ICMI

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of the International Commission on Mathematical Instruction

No. 14

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Secretariat
Centre for Mathematics Education
University of Southampton
Southampton, SO9 5NH
England

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Editors: Keith Hirst and Geoffrey Howson
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University of Southampton
Southampton, SO9 5NH
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COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE (INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

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H. HOGBE-NLEND, IMU Representative at CTS-ICSU, Université de Bordeaux — 1, UER d'Informatique et de Mathématiques, 351 Cours de la Libération, 33405 Talence, France.

EDITORIAL NOTE

In Bulletin 13, the first to be published under our editorship, we explained that all the contributions had been solicited or supplied by the editors. We went on to express the hope that in future readers would take the initiative and send us material which they thought would be of interest to the international community of mathematics educators which the Bulletin seeks to serve. We are most grateful to those readers who accepted our invitation and hope that others can be persuaded to follow their example.

This issue sees the first of our series of articles on institutions for the development of mathematics education and the second of the articles on periodicals. It also contains the first instalment of a history of ICMI's activities, one specially written to commemorate ICMI's 75th birthday. This article will be continued in Bulletin 15, in which we also expect to include descriptions of the recent Warsaw symposium, the work of the International Study Group on the Relations between the History and Pedagogy of Mathematics and the German periodical Zentralblatt für Didaktik der Mathematik.

By now the new form and aims of the Bulletin should be becoming apparent. Remember that those aims will only be achieved if our readers are willing to become our writers!

KEH, AGH.

NOTE

The production of this Bulletin has been greatly facilitated by a generous grant from UNESCO.

ICME 5

Adelaide, Australia 24-30 August 1984

The second and final announcement concerning the Fifth International Congress on Mathematical Education has now been published. Copies may be obtained from

ICME 5
GPO Box 1729
Adelaide 5001
South Australia
Australia

The announcement includes forms for

registration
accommodation
submission of abstracts of short communications.

Early submission of these forms (by airmail) is requested. As an encouragement to 'early birds', the registration fee for those registering and paying on or before 29 February 1984 will be $(Austr)140. The regular registration fee will be $175 and for those paying on or after 1 July 1984 the fee will rise to 195 Australian dollars.

An outline of the programme was given in Bulletin 13. We can now add the names of the plenary speakers. They are:

Professor Ubiratan d'Ambrosio of the Universidade Estadual de Campinas, Brazil, will talk on the socio-cultural basis of mathematics education.

Professor Jeremy Kilpatrick of the University of Georgia, USA, will examine thinking about thinking as a theme in mathematics education.

Professor A.P. Yushkevich of the Institute for the History of Science and Technology of the Academy of Sciences of the USSR will talk on the role of the history of mathematics in mathematics education.

Professor Renfrey B. Potts of the University of Adelaide, Australia, will talk on discrete mathematics and its significance in mathematics education.

There will also be a number of pre- and post- congress meetings. In particular, the International Group for the Psychology of Mathematics Education will hold its 8th Annual Conference in Sydney from 16-19 August and a one-day conference on the theory of mathematics education is planned for Adelaide on 31 August.

We hope to provide further details of these and other meetings in Bulletin 15.

ICMI EXECUTIVE COMMITTEE

The ICMI EC met recently in Warsaw on the occasion of the postponed International Congress of Mathematicians. We give here a brief report on some of the items discussed which we hope will have special interest for our readers.

ICMI is very pleased to welcome as new members Costa Rica and Mozambique. For some years now it has been possible for countries which are not members of IMU to join ICMI and Costa Rica and Mozambique have taken advantage of this arrangement. We hope shortly to publish short accounts of the educational systems and of work in mathematics education in these two countries.

The Warsaw ICM provided the EC with an opportunity to discuss how ICMI might best contribute to the ICM to be held in Berkeley in 1986. It was thought that it would not be appropriate to mount a symposium on the same scale as that at Warsaw to run concurrently with the ICM. However, we are keen that ICMI should make a strong contribution and that time should be devoted to consideration of educational matters, particularly at the higher education level.

One way in which ICMI hopes to contribute is by presenting the results of its first 'study' - on mathematics, computers and computation. This is one of four studies which ICMI is hoping to undertake. Here are their titles together with brief indications of their content.

Mathematics, computers and computation

To what extent has the coming of computers changed conceptions of what constitutes mathematics (e.g. the concepts of proof and of a 'solution' to an equation) and what is desirable knowledge for graduates to possess? Possible changes in views on mathematics and of undergraduate curricula will be considered, along with the role which computers can play in the teaching and learning of 'traditional' mathematics such as analysis, geometry, probability and mechanics. (This study will be mainly concerned with higher level education but the implications for students in their last two years at school will also be considered.)

Mathematics and Cognition

Recent developments and theories concerning how mathematics is learned will be reviewed. Attention will be paid to such aspects as visual perception, problem solving, reasoning, mental imagery, remembering, language and the role of activity. Priorities for further research will be identified and the implications for teaching of recent findings will be set out.

School mathematics in the 1990s

What are the major problems likely to occur in school mathematics in the 1990s? What place will the subject hold in a general education and how is its teaching and learning likely to be affected by changes in society? What decisions will face mathematics educators (and others) and what actions should we now be taking to ensure that the needs of the 1990s are met?

Mathematics as a service subject

As more and more disciplines employ quantitative methods and mathematical models, the demand for 'mathematics as a service subject' grows. What kind of mathematics is now most appropriate for the various groups of users (physical scientists, engineers, natural scientists, social scientists, ...) and how should it be taught/learned? Who should teach it? How will students most successfully acquire concepts, necessary techniques and the skills of mathematical modelling? (This study, which will be undertaken with the cooperation of ICSU - CTS, will be directed at the last two years of schooling and at undergraduate level.)

Each study will be built around an international seminar and will be directed towards the preparation of a published volume intended to promote and assist discussion and action at regional, national or institutional level through:

- the identification of key problems within the subject area and the provision of a framework which will facilitate further study, research, development and/or decision-taking,
- the provision of up-to-date accounts of relevant thought, research and practice.

It is intended to seek funds for these studies from industrial and educational trusts, and, of course, from UNESCO which has done so much to support the work of ICMI.

Already we have received some support from UNESCO and also a generous contribution from IBM Europe. We do ask readers to let us know of any likely sources for funds in their countries. Any assistance in obtaining finance would be most welcome. Since there are many other topics within mathematics education which ICMI could profitably study the greater financial backing we can obtain the better!

The support we have already received, plus a generous guarantee from IMU, has enabled the EC to make firm plans for the computer study. The following programme committee has been appointed:

J.-P. Kahane (France) Chairman
R.F. Churchhouse (UK)
A.P. Ershov (USSR)
J. van Lint (Netherlands)
A. Ralston (USA)
M. Yamaguti (Japan)
F. Pluvinage (France) Secretary and administrator.

Four members of the committee were able to hold a preliminary discussion in Warsaw and a planning meeting will be held in late 1983 or early 1984. The seminar will take place in Strasbourg in April, 1985 and it is hoped that the report will be published in April, 1986 so that it can be presented and discussed at the Berkeley ICM in August that year.

Of course, before the 1986 ICM we shall have held ICME 5 in August 1984 at Adelaide. The EC was most interested to have an account from Dr. Newman on how the plans for ICME 5 were progressing. One topic considered was that of interpretation facilities. The problem of language is an important and rather intimidating one. The provision of translation/interpretation facilities is very expensive and only really becomes economically viable for linguistic groups of 200 or so in size. We are expecting that number of Japanese speakers at Adelaide. It will help the Congress organisers enormously if others with linguistic difficulties will register early so that the extent of the problem can be gauged and action taken whenever possible.

Although ICME 5 is still a year away, the EC had also to consider questions associated with its successor ICME 6. It is vital that a decision concerning the venue of ICME 6 should be taken at Adelaide so that planning can commence immediately. We hope that countries will consider the possibility of putting in a bid to host ICME 6. The officers would be pleased to give any information or help they can to countries who do so. The EC also felt that attempts should be made to ease the burden which falls

on the host country by designing an organisational structure which could be employed at all ICMEs and which would ensure that high professional standards and wide international representation were attained. A sub-committee was established to make proposals to the EC and we hope to report further on this in a forthcoming Bulletin.

Finally, a most successful informal meeting was held with representatives from many countries - some the official 'national representatives', others attending on their behalf. In particular, we were very pleased to welcome many representatives from developing countries, especially from Africa. The question of the part to be played by national representatives at the four-yearly ICMI assemblies was raised and this again is a point on which we hope there will be more to report in an early issue of the Bulletin.

A.G. Howson.

REGIONAL MEETINGS

We regret that the Sixth Inter-American Conference on Mathematics Education announced for November, 1983 has had to be postponed because of financial problems.

The ICMI-sponsored meeting of the Southeast Asian Mathematical Society will be held at Prince of Songkla University, Haad Yai, Thailand in May 1984. The principal theme is "Mathematical Education in the Computer Age'. ICMI is represented on the planning committee by Professor Nebres and our President, Professor Kahane, will be one of plenary speakers. The Conference Secretary is Dr. S. Nualtaranee, Department of Mathematics, Chulalongkorn University, Bangkok 10500, Thailand.

NATIONAL REPRESENTATIVES

We give below some amendments to the list of National Representatives printed in Bulletin No.13. A complete revised list will be published in Bulletin No.15.

ARGENTINA	Professor N.D. Patetta, CAECE, Ave. de Mayo 1396, 1085 - Capital Federal, REPÚBLICA ARGENTINA.
AUSTRIA	Professor F. Schweiger, Institut für Didaktik, Universität Salzburg, Petersbrunnstrasse 19, A-5020 Salzburg, AUSTRIA.
COSTA RICA	Professor B. Montero, Asociación Matemática Costarricense, Apartado 5186, San José, COSTA RICA.
GERMAN DEMOCRATIC REPUBLIC	Professor K. Weber, Akademie der Pädagogischen Wissenschaften, Otto Grotewohl-Str., 1080 Berlin, GERMAN DEMOCRATIC REPUBLIC.
ISRAEL	Professor J. Gillis, Department of Mathematics, Weizmann Institute of Science, Rehovot 76100, ISRAEL.
ITALY	Professor V. Villiani, Dipartimento di Matematica dell Università, Via Buonarroti 2, 56100 Pisa, ITALY.
PORTUGAL	Not known.
SOUTH AFRICA	Professor P.G. Human, Faculty of Education, University of Stellenbosch, Stellenbosch, SOUTH AFRICA 7600.
USA	Professor D.M. Hill, Department of Mathematics, Florida A and M University, Tallahassee, FL 32307, USA.

SEVENTY FIVE YEARS OF ICMI*

This year ICMI celebrates its seventy-fifth birthday. Its life has not always been easy - on two occasions it has of necessity lapsed into a coma - yet it has a proud and interesting history which reveals much about changing attitudes in mathematics education. Even more importantly it has a challenging present and future.

The idea of an International Commission to enquire into mathematical education was first floated in 1905 by the American, David Eugene Smith, in L'Enseignement Mathématique, the revue founded in 1899 by Henri Fehr and Charles Laisant. A formal proposal was considered at the Fourth International Congress of Mathematicians held in Rome in April 1908 and it was resolved to establish the Commission internationale de l'enseignement mathématique (CIEM or, as its anglicized form now is, ICMI). The first President was the great German mathematician, Felix Klein, and the first Secretary-General, Henri Fehr. Klein was to act as President until his death in 1925; Fehr had an even longer association with ICMI, for he served as Secretary until the outbreak of the Second World War. When ICMI was reconstituted in the 1950s, Fehr was made 'Président d'honneur' for life. Unfortunately, he died later that year aged 84.

The reasons for the formation of ICMI at that particular period are not hard to perceive. The educational systems of the major countries of Western Europe and North America had expanded during the early years of the century, new technologies set new demands, and innovators had attempted to carry out significant reforms of the (grammar) school mathematical curriculum. In Germany Klein gave the lectures now known to us as Elementary Mathematics from an Advanced Standpoint, in France a government decree of July 1905 invited 'teachers to follow a method entirely new in geometry', and in England, as a result of the efforts of John Perry and others, Euclid's rule came to an end (not the spirit of Euclid, which Dieudonné was later to deplore, but the use of his Elements). Perry, indeed, wanted far more than just the reform of

* The views expressed in this article are those of the author, Dr. Howson, and not necessarily those of the ICMI Executive Committee or its National Representatives. A fuller version of this account complete with bibliographical references will appear shortly in Educational Studies in Mathematics.

geometry. He laid stress on making mathematics useful and on linking its teaching with that of science and engineering: he argued for 'utility' rather than 'rigour', laboratory-based experience rather than abstraction. His influence was world-wide, ranging from the USA to Japan, whilst German educators coined the term Perryismus. The times then were not dissimilar to the 1960s and there was a call for an international exchange of experience and views.

The Commission met for the first time in Cologne in September 1908, each country being invited to send a small number of delegates (thus, for example, the USA was represented by Smith, W. Osgood and J.W. Young). Nineteen countries were deemed to be 'participating' and a further fourteen were listed as 'associated countries'.

One of the first decisions of the Commission was to reject the terms of reference presented to it in Rome: to study programmes and methods of teaching to be found in the secondary schools of the various countries. To study secondary schools in isolation was, the Commission thought, impossible - ICMI had to consider the teaching of mathematics in all types of schools, including primary and vocational, and also in universities and other institutions of higher education.

Smith had in his Rome address listed certain questions which at that time greatly concerned the American teacher. Seventy-five years on they have acquired a familiar ring:

What have been the results of attempting to remove the barrier between such topics as algebra and geometry, or to teach the two simultaneously [i.e. in the same grade], and are we prepared as yet to make any recommendation on this matter?

[10th grade Geometry, which E.H. Moore opposed in his AMS Presidential Address in 1902, is still a matter for debate in the USA. In many countries, e.g. the USSR, it is still thought advantageous to separate 'Algebra' and 'Geometry' within the school mathematics course, each with its own timetable allocation and textbooks.]

What is the safe minimum of Euclidean geometry, the calculus and mechanics?

What position should the secondary schools take with respect to the nature of applications and the relations of applied to pure mathematics?

What should be the relative nature of the courses in the secondary schools for those who do not intend to proceed to the

universities, and for those who do intend to do so?

[That these and other questions posed by Smith are still with us is not necessarily a cause for pessimism, for it is in the nature of mathematics education that questions such as the last must continuously be asked. Changing social and mathematical contexts ensure that any 'solution' can only be a temporary one.]

It was to the study of these and similar questions that the founding fathers (for women have never been represented on ICMI's major committee) turned. Several international meetings were held between that at Cologne in 1908 and the ICM at Cambridge in 1912. At some of these general topics were discussed; at others, for example that in Milan in 1911, attention was concentrated on specific questions - in that particular case on 'What mathematics should be taught to those students studying the physical and natural sciences?, 'What is the place of rigour in mathematics teaching?' and 'How can the teaching of the different branches of mathematics best be integrated?' Once again, we have three questions equally relevant today.

The main work of the Commission at this time was, however, the preparation of a vast survey of teaching practices in member countries. Each participating country appointed a sub-commission to prepare national reports, often in many volumes, and the result was outstanding both in terms of quantity and quality. Thus, for example, the French report ran to five volumes and that of the USA to eleven. The British contributed only two volumes, but the first of these had over 600 pages! Certainly, nothing on the same scale had been attempted before, or has been attempted since. Moreover, not only did countries comment on their own systems, but, for example, as part of the German contribution G. Wolff wrote a fascinating account of secondary education in England, Der Mathematische Unterricht der Höheren Knabenschulen Englands, which still remains a model of a successful comparative case study. That its delayed publication should have taken place in 1915 when the two countries were locked in battle is just one further bewildering and poignant fact to be recorded from those years.

In addition to receiving the various reports, the 1912 Congress paid special attention to two other problems on which it had instigated surveys, 'The mathematical training of the physicist' and 'Intuition and experiment in mathematical teaching in the secondary schools'.

Encouraged by this extremely fruitful four years' work, the Commission chose two further topics to be studied prior to a meeting to be held in 1914 in Paris. One was the position occupied by calculus in the secondary schools, the second 'The place of mathematics in higher technical education', and the

speakers included Klein and Borel. The questions which the Sub-Commission on Calculus put to the member countries were extremely well devised and many present-day lecturers to first-year university students would still welcome information on, for example, what proportion of schools consider functions of several variables, discuss the remainder term in Taylor's Theorem, introduce the integral as the limit of a summation or as a primitive function, call attention to non-differentiable functions, and deal with irrationals logically and systematically (which is not the same as rigorously).

In six years, then, ICMI had covered an enormous amount of ground; as H.F. Baker wrote in his obituary of Klein, it 'had amassed an amount of information beyond belief'. Alas, the First World War was to bring these extremely productive years to an end. The ICM planned for Stockholm in 1916 was cancelled and when the ICMs were resumed in 1920 (at Strasbourg) little time was devoted to consideration of pedagogical matters. It was not until the 1928 ICM in Bologna that ICMI was formally reconstituted, although in the intervening years Fehr attempted from Geneva to keep the sub-commissions active and in touch. As we have already observed some nations still published reports during the war years, notably Germany and the United States before it, too, entered the conflict. The latter, indeed, produced a series of four volumes which compared mathematics curricula and teacher-training programmes as they were described in the various national reports produced for ICMI in 1912.

In many ways it is frightening to contemplate the work which the Commission successfully completed in those early years, to see how it so quickly identified key questions and started to collect data concerning them, and to realise how little progress may appear to have been made in the seventy years since. I have already referred to '10th grade geometry'; one can also think, say, of the slow rate at which calculus has penetrated US High Schools. Yet, when considered in more detail, one sees that the Commission usually took what we should think of today as a one-sided view of mathematics education. It was very concerned with subject matter and to a lesser degree with how that content was presented. The psychological problems of learning mathematics had yet to be given any deep consideration. It was also the case, as Baker pointed out, that the Commission was more successful in amassing information than in analysing and interpreting it - he expressed the hope that one day the material collected 'may be utilized'.

When the Commission was re-established at Bologna in 1928, the officers were chosen from those who had been active in ICMI prior to the First World War. The choice of Smith as President was interesting in so far as he, unlike other Presidents, was not highly distinguished as a mathematician, being, in fact, the holder of a

chair in mathematics education (at Teachers College, Columbia University). Guido Castelnuovo and Hadamard resumed their places on the Central Committee, now as Vice-Presidents. In 1932 Hadamard succeeded Smith as President and remained in that office until the outbreak of the Second World War caused ICMI to relapse into its second 'coma'. Although Castelnuovo retired from the Committee in 1932, his daughter, Emma, has ensured that the name has continued to feature prominently in ICMI affairs.

The years between the two World Wars were not, however, particularly momentous ones for the Commission. The re-establishment of ICMI coincided with a world-wide depression and there was little interest shown in educational innovation and expansion. Such changes as took place were usually low key - educational systems grew slowly, Gestalt psychology became established but, like the writings of the young Swiss psychologist, Piaget, failed to have any great influence on classroom practice. Smith had retired from Columbia in 1922, and the two Vice-Presidents of ICMI were both well into their sixties. The war had apparently resulted in a 'missing generation' and the ICMI activities wore a tired look. At Bologna it was agreed that a survey on the training of secondary school mathematics teachers would be carried out prior to the ICM to be held in Zurich in 1932. The survey was planned with the thoroughness which distinguished the pre-1914 work. For example, a section devoted to the professional training of teachers sought information on such matters as - did they receive university courses in the methodology of teaching mathematics and if so from what type of person (a university professor or a secondary-school teacher); were they obliged to follow general courses in pedagogy and psychology, and if so how useful were such courses thought; what opportunities were they given to develop practical (laboratory) skills, how much teaching practice was provided and at what stage of their training; how was teaching practice supervised and assessed? Reports were presented at Zurich from fourteen countries (including the newly-created Czechoslovakia, Poland and Yugoslavia). There it was agreed that the topic for the 1936 Oslo ICM should be 'Present trends in the teaching of mathematics'. On that occasion twelve countries presented reports including Japan, the only non-European country other than the USA to participate actively in the inter-war ICMI activities (which would appear to have been largely confined to the two surveys described). As Fehr wrote, such surveys attempted to provide a purely objective view of the current state of mathematics teaching; the Commission did not wish to impose any international uniformity, but only to illuminate new trends and to help facilitate progress - a principle to which ICMI still adheres.

(To be continued)

INSTITUTIONS FOR MATHEMATICS EDUCATION

1. THE IREMs

My purpose here is to give readers a basic idea of the working of the French IREMs (Instituts de Recherche pour L'Enseignement des Mathématiques - Institutes for Research in Mathematics Teaching). I shall make use of my several years experience as a collaborator with, and as director of the Strasbourg IREM; but what I shall describe is essentially true of all the IREMs.

There are in France 25 IREMs, that is to say, with a few exceptions, one per 'Académie'. (France is divided, for the purposes of educational administration into regions called académies; a typical example that of Strasbourg, which is of average size, comprises about 200 secondary schools (130 Collèges and 70 Lycées) with 1200 mathematics teachers.)

The Strasbourg IREM occupies ten or so offices in the Department of Mathematics of Strasbourg University. It includes a secretariat (two persons), a specialised library, and a room for microcomputers.

In order to describe what an IREM does, I am taking as an example the year 1976-77 when material conditions (i.e. financial and scientific resources) were maximal. That year the team of teachers (or 'animateurs'*) of the IREM consisted of 7 members of the university mathematics department, a university astronomer, two psychologists and 13 secondary school teachers: all of whom devoted only a part of their time to the work of the IREM (from one third to a half). The work could be divided out under three headings:

1) The continued training of teachers (in-service education)

About 300 teachers in the region (académie) participated, for 3 hours a week for a period of 25 weeks, in the

* This word with its overtones of advising, enlivening and encouraging would seem to lack an obvious English equivalent [trans].

working groups organised by the IREM. These groups were of three types:

(a) Continuing academic education (e.g. transformation geometry, introduction to informatics, the history of mathematics, astronomy, computational methods).

(b) The practice of teaching (e.g. exercises of intelligence and imagination in classes at various levels, the teaching of mathematics in technical schools, cooperation between mathematics and physics teachers, ...).

(c) Pedagogical reflections (e.g. methods of evaluation, the influence of language in the teaching of mathematics, initiation into team teaching, ...).

The groups (of 10 to 20 persons) met in various towns of the region in order to limit the amount of disturbance involved.

2) Research and publications

Examples: Continued observation of classes in collèges; the study of cognitive processes in mathematics by means of 'questionnaires à modalités'*; the editing and publication of two books - one a collection of exercises of incidence geometry, the other a manual for the 6th class of the collèges (pupils aged 11 to 12).

3) Activities intended to stimulate

These are permanent: the IREM issues periodically a journal containing mathematical and pedagogical information; it also organises each year a 'mathematical rally' for pupils in the senior classes of the Lycées (a competition of the Olympiad type) which has met with great success, attracting from 500 to 1000 participants.

Now let me give some information concerning administrative and financial matters.

Each IREM is attached to a university; its director is a university person nominated for a limited period (2 or 3 years in practice) by the Ministry of Education on the suggestion of the President of the university and the Rector of the académie.

For its working resources it depends upon:

1) the university for its location, its secretarial staff, and the university animateurs,

* see Proc. ICME IV pp 465-7.

2) the Ministry of Education for its recurrent budget and those animateurs teaching in the secondary schools.

The relations with the Ministry are taken care of through the existence of a National Commission for the IREMs which consists of the IREM directors and representatives of the central administration. This commission meets at least once a year and is responsible for distributing the Ministry's financial grant between the IREMs (roughly in proportion to the size of the various académies).

I now wish to return to the way in which the team of animateurs function, for this is the keystone of the system. This team has the following characteristics:

- it is formed at any one time of university teachers, who therefore have experience of research in mathematics, and of teachers at all levels of secondary education.

- it is changed each year; there are no 'permanent' animateurs.

- the animateurs devote only a part of their time to the work of the IREM; they continue to teach in their home institutions.

- the animateurs have no 'hierarchical power' over the teachers who take part in the work of the IREM (this is in contrast to the national inspectors of schools, who in France are responsible both for stimulating and advising teachers and also for assessing their performance).

The 'part-time' principle has led to important difficulties of replacement, for it involves negotiation with the secondary schools and the regional educational administration. In practice it unfortunately often happens that an animateur must continue in full-time teaching at his school, and do his work for the IREM, and be paid for this, as an additional task. So far as university personnel are concerned, the renewal of animateurs has always been difficult; in particular, few active mathematicians are as yet truly interested in the IREMs. On the other hand, the research teams in didactics which have been formed in the IREMs are now extremely active (we note amongst others: Bordeaux, Grenoble, Marseilles, Paris, Strasbourg, ...).

From the example which I cited earlier, one can see that the continuing education of teachers is a very important

part of the work of the IREMs. It is important to make clear that from the creation of the IREMs and up to about 1978, the government assumed responsibility for this work in the following sense: the hours of training (at the IREM) were considered as hours of supplementary work and were therefore paid; in this period, the 'stagiaires' (i.e. the teachers who assisted at a working group) received as payment a sum corresponding to 1½ hours of overtime per week. In order to give an idea of what was involved, in 1976-77 the activities of the Strasbourg IREM represented 480 weekly teaching hours (that is equivalent to about 25 teaching posts); this 'hourly budget' was laid down each year by the Ministry of Education. This policy was a follow-up of the reform of the mathematics syllabus, for which many teachers had not been fully prepared. (This was moreover not a true policy of continuing education, but rather an attempt to palliate the deficiencies of the initial training of many mathematics teachers.) Since 1978 the government has gradually withdrawn this budget; in-service education activities of the type described still continue, but on a much reduced scale: on the other hand the research effort has increased.

To the reader who wishes to know more, I recommend the following publications:

Bulletin Inter-IREM no. 17 (1979): Les IREM; missions et activités,

Bulletin Inter-IREM no. 22 (1983): Catalogue des publications des IREM.

(These booklets can be obtained on request from l'IREM de Lyon, Université Claude Bernard, Bd du 11 Novembre 1918, 69622 Villeurbanne Cédex, France.)

Jean Martinet,
Département de Mathématiques,
7, rue René Descartes,
67084 Strasbourg Cédex,
France.

JOURNALS ON MATHEMATICS EDUCATION

2. RECHERCHES EN DIDACTIQUE DES MATHEMATIQUES

RDM appeared as a product of the very fruitful activity of reflection and research about mathematics teaching which took place in the sixties and in the early seventies. In France, particularly, within the well known IREMs, research groups were formed which attempted to improve a scientific approach to educational facts and phenomena. One manifestation of all this work is the identification of more rigorous and more fruitful pedagogical problems and methods and of basic notions in the organization of research on mathematics education : theory of didactical situations, educational transfer, conceptual fields, etc...

RDM publishes fundamental articles about experimental studies on the conditions of the acquisition of mathematical knowledge by children in classroom situations. These papers contain outcomes of research, reports about pedagogical problems, and/or methodology. The journal also includes proceedings of important meetings and reviews of books relating to mathematics education.

RDM has one volume of 3 issues per year. Each issue contains about 3 or 4 articles. The following titles will show the variety of the mathematical notions and of the level of teaching concerned:

Guy Brousseau : Problèmes de l'enseignement des décimaux
Problems of the teaching of decimal numbers
Volume 1, No.1
Volume 2, No.2

Schubauer-Leoni M.L., Perret-Clermont A.N. :
Interactions sociales et représentations symboliques dans le cadre de problèmes additifs
Social interactions and symbolic representation in the case of additive problems
Volume 1, No.3

Michèle Artigue, Jacqueline Robinet :
Conceptions du cercle chez les enfants de l'école élémentaire
Children's conceptions about circle (primary school)
Volume 3, No.1

Nicolas Balacheff :

Preuve et démonstration en mathématique au collège
Proof and demonstration in the secondary school

Volume 3, No.3

James M. Moser :

The emergence of algorithmic problem solving behavior

Volume 3, No.2

The last issue of the journal was entirely devoted to a series of papers about the teaching of volume at secondary school level. The conference of the 5th meeting of the "Psychology and Mathematics Education" group held in Grenoble in July 1981 was reported in volume 2, No.2 and No.3. From volume 5 onwards, RDM will inaugurate a regular account of the history of mathematics teaching. It will report work which can provide an important contribution to the understanding of educational phenomena. The articles submitted are reviewed by two readers (members of the editorial board) one of whom is of a different nationality from the author. Of course, the journal keeps the right to request articles from specialists in a specific field. RDM publishes articles in French, English or Spanish, according to the international standards. The subscription for one volume is 170 FF/$ 25 and 240 FF/$ 40 for institutions. Subscriptions should be addressed to the publisher.

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ICME IV

The Proceedings of ICME IV held at Berkeley in 1980 have now been published by Birkhäuser. This large volume contains some two hundred papers on all aspects and levels of mathematics education and is clearly a must for any library. (ISBN 3-7643-3082-1)

ANGLO-SOVIET SEMINARS ON MATHEMATICAL EDUCATION

An English version of the Proceedings of the second seminar (see Bulletin 13, pp.13-14) is now available and may be obtained on request from The British Council, STED, 10 Spring Gardens, London SW1A 2BN England.

MATHEMATICS AND LANGUAGE

In Bulletin No.13 we invited readers to send us brief descriptions of what was being done in their countries to study the way in which mathematics and language interact and to alleviate identified problems. Below we print three accounts from Australia, India and Lesotho. We are most grateful to their authors and invite readers from other countries working in this area to send us further accounts for publication in later Bulletins. (Please try to ensure that these reach Southampton before the end of 1983.)

AUSTRALIA

There is a very strong interest in the interaction of language with mathematics education in Australia. Specific concerns include the readability and comprehension of mathematical text, communication in the classroom, cognitive functions of language in mathematical development, cultural factors influencing mathematics learning, and the learning of mathematics in English as a second language. This list is not of course comprehensive but it does show the wide ranging interests of different researchers. The following paragraphs focus on a few aspects of ongoing work.[1]

Firstly, there has been some very productive work in problem solving. Newman has published a 'language of mathematics kit'[2] based on her research which identifies primary children's performance strategies on written mathematical tasks. The classification of these strategies enables classroom teachers to diagnose specific weaknesses of children in problem solving and plan remedial action. Secondly, some recent clinical studies by Lean and Clements in Papua New Guinea have shown a clear link between a preference for the use of language as a tool for thought and success in problem solving at the secondary/tertiary levels. They have also made some interesting observations on the ways students solve spatial tasks, and opened up promising avenues of approach on the interplay of language and visual imagery in problem solving.

In tribal Aboriginal communities, Harris has been studying the language of measurement. She has produced helpful resource materials for teachers in Aboriginal schools.[3] Harris' work supports the contention that concepts of number, space, time and money are very much a matter of culture.

Finally, there is a useful contribution being made in the area of learning mathematics in English as a second language. A great deal of this work has been done in Papua New Guinea (Clarkson, Clements, Jones and Lean to name just a few researchers) but increasing attention is being paid to Aboriginal children and children of migrant families who have settled in Australia. The mathematics register of English is coming under closer scrutiny and some work is being initiated into the role of logical connectives in deductive reasoning. The possible cognitive advantages of bilingualism for deductive reasoning in mathematics have been recently investigated by Dawe. Work in this area is continuing.

1 For a recent review of the Australian literature see Clements,M.A. 'Language Factors in School Mathematics', Australian Review of Applied Linguistics, 5, 1, 1982.

2 Newman, A. The Newman Language of Mathematics Kit, Harcourt Brace Jovanovich Group, Sydney, 1983.

3 Harris, P. Mathematics in Aboriginal Schools Project Series, Northern Territory Dept. of Education, Darwin, 1983.

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INDIA

Even though mathematics has a language of its own, its instruction through the language of the mother tongue in a country like India often poses some problems. The terminology in the language of instruction in respect of mathematical terms may often be inadequate to convey the mathematical import of the terms concerned. As in English, there are some mathematical terms, belonging to the language of instruction, that are used in common parlance. The difficulty ensues when a learner, conversant with such terms as part of common parlance, starts handling the same terms in mathematical contexts. Another kind of difficulty arises from the use of mathematical terms in a situation where there has to be the necessary use of the language of instruction, e.g. 'verbal problems' in mathematics. This is also reflected in the performance of a sample corresponding to a terminal examination. The actual case study undertaken by Das and Sinha shows that there exists a genuine problem of translating common language into the language of mathematics or vice-versa, indicating the weakness,in the language, of vocabulary with mathematical overtones even if the language be the mother tongue. There has also been a case study by Das and Sinha for a very selected group of students who even have English as the language of instruction. The same concern is expressed in the case of various languages of instruction in a multi-lingual country like India. It is, therefore, increasingly realised that the basic issue underlying such problems, namely, the interaction between the language of instruction and mathematical education, should be studied in depth. The series of papers by Das and Sinha and the Ph.D. thesis of Das are attempts in recent times in India in this direction. These also deal with the conceptual framework necessary for obtaining better understanding and insight into the problems concerned.

Some more model studies involving wider samples in complex situations are being undertaken. At the national level a separate session was devoted to deliberations on this theme at the National Conference on Mathematical Education organised by NCERT. As a follow up to this conference and other activities that have already been initiated, a National Seminar cum Workshop on this theme is being planned sometime towards the end of this year. The proceedings of the previous conferences of this type, particularly at Nairobi and Accra,are being kept in view. Various aspects of linguistics, particularly sociolinguistics and psycholinguistics would be taken up as lead presentations. The issue of bilingualism and multilingualism vis-a-vis mathematical education will certainly be a focus in the proposed activity. The language of the teacher in the classroom will also be attended to. Mathematical linguistics and mathematical symbolism will be considered in the proper perspective. The problems of transfer of concepts, problems of concept formation and language as also learning abilities through language will also be taken up. Some project oriented efforts are under way so as to reinforce the findings on case studies pertaining to the above areas. The specific problems in two major Indian languages viz., Bengali and Hindi are being undertaken by Das, Ray Chaudhury, Choudhury, Mukherjee, Yadav and Sinha. Banerjee's work relates to the same theme but essentially geared to applications and applicable mathematics. Chel and Ghosh analysed answer scripts of a public examination to find out linguistic difficulties connected with problems on equations, graphs and geometry. Datta's concern is linguistic aspects of programmed texts in geometry.

Under the auspices of Association for Improvement of Mathematics Teaching three working groups have been set up recently, each in respect of Bengali, English & Hindi, the purpose being

i) to analyse points of "interference" of the language of the medium of instruction in the learning of mathematics.

ii) to prepare a booklet of guidance for teachers in order to increase their awareness of language-related issues in the learning of mathematics.

iii) to analyse new vocabulary introduced in the standard mathematics texts in each of the first 2 or 3 years of secondary education.

iv) to complete a glossary of such terms for use by teachers.

The work of these groups is expected to be completed by the end of September 1983 and would provide the basis for deliberations of a wider group scheduled to meet in October, 1983.

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LESOTHO

A good Lesotho High School, Form A class in 1979: two months from the start of term.

Maths Teacher: "What is this called?"

Pupil: "Yes"

About the same time Priscilla Jones at the National University of Lesotho was determining the working vocabulary of such pupils.. She found they have a common English vocabulary for reading of about 850 words. The result was a week long workshop for 60 people in January 1980 to consider this problem and make recommendations.[1]

Teachers and publishers were alerted to the seriousness of the problem of communication in learning mathematics. This was especially impressed on the writers of a new series of secondary mathematics texts being written and tested locally.[4] A further workshop was planned and before this, tests were set to some 550 pupils to determine their responses to questions asked in mathematical symbols and the same questions in words.

The second Workshop was held in January 1981[2] and out of this came:

i) 3 Resource books for teachers,the mathematics one being of a glossary of mathematical terms for Secondary forms 1 and 2 simply explained.[3]

ii) Further testing of over 2000 pupils to add to the knowledge of the first tests. E.g. Why use: "What is the difference between 7 and 4?" When you mean "7-4 = ?" The first question produced answers:

$7 > 4$; 7 is odd, 4 is even; 5,6.

Children learning mathematics in their first language find it hard enough. Imagine what it is like learning (or teaching) in a foreign language where you have a small vocabulary, a very limited command of tenses and structures and your textbooks make no allowance for this. An analysis of a typical first year text in use in Lesotho showed that about 60% of the verbs used were in tenses unknown to the pupil; add to this long sentences, subordinate clauses, logical connectives, many words unknown and the text becomes next to useless.

Yet with very large classes (60-80 is not uncommon in Primary and 40-50 in Secondary schools); very little equipment; often inexperienced or untrained teachers, or instead expatriates speaking an unknown dialect with many colloquialisms, a clear understandable textbook is essential to the child who needs help or in particular has been absent.

REFERENCES

1. Language in the Mathematics and Science Lesson
   Part 1. Proceedings of the Workshop
   Part 2. Materials produced Edited:
   A. Hurrell, J.B. Selwyn, P.A. Whittle, N.U.L. 1980.

2. Report on the Second Workshop on Language in the Mathematics & Science Lesson. Edited: A. Hurrell, N.U.L., 1981.

3. Teachers' Resource Book 3: Language in Mathematics Lessons Edited: A. Hurrell, N.U.L., 1981.

4. Macmillan Secondary Mathematics Course for Schools. Edited: J.E. Hay, E.D. Bicknell 1984-87.

5. A Selection of Test Results and Recommendations for Primary Teachers. A. Hurrell, N.U.L., 1983.

6. Language and Learning Problems in Basic Mathematics. A. Hurrell, N.U.L. 1983.

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THE SECOND INTERNATIONAL STUDY OF SCHOOL MATHEMATICS

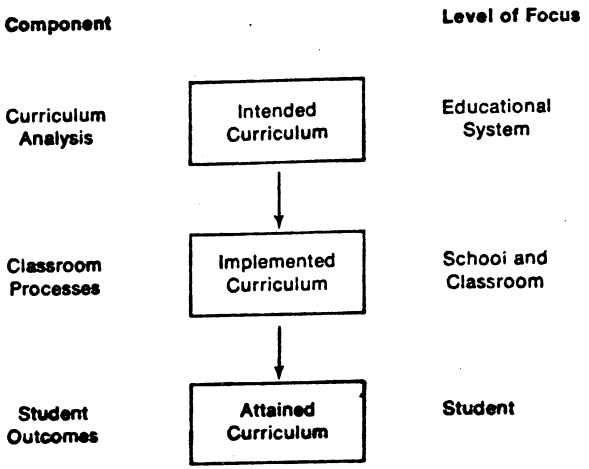
During the past twenty years, two international studies of school mathematics have been carried out by the International Association for the Evaluation of Educational Achievement. The first study, which took place in 1964, involved twelve countries and has been reported by Husén (1967). The second, to be completed by mid-1984, involves twenty-four countries, eleven of which were in the first study. (See Appendix A for list of countries and national institutions responsible for the surveys.)

In each participating country, the second study has been carried out by a nationally recognized research institution in close cooperation with a national advisory committee of specialists in mathematics education and research methodology. International coordination has been carried out by the Department of Education, Wellington, New Zealand. The substantive aspects of the study are the responsibility of the International Mathematics Committee, headquartered at the University of Illinois, Urbana, USA.

Components of the Second International Study of Mathematics

The Second International Study has three components, as represented in the figure below:

THREE ASPECTS OF A CURRICULUM



Curriculum analysis

An analysis of the context of the curriculum will provide information on the structure of school systems (e.g. whether they are selective or comprehensive), the degree of control (e.g. whether the educational system is centralized or decentralized), the retentivity of the system and other details which will help provide a backdrop for the other phases of the Study.

An analysis of the content of the mathematics curriculum is being performed in order to detect patterns of importance given to various topics. Changes in the content of the curriculum during the past twenty years will also be examined.

Classroom processes

What Mathematics Do Teachers Teach?

How much mathematics is taught in a country? Teachers completed reports which indicated whether or not students were taught the skills necessary to answer correctly the international test items. This information is referred to as "opportunity-to-learn" data.

How is Mathematics Taught?

Through the use of detailed questionnaires, information is being gathered on how mathematics is taught. For example, in Japan the recently released national report stated that with respect to the geometry in the junior secondary school, many teachers wish to make the teaching of geometrical figures practical by making or using various objects and refraining from too formal a treatment of the content.

As the data from the various countries is compiled, it is expected that the resulting "international portrait" of mathematics teaching will be a valuable source of information for research and instructional purposes.

Student outcomes

Much effort has gone into developing a battery of achievement tests appropriate for the target groups of students in the countries. These tests represent quite a refinement of the tests used in the first study, although many items from that study were kept to make possible some comparisons between the first and second studies. A battery of attitude tests has also been developed.

Populations being studied

Two target populations are being studied. Population A is students who are in that year of school at which the majority of students are 13 years old by the middle of the school year.

Population B consists of students who are in the terminal year of secondary school and who are studying mathematics as a substantial part (approximately five hours per week) of their academic program.

Comparable populations were also used in the first IEA study, so it will be possible to get an international perspective on changes in school achievement between 1964 and the present. Doubtless, there will be greater changes in some countries than in others. A probing of the wealth of information provided by the studies should help account for those changes.

The Second International Mathematics Study and ICMEs

Planning for the Second International Mathematics Study began in 1976. In order to review these preliminary plans, a working group was formed at ICME3 in Karlsruhe. The feedback from this group was very useful in refining the study. At ICME4, in Berkeley, progress reports were given on the curriculum analysis and classroom process phases of the study. It is planned that the study will be completed in time for reporting at ICME5 in Adelaide.

For further information

Other countries interested in the Second International Mathematics Study may have the opportunity to take part in a "second round". Further information may be obtained from:

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Development Division
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(Mark envelope "PERSONAL")

Appendix A

Second IEA International Mathematics Study Participating Countries

AUSTRALIA*	Malcolm Rosier, Australian Council for Educational Research, Hawthorn, Victoria, AUSTRALIA.
BELGIUM (Flemish)	Christiana Brusselmans-Dehairs, Seminaire en Laboratorium voor, Didactiek, Gent, BELGIUM.
(French)*	George Henry, Université de Liège au Sart Tilman, Liège, BELGIUM.
CANADA (B.C.)	David Robitaille, University of British Columbia, Vancouver, CANADA.
(Ontario)	Les McLean, Ontario Institute for Studies in Education, Toronto, CANADA.
CHILE	Maritza Jury Sellan, University of Chile, Santiago, CHILE.
DOMINICAN REPUBLIC	Eduardo Luna, Universidad Catolica Madre y Maestra, DOMINICAN REPUBLIC
ENGLAND*	Michael Cresswell, National Foundation for Educational Research, Slough, Berkshire, ENGLAND.
FINLAND*	Erkki Kangasniemi, Institute for Educational Research, University of Jyväskylä, Jyväskylä, FINLAND.
FRANCE*	Daniel Robin, Institut National de Recherche Pedagogique, Paris, FRANCE.
HONG KONG	Patrick Griffin, University of Hong Kong, HONG KONG.
HUNGARY	Julia Szendrei, Orszagos Pedagogiai Intezet, Budapest, HUNGARY.
INDIA	T.J. Rajendra Prasad, Andhra Lutheran College, Guntur, INDIA.
IRELAND	Elizabeth Oldham, Trinity College, Dublin, IRELAND.
ISRAEL*	Arieh Lewy, Tel Aviv University, Tel Aviv, ISRAEL.

IVORY COAST	Sango Djibril, Service D'Evaluation, Abidjan, IVORY COAST.
JAPAN*	Toshio Sawada, National Institute for Educational Research, Tokyo, JAPAN.
LUXEMBOURG	Robert Dieschbourg, Institut Pedagogique, Wolferdange, LUXEMBOURG.
NETHERLANDS*	Hans Pelgrum, Hogeschool Twente, Utrecht, NETHERLANDS.
NEW ZEALAND	David Carpenter, Department of Education, Wellington, NEW ZEALAND.
NIGERIA	Wole Falayajo, University of Ibadan, Ibadan, NIGERIA.
SCOTLAND*	Gerard Pollock, Scottish Council for Research in Education, Edinburgh, SCOTLAND.
SWAZILAND	Mats Eklund, William Pitcher College, Manzini, SWAZILAND.
SWEDEN*	Robert Liljefors, University of Stockholm, Stockholm, SWEDEN.
THAILAND	Samrerng Boonruangrutana, Institute for the Promotion of Science and Technology, Bangkok, THAILAND.
USA*	Kenneth J. Travers, University of Illinois, Urbana-Champaign, Champaign, Illinois, U.S.A.

* Countries participated in the First IEA International Mathematics Study.
The Federal Republic of Germany participated in the First Study, but not in the Second.

Appendix B

The IEA, founded by Torsten Husén of Sweden and Benjamin Bloom of the University of Chicago, is an international, non-profit-making scientific association incorporated in Belgium for the principal purposes of: (a) undertaking educational research on an international scale; (b) promoting research aimed at examining educational problems in order to provide facts which

can help in the ultimate improvement of educational systems; and (c) providing the means whereby research centres in the various member countries of IEA can undertake co-operative projects. The current chairman of the IEA Council is Professor T. Neville Postlethwaite of the University of Hamburg, Federal Republic of Germany. The Mathematics Project Council, responsible for the Second Mathematics Study, is chaired by Roy W. Phillipps of the New Zealand Department of Education. Robert Garden, also of the New Zealand Department of Education, is International Project Co-ordinator for the Study. Kenneth J. Travers is Chairman of the International Mathematics Committee (IMC) which designed the Study and developed the international instruments. Other members of the IMC are: Sven Hilding, Sweden; Edward Kifer, United States; Gerard Pollock, Scotland; Tamas Varga, Hungary; and James Wilson, United States. A.I. Weinzweig, United States, is consulting mathematician for the IMC.

Reference

Husén, T. International Study of achievement in mathematics (Vols. 1 and 2). Stockholm: Almqvist and Wiksell, and New York: John Wiley, 1967.

A REVIEW OF RESEARCH IN MATHEMATICAL EDUCATION

We are always pleased to bring publications of an official nature to our readers attention, particularly when they would seem of international interest.

The three-part work which has just been published under the above title was specially commissioned in connection with the recent Committee of Inquiry into the Teaching of Mathematics in the Schools of England and Wales. It is published by NFER/Nelson (distributed in the USA by Humanities Press, New Jersey) and the three parts are

A. Research on Learning and Teaching (pp 336) ISBN 0-7005-0612-8

B. Research on the Social Context of Mathematics Education (pp 84) ISBN 0-7005-0613-6

C. Curriculum Development and Curriculum Research (pp 72) ISBN 0-7005-0614-4.