of trigonometry are learned in the second year of junior school, various functions in the third year, the concepts of continuity, real numbers, tangent line and the approximate approach in the first and second year of

senior school. Then in the third year of senior school, limit, continuity, differentiation and integration by approximate methods are emphasized.

As above, the designs of "Experimental Textbook" are both conforming to the development of mathematics and concentrating on those four transition points, and will definitely help the students grasp the thread of mathematical thought.

Utilising the above ideas, the "Experimental Textbook" serves three sequences of courses as text: Algebra (including probability and statistics), Geometry and Analysis which are spirally arranged at junior and senior levels.

From the experiments on "Experimental Textbook" we have obtained the following insights.

(1) In school mathematics courses, we should integrate the structure of knowledge with that of cognition. How can we realize this integration rationally? We try to do it by concentrating on four transition points. It has been proved by our experiments that this is successful in "Experimental Textbook".

(2) In school mathematics courses, we should integrate teaching students knowledge with training their ability. How can we do that? We know from our experiments that first we should enable the students to fully grasp the theoretical system of mathematical knowledge, and secondly we should emphasize teaching the fundamental mathematical ideas and methods.

(3) In school mathematics courses, we should integrate the completeness with development. This means that the content of school mathematics itself should be completed but shouldn't be closed. This is to say the text should be a developing system.

Besides the above experiment there are experimental textbooks for junior middle school: "Algebra and Elementary Functions" and "Geometry", written by the Central Pedagogy Institute jointly with several teachers' colleges: the experimental textbooks for junior school of education system "5-4-3", written by Beijing Normal University and its three attached middle schools: the experiment on "Experimental Textbook of Junior School Mathematics", carried out by Fusin Middle School in Shanghai: the "Experimental Textbook of Senior School Mathematics", written jointly by the Teacher's Training College of Xuhui and Hongkou District in Shanghai, and several other experiments on textbooks of mathematics. To save space I won't explain them in detail.

III. Thirdly, I would like to briefly discuss the experiments on reform in teaching methods.

Recently the main teaching method widely adopted in our country is still the traditional lecture. But there have appeared more and more experiments on reform in teaching method. The aim of these experiments is to foster students' abilities, especially creativity. The following are some of the varieties of teaching method tried out in our country.

(1) The experiment of self-study under teacher's guidance.

This is a big experiment carried out by the Psychology Institute of Academia Sinica in cooperation with local Education Departments. The aim of this experiment is to find a good way to develop the students' capacity for self-study. Experimentalists keep the mathematical materials the same as those in the unified textbook but have used a lot of new procedures in teaching method. Now they have written three volumes of self-study books: textbook, exercisebook, and testbook. The students study them under the guidance of teachers.

At the beginning when writing these books they followed the principles of "programmed learning": "small steps" and "feedback in time" designed by the American Psychologist B.F. Skinner. Then from their own practice they found that it's better to change "small steps" to "appropriate steps" and that they should pay attention to the leading role of teachers. They then formulated the following as their principles of writing self-study books.

(1) Take the appropriate steps (gradually from short to long) to explain the text so as to suit the students' background, to develop students' ability of learning and to advance their motivation.

(2) Give various problems as examples (avoid mere repetition) so that the students are able to analyse problems correctly and carry out applications freely.

(3) Give multiple choice questions to help students understand the key points deeply and immediately.

(4) Adopt the "principle of good preparation" when arranging exercises. That is to say, an exercise students are doing will be inspired by what has preceded it. Making such arrangements is a requirement for cultivating continuity of thought for students.

(5) As the students are doing exercises, ask them to check answers and correct mistakes themselves so that they realize "feedback in time".

(6) Ask students to present answers in detail and then gradually in simple words, to foster their reasonable and efficient thinking.

(7) Pay attention to the normalization of representation. Ask students to give reasons for every step. This will make them do everything precisely.

(8) According to the "principle of operation" in self-study, ask the students to systematize the knowledge they have learnt and sort out the methods they have used, so as to develop their potential for exploration through their finding the unknown from operating with the known.

(9) Make use of various problems of kinds usually called "observe and reverse association" to improve the flexible reversion of the psychological process.

They have adopted following measures:

(1) Four stages of teaching and the corresponding class routine.

(i) The first stage. It takes a week or ten days to give students instructions on their reading and doing exercises. The class routine in this stage is to lead the students doing intensive and extensive reading, doing independently problems as exercises, and checking answers on their own. In the end, the teacher will give a summary of what the students have learnt.

(ii) The second stage. It takes a month or so to train the students to get used to self-study. The class routine is to make the students do self study and self-check for at least 30 (or 35) minutes without any interruption. In this stage, the "program of self-study" and "outlines for summary and test" should be provided to the students in advance.

(iii) The third stage. What the teacher should do in this stage is only to tell students the sections which will be covered in this class period and to point out what the students should pay attention to. After the students spend 35 minutes in reading the text, doing exercises, checking answers, taking notes and summarizing materials, the teacher will spend 10 minutes in checking students and giving a summary.

(iv) The fourth stage. After one semester of self-study when teacher and students are used to their job, the teacher may begin his or her research work on teaching and learning.

(2) Four targets of measuring self-study experiment.

They are achievement of study, ability of learning, transfer of capacity (and habits) and overall development.

The comparison of achievement of study is mainly based on the report of a test (mid term and final). The ability of learning is measured in the following way. The students are asked to read independently a piece of mathematics which they haven't learned before and then to solve problems. The teacher will assess the problems as well as the right answers and wrong answers the students have produced.

Similarly a teacher can perform the measurement on transfer of capacity by giving students a piece of text in another subject e.g. physics. The overall development should be measured by school course records.

(3) Evaluation of the self-study experiment.

Having measured by the four targets mentioned above, out of more than 400 experimental classes, 90% are overall better than or the same as the control classes. In particular the classes with lower entrance scores are much better in achievement of study and the classes with higher entrance scores in ability of learning.

(4) The factors leading to success.

(i) Keep students occupying the principal position and keep the initiative in the students' hands. This requires the students to find their own study pace, to use their senses comprehensively such as eyes, ears, hands, mouth and mind so they search for what they are interested in.

(ii) Make the textbooks play a dynamic role. The textbooks are written according to the principles mentioned above and will definitely lead the students to having strong ability.

(iii) Ask the teacher to play a leading role correctly. The teacher should not restate the text that the students could understand themselves and should not do the thinking instead of the students. What the teacher should do is to teach students in accordance with their aptitude.

(2) The experiment on a guided discovery method of teaching.

The character of this method is as following. According to the characteristics of the structure of teaching and level of students' thinking, knowledge and ability, the teacher divides the teaching materials into several "discovery processes", and then following the laws of cognition of students and the inherent specialities of scientific knowledge guides students to research problems, to summarize the laws by reading, observation, experiment, thinking, discussion, attending lectures, etc. to achieve the aims of acquiring knowledge, fostering abilities and overall development.

This method includes three key components:

(i) dividing "discovery processes", defining the requirements of teaching;

(ii) well organizing teaching materials, actively guiding students into discovery activities;

(iii) setting a favourable situation for discovery learning.

(3) The experiment on "discussion" teaching methods.

The Yu Cai middle school in Shanghai has tried out a discussion teaching method called "tea house teaching method". In mathematics classes, they adopt such methods as "reading, discussing, exercising, attending lectures". In class the teacher guides students to read the text, to do exercises, then introduces students to discussion of problems and answers the difficult questions raised in discussion, finally giving comments on the discussion. Instead of thorough and deep explanation by the teacher this method emphasizes thorough and deep discussion by students.

(4) The experiment on the teaching method "probe and feedback".

This method has the aim of fostering students' ability to acquire and apply mathematical knowledge. The teacher should organize the teaching materials in such way as to enable students to learn by probing step by step. At the same time, he should pay attention to feedback from the learning results to make the learnt knowledge and ability strong. Here are its concrete steps:

(i) the teacher will inspire and guide the students and set the resources of problems for students;

(ii) the students start to probe and to get results;

(iii) the students summarize the results;

(iv) do exercises by various examples;

(v) get feedback of the effect of probing promptly and regularize feedback.

(5) The experiment on the unit structure method of teaching.

This is a method, such that according to the inter-relationship of content, the teacher divides the teaching materials into various structured units and then teaches them unit by unit. This method is favourable for students to acquire systematic knowledge. It includes the following stages:

1) The first stage is to learn the fundamental knowledge of a given unit. At this stage the first step will be to guide the students to analyse, to compare and to arrange the fundamental knowledge. The second step - to find out the laws, and to formulate the structure of knowledge. The third step - primarily to apply the fundamental knowledge to examples and exercises.

2) The second stage is to drill basic mathematical skills.

3) The third stage is to review and strengthen learned knowledge.

Generally, we hold that the teaching method serves the aim of teaching. Therefore one should choose the most suitable teaching methods to gain optimum effect.

To conclude this article, I would like to point out that there now appears a tendency to improve methods of evaluation and to modernize teaching aids. For example, the system of college entrance examinations will be changed. Last year, Shanghai and Guangdong separately had their own entrance examinations. This year Beijing will do so too. Some provinces or cities will try out "standardized examinations". A few schools have begun to try out computer education and computer assisted instruction. But it is just beginning. In these areas we are lagging behind and have to learn from progressive countries.

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ACKNOWLEDGEMENT

This Bulletin has been prepared with the help of a grant from UNESCO.

NOTE