ICMI

Bulletin

of the International Commission on Mathematical Instruction

No. 16

June 1984

Secretariat
Centre for Mathematics Education
University of Southampton
Southampton, SO9 5NH
England

The International Commission on Mathematical Instruction

Bulletin No. 16

JUNE 1984

Editors: Keith Hirst and Geoffrey Howson
Centre for Mathematics Education
University of Southampton
Southampton, SO9 5NH
England

COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE (INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

EXECUTIVE COMMITTEE, 1983-1986

President :

J.-P. KAHANE, Université de Paris-Sud, Centre d'Orsay, Mathématique — Bâtiment 425, 91405 Orsay Cedex, France.

Vice-Presidents :

B. CHRISTIANSEN, Royal Danish School of Educational Studies, Department of Mathematics, Emdrupvej 115B, DK-2400 Copenhagen NV, Denmark.

Z. SEMADENI, Instytut Matematyczny, Polskiej Akademii Nauk, ul. Sniadeckich 8, 00-950 Warszawa, Poland.

Secretary :

A. G. HOWSON, University of Southampton, Faculty of Mathematical Studies, Southampton S09 5NH, United Kingdom.

Members :

S. F. NEBRES, Mathematical Society of the Philippines, c/o Ateneo de Manila University, P.O. Box 154, Manila, Republic of the Philippines.

M. F. NEWMAN, Department of Mathematics, Institute of Advanced Studies, Australian National University, P.O. Box 4, Canberra, ACT 2600, Australia.

H. O. POLLAK, Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974, U.S.A.

Past President :

H. WHITNEY, Institute for Advanced Study, Princeton, New Jersey 08540, U.S.A.

Ex-officio Members :

J. MOSER, President of IMU, Eidgenössische Technische Hochschule, E.T.H. Centrum — Mathematics, CH-8092 Zurich, Switzerland.

O. LEHTO, Secretary of IMU, University of Helsinki, Department of Mathematics, Hallituskatu 15, 00100 Helsinki 10, Finland.

H. HOGBE-NLEND, IMU Representative at CTS-ICSU, Université de Bordeaux — 1, UER d'Informatique et de Mathématiques, 351 Cours de la Libération, 33405 Talence, France.

ICMI GENERAL ASSEMBLY

A meeting of National Representatives will be held at 19.30 on Sunday, 26 August, 1984 in Adelaide, South Australia.

A G E N D A

1. Welcome to delegates.

2. Report on ICMI activities.

3. Presentation of Financial Accounts.

4. Proposals for the planning of future ICMEs.

5. Composition of the next EC and of the committees concerned with the planning of ICME 6. (Methods by which names can be submitted for consideration.)

6. The role of National Representatives.

7. Any other business.

Note

This meeting will be a closed meeting for National Representatives or their deputies.

An open meeting on ICMI will be held at 20.00 on Wednesday, 29 August, 1984. This will include a report on ICMI activities and a discussion on the possible role and contributions of National Sub-Commissions.

An announcement will be made at Adelaide concerning the rooms in which the two meetings will be held.

SEVENTY FIVE YEARS OF ICMI*

(continued)

Stockholm was to inaugurate a most important stage in ICMI's history; one in which, with the help of UNESCO and other bodies, it was to initiate a number of noteworthy activities. Work began on Volume 1 of New Trends in Mathematics Teaching (UNESCO, 1967), consisting of articles presented to various congresses, short accounts of other international meetings and details of curriculum projects and journals devoted to mathematics education. It is tempting to quote at length from some of the papers, but there is little to be gained from doing this - provided, of course, that nowadays educators and mathematicians have learned the lessons of the sixties: that hopes cannot always be translated into practice, that more can be taught than can be learned, that motivations for teaching do not automatically translate into motivations for learning,..... .

The number of international meetings increased rapidly. Within the academic year 1964-5, ICMI was involved in four major colloquia: at Frascati, Italy on 'Mathematics on entry to university; the present situation and that which is desirable'; at Utrecht, Netherlands on 'Modern trends in secondary school mathematics teaching'; at Dakar, Senegal, on 'The teaching of mathematics in relation to that of other sciences' (the first ICMI-sponsored meeting to be held in Africa); and at Echternach (Luxemburg) on 'The influence of mathematical research on teaching'.

Perhaps, one quotation from a description of the Utrecht meeting merits particular attention:

> 'The crucial point of the colloquium was the confrontation of the ideas provoked by the lectures and the interventions of Professor A. Wittenberg, who stressed particularly the necessity of a precise pedagogical conception of the reform. He also underlined the dangers connected with a formal modernization which does not have an adequate basis in a clear consciousness of the objectives, the means of getting results, and a clear conception of education in general'.

Regrettably, mathematics education was soon to lose Wittenberg and with him one of the few knowledgeable, constructive and serious critics of the contemporary reforms.

Yet much more was happening within ICMI than the setting out of suggestions for what should be taught in secondary schools. There were those who drew specific attention to the need to generate

mathematical activity amongst pupils - not merely to catalogue the mathematics to be taught. This interest was reflected in the choice of the first topic for discussion at the 1966 ICM in Moscow: 'The development of mathematical activity in pupils. The role of problems in this development'. The other two topics had titles more in keeping with previous surveys: 'The use of the axiomatic method in secondary school teaching' and 'The mathematical training of university physicists - is there a need for a separate course or not?'

During this period there were also changes in the way ICMI functioned and even more drastic proposals for what might happen. First it was agreed that a country which was not yet ready or equipped to take its place as a member of the International Mathematical Union might nevertheless become a member of ICMI. The first two countries to take advantage of this were Luxemburg and Senegal. This possibility still exists and indeed during May, 1983 Costa Rica and Mozambique were welcomed as new members of ICMI. Secondly, the Inter-American Committee for Mathematical Education which, with ICMI support, had organised its first Congress in Bogota in 1961 asked in 1965 to be officially affiliated to ICMI and to be recognised as a regional committee; a recognition which was granted with effect from July, 1965. Together, IACME and ICMI planned, with UNESCO's aid, to mount a congress in Lima, Peru in 1966. It was noted, however, that the financial resources generally available to ICMI were insufficient to meet its needs. Moreover, the future tasks which ICMI faced could not be accomplished with its structure as it then was (and still remains). Across the world there was a constant demand for information and the exchange of knowledge; this need could only be met through the establishment of a permanent secretariat, adequately financed and led by a competent specialist who devoted a major part of his time to ICMI duties.

The call for a permanent secretariat was repeated by the new executive committee when it took office in 1967. This was but one of the many suggestions put forward at a meeting of ICMI held in Utrecht under its new President, Hans Freudenthal. Some, offered by the various sub-commissions, were quickly dropped, for example, that ICMI should write pilot texts for use in schools; some, such as the need for ICMI to identify the importance of the computer at all educational levels, are still being acted upon - and because of improved technologies will demand continual review; others, such as the organisation of international exchanges of mathematics teachers, remain good ideas which ICMI's structure prevents it from putting into practice.

Two suggestions were, however, to be accepted with marked effect. Freudenthal returned to the attack on the traditional ICMI reports presented at the ICMs. What was required, he argued, was a congress devoted solely to mathematics education, held in a different year to

the ICM, at which invited talks could be given and opportunities for personal contributions presented. The idea was accepted and Maurice Glaymann proposed that the first such congress should be held in 1969 in Lyons, France.

André Revuz raised the problem of L'Enseignement Mathématique which had always been ICMI's official journal and which seemed on too high a level for secondary-school teachers. The need for other publications, possibly in several languages, was discussed, and a committee was established to consider the problem. In the event ICMI took no further action, but in the following year, Freudenthal himself launched Educational Studies in Mathematics and in the years immediately following it was this journal rather than L'Enseignement Mathématique which had the stronger links with ICMI.

The first volume of Educational Studies in Mathematics indeed consisted of the papers presented at an ICMI-sponsored colloquium held at Utrecht in 1967 under the title 'How to teach mathematics so as to be useful'. ESM also published the recommendations of another important ICMI-sponsored meeting held that year in Lausanne on 'The coordination of the teaching of mathematics and physics'. Volume 2 was again largely devoted to ICMI-related matters, including the papers presented at the First International Congress on Mathematical Education held in August, 1969 at Lyons.

The Lyons congress was a landmark in ICMI's history. Over six hundred mathematics educators from forty-two countries met in an unprecedented fashion. The lack of precedents was to tell against the effectiveness of the congress, for its format did not encourage discussion and active participation. The meeting was built around twenty one-hour plenary lectures supplemented by a number of short (15-minute) contributions by congress members. Last minute attempts to arrange discussions met with mixed success - the members flocked to them in greater numbers than the rooms could accommodate! The discussions also revealed to its full extent what still remains a major problem for those organising international meetings on mathematics education - that of language. Simultaneous translation facilities can be made available for plenary sessions, but their cost is such that it can never be feasible to offer them in any but one hall. Yet mathematics education makes enormous linguistic demands of the speaker and hearer. We lack the international terminology and vocabulary of the mathematician and cannot resort to readily recognised and comprehended symbolism. So much depends on the precise and varied use of language - nuances that are not properly understood can so easily become trite, shallow statements when they are the victims of inadequate translation.

Lyons also had importance in that - perhaps unwittingly - it ushered in a period in which the national sub-commissions were asked to do less and emphasis within ICMI shifted from them to individuals. This,

like the vast majority of all changes, produced both good and bad effects. Professionalism ultimately hinges on the performance of individuals and it must be admitted that not all national committees were representative or active. Yet the active sub-commissions were able to involve another stratum of educators in the work of ICMI - which was not represented at the ICM and cannot always be present at ICMEs.

An attempt actively to involve sub-commissions was made at the ICME which followed Lyons, that at Exeter in 1972 which attracted almost 1400 members from 76 countries. There emphasis switched from the 'set' lecture - there were only seven of these - to forty or so working groups and to national presentations. Seventeen national sub-commissions accepted the invitation to mount presentations; some, for example, those of the hosts, the United Kingdom, and of the USA, formed almost mini-conferences in themselves, whilst others were more modest 'exhibitions'. Exeter, then, had the air of a 'World Expo' of mathematics education. Obvious consequences were that the congress was somewhat diffuse, members were faced with a multitude of alternatives at every session only one of which they could attend, and that some activities and presentations lacked support. A reaction inevitably followed at ICME 3 but by that time other important developments had taken place within ICMI.

The first of these was the appearance in October 1972 of ICMI Bulletin No. 1, an immediate response to requests made at ICME 2. Since then the Bulletin has established itself as a useful means of communication. Bulletin No.1 listed a number of forthcoming ICMI symposia: in Luxemburg, Hungary, Poland, Kenya, Japan, Denmark (later moved to Federal Germany) and India. Of these, that in Kenya deserves particular mention. The theme of the conference, which was very generously subsidised by UNESCO, was 'Interactions between Linguistics and Mathematical Education'. It was a theme which clearly had - and still has - considerable meaning for African countries, many of which teach mathematics in English or French rather than the students' mother tongues. However, what so distinguished the seminar was the meticulous manner in which it was planned and prepared, the way in which relevant specialists from outside mathematics education were involved, its duration (eleven days), and the spread of its membership. All factors which, alas, are closely governed by the amount of funding available!

UNESCO's assistance was also to play a major part in shaping ICME 3 in 1976 at Karlsruhe, Federal Germany. As I have already hinted, ICME 3 differed in many ways from its immediate predecessor. Again, little emphasis was laid on full plenary sessions, but on this occasion the congress was built around thirteen sections covering most aspects of education, each corresponding to a chapter in what was to be published as New Trends in Mathematics Teaching Vol 4 (UNESCO,1979). The section themes provided a useful framework around which to

consider mathematics education and indeed a somewhat similar design is to be used at ICME 5. However, at Karlsruhe the aim of producing a book, perhaps, loomed too large and many members (the number now having grown to 1831) felt that they had insufficient opportunity to contribute. Once again there was to be a swing of the pendulum, and the Berkeley ICME of 1980 offered over 400 speakers drawn from 100 different countries to a membership of more than 2000, as well as a variety of discussion groups and poster sessions.

Some of the working groups established at Exeter continued to meet at Karlsruhe and this led to another interesting development, for it was agreed that the International Group for the Psychology of Mathematical Education (PME) and the International Study Group on the Relations between the History and Pedagogy of Mathematics should become independent groups affiliated to ICMI. PME mounted its own congress in Utrecht, Holland in 1977, the year following Karlsruhe, and has since met annually at a wide variety of venues.

The vast differences in the forms which the ICMEs have taken could be attributed to the national characteristics of the hosts. This, however, whilst not entirely to be dismissed is perhaps too facile an explanation. We have, in fact, witnessed four attempts - and plans for a fifth are now well-advanced - to deal with extremely difficult problems inherent in the nature of mathematics education. First, it is essential that standards of professionalism are established within our discipline and on that account ICMEs must be showcases in which work at an approved level is displayed. They cannot be allowed to become truly 'representative' until the general standards of research and thought have been raised. However, mathematics educators, unlike mathematicians, cannot speak universal truths. The context - social and mathematical - within which they work individually is usually such that their results and thoughts are not readily transferable. There is a need for translation, for mediation and for adaptation. Such actions can only take place as a result of questioning and discussion. ICMEs then must offer opportunities for interaction and the exchange of knowledge and ideas. There is a need to encourage participation, allied to that of establishing professional standards. It is in the weights that have been assigned to these two objectives and the procedures designed to attain them, that the ICMEs have shown the greatest measures of divergence.

Such thoughts, however, lead one more to the consideration of ICMI's future than its past. As we have seen, in its first 75 years ICMI has attained much, and, of course, there are many achievements - for example, the 1978 survey on 'Change in Mathematics Education since the late 1950's - Ideas and Realisation' - which I have neglected to mention. The two decades from 1960 to 1980 were years which saw considerable activity - indeed, it was a major problem to keep track of the many activities taking place and reports which were generated. Now there is less public money available for symposia and travel: there

is a need, therefore, to utilise those resources which we have, finance, information, goodwill and man-power, with particular care.

The status of mathematics education has fluctuated considerably, too, in those decades. During the 1960s governments welcomed the pleas of educators for 'mathematics for all'. In the 1970s disillusionment set in and the 'back to basics' movement began: much was heard of 'minimal competencies'. We are now in a relatively quiescent period so far as governmental pressures on mathematics educators are concerned. Yet vast changes are taking place in society and in technology which demand responses from us. It is with such considerations in mind that the Executive Committee of ICMI proposed to organise 'studies' on the impact of the computer on mathematics and on its teaching in higher education; on current knowledge of cognition and of how teachers of mathematics might respond to this; on probable changes in education and the part within it that mathematics occupies which are likely to result from changes in society; and on mathematics as a service subject in higher education. The aim of such studies is not merely to give surveys of what is best in current practice or most up-to-date knowledge, but to provide frameworks within which national and regional discussion can take place. We hope, therefore, that local discussions and work will supply input for, and benefit from the output of, these studies. They will provide therefore both opportunities for participation and also for the establishment and reinforcement of professional standards within a discipline which each year becomes more demanding and more important.

A.G. Howson.

* The views expressed in this article are those of the author, Dr. Howson, and not necessarily those of the ICMI Executive Committee or its National Representatives. A fuller version of this account complete with bibliographical references has appeared in Educational Studies in Mathematics, 15 (1984), 75-93.

JOURNALS ON MATHEMATICS EDUCATION

4. FOR THE LEARNING OF MATHEMATICS

"The journal aims to stimulate reflection on and study of the practices and theories of mathematics education at all levels; to generate productive discussion; to encourage enquiry and research; to promote criticism and evaluation of ideas and procedures current in the field. It is intended for the mathematics educator who is aware that the learning and the teaching of mathematics are complex enterprises about which much remains to be revealed and understood."

This statement printed on the inside cover of each issue gives some idea of FLM's ambitions. The journal is international, printing articles from anywhere in the world, and takes a multidisciplinary approach to its subject matter, printing articles written from any appropriate viewpoint - mathematical, pedagogical, sociological, etc. Its appeal is to reflective, enquiring and critical readers, whether they are teachers, teacher educators, researchers, administrators, or whatever.

Of course FLM's achievements are only a rough approximation to its aims. Nevertheless in the three volumes published so far, containing upwards of 60 articles, can be found a fair proportion of stimulating articles by leading mathematics educators. For example:

Jeremy Kilpatrick writes about the effectiveness of research in mathematics education (2, 2), Gérard Vergnaud discusses the contribution of cognitive and developmental psychology to mathematics education (3, 2), and Herbert Ginsburg argues the validity of the clinical interview as a research method (1, 3 and 2, 1).

Ken Clements surveys what is known about visual imagery (2, 2 and 2, 3), Jens Holger Lorenz reviews the research on teacher-student interactions (1, 2) and Brian Greer summarises the yield of cognitive psychology in studying mathematical thinking (1, 3).

Philip Davis outlines the need for a philosophy of computation (3, 1) and David Tall describes the genesis of a mathematical discovery (1, 2).

Hans Freudenthal suggests the benefit of a knowledge of history to the teacher of mathematics (2, 1) and Jean Dhombres shows how the messages of history and pedagogy may be in conflict (2, 2).

Efraim Fischbein considers intuition and proof in mathematics (3, 2), Michael Otte looks at the characteristics of mathematical texts (3,3), and Stephen Brown continues his exploration of problem-generating (1, 3).

Joseph Agassi relates mathematics teaching to training for freedom (2, 3), Paul Wolfson discusses the possible effects of Lakatos' philosophy (2, 1) and Willem Kuyk sketches a new theory of mathematics learning (3, 1).

And so on .....

Articles are printed in French or English, the great majority in English. The Editor may solicit articles from particular authors; unsolicited articles are subject to a peer review procedure.

The journal was launched in 1980. The first three volumes of three issues each were published in 1980-81, 1981-82 and 1982-83. Future volumes will be published during calendar years beginning with Volume 4 in 1984. Subscriptions are (U.S.) $18.00 per volume for individuals and (U.S.) $24.00 for libraries. Subscriptions, requests for information and material for publication should be sent to either of the following addresses.

Editor

David Wheeler
Mathematics Department
Concordia University
Montreal, Quebec
Canada H4B 1R6

Publisher

FLM Publishing Association
4336 Marcil Avenue
Montreal, Quebec
Canada H4A 2Z8

PROCEEDINGS OF ICME4

Because of changes of address and in some cases the fact that the Congress organisers had no forwarding address, a number of participants at the 1980 Berkeley ICME have still to receive their copies of the Proceedings. These can be obtained from Dr. H.O. Pollak, 40 Edgewood Road, Summit, New Jersey 07901, USA.

Please make this information available to anyone you know who has still to receive his/her copy of the Proceedings.

INSTITUTIONS FOR MATHEMATICS EDUCATION

2. THE LABORATORY OF MATHEMATICS EDUCATION OF THE ACADEMY OF PEDAGOGICAL SCIENCES, U.S.S.R.

The purpose of this article is to show the designation and functions of the Laboratory of Mathematics Education (Scientific Research Institute for Content and Methods of Education); its role in the development of the mathematics education in the USSR.

The Academy of Pedagogical Sciences of the USSR serves as a centre which coordinates and integrates close scientific research, demanding the combined efforts of specialists in different fields: didactics, methods, psychology, physiology, hygiene. It makes it possible to raise and solve the major educational problems.

In the system of research, which is being developed under the guidance of the Academy, one can point out some basic lines. Among them there are fundamental and applied research into the content (substance) and methods of education, which are within the competence of a specialized Institute of the Academy, named "The Scientific Research Institute for Content and Methods of Education". The Laboratory of Mathematics Education, one of the largest ones in the Institute, deals with the problems of mathematics education in secondary schools.

At present the Laboratory consists of high skilled scientists. The team of collaborators (24 members) includes two professors, doctors of pedagogical sciences, and 19 candidates of pedagogical sciences. In addition, a number of representatives from the different united republics constantly work on probation in the Laboratory. They improve their professional skills, study modern research on methods of teaching mathematics, and fix their individual tasks for research-work. The work on probation may last from one month to 2 years, depending on its purposes.

The Laboratory of Mathematics Education provides postgraduate studentships. So the laboratory becomes a kind of centre where training of highly skilled researchers in methods of teaching mathematics is being accomplished.

The team of collaborators at the laboratory concentrate their efforts upon the solving, both theoretically and practically, of problems such as: to work out the curriculum, to develop the system of teaching resources, to improve the existing system of education, to find out new prospects. Here we shall consider in more detail research on the above problems.

Theoretical and practical elaboration of the secondary school curriculum. The curriculum in any subject is a normative document, which has two main functions: to fix the substance of compulsory education and to organize the educational process. All the existing curricula combine both functions. But to some extent, the functions named are contradictory. The curricula are mostly oriented to one of the functions, and so the fulfilment of the other is rather complicated.

As the range of the purposes of mathematics education is wide enough, it is necessary to bring into life the main functions of the curriculum, to define ways and levels of its practical use. In this connection it has been realized, that the mathematics curriculum complex[1] is to be worked out. The complex should consist of normative documents with different functions. Each document should provide a certain component of school mathematics education.

The most essential components of the curriculum complex have been developed and approved, their functions being:

- to fix the minimal-and-compulsory content of mathematics education in school. It is important to make clear, that content is described in accordance with the logic of the basic scientific lines and is distributed according to education levels, which are detachable from the course of mathematics in the secondary school;

- to describe how to organize the teaching process, i.e. to distribute the content of education to classes and themes, and to fix approximately the teaching time.

The other components of the curriculum complex, which are also needed for the effective teaching of mathematics in compulsory secondary education, are to be finished very soon.

Their functions should be:

- to give the expected results of mathematical training level of schoolchildren (with an adequate procedure for its control);

- to give the detailed explication of educational aims of teaching mathematics with regard to the process of their achievement.

Theoretical and practical development of a system of mathematics teaching resources. Mathematics teaching in the secondary schools of the USSR is being carried out with the help of a mathematics

[1] The Soviet methodologists were the first to propose the idea of such a complex. The complex is destined for the improving of mathematics education in school.

teaching resource system. The system consists of a textbook, teachers' book, a book containing the self-dependent works and control works, tables, film strips, etc. The textbook is the most important of all the above items. It includes the content of education (which is defined in the curriculum) in accordance with a certain methods system. The content and the structure of the rest of the teaching resources depend considerably on the system of the textbook.

The problem of the school textbook as the main teaching resource has become very urgent because of the switch-over from the 8-year education to general secondary 10-year education on the one hand, and of the reform of the content of secondary school mathematics education, which took place in 70's in the USSR, on the other.

The Laboratory of Mathematics Education is involved in research into problems concerning the textbooks. The research has two major aspects. The task of the first aspect is to work out an optimal methods system on the basis of which the new course of mathematics for secondary schools could be created. As an example we may mention the methods system of an algebra textbook for grades 6-8. This system supposes the unification of contents in accordance with the group of basic notions; wide use of visual images; realization of the activity approach in education, etc. The exact formulation of the methods system of school textbooks is needed for the adequate creation of the other teaching resources.

The task of the second aspect is to define the fundamental functions of the school textbook; to improve its structure. The existing textbooks undertake the task of the universal teaching resource. The textbook reveals the logic and succession of the subject, the volume of material and the level of its acquisition, as well as the methods of teaching and education. These functions, which are completely in the competence of the school textbook nowadays, should be evenly distributed among the other teaching resources. When distributed, the most effective realization of the functions will be achieved.

Another problem, closely connected with those mentioned above, is the evaluation of the mathematics textbook quality. The problem seems to be among the most complicated in the pedagogical sciences. The Laboratory has developed the procedure of comparative analysis of mathematics textbooks, providing two kinds of comparison: theoretical and experimental. The theoretical analysis will provide comparative information on a number of parameters relating to the mathematical and pedagogical characteristics of the textbooks. During the procedure of comparative analysis, the final and current results of teaching mathematics with the use of different textbooks are being compared.

Improving the existing system of mathematics education. In the 70's the most radical reform of school mathematics education took place in the USSR. The purpose of the reform was to improve the subject structure, to add new material important for the general education of students; to equip them with the general mathematical methods for solving problems. There were quite a number of difficulties in the realization of the reform. The Laboratory has elucidated both the advantages and disadvantages of the newly created course of mathematics. It has also been shown how to eliminate the disadvantages found: to avoid extra information in teaching, to increase the applied and practical orientation of teaching, to improve the methods system of teaching mathematics.

When describing the ongoing research projects in the Laboratory of Mathematics Education, it is necessary to accentuate the particular features of the Laboratory's activity. On the one hand, the Laboratory is maintaining a close contact with the Mathematical Section of the Acadamy of Sciences of the USSR; on the other hand, the constant cooperation with the mathematics teachers, with school practice.

Professional mathematicians take part in the selection of the substance of school mathematics education, in evaluation of existing and experimental textbooks, in improving the curriculum and the textbooks. The Mathematics Section of the Academy of Sciences of the USSR has formed the Commission on School Mathematics Education Reform. The Commission to a certain extent regulates the activity of the Laboratory. After the death of acad. I.M. Vinogradov, acad. L.S. Pontrjagin has become the Head of the Commission. Many well-known scientists-mathematicians take part in writing the textbooks for the secondary school. They are acad. A.N. Kolmogorov, acad. A.N. Tihkonov, acad. S.M. Nikolsky, acad. A.V. Pogorelov, acad. D.K. Fadeev, acad. A.D. Alexandrov.

The connection of the Laboratory with the secondary schools is expressed in the constant study of schoolchildren's knowledge, in supervision over the teaching process in mass and experimental schools, in regular discussion with the teachers, methodologists and others who collaborate with the educational bodies about the teaching process. It helps to receive different information, to study, to describe and summarize the progressive pedagogical experience. The staff of the Laboratory is not numerous enough to ensure the amount of work enlisted. So the Laboratory takes special measures to provide the work on the required level. For example the Laboratory uses the assistance of contributors from Scientific-Research Institutes for Pedagogy, teachers' re-training Institutes, Pedagogical Institutes, Universities from the different regions and United Republics of our country. The members of the Laboratory have to constantly write and publish the materials providing help to mathematics teachers in their every-day work. The materials are available

in the journals: Mathematics in school, Soviet Pedagogy, Teacher's Newspaper, and the periodical Teacher's Library, which is edited by "Prosveshtchenie" publishing house, etc. The Laboratory team lectures on the pressing problems of mathematics teaching at the all-union, regional, republic, city and district conferences. With us there is a rather particular form of summarising and popularizing the best of teacher's experience - the Pedagogical Readings. They are teachers' symposia, organized in a certain sequence on different levels, the highest of which is the all-union symposium. The Laboratory of Mathematics Education participates actively in the selection and preparation of materials, destined for the Pedagogical Readings.

V.M. Monakhov
V.V. Firsov
S.B. Suvorova

Institute for Scientific Research into
Content and Methods of Instruction,
Academy of Pedagogical Sciences
of the USSR.
5/16 Makarenko Street,
MOSCOW.

NATIONAL REPRESENTATIVES

We have been informed of the following change of address:

Professor J.N. Kapur
Department of Mathematics
Indian Institute of Technology
IIT Post Office
KANPUR-208016 U.P. INDIA

MATHEMATICS AND LANGUAGE III

NIGERIA

Several authors have attributed the language problem as one of the major causes of poor understanding of mathematics among Nigerian primary and secondary school students, see Lassa (1980) Abayomi (1980), Ale (1981), Kalejaiye (1982), Ali (1982). In his statement of the problem Ali (1982) opinioned that "English Language problems experienced by Nigerian Secondary School mathematics students in their study of mathematics, have continually contributed significantly to their poor achievement in their studying, understanding, and the applying of mathematics concepts as well as poor attitudes toward mathematics". Surprisingly however, Ali discovered in his research that in higher forms, "Nigerian secondary school students taught mathematics in English will demonstrate significantly more favourable attitudes towards mathematics compared to their counterparts taught similar mathematics in the dominant local language in a particular geographical location". The reason for this is not far-fetched, since at higher level, their local languages have not been developed to meet the teaching and learning of mathematics.

The case in the primary school is somehow different and of much concern. Lassa (1980) has categorically stated that "when English is the language of education, the majority of pupils were not able to exercise their conceptual potentials. On the other hand, the vernacular (Hausa) were more fruitful medium of enhancing the language thought interaction". The present situation is that in almost every part of the country, mathematics is taught in the local language in the first two years of the primary school, although in some few rural areas, it goes beyond two to say four or five years. In the more enlightened urban areas, the teaching of mathematics starts right from the first year. But no matter when they start teaching the pupils mathematics in English, there is the usual problem of the teacher not reaching the pupils effectively and the pupils not being able to communicate their thoughts with the teacher. There are cases of pupils who do not understand a word of the English language being used. The majority of the pupils are only able to pick up some few words of English being used, and in the mathematics class they inevitably run into the problem of mathematical syntactics and concepts. Unfortunately many therefore lose interest right from the onset. The general practice in many schools however, is the simultaneous use of both the local and English languages. Although this has its problems, most pupils feel more comfortable with this practice.

By and large, the most serious problem in this area of language and mathematics learning in Nigeria is the non-development of local languages in mathematics. There are practically no books in mathematics in any local language beyond the first two or three years of primary school. And it appears that there are no efforts being made to improve this situation. Certainly as Nigerian researchers in this area agreed, excellent achievements in mathematics can be considerably improved through the use of fully and purposefully developed local languages, Mohammed (1978), Usong (1979).

REFERENCES

1. Abayomi, F. (1980) "Relationships between some selected Nigerian Languages and Interest in Mathematics among Ogun State Secondary School Mathematics Students" Journal of Maths Educ. Research 3.

2. Ale, S.O. (1981) "Difficulties facing Mathematics teachers in developing countries" Educational Studies in Mathematics, 12 (479-489).

3. Ali, A. (1983) "Language problems in studying, understanding and applying maths concepts among Nigerian secondary school students" Paper:Mathematics Ass. of Nig. Conf., Univ. of Calabar - Sept. 18-22.

4. Kalejaiye, A.O. (1983) "Evaluating Junior Secondary School Algebra New Syllabus" Abacus - Journal of Math. Ass of Nig. Vol. 16; 46-62.

5. Mohammed, B. (1978) "Maths Achievement and the Language of Instruction: The Sudanese Case Study" Centre for Nigerian Languages, Bayero University, Kano. Mimeo.

6. Lassa, P.N. (1980) "The problems of teaching and learning mathematics in a second language. Africa experiment" Short Communication ICME IV Berkeley U.S.A.

7. Usong, F. (1979) "Teaching Mathematics in Efik for Improving Students Attitude toward Maths" Ph.D. Dissertation. Univ. of Nebraska.

S.O. Ale
Federal University of Tech.
Minna, Nigeria

UNITED KINGDOM

The theme of Mathematics and Language is one which is being actively pursued in many different ways throughout the U.K. The two major professional associations for mathematics teachers, the Association of Teachers of Mathematics and the Mathematical Association, have each established language and mathematics working groups. Also, a substantial amount of valuable work is done at a local level by teachers' groups; these are often co-ordinated at a Teachers' Centre by the Local Education Authority's advisers and inspectors. Furthermore, research and development is being carried out in Universities, Colleges and Polytechnics by both staff and research students; this work often becomes the basis of seminars at the meetings of bodies such as the British Society for the Psychology of Learning Mathematics or the Mathematical Education Section of the National Association of Teachers in Further and Higher Education.

It is not the intention of this article to review all the activities mentioned above. Instead, I will describe the work of the Language and Reading in Mathematics Group, a group to which I have belonged since its early days. Perhaps members of other groups would like to send in reports for publication in the Bulletin, too.

The L.R.M.G. started in 1978 as a discussion group at B.S.P.L.M. meetings. It soon became an independent group, helped financially by the Mathematical Education Section of N.A.T.F.H.E. Its membership has of course fluctuated over the years, but consists of people in all walks of professional life: teachers, advisers, authors, lecturers and researchers. The principal interest of the group is, as its name implies, the language of mathematical text.

Concern for written instruction in mathematics grew in the climate which followed the publication of the Bullock Report (1) in 1975 and which saw the increased use of the readability formulas developed in the U.S.A. Also classroom practices in both primary and secondary schools had reached a stage which made much use of textbooks and worksheets and very often created situations where the pupil had to learn mathematics solely by reading.

One could speculate whether this emphasis on learning from the printed page had always been present, or whether it was a new development, perhaps encouraged by the individualised learning approaches devised for mixed ability teaching. However, concern certainly became widespread that children were often being held back in their mathematical development by a failure of books to communicate clearly rather than a failure of children to understand the mathematics involved.

Interest in communication rather than content led the L.R.M.G. to consider the readability formulas developed to analyse ordinary text (2) and the one developed by Kane, Byrne and Hater (3) for mathematical text. Such formulas proved to be a major disappointment. The formulas all have quite pronounced weaknesses, particularly when used in mathematics. In addition, their use, though sometimes fascinating, does not seem to help with the professional problems faced by teachers and writers. It is very thought-provoking to be told by a formula that a certain textbook for 8 year olds has a reading age of 15. But the formula does not help the author to communicate more clearly nor does it help the teacher to know where and how to give the support needed by the pupils.

The L.R.M.G. investigated a broad spectrum of factors contributing to the readability of a mathematical text, most of which were almost impossible to quantify. The group published a working paper, Children Reading Maths, in 1980 which though not a research paper in the normal sense, has proved to be of practical help to many people working in this field. In particular, the approach adopted has been of value to teachers' groups who have sought ways of looking at textbooks which would help them in their work. Members of the group have made contributions based on the group's approach to various publications (see for example refs (4) and (5)). The working paper has been substantially revised since its initial publication, and a new book, Children Reading Mathematics (6) is to be published later this year.

The approach adopted by the group is one which suggests that the informed judgements of a teacher or writer can be used as the basis for finding how to cope with pupils' reading difficulties. The approach has involved looking at such things as vocabulary, syntax, symbolism, graphic conventions, spatial layout and overall organisation. Though these aspects of text are not really quantifiable, they are, broadly speaking, observable features of the printed page. Recently, group members have been interested in identifying communication problems which remain after the readily observable reading obstacles have been removed. Work with children reveals areas of ambiguity and lack of communication caused more by the nature of the information presented rather than the way it is written and displayed. Also, current interests of the group include: ways of helping children to become more skilful in reading; and the ways in which a teacher can use a book in the course of a lesson.

The group's work is presently continuing. Though particular activities change from time to time, the wish to consider communication processes rather than mathematical content in a practically relevant way remains a constant theme.

REFERENCES

1. Bullock, A. (1975) A Language for Life, H.M.S.O.

2. Harrison, C. (1980) Readability in the Classroom, C.U.P.

3. Kane, R.B., Byrne, M.A. and Hater, M.A. (1974) Helping Children Read Mathematics. New York American Book Co.

4. Harvey, R., Kerslake, D., Shuard, H. and Torbe, M. (1982) Language Teaching and Learning: Mathematics. Ward Lock Educational.

5. Rothery, A. 'Readability in Maths', in Gillham, B. (ed). The Language of School Subjects to be published by Heinemann Educational Books.

6. Shuard, H. and Rothery, A. (eds) (1984) Children Reading Mathematics John Murray (publication expected in May 1984).

Andrew Rothery
College of Higher Education
Worcester

U.S.A.

In the United States of America there is a significant number of individuals who are interested in aspects of mathematics and language at a variety of levels. Among such persons, I have selected ten, condensed their information and listed it here in the hope that others in the international community will contact them in order to further exchange ideas.

Grace Burton (Dept. of Curricular Studies/UNC/Wilmington NC 28403) has quite varied interests. She encourages the teaching of mathematics at the early childhood level through the use of a language experience approach and uses this approach in her college classes as well. She is also enthusiastic about the translation of number sentences into verbal descriptions and vice versa. Closely related topics with which she is currently concerned are ways to use writing as a vehicle for learning, sex-fair language and the use of metaphor as an educational tool. Among her articles in various stages of publication are "Understanding Word Problems" and "Writing as a Way of Knowing in the Math Methods Class."

Gerald Goldin (Dept. of Mathematical Sciences/NIU/DeKalb IL 60115) is a mathematical physicist who is also involved in research on the psychology of mathematical problem solving. His publications include "Variables Affecting Word Problem Difficulty in Elementary School Mathematics", "Syntax, Content, and Context Variables Examined in a Research Study", "Levels of Language in Mathematical Problem Solving" and "Mathematical Language and Problem Solving." The latter appeared in a special issue of Visible Language (Vol. XVI, No. 3, summer 1982) devoted to understanding the symbolism of mathematics. Inquiries concerning the possibility of obtaining this issue, which includes articles from England, France, Canada and Belgium as well as the USA, may be made to Visible Language/Box 1792, CMA/Cleveland OH 44106.

Hadas Rin (Dept. of Education/UC-Davis/Davis CA 95616) is interested in the meaning of "understanding mathematics" and the possible existence of something we call "understanding" which is different from the ability to solve standard exercises. In the communication of mathematics he has found that specialized linguistic skills are implicitly needed in math, but are not systematically included in instructional design. He is also seeking ways to incorporate these issues into teacher training programs. Among his publications/presentations are "Tutoring in a First Deductive Mathematics Course" and "Linguistic Barriers to Students' Understanding of Definitions."

Carol Novillis Larson (Dept. of Elementary Education/UA/Tucson AZ 85721) is interested in the relationship of concept and language acquisition in the area of rational numbers, the processes involved in reading mathematical material and psycholinguistic research on young children's acquisition of relational and dimensional adjectives. Her publications include "The Role of Language in Teaching Beginning Fraction Concepts" and "Linguistic Aspects of More/Less and Dimensional Adjectives Relevant to Diagnosis and Instruction."

Professor Emeritus Dora Skypek (Division of Educational Studies/ Emory Univ/Atlanta GA 30322) lists "Teaching Mathematics: Implications from a Theory for Teaching the Language Arts" among her publications. She has also given numerous talks on mathematics and language over the years and her interests in "keeping up with what's going on" are not diminished.

A second group of Americans consists of those with particular interests in certain subcultures (e.g. African, Asian, Hispanic and Native American). Genevieve Knight (PO Box 6613 Hampton Institute/Hampton VA 23668), Helen Cheek (104 Gundersen/OSU/ Stillwater OK 74078) and Gilbert Cuevas (Bilingual Education Mathematics Project/UM/Coral Gables FL 33124) have all worked to increase the participation and achievement of underrepresented groups in mathematics. Dr. Knight is interested in the effect of language on the acquisition of mathematical knowledge for young children and on the appropriate language of instruction, while Prof. Cheek has written a K-3 curriculum for a bilingual mathematics program using a language experience approach. Dr. Cuevas has research interests in the role a second language plays in mathematics instruction/learning, the development of mathematical concepts among language minority students: cultural factors, the assessment of achievement in mathematics for language minority students: Measurement biases and the relationship between cognitive styles and mathematics learning among culture different students. His publications in the area include "The Effects of Test Language and Mathematical Skills Assessed on the Scores of Bilingual Hispanic Students" and "Mathematics Learning in English as a Second Language."

Lehi Smith (Mathematics Dept/ASU/Tempe AZ 95287) has given several papers at professional meetings. His "Mathematics Education in an American Indian Culture" is noteworthy, in my opinion, for the sensitivity he displays towards this subculture in treating delicate aspects of serious problems in mathematics that far too many Americans have simply written off as being intractable.

Sau-Lim Tsang (Executive Director/ARC Associates, Inc./310 Eighth Street, Suite 220/Oakland CA 94607) has designed and supervised the work of fifteen curriculum writers, evaluators and graphic artists on a project for the development of curriculum materials for Chinese immigrant students. Among his publications/presentations are "A Study of the Effect of the Language and Cultural Content of Mathematics Word Problems on Chinese Students" and "Mathematical Learning Styles Of Chinese Immigrant Students: Final Research Report."

My own informal interest in mathematics and language stems from teaching and consulting trips to Francophone Africa spanning a period of three decades and my current position where I have been for twelve years as professor of mathematics at a Traditionally Black University (i.e. 90% of our students are Blacks. They attend by choice: not by law as it was several years ago in the South.) Among our very weakest students there are some with obvious linguistic difficulties such as the multiple meanings of "more." The most striking example, however, that I have ever encountered occurred during my work among the Batetela tribe of central Zaire. In p v q, their word for "or" (kano) heavily emphasizes the probability that p is true while allowing the faint possibility that q might be true. To put it mildly, the union of sets A and B is very different from the union of B and A.

I look forward to meeting many of you at the Congress in Australia. I fear that too often our work is done in isolation and is fragmented. One of my colleagues mentioned above wrote that his work is piecemeal and difficult. The reality of the problems in mathematics and language is certain, he continued, but the enormity and complexity are staggering. Together, however, we can make progress.

Don Hill, Chairman
US Commission on Mathematical Instruction
Florida A & M University--- Mathematics
Tallahassee, FL 32307

PSYCHOLOGY OF MATHEMATICS EDUCATION

(P.M.E.)

The Eighth Conference of P.M.E. will take place in Sydney Australia, from 16-19 August, 1984. P.M.E. is an officially recognised sub-group of the International Commission on Mathematical Instruction, and developed at the 3rd I.C.M.E. out of the research interests of those active in the field of psychology of learning mathematics. P.M.E. holds annual Conferences, every fourth of which coincides with an I.C.M.E. and is held in the same, or a nearby venue. Hence the 8th P.M.E. is located in Australia prior to I.C.M.E.5. The opportunity is being taken to make I.C.M.E.5 participants aware of the interests, developments, concerns and foci of those researching in the psychology of mathematics learning by running a series of four sessions which will offer insight into both the nature of P.M.E. research and the methodologies usually applied. The four sessions are:-

1. What do we learn from analysing students' work and interviews with students? A range of protocols will be provided for analysis.

2. Long-term evolution of students' conceptions; levels of understanding; illustrations in one domain: early number concepts in the process of learning.

3. How students' conceptions conflict and change in the process of learning. Videotapes and transcriptions will be used as a basis for analysing interaction, with situations, other students and teachers, from different standpoints.

4. The nature of mathematical thinking: intuition, operations, discovery and proof. A panel presentation.

It is hoped that the considerable advances which have been made in understanding the psychological bases of certain aspects of mathematics learning will be demonstrated to ICME participants in order both to inform and also to make results available. Participants, especially practitioners, will have the opportunity to work with transcripts and videos in the mode of researchers but in such a way as to see the applicability of research results to current teaching concerns.

Those interested in participating in PME 8 prior to ICME5 can obtain further details from:

Dr. Beth Southwell, P.O. Box 260, Lindfield,
New South Wales, 2070 AUSTRALIA.

Dr. Leone Burton
Secretary, PME

MATHEMATICS EDUCATION IN ASIA AND THE PACIFIC

(National Institute for Educational Research,
5-22 Shimomeguro 6 - Chome, Meguro-ku, Tokyo, Japan)

The Final Report of a three-week workshop mounted by the NIER of Japan and the Asian Centre of Educational Innovation for Development, UNESCO, Bangkok in October 1983 has just been published.

The Workshop was attended by some twenty educators drawn from Australia, Bangladesh, Indonesia, Japan, Malaysia, Nepal, New Zealand, Pakistan, Philippines, Singapore, Socialist Republic of Viet Nam, Sri Lanka and Thailand. Its objectives were

(1) To review and analyze the present state of the art of mathematics education;

(2) To identify crucial current issues and problems of mathematics education in the region;

(3) To synthesize innovative experiences and approaches developed in this area; and

(4) To develop strategies for future plans in mathematics education.

The Final Report contains a valuable comparative survey of educational opportunities and curricula in the participating countries. These serve to highlight some of the demographic problems faced by developing countries e.g. in the Philippines over 17 per cent of the total population is enrolled in the six-year primary/elementary schools. Yet these are only attended by between 50 and 75 per cent of the age cohort (For comparison, the full cohort which attend similar schools in Japan form about 10 per cent of the total Japanese population.) The Report also presents synopses of the official objectives for mathematics education in the various countries - the differences in which would not always appear to be reflected in the national curricula.

Other chapters deal with 'Problems and Issues', 'Innovative Experiences' and 'Suggestions for Improving School Mathematics Education'.

An appendix gives the impressions of the Workshop participants on the ICMI-JSME Regional Conference which they attended immediately after the Workshop (see ICMI Bulletin No. 15).

This is a very valuable account of the Conference seen through 'developing country' eyes and it succinctly raises many important issues.

TEACHING MATHEMATICS AT UNIVERSITY

'Teaching Mathematics at University' is the title of a new course to be offered by and at the Centre for Mathematics Education, Southampton University. The ten-week course has been especially designed with the needs of young university staff from developing countries in mind.

The course aims to help young members of staff gain professional skills, and to increase their awareness of relevant issues and of sources of assistance. It is intended for those who have successfully completed a postgraduate (doctoral or master's) degree course in mathematics and who have a good command of written and spoken English. Between four and eight participants will be enrolled for each course.

The Overseas Development Administration and the British Council have recognised the course as qualifying for British Government Technical Cooperation Training awards. Thus in certain circumstances fees, international airfares, a stipend and other allowances are paid.

Although the direction of the course is solely the responsibility of the Centre for Mathematics Education, Southampton University, the proposals have received the full approval and support of the Chairman of the International Mathematical Union's Commission on Development and Exchange and of the President of the International Commission on Mathematical Instruction.

It is intended to hold the course annually between October and December.

Further information concerning the course and application forms may be obtained from the Secretary, Centre for Mathematics Education, Southampton University, Southampton, SO9 5NH, England.

ACKNOWLEDGEMENT

This Bulletin has been prepared with the help of a grant from UNESCO.

NOTE