



**PROCEEDINGS OF THE
27TH ICMI STUDY CONFERENCE
MATHEMATICS EDUCATION AND
THE SOCIO-ECOLOGICAL**

Editors: Kate le Roux, Alf Coles, Armando Solares-Rojas, Arindam Bose, Catherine P. Vistro-Yu,
Paola Valero, Nathalie Sinclair, Mariam Makramalla, Rochelle Gutiérrez, Vince Geiger, Marcelo
Borba.

Ateneo de Manila University, Philippines,
January 22nd to January 25th 2025

**THE 27th ICMI STUDY:
MATHEMATICS EDUCATION AND
THE SOCIO-ECOLOGICAL**

CONFERENCE PROCEEDINGS

Edited by: Kate le Roux, Alf Coles, Armando Solares-Rojas, Arindam Bose, Catherine P. Vistro-Yu, Paola Valero, Nathalie Sinclair, Mariam Makramalla, Rochelle Gutiérrez, Vince Geiger, Marcelo Borba.

Copyrights: © 2025 left to the authors.

Contents: The content of each paper in this volume is provided by the respective authors, with whom the responsibility for the content remains. The opinions are those of respective authors. The editors do not assume responsibility for the contents.

Published by: Philippine Council for Mathematics Teacher Educators (MATHTED) Inc., Department of Mathematics, 3/F Science Education Complex, Building A, Ateneo de Manila University, Katipunan Avenue, Loyola Heights, Quezron City, 1108, Philippines (tel: +63 2 8426 6125); and the International Commission on Mathematical Instruction (ICMI), Hausvogteiplatz 11A, 10117 Berlin, Germany (tel +49 30 2037 2430).

Cite as: Le Roux, K., Coles, A., Solares-Rojas, A., Bose, A., Vistro-Yu, C. et al. (Eds.). (2025). *Proceedings of the 27th ICMI Study Conference (Mathematics Education and the Socio-Ecological)*. MATHTED and ICMI.

Permanent URL: <https://www.mathunion.org/icmi/icmi-study-27>

ISBN: 978-621-06-2470-0

INTERNATIONAL PROGRAMME COMMITTEE

Co-chairs

Alf Coles (University of Bristol, United Kingdom)

Kate le Roux (University of Cape Town, South Africa)

IPC Members

Marcelo Borba (Sao Paulo State University, Brazil)

Arindam Bose (Tata Institute of Social Sciences, India)

Vince Geiger (Australian Catholic University, Australia)

Rochelle Gutiérrez (University of Illinois Urbana-Champaign, United States of America)

Mariam Makramalla (University of NewGiza, Egypt)

Armando Solares-Rojas (Cinvestav, Mexico)

Nathalie Sinclair (Simon Fraser University, Canada)

Paola Valero (Stockholm University, Sweden)

Catherine P. Vistro-Yu (Ateneo de Manila University, Philippines)

Ex-officio members

Frederick K.S. Leung, ICMI President 2021-2024 (The University of Hong Kong, Hong Kong SAR)

Merrilyn Goos, ICMI President 2025-2028 (University of the Sunshine Coast, Australia)

Jean-Luc Dorier, ICMI General-Secretary (University of Geneva, Switzerland)

LOCAL ORGANIZING COMMITTEE

Catherine P. Vistro-Yu, Chair (Ateneo de Manila University, Philippines)

Angela Fatima H. Guzon, Co-chair (Ateneo de Manila University, Philippines)

Lester C. Hao, Co-chair (Ateneo de Manila University, Philippines)

Maria Alva Q. Aberin, Finance Officer (Ateneo de Manila University, Philippines)

Errol Matthew C. Garcia, Events and Logistics (Ateneo de Manila University, Philippines)

CONTENTS

PREFACE	11
ACKNOWLEDGEMENTS	13
PLENARY LECTURE CONTRIBUTIONS	14
MORE-THAN-HUMAN MATHEMATICS AND THE POWER OF SPECULATIVE THOUGHT	15
Elizabeth de Freitas	
WHAT CLIMATE ACTION MIGHT MEAN FOR MATHEMATICS EDUCATION	17
Fr Jose Ramon Villarin SJ	
INSIDE THE SOCIOCULTURAL OCTAGON: SOME DIMENSIONS OF A'UWË/XAVANTE ETHNOPEDAGOGY	18
Adailton Alves da Silva	
INDIGENOUS SOCIO CULTURAL ECOLOGICAL SPIRITUAL REALITIES	20
Pedro Walpole SJ	
THEME A - AIMS OF MATHEMATICS EDUCATION	21
THE MAN WHO COUNTS: VALUES IN MATHEMATICAL MODELING	22
Chiara Andrà, Giovan Albano, Andrea Amico, Domenico Brunetto, Maria Polo	
MATHEMATICS EDUCATION'S RESPONSE TO THE QUESTION, "HOW DOES IT FEEL TO BE A PROBLEM?"	30
Peter Appelbaum	
CLIMATE CHANGE, MATHEMATICS EDUCATION, AND THE EPISTEMIC FALLACY	38
Eleany Barrios-Borges, Francisco Cordero	
CRITICAL MATHEMATICAL CONSCIOUSNESS AND LEARNING MATHEMATICS FOR SUSTAINABLE FUTURES	46
Richard Barwell	
THE (FUTURE) CO-EVOLUTION OF SCHOOLING AND MATHEMATICS EDUCATION	53
Jay Bradley	

RACE AND THE SOCIO-ECOLOGICAL TURN	61
Erika C. Bullock	
AIMS FOR A SOCIO-ECOLOGICAL TASK DESIGN: STEPS TO A TRANSFORMATIVE LEARNING OF MATHEMATICS?	68
Alf Coles	
MODELING AND WORLDING: HOW TO IMAGINE MORE-THAN-HUMAN MATHEMATICS	76
Elizabeth de Freitas	
RETHINKING MEASUREMENT IN SOCIO-ECOLOGICAL TERMS	84
Elizabeth de Freitas, Nathalie Sinclair	
COMPOSITIONS WITH MATHEMATICS EDUCATION, PHILOSOPHIES OF DIFFERENCE AND CULTURALLY DIVERSE GROUPS IN SEARCH OF OTHER POSSIBILITIES FOR A HUMANITY COMMITTED TO SOCIO-ECOLOGICAL CONCERNS	92
Cronopies+ Collective, Roger Miarka	
RE-VALUING, DE-CENTRING, UN-LEARNING: ENVISAGING A MATHEMATICS EDUCATION FOR SOLIDARITY AND HOPE	100
Hilary Povey, Corinne Angier	
MATHEMATICS EDUCATION IN THE MATERIAL WORLD: WORK, LIVELIHOODS AND ECOLOGICAL JUSTICE	108
Anita Rampal	
SOCIO-ECOLOGICAL CONCEPTS OF POSITIVE DEVIANCE OF ETHNOMATHEMATICS AS A PROGRAM	116
Milton Rosa, Daniel Clark Orey	
SOCIO-ECOLOGICAL COMPLEXITIES IN ISRAEL? STORIES AND SILENCES IN ISRAELI NATIONAL MATHEMATICS TESTS	124
Laurie Rubel, Nadav Ehrenfeld, Gil Schwartz, in collaboration with Tommy Dreyfus, Anna Sfard	
TROUBLING THE AIMS OF MATHEMATICS EDUCATION IN THE SOCIO-ECOLOGICAL	132
Paola Valero, Francesco Beccuti	
IMPLICATIONS OF ANTHROPOCENE AND GAIA IN MATHEMATICS EDUCATION	140
João Viola, Júlio Faria Corrêa	

THEME B – SCALES OF MATHEMATICS EDUCATION	148
STUDENTS’ SOCIO-ECOLOGICAL GESTURES IN DRAWINGS	149
Aehee Ahn	
LIFE IN A DROP OF WATER: A PARTICIPATORY DESIGN PROJECT TO TACKLING EDUCATIONAL POVERTY	157
Giancarlo Artiano, Emilio Balzano, Maria Mellone	
BUILDING CONNECTIONS ACROSS SCALES TO STIMULATE AND REALIZE TRANSFORMATION IN MATHEMATICS EDUCATION	165
Carrie Bala, Brynja Kohler	
POLICY ENGAGEMENT: INTERNATIONAL CURRICULA, THE SOCIO-ECOLOGICAL AND MATHEMATICS	173
Mark Boylan	
CROSS-CULTURAL EXPERIENCES IN CURRICULUM INNOVATIONS: VOICES FROM ASIA	181
Mary Jane A. Castilla, Heidi Lynn P. Faustino, Melodee T. Pacio, Ram Krishna Panthi, Veronika Fitri Rianasari, Catherine P. Vistro-Yu	
SCHOOL [MATH] CURRICULUM: TOWARDS AN ETHICAL-POLITICAL PERSPECTIVE SITUATED IN THE ‘TERRITORY’	189
Oscar Charry, Carolina Tamayo, Diana Jaramillo	
MATHEMATICS LANGUAGE-KNOWLEDGE DISPARITIES, DE/COLONIALITY AND THE SOCIO-ECOLOGICAL IN THE PHILIPPINES	197
Ivy Estrella, Paola Valero, Hendrik Van Steenbrugge	
RESTOR(Y)ING MATHEMATICS: AN EXPLORATION IN KANAKA MAOLI TARO FARMING	205
Rochelle Gutiérrez	
CONNECTING TO INDIGENOUS WAYS OF KNOWING AND BEING IN MATHEMATICS BY LISTENING TO THE VOICES OF STUDENTS	212
Jodie Hunter, Julia Hill	
CARTOGRAPHIES FOR (RE)STORYING SCALES OF SOCIO-ECOLOGICAL INTERACTIONS IN MATHEMATICS EDUCATION	220
Kate le Roux, Siddique Motala	
CONTRIBUTIONS FOR A SOCIO-ECOLOGICAL PERSPECTIVE WITHIN THE FRAMEWORK OF A CRITICAL STOCHASTIC MODELING SCENARIO	228
Adriana N. Magallanes, Cristina B. Esteley	

WHO OWNS THE NARRATIVE: REFLECTIONS ON SOCIO-ECOLOGICAL INTEGRATION OF MATHEMATICS EDUCATION FROM THE EGYPTIAN CONTEXT	236
Mariam Makramalla	
AUTHENTICITY AND MATHEMATICAL CONTENT IN CONTEXTS OF MARGINALIZATION: A PROSPECTIVE TEACHER'S SHIFTS	243
Sigal-Hava Rotem, Alf Coles	
EXPLORING CHILDREN'S SPATIAL KNOWLEDGE THROUGH MAP MAKING FOR REPRESENTING THEIR TERRITORY	251
Armando Solares-Rojas, Tatiana Mendoza von der Borch	
NETWORK OF COLLABORATIONS AND SOCIAL PRACTICES TO ENGAGE STUDENTS IN SOCIO-ECOLOGICAL ISSUES OF PEATLANDS	259
Lisa Steffensen	
THEME C – RESOURCES OF AND FOR MATHEMATICS EDUCATION	267
BIO AND MATH IN SOCIAL SCIENCE: USING BIOLOGY AND MATHEMATICAL CONTENT IN COMICS TO INTRODUCE SOCIO-SCIENTIFIC ISSUES	268
Fadhlan Muchlas Abrori, Zsolt Lavicza, Branko Andjic	
VIDEO FESTIVAL, MATHEMATICAL EDUCATION, AND SOCIO-ECOLOGY	276
Marcelo de Carvalho Borba, Fernanda Martins da Silva, Aldo Peres Campos e Lopes	
MATHEMATICAL MODELLING AS A KEY PRACTISE FOR PROMOTING LEARNERS' COMPETENCIES TARGETING THE TRANSFORMATION OF GLOBAL SUSTAINABILITY	284
Rita Borromeo Ferri	
DIGITAL VIDEOS AND EDUCATION FOR SUSTAINABILITY: A POSSIBILITY FOR THE DEVELOPMENT OF SOCIO-ENVIRONMENTAL AWARENESS	292
Geciara Carvalho, Marcelo Borba	
PROBLEM-BASED LEARNING AND MATHEMATICS IN HIGHER EDUCATION TO RAISE AWARENESS ABOUT ENVIRONMENTAL DISASTERS	300
Denner Dias Barros, Johnny Nazareth dos Santos	
CRITICAL MATHEMATICAL THINKING: A FRAMEWORK AND ILLUSTRATIVE EXAMPLES	307
Vince Geiger, Thorsten Scheiner, Kim Beswick, Jill Fielding, Merrillyn Goos, Gabriele Kaiser, Katherine Fernandez, Sabahat Arwan	

EXPLORING CRITICAL MATHEMATICS EDUCATION IN TEACHING STATISTICS IN THE PHILIPPINES	315
Eric Loyd Hilario, Ma. Nympha Joaquin	
“GREENWASHING”, DISINFORMATION, OR CLIMATE PROTECTION? HYBRID TASK DESIGN WITH A FOCUS ON THE COMPETENCE OF DATA-BASED ARGUMENTATION	323
Sebastian Kuntze, Jens Krummenauer	
DO THEMES OF ENVIRONMENTAL COLLAPSE GUARANTEE A SOCIO-ECOLOGICAL DISCUSSION?	331
Aldo Lopes, Marcelo Borba	
CITIZENSHIP DEVELOPMENT THROUGH VALUE-BASED CONSENSUS BUILDING IN MATHEMATICAL SCIENCES EDUCATION	339
Mitsuru Matsushima, Keiichi Nishimura, Rintaro Ueda	
WATER RESOURCES AND WATER MANAGEMENT: A SOCIAL-POLITICAL-CULTURAL CHALLENGE WHICH MATHEMATICAL MODELLING APPROACH HAVE POTENTIAL TO ADDRESS	347
Abolfazl Rafiepour	
RESOURCES FOR INTEGRATING MATHEMATICS AND SUSTAINABILITY CHALLENGES: A PERSPECTIVE OF THE GREEN MATHEMATICS FRAMEWORK	355
Soma Salim, Katie Makar, Jana Višňovská	
IMPLEMENTATION OF MEDIA AND DATA LITERACY IN STUDENT TASKS – AN ANALYSIS OF DIFFERENT TASKS	363
Hans-Stefan Siller, Nina Unshelm, Vincent Geiger, Sarah Digan	
DIALOGUES, CONFLICTS, AND ENVIRONMENTAL JUSTICE	371
Ole Skovsmose, Ana Carolina Faustino	
RECONTEXTUALIZING ENGAGEMENT WITH DATA VISUALIZATIONS IN THE CONTEXT OF THE SOCIO-ECOLOGICAL	378
Eva Thanheiser, Laurie Rubel, Beth Herbel-Eisenmann, Courtney Koestler	
CREATING A LEARNING EXPERIENCE TOWARDS AGENCY FOR CRITICAL CITIZENSHIP: A SOCIO-CRITICAL MATHEMATICAL MODELLING PERSPECTIVE	386
Jimbo Juanito B. Villamor, Catherine P. Vistro-Yu	
MATHEMATICAL DRAMATIC CODIFICATIONS AS ECOPEdagogical RESOURCES	394
Katherina von Bülow	

REFLECTIONS ON ABSTRACTION IN A WORLD WHERE ATTACHMENT TO PLACE MATTERS	402
David Wagner, Annica Andersson, Sacha DeWolfe, Julianne Gerbrandt, Hilja Lisa Huru, Anita Movik Simensen, Elisabeth Wirkola	
THEME D – MATHEMATICS EDUCATION FUTURES	410
A KIND OF MATHEMATICS THAT HELPS ME TELL AND HEAR STORIES	411
Yasmine Abtahi	
MATHEMATICAL AI-MODELING IN THE SOCIO-ECOLOGICAL: FUTURING CRITICAL MATHEMATICS EDUCATION	416
Christian H. Andersson, Paola Valero	
ENHANCING MIDDLE SCHOOL STUDENTS' MATH ENGAGEMENT AND SOCIO-ECOLOGICAL AWARENESS THROUGH PROJECT-BASED LEARNING	424
Mrinal Jyoti Baruah, Kumari Shalini, Arindam Bose, Saurabh Singh	
THEORETICAL AND METHODOLICAL ASPECTS OF TRANS-DISCIPLINARY EDUCATION: MATHEMATICS AND THE SOCIO-ECOLOGICAL	432
Lisa Björklund Boistrup, Magnus Ödmo, Ann-Sofi Røj-Lindberg, Mats Braskén	
ETHNO-SUSTAINABILITY USING MATHEMATICS: MAPPING OUT A FUTURE LANDSCAPE FOR MATHEMATICS EDUCATION	440
Arindam Bose, Catherine P. Vistro-Yu	
TERRAPOLIS STORIES: WEAVING IDEAS ABOUT DIGITAL TECHNOLOGY AND THE SOCIO-ECOLOGICAL	448
Lucely Figueroa-Suárez, Anna Chronaki, Ulrika Ryan	
UTILIZING THE INHERENT MATHEMATICAL KNOWLEDGE SYSTEMS OF INDIGENOUS COMMUNITIES IN ADDRESSING SOCIETAL AND ECOLOGICAL ISSUES FOR SUSTAINABLE FUTURE	456
Frankie Fran, Catherine Vistro-Yu	
INTEGRATING EARLY ALGEBRA AND THE SOCIO-ECOLOGICAL	464
Jenny Fred, Paola Valero, Hendrik Van Steenbrugge	
SOME RESPONSE-ABILITIES WITH MATHEMATICS EDUCATION IN THE CARE AND PROTECTION OF MOTHER EARTH	472
Carolina Higuaita-Ramírez, Marcos Yule-Yatacué, Luis Yimer-Pavi, Roger Miarka, Paola Valero	

ABENG FOR CLIMATE DISARMAMENT, REPARATIONS & MULTISPECIES’ FLOURISHING	480
Steven Khan	
NETWORKING ETHNOMATHEMATICS, EMBODIED COGNITION-NEW MATERIALISM, AND MATHEMATICAL MODELING TO REIMAGINE PROFESSIONAL DEVELOPMENT	487
Jessi Lajos, Jessica Benally, Sindura Kularajan	
RECIPROCITY, FEEDBACK LOOPS, SYSTEM DYNAMICS AND THE EDUCATION REFORM IN MEXICO	495
Quitze Morales Meléndez, Ramiro Carrillo-Catalan, Marcela Santillán Nieto	
EMPIRICAL DIFFERENTIATION OF STUDENT COMPETENCIES IN ESD	503
Hans-Stefan Siller, Katrin Vorhölter, Janina Just, Anna Orschulik, Catharina Zieriacks	
A SOCIO-ECOLOGICAL STEM TASK: RE-ATTACHING PRESCHOOLERS TO NATURE	511
Elizabeth Suazo-Flores, Christine McGrail, Amber Adgerson	
APPENDIX: DISCUSSION DOCUMENT	519

PREFACE

A five-year International Commission of Mathematics Instruction (ICMI) study is designed to represent ‘the state of the art’ in an area of mathematics education, but takes a further step: it builds community towards harnessing current knowledge for further knowledge contributions and anticipates new possibilities.

As a named topic in international mathematics education forums, ‘Mathematics Education and the Socio-Ecological’ is relatively new compared to many others in the field. Yet there is wide recognition of the urgent need for thought leadership to consider what is and might be the role of mathematics and mathematics education in multiple, intersecting, social, political, and ecological issues such as climate change, poverty, inequality, health crises, discrimination, and marginalization. There is also increased attention to how traditions such as critical mathematics education, decoloniality, ethnomathematics, feminist and new materialist thought, Indigenous ways of knowing, and mathematical modelling offer resources for research and practice in the interdependencies of mathematics education and the socio-ecological.

These published ICMI Study 27 Conference Proceedings represent a significant step in the ICMI Study 27 process of community building and knowledge production. Prior to the announcement of the Study, a one-day, online, ICMI symposium – the first such ICMI event – brought together 170 participants from diverse contexts to discuss work in this area of interest, in sites such as classrooms, universities and community spaces, in interdisciplinary relations with other subjects, and including issues such as teacher education, research practice and policy (see the [proceedings](#) here).

The ICMI Study 27 Mathematics Education and the Socio-Ecological Discussion Document (Coles at al., 2024), was published in January 2024, an edited version is the Appendix (p.519). The discussion document was conceptualised and authored by the 13-member International Programme Committee (IPC) listed below. The Document invited empirical and theoretical submissions (of eight pages) from across the globe, of those working on issues in mathematics education that span social (including political) and ecological (environmental) problems, and related ethical concerns. Collaborations involving researchers, teachers, teacher educators, policy makers, and other stakeholders from diverse contexts and sites of education were encouraged. Authors were tasked with responding to one of the four study themes: (A) Aims of mathematics education; (B) Scales of mathematics education; (C) Resources of and for mathematics education; and (D) Mathematics education futures. These themes were conceptualized and are led by: Paola Valero and Nathalie Sinclair (A); Mariam Makramalla, Rochelle Gutiérrez and Armando Solares-Rojas (B); Marcelo Borba and Vince Geiger (C); Arindam Bose and Catherine Vistro-Yu (D).

The paper review process was aligned with the orientation of ICMI Study 27 towards fostering a knowledge community that offers thought leadership on and opens space for new directions in mathematics as it relates to the socio-ecological across local and international levels. In an open review process, each submission was reviewed by two or more members of the ICMI Study 27 IPC, all of whom are recognised internationally as experts in this area of mathematical education. Authors

received detailed, constructive feedback on their papers, with a particular focus on the (potential) contribution of the paper to thinking about mathematics education *and* the socio-ecological.

Following the review process (which included, for some authors, the option of submitting a revision for further review by IPC members), all the papers that follow were accepted. These are presented alphabetically by Theme in this volume. Copy editing was overseen by Alf Coles and Kate le Roux and supported by Arindam Bose and Armando Solares-Rojas. We recognize these different kinds of contributions in reverse alphabetical order, in the listing of editors of this document.

As co-chairs of ICMI Study 27 we express sincere gratitude to many in the mathematics education community, both local and translocal. We thank the ICMI Executive Committee for affording the space for this important and timely work, as well as the administrative support offered by ICMI. We thank the ICMI Study 27 IPC for their generosity – offering their time and expertise to provoke and push our thinking about mathematics education in interdependence with the socio-ecological. We thank the Study Conference Local Organising Committee (listed below) for creating the space at Ateneo de Manila University, Quezon City, Philippines, for conversations across diverse contexts towards new directions for research and practice.

Alf Coles and Kate le Roux

ICMI Study 27 Co-chairs.

References

Coles, A., le Roux, K., Borba, M., Bose, A., Geiger, V., Gutiérrez, R., Makramalla, M., Solares-Rojas, A., Sinclair, N., Valero, P., Vistro-Yu, C., Leung, F.K.S., & Dorier, J-L. (2024). *ICMI Study 27 Discussion Document: Mathematics Education and the Socio-Ecological*. ICMI.

ACKNOWLEDGEMENTS

We are grateful to the following institutions for their financial and material support for the 27th ICMI Study Conference. Thanks to them, we were able to have this international conference with participants from across the globe.

1. Ateneo de Manila University

- a) Mathematics Department, School of Science and Engineering
- b) Math Ed Eagles
- c) Dean's Office, School of Science and Engineering
- d) Office of the Vice President for Higher Education
- e) Office of the Assistant Vice President for University Partnerships and Internationalization
- f) Office of the President
- g) Ateneo Institute of Sustainability
- h) Ateneo Art Gallery
- i) Manila Observatory

2. Philippine Council of Mathematics Teacher Educators (MATHTED) Inc.

3. Ecology and Jesuits in Communication

We also express our appreciation to Lena Koch (IMU Secretariat) who assisted us with such efficiency at every step along the way.

PLENARY LECTURE CONTRIBUTIONS

MORE-THAN-HUMAN MATHEMATICS AND THE POWER OF SPECULATIVE THOUGHT

Elizabeth de Freitas

Adelphi University

The Anthropocene is an epoch of radically shifting species awareness, equally if not more revolutionary as the destabilizing impact of Darwinian evolution in the 19th century. Under a new climatic regime, humans have become earthbound in altered ways, increasingly aware of how historical scales (late capitalism, American imperialism, the digital age) intersect with the geologic time of the planet. What kind of mathematics education is appropriate for these times? Which mathematics curricula might be called for, as the earth convulses under new climatic regimes? Should this be a mathematics well suited to studies of extinction? A mathematics of death and decomposition? A mathematics of entropic systems that track the dissolution of familiar climate patterns? How should we prepare future generations for an increasingly inhospitable and chaotic planet? When we consider the unstoppable force of global capital and extractive imperialism, and the rise of xenophobic and racist nationalisms reshaping the geo-political space, how might mathematics education push back at both apathy and nihilism in what seems an increasingly ‘post-truth’ computational climate-culture?

Over twenty years ago, the philosopher of science Donna Haraway (2003) coined the term “natureculture” to describe how nature and culture are never mutually exclusive or ontologically separable, and she wrote extensively on “companion species” and “sympoiesis” to characterize the complex relationality of socio-ecologies. Her work in the 1980s was part of the emerging field of science and technology studies (STS), building on the scholarship of Thomas Kuhn and Michel Foucault among others, revealing how power and politics shape scientific knowledge production. Feminist and Black philosophers of science like Evelyn Fox Keller and Sylvia Wynter contributed to the field, exposing science for its racialized, gendered, and classed bias. Scholars like Bruno Latour showed how scientific practice was a highly political affair of consensus building around controversies. And yet STS was not anti-science, but interested in exposing the actual material-political practices of scientists. According to STS, science populates the world with diverse non-human agencies (gluons, amoebas, gravitational waves, etc.) and traces the relational links between them. Rather than treat matter as inert and passive, “scientists work by negotiating with nonhuman entities, and by entering into alliances with them” (Shaviro 2015, p.11). As such, science has the potential to cultivate pluralism and diversity through increasing awareness of the non-human agencies that make up the world; this process is not simply a new way of talking or “socially constructing” the world because such an understanding would re-center the human as the interpreter/maker of all that matters. Latour (2017) asserts that agentic powers are “*a property of the world itself* and not only a phenomenon of language *about* the world” (p. 69). In other words, the terrestrial becomes a political actor in its own right and refuses to be exhausted as a resource or metaphor. As STS scholars Mira Hird and Katherine Yusoff suggest, this approach reveals how science works through forms of attachment, dependency, trust, and diplomacy (Hird & Yusoff, 2019).

This presentation will take up this scholarship, and extend its more-than-human emphasis to mathematics. Human mathematics is historical, political, and embodied, but it is also a more-than-human achievement – and in that sense, it is never entirely human. What would a turn to the more-than-human look like in mathematics education, and how might it become part of an ethics of care and cross-species relationality? I suggest that the speculative gesture of mathematics, which informs our climate models, is not exceptional or unique to humans. Mathematical speculation is a practice found in mathematical activity of all kinds, be it human or otherwise. I come to the conference asking: How might the turn to more-than-human mathematics disrupt ongoing delusions of human exceptionalism and cognitive grandeur associated with mathematics? This question is intended to support an image of mathematical practice as diverse, material, conceptual, but also posthuman or more-than-human, and linked across different locations and different forms of life. If we claim that mathematics is only an expression of human desire, and predominantly an expression of white/male/European idealism, we actually *reinscribe* human exceptionalism, albeit historically situated, and deny nonhuman others any mathematical nature or propensity. This would be like denying sign-making-facility and worlding capacity to forests, crows, ants, spiders, etc. Why would our extinction spell the end of patterns and persistent structural relations or the conceptual and imaginative thinking of nonhuman animals and nonhuman collective intelligences? Speculative thought and experiment – which are never innocent - are a way to help students appreciate that science/math is not only an exercise in disenchantment and rationalization/instrumentalization, but is also a mode of affective engagement that is deeply passionate and risky.

References

- Haraway, D. (2016). *Staying with the trouble: Making Kin in the Chthulucene*. Duke University Press.
- Kragh, H. (2015). Why trust a theory? Accessible at <https://www.whyytrustatheory2015.philosophie.uni-muenchen.de/program/index.html#kragh>
- Hird, M. and Yusoff, K. (2019). Lines of shite: Microbial-mineral chatter in the Anthropocene. *In*: R. Braidotti and S. Bignalle, eds. *Posthuman Ecologies: Complexity and Process after Deleuze*. Rowman & Littlefield, 265–282.
- Latour, B (2017). *Facing Gaia: Eight lectures on the new climatic regime*. Polity Press.
- Povinelli, E. (2016). *Geontologies: A Requiem to Late Liberalism*. Duke University Press.
- Shaviro, S. (2015). *Discognition*. Repeater.

WHAT CLIMATE ACTION MIGHT MEAN FOR MATHEMATICS EDUCATION

Fr Jose Ramon Villarin SJ

Manila Observatory

Simply defined, climate is the average of weather. Globally, this average does not shift appreciably over certain time scales. However, the observed and predicted shifts in magnitude and rate of change in this century are urgent cause for concern. This concern is compounded by the increasing prevalence and unpredictability of devastating outlier or extreme events that arise from shifts in the climatological mean. A globally warmer planet is bound to be a more volatile and uncertain world.

Simulating the key variables that describe the continuum of atmosphere and ocean by representing these with coupled dynamical equations, applied to discrete points on a grid, is a perennial challenge in climate diagnosis and prognosis. This is just to say that predicting non-linear shifts and feedback loops, such as those found in climate change, is possible but limited.

Given what we know thus far, uncertainty however is not an excuse to defer climate action. Communicating climate change can be challenging because of abstract mathematical concepts such as "average", "anomaly", "per" and the like. Culture and language play a critical role in making sense of climate change and moving people to do something about it.

Moreover, reducing climate risk entails quantitative assessments of such risk. Quantifying risk brings us to more complex territory, when we need to take into account the interplay of non-physical variables such as socio-ecological exposure and vulnerability. The interplay of these variables is usually expressed arithmetically and is the basis of the current heuristic framework we use in risk assessment and reduction. This risk framework can provide good, if tentative, guidance for climate action but can also be improved surely with the inclusion of climate justice concerns and continuing validation on the ground.

INSIDE THE SOCIOCULTURAL OCTAGON: SOME DIMENSIONS OF A'UWĒ/XAVANTE ETHNOPELAGOGY

Adailton Alves da Silva

Universidade do Estado de Mato Grosso (UNEMAT)



Figure 1: Autumn dawn (14/05/10 at 5:16 am) in Caçula village. Godparents during the Wate'wa ceremony in Caçula village (source: author)

This article is, in a way, a continuation of the ideas that I presented/discussed in the article entitled *Ethnopedagogies: socio-educational knowledge without looking for the right way* presented at the main plenary session of the 12th Mathematical Education and Society Conference (MES12) in July 2023, at the Federal University of São Paulo (UNIFESP), São Paulo/Brazil. Here on this occasion, I feel privileged to be able to continue expanding and sharing these ideas with my peers from ICMI Study 27 - International Commission of Mathematics Instruction (ICMI).

In MES12, the main idea was to discuss/reflect on the conception and perspective of Ethnopedagogy of some culturally distinct peoples. In this manuscript, the main objective is to verticalize this discussion/reflection in order to highlight the **cosmological**, **socio-ecological** and **socio-educational** dimensions of just one of these peoples, in other words, to show how these dimensions consolidate this way of generating, systematizing and disseminating their knowledge.

In this sense, we can also say that each culturally distinct people builds its Ethnopedagogy based on its way of conceiving and reading the world. It is a construction that seeks to lead children, young people and adults through the process of belonging to their own culture, and so, logically, each of these readings is done in a very different way, but in line with these dimensions.

Specifically, what I want to bring to this discussion and reflection is the dynamics and the way in which the Ethnopedagogy of the A'uwē/Xavante people is disseminated, that is, how it is put into operation for the benefit of their cultural dynamics from the **cosmological**, **socio-ecological** and **socio-educational** dimensions. Thinking about the ethnopedagogy of these people means thinking about how they produce, systematize and disseminate values, principles, truths and meanings within that group, and between that group and other specific groups. We can say that A'uwē/Xavante Ethnopedagogy is a web, a weave that is woven into the daily social relations of individuals and groups. In other words, it is a web and wefts that are ordered and orderly in terms of meaning and guidance for conduct and relations between men and nature (Silva, 2013).

1 - The Cosmological Dimension - construction of the A'uwẽ/Xavante world

Under construction....

2 - The Socio-Ecological Dimension - space and place A'uwẽ/Xavante

Under construction.....

3 – The Socio-Educational Dimension - the construction of being A'uwẽ/Xavante

Under construction.....

Not to conclude, not to finish, just to record the dynamics

Under construction.....

References

Silva, A. A. da. (2013). Os Artefatos e Mentefatos nos Ritos e Cerimônias do *Danhono*: por dentro do octógono sociocultural A'uwẽ/Xavante. Rio Claro –SP.IGCE – Educação Matemática, (Tese de Doutorado).

INDIGENOUS SOCIO CULTURAL ECOLOGICAL SPIRITUAL REALITIES

Pedro Walpole SJ

Institute of Environmental Science for Social Change

Sharing the experience of living with the Pulangiyēn of Bukidnon, their culture, language ecology, spirituality and vision for the future.

Beyond preaching to the choir, in today's world while there is nominal respect in the sense of human dignity acknowledged, in reality, Indigenous communities are isolated if not otherwise sidelined from the fast pace of society based on an ever-competitive economic model of existence and personalized and privatized and sense of wellbeing. Society, we, don't have time to learn anything from those in the margins, along with the urban poor, what could we learn? In today's model they are a failure and it's probably "their own fault," if we analyze our attitude of getting on with life.

Culture is most beautiful, both in its external displays from dance to food, but also landscape – domain. When we have time, it fascinates the mind in seeking to understand the heart of a people never fully defined. Language takes a lifetime if I have not grown up as a child in the community, interesting and perplexing the effort is to first form the image/thought within.

Cultural ecology is like the mycorrhizal filaments of a forest floor; it is the tactile (meaning all senses) connectivity that operates instinctively with what is around ever sensitive (and grateful) of the coexistence. Another word for this in the personal context is spirituality, and connecting to share a vision for the future.

An education using the mother tongue of the Pulangiyēn and focused on the ancestral domain sustains this ecology. From their traditional practice of cultivation comes the knowledge of regeneration, the actualization of a sustained clean water system and a biodiversity rich yet threatened. Culturally, the experience is that of welcome, reconciliation, offering, peace and thanksgiving. Core questions face all youth: who am I, and again, where do I belong? It is a call for listening, accompanying and a leadership of service.

THEME A - AIMS OF MATHEMATICS EDUCATION

THE MAN WHO COUNTS: VALUES IN MATHEMATICAL MODELING

Chiara Andrà¹, Giovannina Albano², Andrea Amico¹, Domenico Brunetto³, Maria Polo⁴

¹University of Eastern Piedmont, ²University of Salerno, ³Politecnico di Milano, ⁴University of Cagliari

We showcase an example of a narrative task designed to question the use of (simple) models in mathematical activities at school, with the potential to address different ways of intending equity in distributing resources. We recall some of the contributions in the field of mathematics education that raise the issue of considering the socio-ecological. We present the activity and we discuss its features. Then, we show and discuss some protocols. The first lens of analysis for the protocols employs values as they have been conceptualized in the affect-related research in mathematics education. Secondly, we employ the lens developed within research in mathematical modeling and thirdly we speak back to the research in socio-ecological issues in mathematics education.

INTRODUCTION

Gutiérrez (2017) prompts researchers to think differently about the relationship between mathematics, humans and the planet and proposes to abandon the view of mathematics as being at the service of economy and as being able to solve anything with the aid of big data and software. Following Skovsmose (2021), we believe there exists no knowledge that could entirely describe our universe and the human relations with it and Santos (2007) further interprets the overreliance on a single knowledge as an authority as the main reason for domination of a certain culture over the others. We are interested in how values come to play a part in the process of mathematical modelling.

THEORETICAL FRAMEWORK

We start our overview of literature with Gutiérrez's (2017) observation that each individual has knowledge and ignorance that is unique. Thus, different knowers should be thought of as interdependent, as well as different forms of knowing all being legitimate and partial. In fact, we should abandon the paradigm according to which accepting another's knowledge means to negate one's knowledge (Santos, 2007), to embrace a reciprocal view of learning that is understood as *towards* the other rather than *for* the other. Gutiérrez (2017) suggests we consider three values that are part of different indigenous cultures, and that can be summarized as: (i) to see oneself in other living beings and to produce knowledge that benefits the others rather than considering knowledge as performance that benefits only oneself; (ii) to see individuals as having different strengths and needs, and thus needing to rely on others for what they lack; (iii) to value tensions within multiple realities, seeing them as bridges to cross borders with love, compassion and conciliation. Connecting these values to school practice, Gutiérrez (2017) suggests allowing students time to learn how to *relate* with other humans and other-than-human, in order to abandon the practice of *privileging* rule following over rule breaking, abstraction over context, mind over body, logic over intuition.

Coles, Solares-Rojas and le Roux (2024) invite researchers in Mathematics Education to think about gestures as forms of activism that enable teachers and researchers to re-evaluate priorities and practices. The authors bring the example of a Museum about the past, the present and the future of a river that had been seriously polluted, as a gesture that had the meaning of humbly listening to the complex, yet vital, entanglement social and ecological issues, and of attending to the precarity of the

vulnerability of being at the margins, of being incomplete and of being in need of mutual relations beyond mathematics, recalling to a certain extent Gutiérrez's (2017) values. In our view, in fact, the gesture of the museum allows for mutual integration of different forms of knowing and sensing the river, forms that are partial and incomplete but benefit the community, and for valuing the tensions among different ways of either exploiting the river or living with it. In our view, the take-home message of Coles et al.'s (2024) contribution is about the contextuality of mathematics, it being also a political gesture, which entails also a loss of certainty and centrality.

Coles and Helliwell (2023) suggest the use of Critical Mathematics Education to reflect on fallibility of mathematical models, and to abandon the apolitical context-free view of mathematics. The researchers suggest using mathematics to raise consciousness, to read the relations of power, to understand the inequalities and the discriminations, and to write a new world with mathematics, which develops social and individual agency. Coles and Helliwell (2023) also suggest reflecting on the role of discourses, and to question what is accepted as normal. Furthermore, they invite us to engage with contradictions, abandoning assumptions like “there are answers to all problems”, “there is a correct way of doing something”, or “real life can be simplified by a model” (p.11).

Research on mathematical modeling, within mathematics education, is also changing. Blum (2015) proposes the modeling cycle as a schematic diagram which is made of seven steps within and between the real world and mathematics. The cognitive dimension of such a model has been enriched by considering further dimensions (e.g., metacognition and creativity) and socio-cultural differences between the real world and the mathematical world (Freijd & Vos, 2022). Other scholars (see Maass et al., 2022) propose a shift from performance to competency, underlining the importance of the specific, contextual situation in which modelling is used. For example, with respect to the first two steps in Blum's model, which are within reality and are labelled understanding and simplifying, Bassi and Brunetto (2023) outline the importance of making assumptions, to recognise and name the quantities that influence the situation, to see relations among them. The third step, which is between reality and mathematics and is called mathematising in Blum's model, and involves recognising the complexity before simplifying and to represent the situation graphically.

Many values emerge between the lines of the contributions that we have briefly recalled in the previous paragraphs. In order to be specific to the theoretical considerations that emerge in the first part of our theoretical framework, we dwell on the values that are at the core of the aforementioned studies and we start with mathematical values (see Bishop, 1988): Bassi and Brunetto (2023) recall the opposing pairs of performance versus competency, simplification versus complexity, as well as the focus on relations within mathematical modeling; Coles et al. (2024) point to (loss of) certainty and centrality of mathematics. Other values emerge and are related to mathematics education and/or to values that teachers can foster also beyond the discipline itself, and are: rule breaking, tiedness to context, importance of the body and intuition (Gutiérrez, 2017); humbly listening, vulnerability, need for mutual relationships beyond mathematics (Coles et al., 2024); understanding inequalities and discriminations, contradictions (Coles & Helliwell, 2023). Values play a central role in both mathematics and mathematics education (Pa & Tapsir, 2013), but very little attention has been given to them for two very different reasons. The first reason is, as Pa and Tapsir (2013) argue, that “teaching [mathematics] is aimed at the acquisition of knowledge, giving minimum emphasis on the values in mathematics education” (p.450). The second reason is, as Seah (2019) puts it, that values

are confused with beliefs and the two constructs are used interchangeably. A thorough understanding of values in mathematics education and of their role is, in our view, of crucial importance for researchers that aim at reflecting and re-evaluating the priorities and practices of the field in order to address the socio-ecological (Coles et al., 2024). We, thus, look at how Seah (2019) defines values: they are deep personal truths and commitments, which evolve from our attending and responding to stimulus phenomena. Values are qualities that are considered important to us, thus a motivating force that prompts us to get what we want, and that are socially mediated. Values have an ethical and moral connotation (Seah, 2019). Values are also distinguished from beliefs, as the latter are what an individual considers as true (Seah, 2019), while the former reflect what is considered as important (Seah, 2019). The research question we aim at answering is: how our understanding of values in affect-related research in mathematics education can contribute to frame activities that are meant to question the use of models to address current socio-ecological issues?

METHOD

We consider an activity developed by Italian researchers some years ago that builds on the tale of “the man who counts” (Tahan, 2019). We firstly report a summary of the story in what follows.

Beremiz and I were approaching the ruins of a small village close to Baghdad, when we saw a poor man in pitiful condition. He was Salem Nasair, a rich merchant in Baghdad. Unfortunately, his caravan had been attacked and robbed. Salem was starving, so he asked for something to eat. I replied that I had three loaves of bread, while Beremiz, the Man Who Counts, had five loaves of bread. Salem begged to share our loaves with him, so that he would give us eight gold coins as soon as we reached Baghdad with him. Since we had 8 days of journey, we agreed to consume only one loaf of bread a day, dividing it in three equal parts. Once we entered Baghdad, a powerful vizier crossed our way and suddenly recognized the rich merchant, who told him the story. The vizier lent the eight gold coins to Salem to compensate us for our generosity. Turning to the Man Who Counts, Salem said: “Here are five gold coins for your five loaves.” Then to me: “And three to you for your three loaves”. Nevertheless, Beremiz, the Man Who Counts, raised an objection: “this subdivision, which may seem simple, is not mathematically correct. Since I gave five loaves of bread, I must receive seven coins. My friend who gave three loaves of bread should only receive one.” But, the vizier and Salem asked for clarification.

The story stops and the participants are invited to figure out why Beremiz made such a proposal. Then, the story ends with this short part:

“But this division,” added Beremiz, “is not perfect in the eyes of the Almighty. Since everyone has made available what they had, we divide the reward in half: 4 coins for each.”

Even if this tale had not been developed to address specifically socio-ecological issues, it allows for the emergence of three mathematical models, which in turn have the potential to refer to issues like pollution, resource exploitation and hunger. For the data analysis, we dwell on the models and the values they elicit, while in the discussion we propose some extensive reflections on the potential of the story for being used with mathematical activities that aim at understanding socio-ecological issues. The first model takes into account the number of loaves each person has contributed (5 and 3 respectively) and then rewards the narrator and Beremiz accordingly. The second model, however, takes into account how much each of the two people actually contributed to feeding Salem Nasair, i.e., considering the thirds of a loaf that were eaten by those who initially owned the loaves (the

narrator divided each loaf into 3 parts, obtaining 9 pieces of bread and giving only one to Salem Nasair, while Beremiz got 15 pieces of bread and gave 7 to the man they rescued). In other words, this model takes into account how much bread each of the two owners has deprived themselves of. It represents a change of perspective with respect to the 5-3 model, as for the 7-1 model to hold, one considers the loaves as commodities sold, putting oneself in the mindset of a marketplace, where one is rewarded for what they sell, not for what they have. The third model considers that everyone contributed to their best and everyone ate the same amount of bread in the 8 days of travel.

The participants to the activity are 65 in-service primary school teachers, in a project that addressed socially disadvantaged children and the so-called summer loss, due to school closure. The participants listened to the story during a webinar, as they come from different parts of Italy, and they were invited to: (i) make a drawing of the story, (ii) justify why Beremiz (i.e., the man who counts) suggested the 7-1 division instead of the 5-3 one, (iii) opt for the division of money they prefer and say why.

The use of drawing is considered in Mathematics Education a strategic tool for learning and problem-solving (Fiorella & Kuhlmann, 2020). We want to promote an approach to using drawing as an integral part of solving a problem. The drawing is not just a support for the mathematical activity. Indeed, drawing is doing mathematics. Moreover, researchers such as Rellensmann, et al. (2017) distinguish 2 types of drawing: the drawing is called *situational* if it represents the real physical object; it is called *mathematical*, if it represents only the relevant mathematical characteristics. In this research, we question this dichotomy and want to promote an approach to drawings in problem-solving activities that does not distinguish between right drawings and wrong or less adequate drawings. A drawing expresses the way in which the subject makes sense of the situation. The activity of drawing, in fact, has two main purposes in our research. The first one is to allow for multimodal ways of both representing and making sense of the story. The second one is to promote, among teachers, a view of mathematics that puts at the center the value of storytelling. The teachers' subsequent mathematical models are closely linked to different values and, therefore, call into question the relationship with mathematics and restore/construct its socio-cultural value.

DATA ANALYSIS

We should firstly stress that the participants drew a lot. Almost one third of the drawings focused on the loaves of bread and on the needy conditions of Salem Naisir (e.g., the left drawing of Figure 1).



Figure 1: Two of the participants' drawings: to the left, one that focuses on the needy situation of the person encountered and on the pieces of bread; to the right, one with mathematical elements.

We comment that the drawing to the left is a situational drawing because it reports real-world objects and focuses on very relevant parts of the story, that are: the encounter with the poor guy and the

possession of the loaves, which is unequal between the two characters. To help a person that is in difficulty and to recognise (or not) the initial difference in how each one contributed, and to which extent, are the values that guide the entire activity. Adopting Gutiérrez's (2017) lens of analysis, we comment that drawings of this sort let the value of seeing the needs of the others emerge. With Bassi and Brunetto's (2024) lens, we see the drawings allow participants to recognize and name quantities.

Other drawings contain elements that can be more directly linked to mathematics, like the one to the right of Figure 1. Beremiz and his friend have 5 and 3 loaves of bread respectively, which might yield one to consider that the 5-to-3 division is the correct and even one. However, with the aid of colours, it is shown how the bread has been divided and how each character contributed to the survival of the group. With Bassi and Brunetto's (2024) lens, we see that also this kind of drawing allows the participants to recognise and name the quantities that are involved in the models, in the phases of understanding and simplifying (Blum, 2015), but they also allow us to see the relations among quantities, in the phase of mathematising (Blum, 2015). Moreover, we notice that the arrangement of the characters and the bread invites the shift of perspective we anticipated in the methodology, namely: in the drawing to the left, the 5 and the 3 loaves are carried in the bag, (resources that are owned), while in the drawing to the right, it seems Beremiz and his friend are selling commodities.

The analysis now focuses on verbal answers to the task, regarding the justification for opting for one of the three models. We again integrate multiple lenses in our analysis. With respect to the choice of the model, a small number of teachers opted for the 7-to-1 one, of which T4 is an interesting example:

T4: The division I prefer is, therefore, the mathematical one (i.e. 7-to-1), because it ignores the inclinations of each individual, the spirit of the group and responds to an objective criterion.

Teacher T4 refers to the model 7-to-1 as “the mathematical one” (as if the other two ones are not mathematical!) and this has a strong connotation in terms of mathematical values. We see, in fact, that the values of certainty, of one and only answer, of simplification, emerge. It emerges the belief that there exists only one mathematical model to explain everything. This participant explicitly says that the value of this model is in ignoring the inclinations of each individual and the spirit of the group, thus the value of mathematics as context-free and universal, i.e., valid for any person irrespectively of their personal features. Moreover, T4 considers this as objective: objectivity being a value, we underline that the feature of objectivity is assigned to this kind of model, which is the most unequal among the ones that are proposed in the story: in fact, the person who had less at the beginning (i.e., 3 loaves) has even less at the end (i.e., 1 coin), and the one who had more at the beginning (5 loaves), has even more at the end (7 coins). Finally, a view also emerges of mathematics as being at the service of economy (Gutiérrez, 2017), as well as a belief that mathematics serves the purpose of picturing reality (Skovsmose, 2021), and the purity of its role is not questioned. Both beliefs support an overreliance to a single model (Santos, 2007), which does not take into account the complexity and the contextuality of the situation.

Another (even smaller) group of participants opted for the 5-to-3 model, including teacher T13:

T13: I prefer the 5-to-3 split, because each coin corresponds to a loaf of bread.

The value of one-to-one correspondence emerges in this very short excerpt from T13. We interpret its shortness as if T13 does not feel the need to justify more, as if it is obvious that it is as it is and,

somehow, it recalls the fast reasoning that Salem did in the tale. It seems that the emerging value is that each one is rewarded proportional to what they have.

A huge majority of participants opted for the 4-to-4 model and the accounts for this choice range from the sake of simplicity (“it's easier to split the coins in half”, T2), to focus on interdependencies (“the trip was possible thanks to both of the characters”, T60), to privileging humanity over computation (“people matter more than numbers”, T12). This last aspect is also argued by T5:

T5: Beremiz ultimately chooses to divide the coins equally with his friend, putting aside his passion for mathematics, numbers and logic. In this case heart and friendship prevail. Reason does not prevail: "my" loaves and "your" loaves! He preferred to think that those were "their" eight loaves and in this way divide the agreed compensation in two.

Adopting Gutiérrez's (2017) words, we can comment that the cases of T12 and T5 still adhere to the practice of privileging, even if they value prioritizing people over numbers, friendship over mathematics, and heart over logic, namely they somehow switch the priorities. However, there also emerge: (mathematical) rules breaking, because T5 stresses that it is not the case for “my” and “your” loaves; valuing friendship; the need for mutual relationships. The same seems to hold for T51:

T51: In my opinion, Beremiz, despite having demonstrated the mathematical correctness of the 7-to-1 division, preferred to divide the coins equally with his traveling companion, because they shared the same experience, the same difficulties and hardships. Dividing the coins differently would have penalized the travelling companion, guilty only of having had a smaller number of loaves available to divide.

In T51's words, we see that the mathematical model is seen as serving the purpose of formatting reality (Skovsmose, 2021), and not only picturing it, because from T51, emerges the need to change one's perspective radically: it is not “what is mine” and “what is yours” (or, it is not what is inside each ones' boundaries) that counts, but it is more important that we are part of this unique planet Earth, so it makes no sense to introduce inequalities. Nevertheless, it is interesting “correctness” is used to identify the model 7-to-1: i.e., T51 stresses it is the only mathematical correct solution, ignoring how this solution depends on hypothesis during the structuring phases of the MM cycle.

The participant T26 further focuses on the values that are promoted at school:

T26: From a mathematical point of view, I like the calculation made initially by Beremiz, who has the ability to go beyond the appearance of 5-to-3 and analyze the situation from different aspects. Beremiz puts into practice what we ask of our students when faced with a problematic situation: find more solutions (if possible) and discuss them.

We can notice, here, some similarities with the experience of the museum as a political gesture (Coles et al., 2024), as the teachers point to the value of seeing reality from different perspectives, to value the possible conflicts and to take the stance of those who are disadvantaged. Inequalities can be understood, different answers considered, uncertainty has a place and assumptions are made explicit.

A last excerpt, which ends the roundup of cases that we show in this paper, raises arguments of opportunism, namely strategies to keep good relationships with men that have power:

T16: After the journey spent together, having seen the friend's condition and the trader's reaction, Beremiz decided to divide the coins in half to maintain good relations with the trader for any future collaborations.

The existence of an excerpt of this sort is relevant in our view, as it prompts us to see that the preference for a more equal model does not necessarily entail a change in beliefs. In fact, the belief of mathematics at the service of economy emerges in T16's words. In our view, this is important as it adds complexity and forces us to go beyond a sort of one-to-one correspondence between models and values, but to consider, for each choice of a model, the values that individuals hold for it.

DISCUSSION AND CONCLUSIONS

We start the discussion by saying that, in this paper, we push ourselves into an analysis that, rather than only juxtaposing the affective lens and the lens of mathematical modeling, integrates them. In this way, the answers to the question about which model is preferred bring out the value choice that guides the choice of the model and is, in some cases, in contrast and, in other cases, in line with beliefs about mathematics (see for example T5 and T2). Moreover, our understanding of values in affect-related research in mathematics education contributes to framing activities that are meant to question the use of models to address current socio-ecological issues. Many elements of the activity may serve this purpose. For example, we have said (and the participants to the study acknowledge this) that the first model takes into account the number of loaves each person has contributed (5 and 3 respectively) and then rewards the narrator and Beremiz accordingly. With respect to the parallel with resource exploitation in a socio-ecological context, this model assigns a reward that corresponds to the initial amount, in a sense: "if you had 3 at the beginning, you got 3 at the end". Adopting Skovsmose's (2021) analysis, this model can serve the purpose of reinforcing the strong belief that mathematics purely describes reality by relying on a simple one-to-one correspondence, which seems intuitive and fair (we have also commented on T13's short statement as an indicator that this model seems so intuitive to not requiring extensive justifications). It could be used, in a school activity, to discuss how fines for pollution among states are established, for example.

The second model represents a change of perspective with respect to the 5-3 model, as for the 7-1 model to hold, one should consider the loaves of bread as commodities that can be sold, rather than as goods owned, and thus put oneself in the mindset of a market place as it is well depicted also in the drawing to the right of Figure 1, where Beremiz and his friend seem to be selling their loaves of bread. In this case, we see that this model describes, in Skovsmose's (2021) terms, another piece of reality, but, as it is expressed by the Vizier's request for clarification at the end of the story, it introduces an even more marked inequality (Coles & Helliwell, 2023), as the one who had 3 got 1 and the one who had 5 got 7. If a teacher had established a parallel with the fines for pollution at the level of States, this model may help reflecting on the fact that poorer countries might be charged more for the fact of being producers for richer ones, thus polluting more than the latter, to provide commodities that do not end in their citizens' hands. If a parallel is made with resource exploitation, one can underline that this model does not take into account the part that a producer consumes in order to keep themselves alive while producing the goods that are required by the market. Inequalities with respect to food availability can be depicted. Thus, a teacher might design an activity that leads to the conclusion that mathematics can shape reality (Skovsmose, 2021).

The third model, finally, considers the fact that everyone contributed to their best and everyone ate the same amount of bread in the 8 days of travel. Some participants' protocols invited us to consider this as a metaphor for "we all live on the same planet", which allows us to establish a parallel with fines for pollution that value the fact that each one of us breathes the same air, thus it makes no sense

to introduce differences on the basis of how much individual States are contributing to it, as if it is a shared responsibility. Of course, this might not be perceived as fair as well, but we believe that the tensions (Gutiérrez, 2017) and possible contradictions (Bassi & Brunetto, 2023) that might emerge have value in themselves, because these three models co-exist and no one is better than the other.

The co-existence of these three different models not only prompts us to reflect on how resources are assigned, as we have been invited to by Coles and Helliwell (2023), but it also raises awareness that there does not exist a model (a knowledge) that describes anything, as Gutiérrez (2017) underlines. Moreover, the certainty of mathematics is shaken (Coles et al. 2024). The three characters of the story are interdependent, as Gutiérrez (2017) argues that knowers are interdependent and each one of them has knowledge and ignorance that is unique.

Acknowledgements

This research has been funded by the Italian Ministry of University and Research (MUR) under the National Project: PRIN 2022, Prot. 2022TWCJAS.

References

- Bassi, C., & Brunetto, D. (2023). Students' mathematical modelling competencies developed in addressing a real-world problem. *Rivista di Matematica della Università di Parma*, 2(14), 317 – 332.
- Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In: *The proceedings of the 12th international congress on mathematical education: Intellectual and attitudinal challenges* (pp. 73-96). Springer International Publishing.
- Coles, A., & Helliwell, T. (2023). The role of mathematics teacher educators in preparing teachers of mathematics to respond to global challenges within their classrooms. *London Review of Education*, 21(1), 1-13.
- Coles, A., Solares-Rojas, A. & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, online first
- Fiorella, L. & Kuhlmann, S. (2020). Creating drawings enhances learning by teaching. *Journal of Educational Psychology*, 112(4), 811.
- Frejld, P. & Vos, P. (2022). A commentary on the Special Issue “Innovations in measuring and fostering mathematical modeling competencies”. *Educational Studies in Mathematics*, 109, 455-468. <https://doi.org/10.1007/s10649-021-10122-4>
- Gutiérrez, R. (2017). Living mathematx: Towards a vision for the future. *Philosophy of Mathematics Education Journal*, 32.
- Maass, K., Sorge, S., Romero-Ariza, M., Hesse, A. & Straser, O. (2022). Promoting active citizenship in mathematics and science teaching. *International journal of science and mathematics education*, 20(4), 727-746.
- Rellensmann, J., Schukajlow, S., & Leopold, C. (2017). Make a drawing. Effects of strategic knowledge, drawing accuracy, and type of drawing on students' mathematical modelling performance. *Educational Studies in Mathematics*, 95, 53-78.
- Santos, B.S. (2007) 'Beyond abyssal thinking: From global lines to ecologies of knowledges', *Revista Critica de Ciencias Sociais*, 80.
- Seah, W. T. (2019). Values in mathematics education: Its conative nature, and how it can be developed. *Research in Mathematical Education*, 22(2), 99-121.
- Skovsmose, O. (2021). Mathematics and crises. *Educational Studies in Mathematics*, 108(1-2), 369- 383
- Tahan, M. (2019). *L'uomo che sapeva contare*. Salani Edizioni.

MATHEMATICS EDUCATION'S RESPONSE TO THE QUESTION, "HOW DOES IT FEEL TO BE A PROBLEM?"

Peter Appelbaum

Arcadia University

Mathematics Education research and practice tends to believe it contributes to improving our world. Even critiques of mathematics education are guided by a belief in such progress toward a better life for all, like a letter from a caring, critical friend. This paper asks us to reflect on the ways that our motivations may serve our desire to believe we are good people more than our apparent goals of social and ecological justice. Three directions are proposed: (1) See ourselves as the problem, and use mathematics to interrogate our fears and vulnerabilities; (2) Use the examination of everyday artifacts of life to explore those fears and vulnerabilities, as opposed to leaping into problem-solving mode; and (3) Learn from all ancestors, human and more-than human -- past, present and future -- instead of studying them as resources, gathering in genuine community.

FEELING GOOD -- BUT THAT IS THE PROBLEM

Mathematics education talk can be seen as a "boomerang discourse" (Spelman 1988), where the focus outward often returns to the speaker, reflecting back on their own identity. Beyond avoiding responsibility, this talk often helps the speaker construct an image of themselves as non-racist and non-colonial. It creates the impression that we, as well-intentioned members of privileged communities, care deeply about global inequalities and the environment. This privilege includes most contributors to this ICMI study – those of us who focus on writing, researching, and innovating in mathematics education, who attend conferences, and who are involved in these academic pursuits.

When this discourse occurs in diverse settings, particularly among people of color or in Indigenous and multicultural assemblies, it often serves as a ritual of moral purification. The goal is to gain affirmation from these communities, reinforcing the belief that mathematics should empower everyone. However, as privileged educators and researchers who are often blind to our own biases (as described by de Sousa Santos in 2007), we may misinterpret these interactions. We either take them as confirmation of our goodness – "See? Ngai thinks I'm a good person" – or, if we do not receive such affirmation, we assume it is because our goodness is not being recognized due to the other's inability to see it – "Carlos is so caught up in his oppression that he can't understand my allyship."

When our participation is focused on ecological justice, I warn of a similar ritual. How are our calls for ecological justice attending to the rights and needs of all of our ancestors, past, present and future, including our more-than-human cousins and siblings – rocks, streams, rainstorms and mountain ranges, oceans and winds? Participation in this 27th Study requires increases to our carbon footprints, ignorance of the energy needs for internet communications, downloading and uploading documents, and more. Personally, I cannot resist participation in this ICMI Study – it resonates deeply with my stated dreams and wishes for everyone on Earth, saving life on our planet, aligning mathematics education with social movements addressing climate change, severe weather, and health, poverty, forced migration, human trafficking, and more. Finally, my personal convictions are connecting with

my professional work in mathematics education. However, I worry that we may reproduce in this realm what many in the fields of social justice have noted: that we often remain ignorant to how our well-intended, good efforts serve ourselves more than the seemingly important goals we articulate.

The Post-Colonial

Mathematics education often centers around a Eurocentric version of school mathematics that preserves the "abyssal line" de Sousa Santos describes as erasing local and Indigenous mathematical practices. In our post-colonial world, this approach to mathematics education prioritizes a version of mathematics that marginalizes others. Mathematics exemplifies this divide, where its teaching and learning often perpetuate existing power structures even after the official end of colonization. Ethnomathematics emerged as a response to this situation, acknowledging the issue but paradoxically also contributing to the continuation of colonial legacies (Appelbaum & Stathopoulou 2023).

Ethnomathematics was initially seen as an explanation for why colonized populations might struggle with school mathematics, offering a perspective that highlighted local cultural practices sharing similarities with Western mathematical concepts. However, it also played a role in the colonization process, as Western school programs co-opted examples of Indigenous mathematics, using them to reinforce the dominance of Western mathematics. This process reflects a broader issue: something is only recognized as mathematics if it resembles what the colonizing culture defines as mathematics. Consequently, many forms of mathematical activity are overlooked or lost because they do not fit this narrow definition.

Ubiratan D'Ambrosio (1977) proposed an Ethnomathematical Program to study the creation and dissemination of mathematical knowledge across different cultural systems, focusing on the mathematics practiced by various cultural groups. This approach revealed two contradictory views: first, that mathematics is objective and external, and second, that its usefulness hides its cultural origins. Critical ethnomathematics now focuses on the political and social justice aspects of the field, challenging the assumptions behind terms like "culture" and "mathematics" and exposing how these concepts can perpetuate colonial structures, especially in education (Appelbaum & Stathopoulou, 2023).

Despite these efforts, discussions and actions within ethnomathematics often fail to challenge the universal dominance of Western school mathematics. This discourse often distracts from the real issue: the fear that drives the persistence of these inequities. For many mathematics educators, engaging with these issues allows them to feel they are addressing important concerns, like racism and colonialism, while maintaining a sense of comfort and safety. This self-assured goodness often leads them to avoid deeper engagement with the messy realities of these issues, believing they are not complicit because they are "good" educators.

The real problem lies in how mathematics education itself is perceived. Being seen as a "problem" is different from "having" a problem. Colonialism turned certain groups into "problem people," and school mathematics has historically labeled entire populations as mathematically deficient or inferior. However, we cannot critically address the issue by simply reversing these roles and labeling mathematics educators as the new "problem." Instead, we need to examine what mathematics education does – how it performs and the impact it has on society. For example, if problem-solving is the organizational structure of a common mathematics curriculum, this implicitly and powerfully

acculturates learners into a Western perspective on the core purposes of mathematics, erasing alternative (non-Western) functions and aesthetics of mathematical thinking while also reducing the more-than-human members of our co-created world to resources without rights and desires.

James Baldwin spoke of "disagreeable mirrors" (Baldwin 1985: 409) when discussing racism and whiteness, and the same applies to mathematics education. The discourse allows educators to avoid confronting their own biases and failings. They may rush to help communities without considering whether their help is wanted or whether they are perpetuating harmful practices. This avoidance of self-reflection helps maintain their sense of moral goodness while preventing them from truly understanding their complicity in perpetuating the problems they claim to address.

To address the real issue, mathematics educators must look beyond the surface and understand the complexity of mathematics education discourse feeding resistance to understanding and change. How many of us: embrace critical ethnomathematics and truly live a life of mathematics education constructed through the co-creation of mathematics communities with the elders of the local community? Re-educate ourselves away from linguistic nominalization that makes mathematics about objects and facts (nouns) toward verbification about processes of becoming, actions upon actions (Borden 2011)? Avoid both the perpetuating lack of awareness of half a century of ethnomathematics that preserves coloniality *and* a rigid reclamation of local and Indigenous ways of practicing counting, measuring, designing, playing or explaining? We need to recognize the fear and anxiety driving this and challenge the assumptions that underlie our work. This means embracing approaches like "two-eyed seeing," which values both local traditions and Western mathematics, allowing learners to gain a broader, cross-cultural understanding of the world (Hatcher et al. 2009). Only by doing so without clinging to a preconceived set of concept and skill objectives can we begin to address the deep-rooted issues in mathematics education and move towards a more equitable and inclusive future.

The Post-Anthropocene

The boomerang metaphor can be applied to our current era, the Post-Anthropocene. Some people still talk about how humans have caused irreversible damage as a geological force (the "Anthropocene"). They circle back to thinking that humans can solve the problems we have created, not yet realizing that the planet is pushing back against us, just as we try to control insects and viruses. This mindset keeps us focused on what knowledge is most important for humans, which often leads to data collection and analysis to convince nations and corporations that their pursuit of profit is harming the planet. We witnessed much of this during the recent pandemic, during which the mathematics of epidemiology and exponential growth became exercises in the understanding of a crisis (Chan et al. 2021). Others believe that communication skills are key to helping young people work together across cultures for social and ecological justice (Mychajliw et al. 2015; Mikulan & Sinclair 2017; Coles 2017). This would turn mathematics education into skills and their use (Somos 2019, Buckley 2022). A third perspective suggests that we should use numeracy and mathematical literacy for practical purposes, such as building shelters after severe weather, designing networks in a crisis, or even using recreational mathematics to build community in times of despair. This would imply mathematics as the language of science and engineering, also of art and imagination, for which there are many examples in the STEM and STEAM literature.

Two additional perspectives may be more useful. The first sees us as newcomers to the Earth, arriving long after the land, waters, plants, and animals. We should learn from these elders rather than focusing solely on human knowledge. What forms of learning might they offer us if we listen (Gutiérrez 2022)? And rather than taking knowledge from the world, we should contribute mathematics to it, honoring our responsibility to the Earth (Karrow et al 2017).

The second additional perspective brings us back to our place, where we are now, not as outsiders, but as part of an eco-community (Wolfmeyer & Lupinacci 2017). We should ask, “How does our presence contribute to a reciprocal or sustainable relationship with the land?” (Abram, in London 2023). Knowing about crises and catastrophes is important, but an obsession with knowledge alone will not change our way of life in time. Instead, we should focus on how we can live in harmony with the land, honoring our sensory experiences and the wisdom they bring. Knowledge is powerful, but we can learn from Indigenous traditions where magicians and shamans use their skills in service to the Earth, ensuring that we give back as much as we take.

In this way, stories become the law of the land (Yunkaporta 2021). “Right stories” regenerate the landscape and all its inhabitants, including humans, as stewards of the Earth. “Wrong stories,” on the other hand, spread curses, illusions, and bad faith, leading to self-destruction. When wrong stories form the basis of our decisions, they lead to disastrous outcomes. But nature imposes limits, acting as a natural immune system against those who misuse the land. Historically, school mathematics has treated mathematics as a set of games and operations within fixed systems. But when these systems go wrong, nature always corrects them. Smarter communities listen to those who remember the “right stories” and learn from them.

This is why I suggest shifting our focus away from curriculum content and towards using it as a tool for living in harmony with our place (Appelbaum 2024). Learning about issues like climate change or social justice can distance us from community engagement, fostering intellectual authority rather than connection. Instead, we should focus on experiences that center allyship in communities, where we explore the relationships between science, fiction, and the more-than-human world. Our technologies of knowing often keep us detached from the living world. But by using academic knowledge to foster conviviality and stewardship, we can maintain the porous boundaries between humans and the rest of nature. Education should teach us to let go of what no longer serves us and embrace the changes happening beneath the surface.

The Post-Anthropocene calls for new ways of living, where we are no longer the primary force of change. We are like immigrants, holding onto the old world, not yet accepting that we are in a new place. In times of dramatic transformation, like during World War II, Margaret Mead (1943) suggested that we teach children to navigate new paths and face challenges we cannot foresee. Immigrant children teach their elders how the new world works. Similarly, a Post-Anthropocene pedagogy should nurture the ability to act independently, questioning the status quo, and finding new ways of learning and living.

WILLFUL IGNORANCE

Mathematics education often shuts down alternative ways of knowing. This can be seen as a form of willful ignorance, which Nancy Tuana (2006) defines as “the condition of not knowing and not wanting to know.” Willful ignorance is not passive; it is an active and complex effort to maintain

ignorance. It is not just about lacking information—like saying, “Sorry, I didn’t know that!”—but rather about deliberately ignoring or dismissing certain facts, like when we focus on comparing countries by their PISA test scores or insist that a month must be spent on the binomial theorem to prepare for Calculus instead of learning from the lands and waters in our place.

Maintaining this ignorance involves guarding the familiar patterns of dismissals and denials that are common in mathematics education. For example, we might acknowledge that Mosques used aperiodic tiling long before Penrose or that African fractals have influenced complex technology for millennia, only to then trivialize these contributions by saying, “Maybe we can use some pictures for motivation in the textbook?”. Similarly, while recognizing the importance of tipping points and modeling for understanding climate change, we might dismiss it by saying, “But years 10-13 are already full with trigonometry and sequences.”

Mathematics education talk represents willful ignorance not because of a lack of knowledge but because it allows us to avoid confronting uncomfortable truths. When I rely on this kind of talk, I am actively choosing to put issues like racism and genocide in the past, dismiss the real struggles of people of color, or prioritize my own comfort over addressing these issues. I might indulge in a “doomer” mindset, believing that my actions are insignificant compared to global corporate impacts on the environment. I stay complacent, attending academic conferences worldwide while conveniently disconnecting past atrocities and current environmental crises from my immediate reality.

FLIPPING THE SCRIPT

The power of Mathematics Education talk remains strong not because it advances careers or brings financial rewards but because it helps us avoid facing our fears. We stay ignorant because we are afraid that the ideas behind critical ethnomathematics might be true, which would mean we would have to start over and learn from the wisdom of nature and our own or other cultures. It is easier not to think about it at all than to think and realize we need to change. Flipping the script on “the problem” is not, as Alison Bailey (2021, p.45) writes, the “scholarly equivalent of an adolescent back-seat quarrel on a long road trip.” Particular people are not “the problem” It is not, “You’re the problem . . . No, you’re the problem . . . No! YOU’RE the problem! It’s a cultural interaction problem! No, it’s a mathematics education problem! No, it’s a psychological-consumer culture-political discourse problem! NO! It’s an industrial complex problem!” The construction of African, Indigenous, Asian, rural or other mathematics learners as “problems”, loss of local mathematical practices as a “problem”, are the other side of a coin that constructs mathematics educators as in a position to solve these problems. Mathematical modelling providing accurate predictions of ecological catastrophe while failing to provide convincing pathways to the solutions to these crises is not a question of whose problem or who is in the best position to solve the problem. It is instead a symptom of mathematics education *as* the problem. But we will not be solving “the problem of mathematics education as the problem” by flipping the script. We cannot simply swap essence for essence. Mathematics education as the problem is not going to find a solution by searching for specific features of mathematics education as the problem – this problem is fundamentally different from historically designated “problems” like “Native insurgencies”, “the Jewish problem”, and other racist-based so-called “problems”. We cannot identify through research specific essences of mathematics education that answer questions. What we need to do instead is to study what mathematics education does. When

the script is flipped, we begin to understand what the performance of mathematics education accomplishes.

The desire to present ourselves as good and innocent is also driven by willful ignorance. But this ignorance has a huge and ongoing cost for Black, brown, Native, and Asian communities, draining any moral credit we think we have. When we rely on willful ignorance, our sense of goodness becomes a form of selfishness that prevents us from genuinely respecting and recognizing others. This undermines the idea that mathematics can help us find solutions to global challenges like climate change, severe weather, and water shortages. The evidence of the benefits of Mathematics Education talk collapses when we refuse to consider other perspectives, including the more-than-human perspectives.

Vulnerability

At the heart of the problem is fear, driven by our vulnerability. We fear because we feel powerless to make a difference. We can either hide our fears behind Mathematics Education talk, or we can embrace our vulnerability and move forward. Instead of talking about Mathematics Education in a way that covers up our fears, let us talk about vulnerability. Let us celebrate vulnerability through mathematics. Vulnerability is not a weakness; it is a state of potential. It makes us open to being hurt, but also to loving, learning, and finding comfort in others. It allows us to experience a range of emotions and to grow from them.

Susan Solomon (2024) describes how we have healed the Earth before and believes we can do it again. She shares stories of people who felt vulnerable and took action because of it. She writes that success begins when an environmental problem becomes personal and visible to the public. This creates momentum, leading lawmakers, diplomats, industries, and agencies to respond, and real change happens when consumer pressure aligns with practical solutions from legislation and regulation. Socio-ecological mathematics could help people understand their vulnerabilities, see how these drive their fears, and take action to solve the problems causing them. To do this, we need to let go of our need for comfort and recognize that fear and discomfort can be valuable sources of knowledge and connection. What if, instead of trying to avoid or reframe fear, anger, shame, and guilt, we treated them as genuine sources of knowledge? Given the thick monsoon of research on making learning of mathematics easier, it is a direct challenge to explore how to complexify mathematics so that learners can enjoy the *jouissance* of uncertainty. What if we understood complexity and uncertainty as the heart of mathematics?

Artifacts of Everyday Life

I propose a Post-Anthropocene Pedagogy that helps us think about everyday objects and experiences in a way that connects mathematics education to the idea of “living well in our place” (Appelbaum 2024). This pedagogy has four phases, adapted from William Pinar (1975), encouraging us to reflect on simple, everyday things and the role of mathematics in understanding them. These phases guide us through looking at the past, imagining the future, analyzing the present, and then creating new ways of learning. The curriculum of the most mundane, everyday artifacts of life can become important entry points into new forms of community.

The first phase is the “regressive” phase. In this phase, a group looks back at their experiences with an everyday object, which can be anything from a tool, a product, or even an idea. By sharing stories

about how we interact with these objects, we build a collective understanding of what we learn from them. Questions like “What is it like to use this object?” and “What does it teach us about who we are becoming?” help us understand the impact of these everyday things. Next is the “progressive” phase, where we imagine the future of the object. What will happen if we keep using it the way we do? What new possibilities might it have that we have not thought of yet? This phase helps us think about how we can collaborate to use the object in different and creative ways. The third phase is the “analytic” phase. Here, we explore the history of the object. How did it come to be a part of everyday life? Who made it? Why does its history matter now? Understanding the origin and development of the object helps us see its impact on the world today—environmentally, culturally, politically, and more.

Finally, the “synthetic” phase encourages us to pull everything together and ask, “What do we do with this knowledge?” Based on what we have learned from the past, present, and future, we brainstorm new educational ideas, activities, and even ways to assess what we have discovered. We reflect on ourselves and our surroundings, asking, “Who are we now, and how can we grow from here?” This phase helps us think about how we can live and flourish in connection with more-than-human life. This whole process of analyzing artifacts is about exploring our experiences. It turns everyday objects into opportunities for growth and change, helping us move forward rather than just looking back.

IMPLICATIONS

A psychoanalytic jewel is to see the problem as the solution (Appelbaum 2008). Such an approach suggests that we use mathematics to understand and appreciate our fears and vulnerabilities, rather than to leap into solving problems from outside of them. Because it is challenging to directly confront our vulnerabilities, studying the artifacts of everyday life in four phases of description, ignorance, and projection, especially those overlooked and mundane artifacts, can be a powerful beginning. These approaches would only generate community with *all* of our ancestors – past, present and future, if we learn from them rather than study them as resources.

References

- Appelbaum, P. (2006). *Embracing mathematics: On becoming a teacher and changing with mathematics*. Routledge.
- Appelbaum, P. & Stathopoulou, C. (2023). History of ethnomathematics: Recent developments, in Romero, S., Serrado, A., Aldon, G. & Appelbaum, P., *The Role of the History of Mathematics in the Teaching/Learning Process*, pp.189-207. Springer.
- Appelbaum, P. (2024). *Post-Anthropocene civic and global education studies: Beyond posthuman thinking*. Springer.
- Bailey, A. (2021). *The weight of whiteness: A feminist engagement with privilege, race and ignorance*. Roman & Littlefield.
- Baldwin, J. (1985). White man's guilt, in *The price of the ticket: Collected nonfiction, 1948- 1985*. St. Martin's Press.
- Borden, L.L. (2011). The ‘verbification’ of mathematics: Using grammatical structures of Mi’kmaq to support student learning. *For the Learning of Mathematics*, 31(3), 8-13.
- Buckley, C. (2022). ‘OK Doomer’ Why a growing number of young advocates are telling climate pessimists to lighten up.” *The New York Times*, March 23, 2022, p. A11.

- Chan, M.C.E., Sabena, C. & Wagner, D. (2021). Mathematics education in a time of crisis—a viral pandemic. *Educational Studies in Mathematics*, 108, 1–13.
- Coles, A. (2017). Habits and binds of mathematics education in the Anthropocene. *Philosophy of Mathematics Education Journal*, 32, <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/index.html>
- D'Ambrosio, U. (1997). Ethnomathematics and its place in the history and pedagogy of mathematics. In M. Frankenstein & A. Powell (Eds.), *Ethnomathematics: Challenging eurocentrism in mathematics education*, 13–24). State University of New York Press.
- Gough, A. (2021). *Education in the Anthropocene*. Institute for Interdisciplinary Research into the Anthropocene. <https://iiraorg.com/2020/11/16/education-in-the-anthropocene/>
- Gutiérrez, R. (2022). A spiritual turn: Toward desire-based research and indigenous futurity in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 379-388.
- Hatcher, A., Bartlett, C., Marshall, A., & Marshall, M. (2009). Two-eyed seeing in the classroom environment: Concepts, approaches, and challenges. *Canadian Journal of Science, Mathematics and Technology Education*, 9, 141–153.
- Karrow, D., Khan, S., & Fleener, J. (2017). Mathematics education's ethical relation with and response to climate change. *Philosophy of Education Journal*, 32, <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/Karrow%20Khan%20Fleener%20submitted.docx>.
- London, S. (2023). The ecology of magic: An interview with David Abram. <http://scott.london/interviews/abram.html>.
- Mead, M. (1943). Our educational emphases in primitive perspective. *American Journal of Sociology*, 48(6), 633-639.
- Mychakliw, A., Kemp, M.E., Hadly, E.A., Using the Anthropocene as a teaching, communication and community engagement opportunity. *The Anthropocene Review*, 2(3), 267-278.
- Pinar, W. (1975). The method of currere. In Pinar, W. (Ed.), *Autobiography, politics and sexuality: Essays in curriculum theory, 1972-1992*, 19-27. Peter Lang.
- Santos, B. DeSousa. (2007) Beyond abyssal thinking: From global lines to ecologies of knowledges, *Review (Fernand Braudel Center)*, 30(1), 45–89.
- Solomon, S. (2024). *Solvable: How we healed the Earth, and how we can do it again*. University of Chicago Press.
- Somos, C. (2019). Greta Thunberg's four simple steps to combat climate change. Sci Tech. *CTV News*. <https://www.ctvnews.ca/sci-tech/greta-thunberg-s-four-simple-steps-to-combat-climate-change-1.4613253>.
- Spelman, E. (1988). *Inessential woman: Problems of exclusion in feminist thought*. Beacon Press.
- Tuana, N. (2006). The speculum of ignorance: The women's health movement and epistemologies of ignorance. *Hypatia*, 21(3), 1-19.
- Wolfmeyer, M. & Lupinacci, J. (2017). A mathematics education for the environment: Possibilities for interrupting all forms of domination. *Philosophy of Mathematics Education Journal*, 32, <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/WolfmeyerLupinacci-submitted.doc>.
- Yunkaporta, T. (2021). All our landscapes are broken: Right stories and the law of the land. *Griffith Review*, 72(1), 1-4.

CLIMATE CHANGE, MATHEMATICS EDUCATION, AND THE EPISTEMIC FALLACY

Eleany Barrios-Borges, Francisco Cordero

CINVESTAV

Our research sheds light on the potential role of mathematics education in addressing the environmental crisis. The traditional reductionist and linear approach to mathematics education could lead to epistemic fallacies in addressing climate change. It is therefore proposed that research be directed towards the use of mathematical knowledge in disciplines that address climate change and that frameworks for mathematics education be established. This study falls under the socio-epistemological modeling category and explores using data (observations) as mathematical knowledge to understand climate change. The different functions and forms of data use are examined, with the hypothesis that estimation can be seen as a resignification in situations of pattern detection.

INTRODUCTION

Climate change is widely discussed in the media, but is this information transparent to the general public? What do people essentially understand about climate change? To gain a better understanding, it is essential first to understand what climate is. Climate refers to the prevailing atmospheric conditions at a location over an extended period. On the other hand, climate change involves changes in temperatures and other weather patterns.

Throughout history, natural changes have been attributable to terrestrial or solar system activities. However, since the 19th Century, abrupt changes in weather patterns have been significantly influenced by human activity. This activity has resulted in the increasing emission of greenhouse gases, which contribute to the phenomenon known as global warming. These climate changes have hurt sea levels, the availability of water resources, forest cover, and the intensity of meteorological phenomena, among other aspects.

The adverse impacts of climate change on the planet are significant, underscoring the urgency of adopting more sustainable lifestyles and implementing ad hoc solutions (Rosa et al., 2022). Examples of these solutions include using renewable energies, reforestation, recycling, reducing energy consumption, and other measures implemented or suggested in various fields.

Since the 19th Century, the sciences have tried to substantiate, describe, and predict climate change. In 1824, Fourier described what is now known as the greenhouse effect. Then, in 1856, Eunice Newton Foote demonstrated that atmospheric carbon variations affect climate. Forty years later, Svante Arrhenius concluded that burning coal accelerates the greenhouse effect. In 1960, Charles David Keeling provided the first evidence of rising atmospheric carbon dioxide levels. In 1988, the Intergovernmental Panel on Climate Change (IPCC), an interdisciplinary team of scientists providing regular climate change assessments to policymakers, was established. This group has consistently warned about human influence on climate and the urgency of drastically reducing emissions in all sectors to limit global warming, given that a 2°C increase in the Earth's temperature would have catastrophic consequences for human life and other systems on Earth (IPCC, 2021).

Crises and mathematics are related in three distinct ways, according to Skovsmose (2021). First, mathematics can picture a crisis, as does classical mathematical modeling. Second, mathematics can constitute crises, i.e., be an intrinsic part of them, as in the case of the 2008 economic crisis. Thirdly, mathematics can format a crisis, referring to the ways of acting derived from mathematical interpretation.

However, looking at the climate change crisis from a complexity perspective shows how mathematics is linked to these three dimensions. On the one hand, they provide models of weather phenomena, disease spread, biodiversity loss, migrations, and social systems, all interrelated and affected by the climate change crisis. On the other hand, mathematics underpins technologies and platforms of production and economy, guiding their functioning and dysfunctions. Moreover, interaction is not considered integration as a whole species. For example, the global north has exploited the global south, depleting its biodiversity and water resources and locating highly polluting industries such as textiles in these territories. These activities generate large amounts of greenhouse gases, the leading cause of global warming and the most obvious manifestation of the current climate crisis. Although climate change predictions are uncertain, they are indications of how to act and, above all, what not to do.

With a framework such as the above, it is crucial to conduct research, innovations, and actions considering mathematics education's current and potential role in climate change issues. However, traditional methods of teaching mathematics in the classroom need to be revised to address the most pressing challenges of the 21st Century. It is imperative that future professionals engage with the most pressing environmental and social issues and that citizens, in general, better understand the implications when making predictions, assessments, or decisions (Caron, 2019).

To achieve this, it is necessary to identify what mathematics is appropriate for these professionals. However, the curricular content generally follows the opposite direction, as they focus on applications of already established mathematical knowledge, for example, applications of integral calculus, such as determining the area of a flat region. According to Galbraith (2015) - from whom we have drawn inspiration to title this paper - this same contradiction is present in the field of mathematical modeling in mathematics education: although its ontological foundations are based on solving real-world problems, this foundation is distorted in mathematics education perspectives that approach modeling as a mere vehicle for teaching a specific topic.

Thus, a problematic issue arises in the mathematics education of future engineers: there must be more clarity between mathematics classes and the crucial aspects of climate change aspects and the associated ecological crisis. Conversely, engineering mathematics classes often address mathematical topics predating the 18th Century (Davis & Sengupta, 2020). Although fundamental, these topics reflect a deterministic paradigm emphasizing causal and linear relationships. In addition, expressions of the use of mathematics in other non-mathematical disciplines or engineering are frequently neglected.

On the other hand, climate change imposes new challenges that engineers must address, requiring skills and knowledge that should be more valued and addressed in their current training. For example, a thorough understanding of the fundamentals of climate science and its impacts is needed (Martin et al., 2022). This also involves adopting complexity theory to understand the intrinsically complex

nature of the phenomenon and the necessary responses (Karrow et al., 2017; Boylan, 2017; Coles, 2017). This holistic approach is fundamental to developing functional knowledge, i.e., peculiar and plural mathematics (Cordero, 2023), that enables engineers to address the challenges of climate change effectively.

In educational mathematics, climate change is approached from at least five perspectives. These include critical mathematics education (Skovsmose, 2023), mathematical modeling that considers sustainability as a context in activities (e.g., Wiegand & Borromeo Ferri, 2023), sustainable mathematics education (e.g., Renert, 2011), ethics and ethics theory (e.g., Ernest, 2024), and the social and political dimension of mathematics education (e.g., Jurdak et al., 2016).

Karrow et al. (2017) raise the question of what role mathematics education should play in the ecological and social challenges of climate change and its response. They suggest promoting an epistemological shift within the field of Educational Mathematics towards an emergentist epistemology proper to complex systems, which considers randomness, unpredictability, iteration and recursion. In addition, they propose the integration of interdisciplinary considerations in the curriculum. This approach of the authors represents a call to change thinking from research to meet the demands of climate change.

These approaches in Educational Mathematics represent a severe and profound commitment; however, an epistemic fallacy may arise when attempting to address climate change issues in the classroom under the traditional reductionist and linear paradigm (Bhaskar, 1975; Bryant, 2011; Galbraith, 2015). This fallacy involves the confluence between ontology and epistemology, i.e., the tendency to confuse the nature of a reality with the knowledge of it. Therefore, it is necessary to research the use of mathematics in other disciplines that address climate change.

Given this problem, it is crucial to study climate change's epistemology as a complex dynamic system. This study aims to deeply understand the interrelated variables contributing to the increase in the Earth's temperature and the nature of these interactions. The uses of mathematical knowledge in the foundations of climate science and its effects are discussed, focusing specifically on the framework proposed by Klaus Hasselmann (1979). This framework is used to systematically compare climate models and purely spatial observations to address the following questions: What circumstances have led the climate to its present state? Is it due to natural variability or increased greenhouse gas concentrations due to human activity? Hasselmann was recognized in 2021 with the Nobel Prize in Physics for his pioneering contribution to understanding complex physical systems.

In the context of the advances in the study of climate change (Hasselmann, 1976), and with the interest of contributing to avoiding confusion between the nature of reality and the knowledge of this in educational proposals, a research question in Educational Mathematics has been proposed for this research: What characterizes the use of mathematical knowledge in the systematic comparison between models and observations, to determine the circumstances that have led the climate to its current state?

The theoretical and methodological bases for addressing the research question are presented below.

ELEMENTS OF THE SOCIAL-EPISTEMOLOGICAL MODELING CATEGORY

Our research is framed in the Socioepistemological Theory of Mathematics Education (Cantoral, 2020) (STME), specifically in one of its research programs, Forgotten Subject and Transversality of Knowledge (Soltsa), which introduces a category of socioepistemological modeling ($\zeta(\text{Mod})$) (Cordero, Mendoza Higuera, et al., 2022). This category acts as a domain ontological and epistemological model for the analysis of the putting into use of mathematical knowledge $\mathbf{U}(\text{CM})$, its meanings, and moments of transversality or resignifications ($\text{Res}\mathbf{U}(\text{CM})$) in diverse human activities and realities.

Domain ontology encompasses various aspects of reality and the nature of mathematical knowledge, including its essential characteristics, such as the type of knowledge (e.g., school, professional, academic) and how mathematical knowledge is related to that of other domains to solve tasks. While epistemology defines how one knows and reasons about a specific reality, each domain ontology is associated with a specific epistemology (Galbraith, 2015). Reality is the everyday life of people in school, different disciplines, work, life, or any other situation where routine uses of mathematical knowledge are expressed (Cordero, 2023).

From the uses and meanings of mathematical knowledge, frames of reference are formed to value in mathematics education the functional justification required by other domains of knowledge and to create relationality between school mathematics and everyday realities that favor the learning of the meanings of mathematics (e.g., Mendoza et al., 2018).

The $\mathbf{U}(\text{CM})$ represents functional mathematical knowledge, i.e., valuable knowledge to people in different life scenarios (academic-school, profession-work, life-people). It is a structure of relationships that, theoretically and methodologically, presents both a local and a global framework (Barrios Borges & Cordero, in press). The functioning and form of mathematical knowledge in a specific situation are analyzed locally. Functioning refers to the organic function of the situation, manifested through the tasks that compose it, while the form is related to the kind of these tasks (Cordero & Flores, 2007). This implies investigating what for and how mathematical knowledge is used in the specific situation. Tasks can be activities, actions, and executions performed within that situation. When the alternation between tasks results in a new performance that debates with the form of use, a $\text{Res}\mathbf{U}(\text{CM})$ is produced (Cordero & Flores, 2007).

At the global level, moments of transversality in the use of mathematical knowledge are identified and characterized by a structure called epistemology of uses or category of uses of mathematical knowledge. This structure underlies the characterization of the use of mathematics in different situations. The epistemological structure is organized in a core situation. It is broken down into meanings, procedures, and instruments that generate a transversal argumentation ($\text{Res}\mathbf{U}(\text{CM})$) in specific situations in different domains and scenarios. However, these transversal elements also have specific components associated with the functional aspects of putting certain mathematical knowledge into use in the peculiarity of the specific situation. (Barrios Borges and Cordero, in press).

The epistemological structure has aspects that explain it, and simultaneously, there are the methodical components that reveal it (Barrios Borges & Cordero, in press). Meanings are reflected through situational arguments, which can be based on connections consistent with definitions and properties and personal interpretations, complete with images and metaphors (Bishop, 1999, cited in Buendía

and Cordero, 2005). *Procedures* are the operations induced by the meanings and concepts that are progressively constructed. Instruments are systems of resources for constructing meanings in the context of interaction (Cordero, 2001).

ANALYSIS OF THE USES AND MEANINGS OF THE USE OF DATA

Klaus Hasselmann (1979) created a framework for systematically comparing climate models and spatial observations (data) to answer the following questions: What circumstances have led the climate to its present state: natural variability or increased concentrations of greenhouse gases due to human activity? Determining the mean atmospheric response in the presence of natural variability presents both model experiments and accurate data with a primary signal detection problem. Often, the length of the data record available to filter the mean signal from the noise could be much higher, either due to the short time history of recorded meteorological data at the hemispheric or global level or the overhead of running high-resolution models that simulate natural variability. The signal-to-noise problem may become one of the most severe limitations in studying atmospheric response to external forcing.

The study of the nature of reality and its knowledge, particularly the role of data as mathematical knowledge, has been a key focus in Hasselmann's work (1979). The category of mathematical knowledge, termed "hypothetical" due to the need for validation of its emergence and its applicability in various human activities and realities, plays a crucial role in understanding the nature of reality. These uses of data and the category of mathematical knowledge provide a solid epistemological foundation for the design of future school interventions.

Hasselmann (1979) states that the problem of identifying the mean atmospheric response to external forcing in the presence of natural atmospheric variability "is essentially one of pattern recognition" since the atmospheric response signal "represents a multi-dimensional vector or field" (p. 251).

For a consistent pattern recognition analysis it is necessary to regard the signal and noise fields as multi-dimensional vector quantities, characterized by appropriate multi-dimensional joint probability distributions, and the significance analysis should accordingly be carried out with respect to this multi-variate statistical field. (...) if a priori hypotheses regarding the general structure of the expected response can be formulated, pattern filtering can be applied to improve greatly the signal-to-noise ratio. (...) such hypotheses can normally be rather easily generated using simple, linearized models. (p. 252)

When analyzing the functions and form of the use of mathematical knowledge (Cordero & Flores, 2007) in Hasselmann (1979), we see that:

Functioning	Forms
Pattern recognition analysis	Consider signal and noise fields as multidimensional vector quantities.
Formulate a priori hypotheses about the general structure of the expected response	Use models
Improve the signal-to-noise ratio (model of the natural variability of the	Pattern filtering

atmosphere/atmospheric response to external forcing).	
Modeling atmospheric response	Multivariate analysis of low-frequency variability

Table 1. Uses of mathematical knowledge in the problem of the signal/noise ratio in atmospheric response studies.

Multivariate analysis of low-frequency variability is a powerful analytical tool used to understand long-term fluctuations. It helps in exploring and understanding the relationships between multiple variables in a model or system, thereby contributing to a better understanding of its behavior and temporal evolution.

On the other hand, the analysis of the epistemology of uses in Hasselmann (1976) infers that:

Kernel Situation	Pattern detection
Meanings	Understanding complex phenomena
Procedures	Identify and describe significant structures in the data
Instruments	Response pattern construction
Argumentation (Res <u>U</u> (CM))	Estimate

Table 2. Uses of mathematical knowledge in the signal-to-noise ratio problem in atmospheric response studies.

CONCLUSIONS

This paper represents a contribution to the interests raised by the international community about the relationality between mathematics education and the global environmental crisis. Research in the field of Mathematics Education is still limited. In this context, open questions arise that need to be explored from different epistemological positions: What is the role of mathematics and mathematics education in the face of the global environmental crisis? How can interdisciplinary work contribute to this field? How can we integrate decision-making, risk, and uncertainty in mathematics education?

This study aimed to address the first of these questions as part of an effort to advance understanding and action in mathematics education. It represents the initial phase of a doctoral project focused on STME. The project seeks to develop frameworks that integrate the socio-environmental crisis into mathematics education, while providing students with authentic mathematical experiences rooted in real-world situations.

It is necessary to complement the Newtonian paradigm of causality, typical in the epistemology of school mathematics, with notions linked to complex dynamical systems proper to an emergentist epistemology. The Newtonian paradigm of causality is a deterministic view of the world where every event is seen as a direct result of a previous event, while an emergentist epistemology is a view that

emphasizes the complex and unpredictable nature of systems, which is more suitable for understanding the uncertain nature of the global environmental crisis.

The study of the uses and meanings of mathematical knowledge in Hasselman's work (1979), as represented in Tables 1 and 2, helps to distinguish between ontology and epistemology when addressing climate change-related situations. A common approach in the epistemology of school mathematics is to present problems contextualized within the framework of the environmental crisis. However, this study highlights that a critical component is pattern detection. In the modern era, problems related to pattern detection are particularly relevant to data analysis, as they offer powerful tools for exploring, understanding, and utilizing information from extensive and diverse data sets. Pattern detection is essential for extracting meaningful insights from complex and heterogeneous data, enabling informed decision-making that can shape the future, identify improvement opportunities, optimize processes, and enhance understanding of intricate phenomena.

Likewise, constructing an epistemology of estimation constitutes a frame of reference different from the traditional one of school mathematics. Estimating involves calculating or approximating an unknown or uncertain value based on limited or incomplete information, which can be useful for decision-making. In contrast to school mathematics, where specific information is typically provided to perform ad hoc procedures and single, exact answers are expected, estimation involves calculating or approximating an unknown or uncertain value based on limited or incomplete information.

References

- Barrios Borges & Cordero (in press). Una perspectiva socioepistemológica de la modelación matemática: estudios de casos con enfoque etnográfico en las ingenierías. En: A. Solares y P. Preciado (Eds.), *La investigación en modelización matemática: un diálogo entre educadores de Latinoamérica y España*. SOMIDEM.
- Bhaskar, R. (1975). *A realist theory of science*. Brighton: Harvester.
- Boylan, M. (2017). Towards a mathematics education for ecological selves. *Philosophy of Mathematics Education Journal*, 32. <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/index.html>
- Bryant, L. R. (2011). *The democracy of objects*. Ann Arbor: Open Humanities Press.
- Cantoral, R. (2020). Socioepistemology in Mathematics Education. In: Lerman, S. (eds) *Encyclopedia of Mathematics Education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_100041
- Caron, F. (2019). Approaches to Investigating Complex Dynamical Systems. In: Stillman, G., Brown, J. (eds) *Lines of Inquiry in Mathematical Modelling Research in Education. ICME-13 Monographs*. Springer, Cham. https://doi.org/10.1007/978-3-030-14931-4_5
- Coles, A. (2017). Habits and binds of mathematics education in the Anthropocene. *Philosophy of Mathematics Education Journal*, 32. <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/index.html>
- Cordero, F. (2023). *Matemáticas, sus usos y significados. Un programa socioepistemológico de la Matemática Educativa*. Gedisa, México.
- Cordero, F.; Mendoza Higuera, E. J.; Pérez Oxté, I.; Huincahue, J., Mena Lorca, J. (2022). A Category of Modelling: The Uses of Mathematical Knowledge in Different Scenarios and the Learning of Mathematics. En M. Rosa, F. Cordero, D. Orey, P. Carranza (Eds), *Mathematical Modelling Programs in Latin America. A Collaborative Context for Social Construction of Knowledge for Educational Change* (pp. 247-267). Springer, Cham. https://doi.org/10.1007/978-3-031-04271-3_12
- Davis, B. & Sengupta, P. (2020) Complexity in Mathematics Education. In: Lerman S. (eds) *Encyclopedia of Mathematics Education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_28

- Ernest, P. (2024). *Ethics and Mathematics Education. The Good, the Bad and the Ugly*. Springer, Cham. <https://doi.org/10.1007/978-3-031-58683-5>
- Galbraith, P. (2015). Modelling, Education, and the Epistemic Fallacy. In: Stillman, G., Blum, W., Salett Biembengut, M. (eds) *Mathematical Modelling in Education Research and Practice. International Perspectives on the Teaching and Learning of Mathematical Modelling*. Springer, Cham. https://doi.org/10.1007/978-3-319-18272-8_28
- Hochmuth, Reinhard (2020) Service-Courses in University Mathematics Education. In: Lerman S. (eds) *Encyclopedia of Mathematics Education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_100025
- IPCC (2021). *Climate Change 2021: The Physical Science Basis*. IPCC
- Jurdak, M., Vithal, R., de Freitas, E., Gates, P., & Kolloche, D. (2016). *Social and Political Dimensions of Mathematics Education. ICME-13 Topical Surveys*. Springer, Cham <https://doi.org/10.1007/978-3-319-29655-5>
- Karrow, D. D.; Khan, S. K.; Fleener, J. (2017) Mathematics Education's Ethical Relation With And Response To Climate Change, *Philosophy of Mathematics Education Journal*, 32. <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/index.html>
- Martin, M. J., S. J. Diem, D. M. A. Karwat, E. M. Krieger, C. C. Rittschof, B. Bayon, M. Aghazadeh, et al. 2022. "The Climate is Changing. Engineering Education Needs to Change as Well." *Journal of Engineering Education* *111*(4): 740–746. 10.1002/jee.20485
- Mendoza Higuera, E. J., & Cordero, F. (2018). La modelación en las comunidades de conocimiento matemático. El uso de las matemáticas en ingenieros biónicos. El caso de la estabilidad. *Revista Latinoamericana de Etnomatematica*, *11*(1), 36-61. <https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/501>
- Renert, M. (2011) Mathematics for life: Sustainable mathematics education. *For the Learning of Mathematics*, *31*(1), 20-26. <https://www.jstor.org/stable/41319547>
- Rosa, M., Cordero, F., Orey, D., Carranza, P. (2022). *Mathematical Modelling Programs in Latin America. A Collaborative Context for Social Construction of Knowledge for Educational Change*. Springer, Cham. <https://doi.org/10.1007/978-3-031-04271-3>
- Skovsmose, O. (2021). Mathematics and crises. *Educational Studies in Mathematics*, *108*(1-2), 369-383. <https://doi.org/10.1007/s10649-021-10037-0>
- Skovsmose, O. (2023). *Critical Mathematics Education. Advances in Mathematics Education*. Springer Cham. <https://doi.org/10.1007/978-3-031-26242-5>
- Wiegand, S., Borromeo Ferri, R. (2023) Promoting pre-service teachers' professionalism in steam education and education for sustainable development through mathematical modelling activities. *ZDM Mathematics Education* *55*, 1269–1282. <https://doi.org/10.1007/s11858-023-01500-8>

CRITICAL MATHEMATICAL CONSCIOUSNESS AND LEARNING MATHEMATICS FOR SUSTAINABLE FUTURES

Richard Barwell

University of Ottawa

I introduce and develop the notion of mathematical consciousness as a way to understand how mathematics is used by people to cause harm to the planetary ecosystem. I outline some key relevant ideas from critical mathematics education as well as the underpinning dialogic principles through which mathematical consciousness can be understood to emerge. I illustrate the various concepts with examples of ecosystemic crises, as well as an example of a scientific paper on modelling wolf populations. Finally, I consider what features might we expect in dialogic critical mathematical consciousness.

That the increasing presence of humans is implicated in rapid, significant and, from a human point of view, dangerous changes to every aspect of the Earth's ecosystem is no longer really in any doubt. There is some uncertainty about whether changes can be made to reverse some of these effects or at least reduce their scale and significance, but even here, there is much pessimism. In some sense, therefore, the recent interest and scholarship arising among mathematics education researchers is a bit late in the day. Nevertheless, there are plenty of good reasons why research within mathematics education is still important and necessary. However dire the situation, as educators, we are preparing the citizens of the future to face this changed and changing reality and we know that one way or another, mathematics will be present: necessary to understand what is happening, and also fundamental to enabling the consumer-industrial society that is driving these changes. In recent years, some important questions have emerged for mathematics education in relation to this responsibility; many such questions can be found in the Discussion Document for this conference. The question I have been exploring for the past few months, and which I take up again in this paper, is, roughly: what kind of mathematical citizen do we wish our students to become? [1, 2]

What is the relevance of this question? The ecosystem challenges we are witnessing are often particularly complex and have similar features. Funtowicz and Ravetz (1993) have pointed to uncertainty, risk and the role of values, all of which arise because of the irreducible nature of complex systems. In the case of planetary ecosystem challenges, these complex systems involve both 'natural' and socioeconomic dimensions. To take one example, the gradual degradation of the Amazon rain forest involves various natural dimensions, such as the web of interrelated species and their lifecycles, as well as socioeconomic dimensions, such as poverty or the high demand for beef in richer countries. There are risks involved in the status quo (e.g., continued deforestation, air pollution, habitat loss), or in taking action (e.g., social unrest). The role of values arises, among other things, in attempting to balance such competing or contradictory risks. Funtowicz and Ravetz further argue that such problems cannot be solved by laboratory science, through which policy-makers are provided ready-to-use solutions based on a narrowly defined and controlled scientific process. Precisely because of the high levels of risk and uncertainty, and the potentially significant impact of any chosen path, a broader form of scientific process is needed. In particular, they argue for the development of processes through which at least some citizens can be involved. The idea here is not that citizens become

scientists, but that citizens have a voice in key decisions in the scientific process, particularly where values become relevant. Deciding what level of risk is acceptable in locating a nuclear power station, for example, is not something that can be answered in entirely scientific terms. More generally, in democratic societies, it is important that citizens understand mathematical and scientific information and are able to meaningfully engage in social and political processes relating to ecosystemic challenges. The question, then, is what kind of mathematics education can prepare learners to engage in these kinds of democratic processes.

In what follows, I first review some key ideas from critical mathematics education, pointing out their pertinence for education for sustainable futures. I then outline a dialogic perspective on consciousness and use it to sketch a notion of critical mathematical consciousness.

CRITICAL MATHEMATICS EDUCATION FOR SUSTAINABLE FUTURES

Throughout his work, Skovsmose (e.g., 1994, 2023) discusses different dimensions of knowing mathematics in relation to society. In his earlier work, he proposes a conceptual distinction between mathematical knowing, technical knowing and reflective knowing. Briefly, mathematical knowing is about how to do mathematics, technological knowing is about how mathematics works, and reflective knowing is about the impact of mathematics in society. These kinds of knowing are described in terms of interrelated competences. Mathematical knowing might be about how to construct a set of equations and manipulate them, technological knowing might be about how to use sets of equations to model the population dynamics in an ecosystem, and reflective knowing might be about the limitations of reducing an ecosystem to a set of equations (for an example, see Barwell, 2023). It seems probable that most of the time in most mathematics classrooms is devoted to developing mathematical knowing and technological knowing. The focus of much work in critical mathematics education is on how to promote reflective knowing in mathematics classrooms.

Much of Skovsmose's work is about how mathematics, embedded in technology of various kinds (we can think of digital systems, digital communications and AI, for example), shapes our lived social realities, through, among other things, the process known as formatting. In this process, for example, mathematical models are used to describe or understand a situation or problem, but gradually come to define it, and then serve as the basis to make decisions about preferred future states. I have written about how models of wolf populations are used to determine preferred population sizes and then to determine quotas to exterminate excess wolves (Barwell, 2023), or about how climate models are used to determine how much pollution we can afford (the idea of a carbon budget) before climate change gets too much (Barwell, 2013).

More recently, Skovsmose (2023) has examined how mathematics plays different roles in relation to crises, such as picturing a crisis, constituting a crisis and formatting a crisis. In this sense, mathematics does not simply picture the climate crisis, it also serves to define it as a crisis and indeed to shape the reality of the crisis. The climate crisis is a good example, in fact, since it is impossible for any individual to directly experience the planetary climate. It is possible to observe changes or experience extreme weather events, but it is only with mathematics that we can even have an understanding of the planetary-level, decades-long changes that are taking place.

Skovsmose (1994) has described these various processes in terms of language games. This framing is significant, since it introduces a degree of relativity to understanding how mathematics functions

in society. There are various ways to talk about the climate – discourses, we could call them – each of which functions in a slightly different way. Within mathematics there are also different language games, which we might organize in relation to mathematical, technological and reflective knowing. Moving between these different language games is an important part of doing mathematics. Mathematical modelling, for example, involves moving between language games of ‘everyday life’ (to talk about the problem in its context), the creation and manipulation of mathematical expressions to construct the model, and the application of these mathematical tools to the problem or situation being addressed. There is always some slippage as we move between different language games. In particular, mathematical models are not exact copies of the real world; they involve selections, simplifications, mathematizations, and so on. Using mathematical models might involve simplification, approximation or redefining the problem. Thinking about mathematics in terms of language games allows Skovsmose to highlight an increasingly important concern with uncertainty. In moving between different language games, information is lost, concepts are transformed and, in the gaps, uncertainty arises. Mathematics cannot solve everything.

In his recent writing, Skovsmose (2023) has highlighted one further aspect of mathematics in society that is relevant to this paper. Taking his cue from Hannah Arendt’s book about the trial of Albert Eichmann, Skovsmose discusses the banality of mathematical expertise:

With this expression, I want to highlight an extremely problematic feature of mathematical expertise: it seems to operate in an ethical vacuum by first of all concentrating on technical features of doing mathematics without reflecting on the possible social impacts of doing mathematics. (p. 116)

How well does mathematics education serve society if what learners get from it is a high level of technical knowing but little awareness of the impact of using mathematics in society in the context of whatever task or problem they are working on. This issue was an important theme in the recent film, *Oppenheimer*, about the US atomic bomb project during the Second World War. Did the mathematicians who developed the bomb have any responsibility for the thousands of deaths in Hiroshima and Nagasaki? Does mathematics education prepare mathematical citizens who are able to think about such questions?

MATHEMATICAL CONSCIOUSNESS

Mathematics does not exist outside of human interaction, which includes face-to-face interaction as well as interaction of broader scales of time and space, such as through print or digital media. I will focus on what is still the most common context in which the learning and teaching of mathematics take place: a physical classroom in which learners engage in activities organized by their teacher. For this paper, I am thinking about the ‘standard’ curriculum found in most parts of the world. At this point, I want to introduce a dialogic perspective to think about this process of socialization through interaction in mathematics classrooms over time. This perspective is based on the work of Bakhtin (1981, 1993) and goes far beyond a theory of the features of human talk [3].

The first key principle I will mention is that learning mathematics can be understood as a process of socialization into particular forms of language (we could also call them language games). This process is not simply one of adopting certain words or formulations. It requires learners to take on the forms of language used in mathematics and use them to make meaning of their own. Where do these forms of language come from? They come from others – most obviously from teachers, but also

from authors of textbooks or worksheets, the curriculum and classmates, among others. In taking on these forms of language, learners are engaged in a kind of struggle:

Language is not a neutral medium that passes freely and easily into the private property of the speaker's intentions; it is populated—overpopulated—with the intentions of others. Expropriating it, forcing it to submit to one's own intentions and accents, is a difficult and complicated process. (Bakhtin, 1981, p. 294)

Learners must grapple with the meanings that language carries and of which we are not always aware. Through interaction with others over time, learners come to use mathematical language to make appropriate meanings, through a process of dialogue between their intentions and those that are carried by the language they encounter. Learners encounter various standard forms of mathematical discourse and to be successful must align with these norms. The forces of standardization in relation to mathematics reflect various ideologies about what mathematics is like and what mathematical interaction should be like.

The second principle I want to introduce is that human consciousness arises through interaction with others – consciousness is itself dialogic. Indeed, for Bakhtin, there can be no consciousness without others:

What is the role of the other in the accomplishment of individual consciousness? Bakhtin starts from the simplest ground: we can never see ourselves as a whole; the other is necessary to accomplish, even if temporarily, a perception of the self that the individual can achieve only partially with respect to himself. (Todorov, 1988, p. 95)

In mathematics classrooms, learners encounter others, such as their teachers, who give them some sense of who they are (or might be), particularly, for the purposes of this paper, who they are as mathematical beings. Further, for Bakhtin, language is central to this process:

All that touches me comes to my consciousness – beginning with my name – from the outside world, passing through the mouths of others (from the mother, etc.), with their intonation, their affective tonality, and their values. At first I am conscious of myself only through others: they give me the words, the forms, and the tonality that constitute my first image of myself ... Just as the body is initially formed in the womb of the mother (in her body), so human consciousness awakens surrounded by the consciousness of others. (Bakhtin, 1970-71, cited in Todorov, 1988, p. 96)

We can transpose this last quotation to a mathematics classroom context and propose that a learner comes to mathematical consciousness through the mouths of their teachers, classmates, textbooks, and so on. I understand this consciousness to be more than 'knowledge' or competencies or skills. By mathematical consciousness, I refer to a person's awareness of their mathematical thinking, of how it can be used, of what it can be used to do, as well as associated values about what are useful or desirable or undesirable things to do with mathematics. Mathematical consciousness includes an ethical dimension, although it may be tacit or implicit: a dialogic notion of mathematical consciousness creates a responsibility (Barwell, Boylan & Coles, 2022), particularly for teachers, since they are implicated in the kind of mathematical consciousness their students may develop.

What might mathematical consciousness be like? This is a question you can ask yourself, since you are a mathematically educated person. In many mathematics classrooms, where the focus is on technical aspects of mathematics, learners might develop a mathematical consciousness that leads them to see the world and to act in the world with mathematics in particular ways. Walkerdine (1988) argued that mathematics classrooms produce a kind of rational child who learns to simplify situations, such as by ignoring contextual features, in order to subject them to mathematical analysis. Let us look

at an example that relates to mathematical modelling. The following quotation is from a paper about a new way to model wolf populations within an ecosystem:

Matrix population models are widely used in ecological modelling. In stage-structured models, individuals are assigned to different stages (e.g., morphological for insects or phenological for plants) and all individuals within a same stage are assumed to be identical. Matrix models are commonly used in demographic studies to obtain estimates of the asymptotic population growth rate, population size and stable stage distribution. When used in decision theory, population matrix models can be combined with an optimization procedure to choose between alternative management strategies. (Marescot et al., 2012, references removed)

We can get some sense of the kind of mathematical consciousness involved in doing this work. Populations of creatures or plants are smoothed out so that individuals are treated as identical. There is no room here for personality or relationships. Complex lifeworlds are simplified into stages. The important features of the population are about growth and size. The authors also make explicit the goal of optimization and its link to population management. Underlying these ways of formatting populations of creatures are various mutually supporting values, such as that populations of creatures like wolves *need* managing, or that humans can (have a right to) determine optimal population size and manage populations accordingly. These values give an illusion of control by humans of the non-human world (made possible by an assumed separation and hierarchy between the two). This control is often used to shape ecosystems in line with human needs, desires or fantasies. We can get a sense of the banality of mathematical expertise, here: through thoughtful, sophisticated mathematical work, wolves, or fish, or mosquitos can be killed. It seems that mathematical modelling alienates the mathematical actor from the objects of the model.

CRITICAL MATHEMATICAL CONSCIOUSNESS

There is perhaps a dilemma underlying my argument so far. The teaching and learning of mathematics, as commonly undertaken, favours the development of mathematical consciousness that instrumentalizes ecosystems and sees the world in human-centric terms. From a critical perspective, such a monologic mathematical consciousness is problematic. At the same time, mathematics is extremely effective, has produced many benefits and is invaluable in understanding the ecosystem crises. I am not convinced that (human) mathematics needs to be changed into something else, although it may turn out to be necessary. Rather, we need to find ways to foster a different kind of mathematical consciousness; one that does not objectify other creatures but sees the world in relational and ethical terms. Skovsmose argued that reflection on mathematics and its consequences is as important as learning the technical aspects of mathematics. In the same way, I argue that critical mathematics education needs to consider the possibilities of a critical mathematical consciousness. This consciousness is, I suggest, dialogically oriented and understands that humans are related to and depend on the well-being of all other species on our planet. If we understand that we are related to wolves (and fish, and mosquitos), we can be open to them as ‘others’ who can contribute to our mathematical consciousness. A critical mathematical consciousness may, in a narrow sense, involve the same ways of being mathematical in the world – but these ways of being are embedded within a broader, relational, dialogic awareness. Through this awareness, learners see the practices and outcomes of mathematical actions within multiple relationships and do not privilege humans to the same extent.

I am not entirely sure how a critical mathematical consciousness can be developed, but I suspect that one path is to involve learners of mathematics in complex problems and to support them to work relationally. Some possible strategies may include:

- working locally, in connection with the land, which is likely to foster a greater affinity for the web of life on which we all depend;
- working on mathematics in relation with other ways of knowing, such as poetry, music, or theatre, which is also likely to encourage awareness of the limits of mathematics;
- working on mathematics as part of relationships with community members who have deep connections with the land – such as, in many parts of the world, Indigenous peoples.

These kinds of strategies may serve to introduce a variety of ‘others’ into the learning and teaching of mathematics through which a more diverse, dialogic mathematical consciousness can emerge. Such strategies are also likely to make visible issues of uncertainty, risk and associated ethical challenges, in a way that monologic mathematics classrooms tend to avoid.

If we succeed in offering a critical mathematics education that leads to critical mathematical consciousness, our learners will become citizens ready to engage with the many challenges facing our society and our planetary ecosystem in the coming decades.

Notes

1. This question is prompted by the title of the book *What kind of citizen?* (Westheimer, 2015) by my colleague, Joel Westheimer.
2. This paper continues a thinking process that can also be seen in papers presented at ICME15 and PME47 (Barwell, 2024a, 2024b).
3. Skovsmose (2023) includes dialogue as a feature of critical mathematics education, although he draws on different influences. Wittgenstein’s ideas about language games, however, have many similarities with Bakhtin’s thinking.

References

- Bakhtin, M. M. (1981). *The dialogic imagination: Four essays* (Eds., M. Holquist; Trans, C. Emerson and M. Holquist). University of Texas Press.
- Bakhtin, M. M. (1993). *Toward a philosophy of the act*. University of Texas Press.
- Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16.
- Barwell, R. (2023). Wolf diminished: mathematics, education and biodiversity. *For the Learning of Mathematics*, 43(2), 2–7.
- Barwell, R. (2024a). Notes on language and consciousness in mathematics education with particular reference to the ecosystem crisis. Paper presented to TSG3.13 at the ICME15 conference, Sydney, July.
- Barwell, R. (2024b). Mathematical consciousness and the ecosystem: An example of mathematical modelling. Oral communication presented at PME47, Auckland, July.
- Barwell, R., Boylan, M. & Coles, A. (2022). Mathematics education and the living world: a dialogic response to a global crisis. *Journal of Mathematical Behavior*, 68, 101013.
- Funtowicz, S.O., & J.R. Ravetz. (1993). Science for the post-normal age. *Futures*, 25(7), 739–755.

- Marescot, L., Gimenez, O., Duchamp, C., Marboutin, E. & Chapron, G. (2012). Reducing matrix population models with application to social animal species. *Ecological Modelling* 232, 91–96.
- Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Kluwer.
- Skovsmose, O. (2023). *Critical mathematics education*. Springer.
- Todorov, T. (1988). *Mikhail Bakhtin: The dialogical principle* (trans.: W. Godzich). University of Minneapolis.
- Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. Routledge.
- Westheimer, J. (2015). *What kind of citizen? Educating our children for the common good*. Teachers College Press.

THE (FUTURE) CO-EVOLUTION OF SCHOOLING AND MATHEMATICS EDUCATION

Jay Bradley

George Mason University

The current state of mathematics education appears to be inadequate for the challenges faced by those located in our planet's most vulnerable communities. Schools demand that teachers of mathematics work towards aims and goals designed to improve the individual computational skills and knowledge of students at the expense of a mathematics education geared toward collective and ethical solutions for global problems. This paper proposes an evolutionary framework for understanding mathematics education that helps better theorize its current state of inaccessibility for the least privileged populations. Evolution helps us to better understand symbiotic relationships between man-made systems and provides a possible pathway of how human cooperation (via education) can lead to more hopeful futures for fragile and marginalized societies. Current and past attempts at equity in mathematics education are contemplated considering our current understandings of symbiosis. The paper concludes with possibilities for new aims for mathematics education that harness the uniquely human capacities of ambition and imagination.

INTRODUCTION

Even before the introduction of *homo sapiens* to the planet, mathematics was used as a tool to shape and remake the world (Rappenglück, 2012). It simultaneously emerged, as technology philosopher Tom Chatfield writes in his book *Wise Animals*, with ways “to think not only about how things are, but about how they *might* or *could* be: about what it might mean to remake both the self and the world” (Chatfield, 2024). Our prehistoric ancestors produced perfectly round objects, crafted exact isosceles triangles, and even reshaped mammoth tusks into cylinders to manipulate the time and space around their surroundings. It is tempting, in the light of such early attempts at technological usage and design, to believe that our experiences with technology in today's world are the inevitable outcome of a journey towards technological mastery for our species. When we observe (or experience) the comfortable lives of the most privileged people on our planet, it is tempting to believe that the pathway to relieving human suffering involves simply getting everyone else to “catch up”. Likewise, in mathematics education, considerable efforts and research have been invested in “mathematics for all” (Martin, 2003) strategies that assume that global problems will be solved and equity in school mathematics will be achieved, when the standardized test performance of those in the less developed global south match those of their counterparts in more industrialized nations.

Meanwhile, the warnings from the latest findings of the Intergovernmental Panel on Climate Change (IPCC, 2022) indicate that our current use of technology may very well lead to our demise. Already, vulnerable areas of our planet are suffering from food shortages and extreme heat because of human-induced climate change. This theoretical paper, using the framework which Chatfield (2024) describes as *the delusion of comprehension*, will contemplate the possibility that our existence is more fragile and uncertain than we might like to believe.

SUSTAINABLE DEVELOPMENT

The United Nations Educational, Scientific, and Cultural Organization (UNESCO) maintains what they describe as a Learning Data Resource Center which, among other aspirations, purports to help nations monitor whether they are meeting the educational goals of the 2030 Agenda for Sustainable Development. Sustainable Development Goal (SDG) Four challenges nations to “ensure inclusive and equitable quality education and promote lifelong learning opportunities for all”. In a policy brief to improve global learning outcomes (Sandoval-Hernandez, 2022), national leaders are encouraged to compare the performance of their own students to students who are performing higher on “global tests” to make sure that their students are making progress in mathematics. Left unsaid by UNESCO, unfortunately, is whether students who are performing higher on “global tests” are indeed on a pathway worth pursuing. It is taken for granted that if students in Uganda, for example, raise their test scores and perform like their counterparts in the United States, that students in both countries (and the world) will be better off. Of course, it is undeniable that the technological advances that countries like the United States enjoy have made the lives of its citizens easier and no one would blame Uganda (or any other nation) for having similar aspirations. Nevertheless, we must ask, are countries where more citizens have access to advanced technologies on sustainable pathways?

In this paper, I will suggest that the evaluation of mathematics education goals in this manner represents the results of incomplete stories that we tell ourselves. The technological achievements of countries like the US are certainly worthy of envy. But what about the knowledge that the US is also, per capita, one of the world’s biggest contributors to man-made climate change? (United Nations Environmental Programme, 2023). Is that not also worthy of consideration as our goals are being constructed? If our current technological achievements are placing the planet on the pathway to making large swaths of our planet undesirable for human habitation, what will placing billions more on that pathway do? Certainly, the technological development of the United States is more complex than that of say, Uganda, but we must not *delude* ourselves into thinking that these nations are both travelling upon some “continuum” of development. Evolution, as philosopher and biologist Godfrey-Smith (2016) writes, is “headed nowhere”. We should not assume that students of mathematics and the nations they belong to are on or are currently following along some predetermined path that we fully comprehend. In fact, we should, as perhaps our ancestors may have even contemplated, attempt to use mathematics to tell fuller stories about our impact upon our surroundings, to direct our imaginations to develop better solutions that minimize negative impacts upon the world and perhaps, create less harmful and more equitable pathways for human development.

THE SCHOOL CREATES THE MATHEMATICS

In *Wise Animals*, Chatfield (2024) analyzes the dynamic relationships between actors located in ecosystems that create new technological advances. These insights are important, I believe, because they give us insight into the current state of mathematics education. While some have pointed out the potential of mathematics to obscure crises and problems (Skovmose, 2019), there is an overwhelming tendency in education to believe that the school mathematics performed in economically successful societies represent the “gold standard” of educational achievement and perhaps, the best “understanding” of how mathematics education should be done. Yet, we know that even before societies were able to comprehend the basics of physics, principles of buoyancy, or wind currents, they were able to create boat designs that were able to sail across vast distances. The existence of a

lake, and the need for efficient ways to reach resources on the other side of the lake may have led to a boat. But over time, the selection pressures of evolution caused boat designs that were ineffective to be lost to the floor of the lake. Some designs thrived because they made their creators happy. Some innovations were lost to unfortunate storms or other acts of natural calamity. Evolutionary progress does not require sudden insights – given time, changes will happen, structures may become more complex, and as we will see, unexpected and unwanted consequences can and will occur.

There is no reason today to believe that the mathematics education that is found in schools around the world exists due to some great understanding of the best way to teach mathematics. Yet, as we noted earlier, the United Nations is asking individual member nations to modify their nation’s practices in ways that ensure their own children will think about mathematics, at least measured by standardized tests, in the same ways as other children. The assumption made is that certain formal modes of mathematics instruction will lead to economic dominance. But what if the need for economic dominance demands modes of instruction that creates a callous disregard for their effects? After all, one common theme in American mathematics education research has been the need for maintaining dominance in world economic affairs (Gutstein, 2009). Even if teachers and students have not *consciously* been working toward such goals, we know, from an evolutionary standpoint, that biological systems can develop complex mechanisms without knowing anything about their objective use. Does the human heart know or understand its role in the complex human circulatory system? Did it “construct” itself in such a way that allowed it to provide an ample blood supply for itself and the other organs of the body? Did it “develop” its tissue in such a way that it is strong enough to beat consistently for a human lifetime without stopping?

Scientific research has shown biological organisms do not exist by themselves but rather survive and thrive by forming deep relationships with other organisms (Dimijian, 2000). Symbiotic relationships between two or more organisms are commonplace in our natural world and are often the basis of each individual organism co-evolving or acquiring adaptations that make their individual survival more likely (Behar & Lauzon, 2023). When the United Nations positions nations as “measuring sticks” for other nations to measure themselves by, it adopts a pre-evolutionary and simplistic *cause/effect* view of the world that is inadequate for current challenges. I suggest that the continued success of inequitable “school mathematics” - despite frequent attempts to eradicate it (Chen & Horn, 2022) - is due to the “successful” symbiosis and co-evolution of “school” and “mathematics” in advanced nations. Of course, this “success” remains problematic for those who aspire for more equitable mathematics education outcomes because successful (in terms of survival) adaptations, as we shall see, are difficult to dislodge.

SCHOOL MATHEMATICS AND THE PURSUIT OF EQUITY

Watson (2008) contrasts the discipline of mathematics to “school mathematics” and finds that school mathematics is a discipline that prioritizes

answers and generalizations rather than structural insight and observation...avoidance [of] uncertain choices... concrete ways, rather than abstract or imaginary ways... inductive over probabilistic or deductive reasoning

It is characterized by injustice and learning opportunities are stratified along lines of class, gender, and race. The primary purpose for a student at any level of school mathematics, Watson argued, was

not to acquire skills and tools to be a professional “mathematician” (or do other professional work) but rather to acquire the ability to do the “next” level or grade of school mathematics.

Generations of mathematics students have asked their teachers “Why do we have to learn this?”. Perhaps if they had better science teachers, they would already know that school mathematics is part of a coevolutionary process that has no fixed destination and no predetermined *point*. “School” mathematics (and the people who participate in it) is trying to *survive* and it, through symbiosis, will shape itself, and change, if necessary, for the benefit of the environment it finds itself coexisting with.

Martin (2019) has described equity related reforms in mathematics education as “fiction” that do little more than maintain marginalized learners in the same position as they were before the reform. His examination of policies in an American context shows that efforts to close the “achievement gap” between privileged and marginalized students through the usage of standardized testing or “tinkering with pedagogy, curriculum, assessment, and standards...” have done little to benefit marginalized students. Kollosche (2018) documented the resistance of mathematics education to reform and suggests that the functions of other elements in the school mathematics ecosystem must be altered simultaneously with instruction for meaningful change to happen. In nature, ecosystems that stay the same from generation to generation with little change are described as being in “equilibrium”. Equilibrium is the tendency of an ecosystem to return to its original state after a disruption. Mathematics has adapted well to the institution of “school” but its inelasticity and inability to continue evolving in school environments is not inevitable nor desirable. Genetic equilibrium is rare in nature but man, who alone has the imagination to reshape nature through “human activities”, can certainly bring it about. If school mathematics is in a state of genetic equilibrium, our hopes for an equitable future may lie in figuring out how to successfully disrupt it.

FORWARD PROGRESS INDEED

The immaturity, or childhood, of a human being is longer than that of almost any other mammal (Hassett, 2017). This extended childhood grants advantages that are simply not experienced in any other species. Children use this extended time to acquire the powerful skills that have allowed our species, for better or worse, to develop an unprecedented ability to transform life on Planet Earth. In fact, menopause is an evolutionary adaptation to ensure that children not only have parents but also have something that no other species has managed – a grandparent able to provide needed guidance and training. The ability to build relationships or to learn how to craft powerful tools are skills that are acquired through nurturing (as opposed to instinct) and for most of the planet, schools are considered institutions where children are expected to acquire the skills that will serve them through life.

Schools are also places where children are taught to make sense of the tool that we call mathematics. It is a place where two powerful factors converge together to help bring us to the socio-ecological crisis that we face today. Human cooperation allows us to outstrip many of the natural constraints that repel other animals from extreme environments and gives us the ability to settle and extract resources from any region of the world. In addition, our species is not satisfied with only what we need to survive. Our ambitions and desire to excel above our fellow humans have also led us to acquire, develop, and manipulate our environment in ways that go beyond meeting basic needs of survival.

INVASIVE SPECIES AND THE ENVIRONMENTS THAT WELCOME THEM

Within the last 2,000 years, humans have designed urban spaces where the common German cockroach thrives (*B. germanica*) and within the last 250 years, humans have provided this pest with the impetus to spread globally (Tang, et. al., 2024) through our transportation networks. Although there are a variety of other species of cockroaches, this cockroach is the most widespread. It has never been found in natural habitats and cannot survive outside human structures. Additionally, it extracts significant social, medical, and economic costs from human environments. *B. germanica* has developed a tremendous resistance to attempts at eradication and easily outcompetes other roach species for survival in our human environments. It really thrives in the buildings of the industrialized global north and can only be found in less industrialized spots of the world near areas of prepared food (Lee & Wang, 2021). Biologists predict that the impact of globalization in the global south will only increase the prevalence of this pest, increase the cost of dealing with its negative effects, and create more need for pest eradication services.

Wang, Lee, and Rust (2021) also predict that as the global south begins to adopt the housing designs of the industrialized north, it should also prepare to see that the problems presented by the German cockroach will only increase. In a paper presented to the NESCent Working Group of the Evolutionary Biology of the Built Environment, the authors noted that traditionally, global housing construction was based on the “climate and needs of their respective locations”. While no one doubts the benefits of introducing indoor heating and cooling to other regions of the world, the *standardization* of the construction process helps to introduce *B. germanica* to new environments where it will continue to be harmful to human health.

LACKING GENETIC VARIATION

Without genetic variation, key mechanisms of evolutionary change cannot operate. As mentioned above, *B. germanica* has competitors but human factors have helped to provide it with its dominance in urban areas around the world. The spread of the German cockroach has been described as a “biological invasion”. Biological invasions are characterized by the loss of genetic variation which include bottleneck effects and founder effects. As noted before, this loss of genetic variation leads to genetic equilibrium.

These realities should bring pause (and humility) to anyone looking to bring standardized mathematical solutions to solve the problems of the planet. If anything has been pervasive or cockroach-like in the mathematics education community, it has been traditional school mathematics. I would argue that our schools are being structured in ways that are making other models of mathematical expression including critical mathematics (Skovmose, 2011), mathematics for social justice (Gutstein, 2006), and the mathematical practices of cultural groups (ethnomathematics) around the world impossible to thrive in. If biologists predict that increased globalization and urbanization will only strengthen the cockroach, why should we not believe that standardized global attempts to solve the mathematics education problem will only strengthen the negative effects of traditional mathematics education (to the detriment of those least likely to deal with its “illnesses”, “contaminations”, and “eradication” costs).

B. germanica thrives on human vulnerability. Without the human need for shelter or the need to prepare food, it is clear (Lee & Wang, 2021) that it could not exist on its own. Its existence creates

the need for its eradication by humans, which creates the need for more pesticide companies, which leads to more economic growth, which leads to new markets, which leads to development of more strategies to eradicate its growth. This reminds one of high stakes testing mandates in mathematics education, which leads to testing companies, which leads to “strategies” to help students compete more successfully, which leads to the need for more testing to measure additional growth and so on.

Cockroaches, parasites, and invasive plant species thrive in areas of vulnerability and inequality. If we desire to influence these systems, as Chatfield (2024) writes, we need to better understand them in terms of mutual dependency. We must understand that these crises arise out of vulnerability and imbalances. The attempts of many school systems to reform mathematics education might be seen as a form of insecticide, with indiscriminate spraying that temporarily “works” but unwittingly gives rise to a climate that ensures more pestilence and more “need for reform” - which will only lead to more damage in its wake.

It has been said that cockroaches could survive the fallout from a planetary nuclear war. If that is the case, then, eradication for them (or school mathematics) might be wishful thinking. We probably have more to learn about how a near-indestructible and unpleasant creature evolved, but it should bring about a realization that “progress” is often only part of the story, and the other part of the human story should be a general sense of learning to work together to mitigate risk and protect those who are at the most vulnerable.

Scientists have learned how and are continuing to learn how to restore genetic variation after genetic equilibrium has been achieved. It is in this spirit that I write this final section and speculate how learning how to (metaphorically) perform genetic rescue, reintroduce lost species, and remove harmful stressors and pests may help mathematics education regain its capability to evolve.

WRITING THE NEXT CHAPTER

It is likely that the aims of the mathematics in our schools have evolved along with the aims of our schools to exist in their current form today. These aims are powerful and, perhaps paradoxically, need/fuel the continuous technological growth without regard for consequence which has helped lead us to our crisis today. They thrive on our vulnerabilities and will likely never be eradicated because human vulnerabilities will always exist. But we can also give other, perhaps even more beneficial “species”, or newly evolved aims of mathematics education, a fighting chance.

In the United States, for example, the sequence of high school mathematical instruction has not changed since the 1950’s. In addition, students study isolated topics (algebra, geometry, etc.) often separated from the surroundings of the world around them. Genetic rescue is the introduction of species variation into static environments to help jumpstart the evolutionary process. For too long, mathematics has enjoyed a symbiosis with types of schooling that discouraged contact with problems of our society that interfered with or questioned our desire to dominate global commerce. Our current inability to act in the face of danger and persistence in acting as if things will never change strongly mirrors the disconnected and antiquated mathematics lessons that many students experience today.

The global desire for the (illusion of) progress has led many countries to abandon Indigenous ways of understanding mathematics, standardize their curriculum, and abandon the usage of native tongues in understanding mathematics (Essien, 2018). I recognize that this is an ongoing problem and many parents, wanting to make sure that their children are not “left behind”, demand that their children be

instructed in dominant Western languages and use Western techniques. Issues related to insecurity will always affect the mathematics classroom, but mathematics education research should prioritize the incorporation of culture into the classroom and not rely on standardized test comparison to close the gap between global haves and have nots.

Just as nuisance pests and invasive plants can thrive in human environments that make no attempt to respect “the climate and needs of their respective locations”, it is likely that undesirable approaches to mathematics will be found in school environments where classroom environments do not respect the surroundings and needs of their students. Another aim of mathematics education should bring about the end of treating mathematics as a neutral activity capable of objective decisions (Skovsmose, 2011).

Finally, we need to recognize our own limitations as humans in creating progress. Our propensity for positive stories has made it seem that the human story has been one of perpetual progress with negative consequences being merely an unfortunate inconvenience along the way. This denies, however, the many aspects of our “story” that remain obscure, hidden or even suppressed. Yes, evolution is not “leading” anywhere but our honesty about the world around us can help us construct more hopeful and more inclusive aims of mathematics education that can coevolve in a technological path better suited to our long-term interests.

References

- Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013.
- Behar, A., & Lauzon, C. R. (2023). Coevolution of insect-gut microbiome. *Frontiers in Ecology and Evolution*, 11, 1242673.
- Chatfield, T. (2024). *Wise animals: How technology has made us what we are*. Picador
- Chen, G. A., & Horn, I. S. (2022). A call for critical bifocality: Research on marginalization in mathematics education. *Review of Educational Research*, 92(5), 786-828.
- Dimijian, G. G. (2000, October). Evolving together: the biology of symbiosis, part 2. In Baylor University Medical Center Proceedings (Vol. 13, No. 4, pp. 381-390). Taylor & Francis.
- Essien, A. A. (2018). The role of language in the teaching and learning of early grade mathematics: An 11-year account of research in Kenya, Malawi and South Africa. *African Journal of Research in Mathematics, Science and Technology Education*, 22(1), 48-59.
- Godfrey-Smith, P. (2016). *Other minds: The octopus, the sea, and the deep origins of consciousness*. Farrar, Straus and Giroux.
- Gutstein, E. (2006). “The Real World As We Have Seen It”: Latino/a Parents' Voices On Teaching Mathematics For Social Justice. *Mathematical Thinking and Learning*, 8(3), 331-358.
- Gutstein, E. (2009). The Politics of Mathematics Education in the United States: Dominant and Counter Agendas. In B. Greer, S. Mukhopadhyaya, A. B. Powell, & S. Nelson-Barber, (Eds), *Culturally Responsive Mathematics Education* (pp. 151-178). Routledge.
- Hassett, B. (2018). *Built on bones: 15,000 years of Urban life and death*. Bloomsbury Publishing.
- IPCC, (2022). *Climate Change 2022: Mitigation of Climate Change. Contribution of Working Group III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change* [P.R. Shukla, J. Skea, R. Slade, A. Al Khourdajie, R. van Diemen, D. McCollum, M. Pathak, S. Some, P. Vyas, R. Fradera, M. Belkacemi, A. Hasija, G. Lisboa, S. Luz, J. Malley, (eds.)]. Cambridge University Press, Cambridge, UK and New York, NY, USA.

- Martin, D. B. (2003). Hidden assumptions and unaddressed questions in mathematics for all rhetoric. *The Mathematics Educator*, 13(2), 7-21.
- Rappenglück, M. A. (2012). Stone age people controlling time and space: Evidence for measuring instruments and methods in earlier prehistory and the roots of mathematics, astronomy, and metrology. In, D. Valls-Gabaud & X. Moussas (Eds). *Proceedings of 5th International Conference of the ESHS, Symposium 28, Scientific Cosmopolitanism as Tracy by Astronomical Instruments*, 473-481. Athens.
- Sandoval-Hernández, A. (2022). Rosetta Stone Policy Brief. Establishing a concordance between regional (ERCE/PASEC) and international (TIMSS/PIRLS) assessments Rosetta Stone Policy Brief Establishing a concordance between regional (ERCE & PASEC) and international (TIMSS/PIRLS) assessments. UNESCO Institute for Statistics. https://tcg.uis.unesco.org/wp-content/uploads/sites/4/2022/06/Rosetta-Stone_Policy-Brief_2022.pdf
- Skovsmose, O. (2019). Crisis, critique, and mathematics. *Philosophy of Mathematics Education Journal*, 35, 1-16.
- Tang, Q., Bourguignon, T., Willenmse, L., De Coninck, E., & Evans, T. (2019). Global spread of the German cockroach, *Blattella germanica*. *Biological Invasions*, 21, 693-707.
- United Nations Environment Programme (2023). Emissions Gap Report 2023: Broken Record – Temperatures hit new highs, yet world fails to cut emissions (again). Nairobi. <https://doi.org/10.59117/20.500.11822/43922>
- Wang, C., Lee, C. Y., & Rust, M. K. (Eds.). (2021). *Biology and management of the German cockroach*. Csiro Publishing.

RACE AND THE SOCIO-ECOLOGICAL TURN

Erika C. Bullock

University of Wisconsin-Madison

The social turn and socio-political turn in mathematics education have interests in equity. The proposed socio-ecological turn in mathematics education holds great promise to bring an ecological interest in mathematics education. This turn challenges the propensity within mathematics education to treat the environment as context rather than an actor in processes of teaching and learning mathematics. In a moment where there is global concern about climate change and sustainability, the socio-ecological turn seems like an important logical step in mathematics education's thinking. However, such a shift is not innocent. This paper problematizes the socio-ecological turn with respect to race. Beginning with the idea that mathematics is part of an alchemy of school subjects dedicated to building the ideal modern citizen through schooling, the paper considers racialized deviance as an axiom in critical mathematics education. Then, it problematizes the socio-ecological turn with respect to race to show that making this turn will not eliminate the racial and colonial logics inherent to much of the logic of other critical mathematics education efforts.

INTRODUCTION

Equity has been a significant thrust in global mathematics education since the 1990s. Over time, different proposals have gone forward in the name of *equitable mathematics education for all*, although it has not been clear with the nature of an equitable mathematics education is or who is included in the “all” (Bullock, 2019). Most of the suggestions have been for interventions related to pedagogy, curriculum, and teacher disposition. The goal, as it appears, is to construct a vision of mathematics education committed to “mathematics for all” (Yolcu, 2019). Equity has been a key idea within the social turn and socio-political turn moments in mathematics education (Stinson & Bullock, 2012) and aligns discursively with the proposed socio-ecological turn.

Equity has been a significant thrust in all aspects of the mathematics education enterprise. Previously, I have argued that equity research in mathematics education is a genre that operates according to implicit ideological and rhetorical rules and assumptions that form how one can think about equity and inequity (Bullock, 2024). One such rule is the axiom of racialized deviance, a logical tool developed by whiteness to establish its dominance and to justify physical, psychic, and epistemic violence against blackness. Du Bois (2014) argues that this deviance is an ontological position of blackness as “problem” (p. 67). Popular solutions to ‘solve’ this problem include altering students’ mathematical mindsets (Boaler, 2016) and correcting teachers’ beliefs (Yow, 2012) and their capacities to ‘notice’ (van Es et al., 2017). Equity, I argue, operates from a sense of benevolence (Jefferess, 2011) that requires its proponents (i.e., future and current teachers, in this case) to assume an egalitarian moral position while taking up a logic that maintains status quo with respect to power and subjection. While this prior argument focused on mathematics education research, the logic that I examine extends through pedagogy, teacher education, curriculum, policy, and beyond.

In this paper, I consider the idea of racialized deviance within the socio-ecological turn that Coles (2022) proposes. The prior argument was made largely in a socio-political frame where I committed

the error that Coles (2023) warns against: treating nature as invisible background to socio-political systems. Engaging the socio-political turn requires that I consider ecological systems as actors within the world of mathematics education. Thus, I ask: *How does a socio-ecological perspective shift our understanding of racialized deviance in mathematics education?*

MATHEMATICS IN THE ALCHEMY OF SCHOOL SUBJECTS

School mathematics is a permutation of mathematics that operates within the alchemy of school subjects (Popkewitz, 2004). The alchemy refers to the idea that school subjects (i.e., school mathematics) conspire with schooling as an institution to build the child into the ideal modern citizen. Education discourse assumes that the disciplines are naturalized in the school subjects and that these subjects are entry points into the disciplines. However, the alchemy positions the school subjects outside the disciplines. Therefore, school mathematics is not a distilled form of mathematics; it is a portrayal of mathematics that has cultural and political functions that extend beyond mathematics itself (Popkewitz, 2004). Within the school, mathematics' role is to teach children to be rational, objective, and logical problem solvers.

There are several inscription devices that must be in operation for the alchemy to function as designed. Latour and Woolgar (1986) define an inscription device as “any item or apparatus or particular configuration of such items which can transform a material substance into a figure or diagram which is directly usable” (p. 51). Within education, inscription devices are discourses and technologies that assemble to transform the child's thought into something that is visible, governable, and measurable (Diaz, 2018; Popkewitz, 2004). These can include models based in psychology, sociology, pedagogy, and other discursive formations.

The purpose of the inscription of the child within the alchemy is to transform them into the ideal citizen. The report, *A Nation at Risk* (NCCE, 1983) articulated the desired image of the ideal citizen that became the goal for educating the child. The desired product was a mature, secure, informed, managed, and self- and society-serving *individual* (serving self first and society second). School mathematics' primary role in this production is to promote cultural practices such as perseverance, problem solving, critical thinking, abstraction, efficiency, objectivity, and precision. Mathematics education literature describes reasons for learning mathematics that connect to these desired practices (e.g., Niss & Højgaard, 2019). Also, the Standards for Mathematical Practice in the Common Core State Standards for Mathematics represent the most recent move within the U.S. mathematics curriculum to make a student's demonstration of desired cultural practice part of the learning goals.

DEVIANCE IN MATHEMATICS EDUCATION

Inclusion and exclusion are governing principles of the alchemy. Inclusion happens via inscription devices. Inscription devices are intellectual tools developed through education research to “render the thought of a child visible and amenable to governing” (Popkewitz, 2004, p. 4). These tools include, but are not limited to, pedagogical approaches, assessment practices, curriculum standards, and content knowledge. They also determine the requisite habits of mind and ways of being for success in the school subjects. Those who meet these expectations are included; those who do not are excluded. Therefore, the inscription devices determine who is capable of learning and who is not, who is successful and who is not, and who is deviant and who is not. The excluded child is said to occupy that state due to their own deviance.

Narratives of the deviant child are familiar:

The child who does not ‘fit’ [...] is the child who lacks self-esteem, has a poor self-concept, and has fewer of the ‘proper’ literacy and numeracy skills. The child who is left behind is one that does not embrace the [...] mode of life that includes lifelong learning, ‘problem solving’, collaboration, and continuous innovation and choice. (Popkewitz, 2007, p. 77)

These indicators of deviance are internal to the child, implying that deviance is produced in the child’s internal world. However, Popkewitz (2007) continues to show that deviance is also formed in the child’s external world:

The fears of moral disintegration and social instability are re-inscribed as psychological qualities of learning, problem solving and self-esteem overlapped with social narratives about the moral disintegration of the community, family and environment, such as single parents and teenage mothers, and ‘recidivists’. (p. 78)

Narratives of the child’s social world are contributory to their deviance. Many of these narratives connect to the “culture of poverty” (Lewis, 1966) discourse that has been used to describe the way of living of many subjugated people groups around the world (McDermott & Vossoughi, 2020). Characteristics of the culture of poverty include family instability, refusal to value or protect childhood, and low self-esteem (Lewis, 1966; McDermott & Vossoughi, 2020). Lewis (1966) establishes those who live under the culture of poverty as deviant by comparing them to “most primitive peoples [who] have achieved a higher degree of sociocultural organization” (p. 23). But the deviant child is not intended to be left in this state. They are to be rescued and restored to the alchemy through better inscription, or better pedagogy.

RACIALIZED DEVIANCE

While my earlier argument that racialized deviance is axiomatic was related specifically to research (Bullock, 2024), my overarching argument is that equity discourse in mathematics education—as the only mechanism that the field has taken up to address race from a social or socio-political perspective—exists based on an axiomatic assumption of racialized deviance. In his 1897 essay “Strivings of the Negro People”, W. E. B. Du Bois begins with a question that the dominant world hesitates to ask outright: “How does it feel to be a problem?” (p. 67). For Du Bois, there is no escape for a Black person from this position as problem. According to Roberts (1994), white supremacy as an ideological position dictates that those who occupy the position of blackness in each racial hierarchy are always “*at least* potential offenders” because “blackness places [one] beyond the bounds of respectability” (pp. 180-181, emphasis added). I see the unspoken question that Du Bois raises combined with Roberts’ assertion that blackness is constructed as always already (potential) offense as affirmations of the deviance of blackness. If those who occupy the position of blackness are ontologically constructed as “a problem”, then blackness deviates from an established norm based in whiteness.

Racism casts upon blackness an assumption of criminality and moral and intellectual deficiency (Roberts, 1994). These assumptions can be traced to biological determinism’s argument that “people at the bottom are constructed of intrinsically inferior material (poor brains, bad genes, or whatever)” and that worth should be assigned accordingly (Gould, 1996, p. 63). Quantification has been a necessary tool of biological determinism because it makes it easier to assign proper status to people and groups, to rank them, and to valorize them (Dixon-Román, 2016). It is through quantification that

“the norm” is established and, as a result, the threshold for deviance is set (DuBois, 2014). Because norms are set relative to those with power, those with little or no power in society “engage in counter normative behaviors” that makes them deviants (DuBois, 2014).

Connecting this idea of racialized deviance to the notions of deviance raised earlier, related to alchemy, offers a different vantage point on the idea of equity. If the path from the uncivilized, unruly child to the ideal disciplined citizen were a walking path, any student who deviates from that path requires a corrective measure to steer them back to it. Equity is one of these corrective measures. Others could include remediation, formative assessment structures, etc. But equity is different. Equity discourse seeks to resolve the malfunctions and missteps that cause a child to deviate based on characteristics unrelated to mathematics or schooling—characteristics such as race, class, and gender. While I focus on race here, there is an argument to be made related to this issue based on other factors.

The equity infrastructure claims to do its work in the name of access and opportunity for *all children*. While this is a noble claim, one must only glance at the corpus of literature to see that equity does not enter the conversation outside of some form of marginalization. Additionally, any research that addresses marginalization in any form is in the equity category, regardless of the researcher’s intention or the nature of the research. The same is true in mathematics teacher education. Equity is presented as an interest for all teachers related to all students, but the energy behind the rhetoric is always connected to marginalization.

QUESTIONS RAISED BY A SOCIO-ECOLOGICAL PERSPECTIVE

A socio-ecological perspective brings up several questions related to the ideas that I have put forth here. First, a socio-ecological perspective directly challenges the individualist ideal that the alchemy describes. Within the alchemy, the individual is the center of learning and the goal of the alchemic process is its perfection as key to individual and collective survival. Coles (2023) posits that centering the individual is an epistemological error because relationality is the basis of how beings survive and learn. Making the shift from an individualist perspective, Coles argues, brings forward community and political, ethical, and ecological contexts in new ways that oblige every member of the living world to answer for the health of the planet.

One question that scholars of race raise related to posthumanist perspectives is related to the primacy of concern for the planet. While we are all inhabitants of this planet and, therefore, stewards of it, it is difficult to reconcile a concern for the planet and all living things—human and non-human—in a world that has not yet accepted answerability to other humans and is, in fact, still debating the question of who occupies the human category. In their critique of plantationocene scholarship, Davis and colleagues (2019) note how scholars like Donna Haraway and others critique anthropocene scholars for not acknowledging the plantation as a particular socio-ecological formation, while they themselves handle the plantation as a space of flattened multispecies existence, neglecting the racial ideologies that govern the determination of human and non-human categorization. Jackson (2020) describes this flattening in mathematical terms:

Slavery’s archival footprint is a ledger system that placed black humans, horses, cattle, and household items all on the same bill of purchase. This ledger’s biopolitical arithmetic—its calculation of humanity—dislocated, depersonalized, and collapsed difference, except in the area of market value. (p. 45)

As Jackson highlights, collapsing difference is actually an erasure of blackness by ~~skirting the ongoing question of humanity~~ in favor of multispecies consideration. I strike through the text in the previous sentence to trouble the thought that this flattening is a simple evasion of the question of humanity rather than a tacit agreement with “a *genre* of the human that reifies Western bourgeois tenets” (Wynter & McKittrick, 2015, p. 9, emphasis original).

The Western genre of Man that Wynter (1979) describes as the standard image of the human is always already white and maintains human status “on the condition that he distanced himself as far as possible from the symbolic negation of manhood and whiteness—the black” (p. 150), which is the deviant.

So, what does de-centering the human *do*? Coles (2022) sees potential in post-humanism’s potential to “help us view our role as humans in a more appropriate place (compared to assuming we are always front and center)” (p. 211), which seems innocuous. However, Coles goes on to assign similar potential to “moves to accord legal, personhood status to animals and to rivers” (p. 211). Where do such moves leave those who occupy the place of blackness whom liberal humanism has “abjectly animalized” (Jackson, 2020, p. 12)? Attempts to nullify the human erase those in blackness, accomplishing the true work of racism—“to render one’s humanity provisional, where the specter of nullification looms large” (p. 16).

RACISM IN THE SOCIO-ECOLOGICAL TURN

If the purpose of school mathematics is to develop aspects of the ideal citizen in conjunction with the other school subjects, then it would seem that the socio-ecological turn is antithetical to school mathematics based on its refutation of individualism. If we follow Popkewitz’s (2004) theory of the alchemy of school subjects, undermining school mathematics at this level would require interrupting capitalism and the entire modern project. Anthias and Asher (2024) argue that the necessary position would have to embrace “non-humanist, non-extractivist, non-modern ontologies, which might [...] offer a blueprint for ‘moderns’ to renounce their own imbrication in an ecocidal system of colonial capitalism” (p. 2). This may also be the only way to resolve the questions of race and racism within the socio-ecological turn raised in the previous section.

Capitalism organizes relations in a way that maintains a need for severe inequality and a provisional sense of humanity (Jackson, 2020; Melamed, 2015). The accumulation of capital requires violences such as colonialism, genocide, and enslavement that also maintain racial hierarchy. Therefore, scholars like Robinson (1983) and Melamed (2015) argue that there is no capitalism apart from racism. Hence, “capitalism *is* racial capitalism” (Melamed, p. 77). Therefore, in order to make the socio-ecological turn, mathematics educators must contend directly with race and racism, something that the field have avoided heretofore despite being replete with racial tension (Bullock, 2019, 2024; Martin, Valoyes-Chávez, & Valero, 2024). Coles and colleagues (2022) describe mathematics education as a humanist project “raised from within the dominant Euro-modern knowledge and education structures” (p. 173). These structures hold racism and white supremacy as core values by “reproduce[ing] and reinforce[ing] racist and colonial assumptions” (Davidson & da Silva, 2022, p. 522).

As an example of these core values, Coles and colleagues (2022) write: “We do not see the socio-ecological as replacing other perspectives, but rather as complementing and building on them” (p.

171). While this is a generous gesture, I question the value of such generosity in the face of global humanitarian and climate crises. In this moment in history, it is incumbent upon those who espouse critical interests in mathematics education to stand in direct opposition to those perspectives that either ignore or reify these crises. Anything less than such direct opposition is an exercise in racial liberalism (Mills, 2017) that stops short of justice. With this acquiescence comes a tacit belief in the disposability of certain people who, in the interest of complementarity, can continue to teach and learn mathematics in conditions that are ecologically lethal. The disposable ones will always be those designated as deviant.

CONCLUSION

The purpose of this paper is to problematize the socio-ecological turn in mathematics education with respect to race. If, as I have argued previously (Bullock, 2024), equity orientations connected to both the social and socio-political turns in mathematics education rely on an axiom of racialized deviance, taking a socio-ecological turn requires a direct consideration of race to avoid replicating the same epistemic error. What is the goal for a mathematics education within the socio-ecological turn? If the socio-ecological turn is essentially the socio-political turn with an added interest in the environment, then the goal is likely similar to that of school mathematics outlined early in this paper: to develop a mature, secure, informed, managed, and self-, society-, and environment-serving *individual*. While this addition may bring consideration to the non-human world of plants and animals, it does not yet account for those who have never been human: the racialized deviant. Attending to this missing element requires direct confrontation with racial capitalism and white supremacy. This early stage of consideration is the right time to make a commitment to a socio-ecological turn that directly resists racism and challenges frameworks that both dismiss the ecological and accept disposability by race.

References

- Anthias, P., & Asher, K. (2024). Indigenous Natures and the Anthropocene: Racial Capitalism, Violent Materialities, and the Colonial Politics of Representation. *Antipode*, online first, 1-22..
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. Jossey-Bass.
- Bullock, E. C. (2019). Mathematics curriculum reform as racial remediation: A historical counter-story. In J. Davis & C. C. Jett (Eds.), *Critical Race Theory in Mathematics Education* (pp. 75–97). Routledge.
- Bullock, E. C. (2024). Racialized deviance as an axiom in the mathematics education equity genre. *Educational Studies in Mathematics*, 116, 333–350
- Coles, A. (2022). A socio-ecological turn in mathematics education: Reflecting on curriculum innovation. *Paradigma*, 43(1), 207–228.
- Coles, A. (2023). Towards a socio-ecological practice of mathematics teacher education. *Avances de Investigación En Educación Matemática*, 23, 19–35.
- Coles, A., le Roux, K., & Solares-Rojas, A. (2022). Towards a socio-ecological perspective of mathematics education. In: C. Fernández, et al. (Eds.). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (pp.171-178). Alicante: PME, Vol. 2.
- Davidson, J. P. L., & da Silva, F. C. (2022). Fear of a Black planet: Climate apocalypse, Anthropocene futures and Black social thought. *European Journal of Social Theory*, 25(4), 521–538.
- Davis, J., Moulton, A. A., Sant, L. V., & Williams, B. (2019). Anthropocene, Capitalocene, ... Plantationocene?: A Manifesto for Ecological Justice in an Age of Global Crises. *Geography Compass*, 13(5), 1–15.

- Diaz, J. D. (2018). *A cultural history of reforming math for all: The paradox of making in/equality*. Routledge.
- Dixon-Román, E. J. (2016). Diffractive Possibilities: Cultural Studies and Quantification. *Transforming Anthropology*, 24(2).
- DuBois, W. E. B. (2014). *The problem of the color line at the turn of the twentieth century: The essential early essays* (N. D. Chandler, Ed.). Fordham University Press.
- Gould, S. J. (1996). *The mismeasure of man: The definitive refutation of the argument of The Bell Curve*. W W Norton & Company.
- Jackson, Z. I. (2020). *Becoming human: Matter and meaning in an antiblack world*. New York University Press.
- Jefferess, D. (2011). Benevolence, global citizenship and post-racial politics. *Topia*, 25, 77–95.
- Latour, B., & Woolgar, S. (1986). *Laboratory Life: The Construction of Scientific Facts*. Princeton University Press.
- Lewis, O. (1966). The culture of poverty. *Scientific American*, 215(4), 19–25.
- Martin, D. B., Valoyes-Chávez, L., & Valero, P. (2024). Race, racism, and racialization in mathematics education: global perspectives. *Educational Studies in Mathematics*, 116(3), 313–331.
- McDermott, R., & Vossoughi, S. (2020). The culture of poverty, again. *Diaspora, Indigenous, and Minority Education*, 14(2), 60–69.
- Melamed, J. (2015). Racial capitalism. *Critical Ethnic Studies*, 1(1), 76–85.
- NCTM. (2000). *Principles and standards for school mathematics*. NCTM.
- Mills, C. W. (2017). *Black rights/white wrongs: The critique of racial liberalism*. Oxford University Press.
- National Commission on Excellence in Education (NCCE). (1983). *A Nation at Risk: The Imperative for Educational Reform*. USA Department of Education.
- Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, 102(1), 9–28.
- Popkewitz, T. S. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Educational Research Journal*, 41(1), 3–34.
- Popkewitz, T. S. (2007). Alchemies and Governing: Or, questions about the questions we ask. *Educational Philosophy and Theory*, 39(1), 64–83.
- Roberts, D. E. (1994). Deviance, Resistance, and Love. *Utah Law Review*, 179, 179–191.
- Robinson, C. J. (1983). *Black Marxism: The making of the black radical tradition*. The UNC Press.
- Stinson, D. W., & Bullock, E. C. (2012). Critical postmodern theory in mathematics education research: a praxis of uncertainty. *Educational Studies in Mathematics*, 80(1–2), 41–55.
- van Es, E. A., Hand, V., & Mercado, J. (2017). Making visible the relationship between teachers' noticing for equity and equitable teaching practice. In E. Schack, M. Fisher, J. Wilhelm (Eds) *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks*, pp.251–270. Springer.
- Wynter, S. (1979). Sambos and Minstrels. *Social Text*, 1, 149–156.
- Wynter, S., & McKittrick, K. (2015). Unparalleled catastrophe for our species? Or, to give humanness a different future: Conversations. In K. McKittrick (Ed.), *Sylvia Wynter: On being human as praxis* (9-89). Duke University Press.
- Yolcu, A. (2019). Research on equitable mathematics teaching practices: Insights into its divergences and convergences. *Review of Education*, 7(3), 701-730.
- Yow, J. A. (2012). Prospective teacher beliefs about liberative and oppressive mathematics teaching practices: a first step toward equitable instruction. *Journal of Mathematics Teacher Education*, 15(1), 83–96.

AIMS FOR A SOCIO-ECOLOGICAL TASK DESIGN: STEPS TO A TRANSFORMATIVE LEARNING OF MATHEMATICS?

Alf Coles

University of Bristol

In this paper I offer theoretical ideas relating to the aims of mathematics education. Given the precarity of life sustaining earth systems, mathematics education is called upon to make new kinds of gestures – gestures which take account of the entanglement of socio-political and ecological endeavours, in other words, socio-ecological gestures. A socio-ecological task design, I argue, might look to the UN’s “High Impact Initiatives” (launched in 2023) which include “transforming education”. After considering a range of meanings for “transforming”, I draw on Bateson’s theory of learning as a guide, which allows a focus on learning and wider system change. This prompts the question of how a socio-ecological task design might occasion transformative learning. The notion of a symbolically structured environment is one attempt to capture features of a transformative learning environment in mathematics – and hence one that has potential as a socio-ecological gesture. I exemplify the possibilities with a task relating to systems of climate change.

INTRODUCTION

This is a theoretical contribution to thinking about the aims of mathematics education, in relation to task (and curriculum) design. There is an urgent need for thinking differently about task design, linked to the radical earth system changes taking place. In the next section, I consider some of the literature around transforming, or transformative education and learning (which is a proposal from the UN for how education needs to change), which leads into the question of what task design could look like, with transformative learning of mathematics in mind. I propose the principles of a symbolically structured environment and show how they could link to gestures which move away from an individualistic view of learning and learners. This contribution aims to address the following question from the ICMI Discussion Document: “A2.1 With the addition of the socio-ecological to the mix, how are mathematics, education, or mathematics education inflected?”. What I mean by the socio-ecological is, at root, a non-separation of nature and culture (Kirby, 2011) – an idea I explore in more detail when discussing task design.

TRANSFORMATIVE LEARNING

When thinking about sustainability, in putting the socio-ecological “into the mix” of mathematics education, the work of the UN, however contested, is hard to ignore, with their Sustainable Development Goals (SDGs). While these can be rightly criticized for their implicit assumptions (e.g., around maintaining a status quo, or being human-centric) they do provide some common language and benchmarks. Haraway (2015), in discussing the term Anthropocene, suggests it is too widely used to be ignored or even replaced but needs to be complicated. It is in a similar vein that I want to consider the UN’s high impact initiative of “transforming education” and aim to complicate it. What might be a suitable aim for mathematics education, if we consider its potential to contribute towards transforming education?

One important departure for complication comes from Valero (2023), who cautions about the manner in which aims of mathematics education can end up re-enforcing modernism's destructive impulse. Valero (2023) points to how mathematics education "stabilizes, reproduces and even legitimizes existing patterns of behaviour" (Valero, 2023, p. 54), including the patterns which have brought life on earth to the brink of extinction. One of the ways in which mathematics education plays this reproductive role is through a focus on individuals. Learning mathematics can seem like the pinnacle of the individual pursuit, with cultural myths of the lone mathematics genius, for example. And through focusing on individuals, we can become blinded to wider systemic injustices. It is with wariness about the dangers of individualization, I now consider the word "transforming" as it is applied to learning, with the related phrase of "transformative learning".

The most cited author writing about transformative learning is Mezirow (e.g., 2008) who defined the concept as pointing to:

[t]he process by which we transform problematic frames of reference (mindsets, habits of mind, meaning perspectives) – sets of assumption and expectation – to make them more inclusive, discriminating, open, reflective and emotionally able to change. (p.92)

An example of a habit of mind could be a bias in our thought and action towards our own ethnicity, class, or species. For Mezirow, transformative learning takes place at the moment that a frame of reference (or habit of mind) shifts. These are therefore pointing to moments of relatively profound change. Mezirow's (2008) approach has been questioned for its overly cognitive assumption (Wolff et al., 2022). Alternatives have appeared, giving a role for affect, or proposing a socio-cultural interpretation of transformative learning (Friedman, 2022). These alternatives share a sense of transformation being relatively profound.

What transformative learning means, of course, depends on what we take learning to mean. But what seems common to the different variations is a sense of trying to get at a distinction between different levels or depths of learning. There is something different about learning a fact such as 7×8 is 56, and the *kind* of learning that means I understand what it takes to make a conjecture in mathematics.

Bateson (1972) articulated different levels of learning, which helps to elaborate on how we might be more precise about the possibility of different forms of learning. His views on learning relate to his thinking on communication, and the notion of "frames"; Bateson (1972) conceived of all communication as occurring within frames that define the kinds of communications that are expected, or allowable:

The frame [...] becomes a part of the premise system [...] the frame is involved in the evaluation of the messages which it contains, or the frame merely assists the mind in understanding the contained messages by reminding the thinker that these messages are mutually relevant and the messages outside the frame may be ignored. (p.188)

One example Bateson cites is the frame, "this is play", e.g., in animal interaction which might be communicated by a puppy "nipping" rather than "biting" its sibling. Successful interpretation of such messages is vital to animal survival and no less with human communication. Bateson continues:

a frame is metacommunicative. Any message, which either explicitly or implicitly defines a frame, *ipso facto* gives the receiver instructions or aids in his attempt to understand the message included within the frame [...]. The converse [...] is also true. Every metacommunicative or metalinguistic message defines, either explicitly or implicitly, the set of messages about which it communicates, *i.e.*, every metacommunicative message is or defines a psychological frame. (p.188)

The notions of frames and metacommunication relate to ideas of learning. Bateson (1972, 1979) developed an abstract classification of levels of learning. A simple change in behaviour (e.g., in response to a stimulus) is “Learning I” and is within the capacity of animals and plants (e.g., it has been shown plants can become habituated to a stimulus and change their response). A change in the *way* in which Learning I occurs is defined as “Learning II”. Relating these ideas to frames and metacommunication, Learning I takes place within a particular frame, and under a consistent set of metacommunicative assumptions. Learning II implies a change in frame, a change in metacommunicative messages. And, as Bateson (1972) noted, Learning II is much less usual than Learning I. When Mezirow (2008) discussed transformative learning, I take this to be equivalent to Bateson’s Learning II – a learning about learning, or a learning about the context of learning. And, importantly, linking learning to communication, it is not tied to an individualistic assumption. Frames and metacommunicative assumptions are established by communities. Opportunities for Learning II both arise in communal contexts and help shape those contexts. In other words, there is a constant interplay between individual learning, and the communities and contexts in which learning arises.

I have argued so far that “transformative learning” is a possible aim for a mathematics education with socio-ecological thinking in the mix. I have then argued that transformative learning implies “Learning II”, a shift in the manner of learning, which is relevant to individuals and communities. Some possibilities for Learning II, that might be relevant here, are awarenesses such as: mathematical statements are only true in relation to a set of assumptions; mathematicians need a shared language; the use of mathematical models has ethical implications; what I think is true may not be what you think is true, and we both might be justified in our beliefs. These are all potential learnings *about* learning mathematics (and are unlikely to be consciously articulated). With these possibilities in mind, I turn to consider a possible approach to task design, relevant to transformative learning.

TASK DESIGN FOR TRANSFORMATIVE LEARNING

One dilemma in thinking about a socio-ecological task design, or curriculum, is how to handle the kinds of “real-world” context associated with issues such as climate change. If mathematics is seen as separate from the “real-world” then it makes sense to ask questions such as how the real-world should be mobilised in the mathematics classroom. Distinctions seem plausible such as whether a lesson or task begins with the mathematics or begins with the real-world. The notion of “wrapper problems” (Stillman, 1998) – where a real-world context is unwrapped to expose the mathematics, that mathematics then being dealt with in isolation from the context – similarly confirms the separation of world and mathematics. And this separation seems aligned to a separation of nature and culture, which is an assumption perhaps widely shared (and unacknowledged) within much mathematics education research. And, of course, there is plausibility in such a separation – working on a tough mathematics problem just *seems* so different to the life of a plant, or the transformations undergone by a rock over time. What would it mean to *not* separate nature from culture (Kirby, 2011)?

An alternative perspective can be found in the monist, entangled philosophical outlook of de Freitas and Sinclair’s (2014) inclusive materialism. In thinking nature/culture together, one implication is that our concepts (including mathematical concepts) are necessarily material (*ibid*). Concepts cease being timeless, static entities which learners can attain (or not). Rather, they arise as part of a dynamic, unfinished, co-evolving and material relationship (e.g., with humans). In arguing for the materiality of concepts, de Freitas and Sinclair (2014) break down a binary that has dominated Western thought

(not just within mathematics education) – a binary they show can be read in the separation of mind/body, spirit/matter – where one side has agency, truth, value and the other is passive and available. And it is just such a binary which then has socio-political implications when it translates into similarly valorised separations of male/female roles, or “civilised”/“uncivilised” people.

This paper can be read as an attempt at thinking with entanglement about task design. In Coles et al. (2024), we articulated three “gestures” towards a socio-ecological mathematics education. These gestures included some of the ideas touched on above – moving away from an individualistic view of learning and moving away from a nature/culture binary. An earlier attempt at thinking through what such gestures mean for task design was in Coles (2023), where I articulated possibilities for task design in which broad questions (such as “How quickly do things change?”; “How do things occupy space?”) might lead to surfacing of socio-ecologically relevant issues, and into mathematical explorations of those issues. These broad questions attempt at entangling mathematical sensibilities with broader socio-ecological concerns. My own thinking here was influenced by a collaboration, described below, on task design (not relating to socio-ecological contexts). I elaborate and illustrate those ideas in the next section and relate them to socio-ecological gestures.

Symbolically structured environments

In Coles and Sinclair (2019), we aimed to articulate a tentative set of principles for designing tasks or learning environments. These ideas emerged out of commonalities we noticed in the teaching of mathematics teachers who appeared to occasion transformative learning (e.g., Madelaine Goutard, Robert Davis, Seymour Papert) and in reflection on our own teaching experiences. We proposed that when working with classes of students, these teachers behaved in quite unusual and powerful ways, to get students acting mathematically, in a sophisticated manner, and in a short space of time. We labelled what was happening as the setting up of a “symbolically structured environment” (SSE). One influence here was the anthropologist Bell (1991), who wrote about processes of ritualisation (in groups or societies), where certain patterned activities can become valued and where important work takes place below the threshold of discourse. The principles for SSEs were as follows:

- (a) symbols are offered to stand for actions or distinctions;
- (b) symbol use is governed by mathematical rules or constraints embedded in the structuring of the environment;
- (c) operations can be immediately linked to their inverse;
- (d) complexity can be constrained, while still engaging with a mathematically integral, whole environment;
- (e) novel symbolic moves can be made. (Coles & Sinclair, 2019, p.470)

A video recording of Bob Davis teaching (from, <https://videomosaic.org>) might offer a glimpse of the meaning of these principles. In the video, Davis is teaching a class of 6- or 7-year-olds. He gets a student (Joe) to hold a bag containing some stones. A second student (Brett) says “go” after which Davis puts in some stones and removes some stones. He records his actions on the board, for example $3 - 3 = 0$ (to indicate putting in 3 stones, removing 3 stones and there then being the same number of stones in the bag, as when Brett said go). In a second example, another student (Nancy) says “go” and Davis puts in 5 stones in the bag and is asked to remove 6. He writes $5 - 6 = -1$ (to indicate putting in 5, removing 6 and ending up with 1 less in the bag, than when Nancy said go). In this task,

the number symbols represent actions of adding or removing stones from the bag (condition a). There are constraints in that stones are first added then removed (b). There is putting in and, the inverse, taking out, being done together (c). It is easy to see how the task could be developed and complexified (d) and, in the video recording, it is a student who prompts taking out more stones than were initially added (e) (which only works because there are some stones in the bag already).

Alluding to the language of the earlier section, what a SSE appears to do is to make available to students some of the “rule of the game” of being successful at mathematics by laying bare some of its metacommunicative aspects. There is nothing explicit mentioned (in the example above) about how negative numbers work (this remains below the threshold of discourse). There is no need to memorise rules or definitions, for example. Yet students get feedback from the learning environment and their peers, which supports them in acting effectively, i.e., acting in a mathematically consistent manner. In other words, the environment is structured in such a way that, as a group, students soon come to be able, themselves, to evaluate each others’ messages (e.g., by re-doing the actions implied by the symbols). Recalling Bateson, “the frame is involved in the evaluation of the messages which it contains” (1972, p.188) and hence, if students come to be able to evaluate their own and each others’ statements, they are gaining access to the framing of learning mathematics. Mathematics ceases to be held and controlled by an external and potentially arbitrary-seeming presence. Therefore, I want to suggest that a SSE is a space that occasions transformative learning. And so, I want to argue, a SSE can be a socio-ecological gesture, whether it is addressing questions of climate change, or dealing with mathematical topics such as negative numbers.

In any mathematics task, we are learning about the world around us (in large part, through picking up the framing, or metacommunicative messages about such learning). A mathematics education may re-enforce the idea of hierarchies of intelligence, for example, or that the world’s resources can be quantified without loss of insight. *Any* mathematics task, therefore, can be seen as making a socio-ecological gesture, by inviting particular modes of engagement with the living world. The question, for task design, therefore becomes: what kinds of metacommunicative messages do we care about, or, what framings of mathematical tasks might point towards an entangled view of the socio-ecological? And such questions are relevant, however narrow or broad the content of that learning. So, while I might think that a topic such as climate change is one that is of particular relevance to the sustainability or regeneration of the living world, I am suggesting that the topic of negative numbers can be equally relevant (and potentially even more so), depending on the metacommunicative framing in which such learning takes place. Introducing climate change and other topics into the curriculum will not, by itself, lead to transformative learning. For example, it would be possible to imagine a mathematics education, including many topics relevant to the living world, but done within a metacommunicative framing that continues to valorise the individual and segregate learners on the basis of class and race (or, other euphemisms such as “ability”). Such a framing would be likely to remain hidden and so might even make harder the possibilities for transformative learning (which would imply coming to some awareness of the framing in which learning takes place).

In the next section, I aim to elaborate on what the principles of a SSE might offer in terms of thinking about a mathematics task or curriculum (aiming at transformative learning) where there is an attempt at linking to ideas of climate change (recognising, as argued above, that such a focus is not necessary for mathematics education, in terms of making a socio-ecological gesture).

Symbolically structured environments and climate change

It seems clear that climate change is an issue that is, and will, exacerbate the risks from other socio-ecological challenges facing the living world. Notwithstanding the comments immediately above, it does seem important to consider whether a mathematics curriculum could have a role in supporting awareness of climate change, e.g., linked to modelling change. I consider, briefly, how the principles of a SSE might support an entry into the mathematics of climate change. In reporting on the ideas below, I draw on an on-going (small scale) collaboration with the UK's Meteorological Office.

The climate can be viewed as flow system, with heat coming in (from the sun) and heat going out (emitted by the earth). Perhaps starting from the question "How quickly do things change?", a realistic climate model can be generated from modelling the heat entering the atmosphere (and how much is reflected back by ice) and the heat escaping the earth (and how much is reflected back by greenhouse gases). In constraining the complexity (condition d), we have essentially a system with four flows (see Figure 1), one IN (A) of which some is rejected (B) and one OUT (C), of which some is replaced (D). A SSE requires symbolization (condition a), with symbol behaviour governed by rules (condition b). Flows in opposite directions offer an intuitive starting point for symbolising actions and their inverse (condition c). The situation can be set up as an extension of the task in the Bob Davis video.

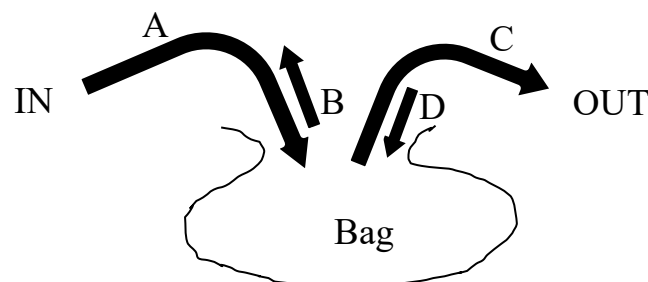


Figure 1: A flow system

To generate the conditions of a SSE, what gets symbolized is vital to consider carefully. In the case of this flow model, there are obvious actions of movement into and out of the container, these can initially be symbolised by numbers, to get a sense of how the system works.

In one turn, we might try to put 5 stones in the bag (A), of which 2 get rejected (B) and take 5 stones out (C), of which 2 get returned (D). This can be symbolised (conditions a, c) as: $(5 - 2) - (5 - 2)$, which would result in no change in the bag. We have a starting point here for a potential SSE. Depending on the age of the children, a range of scenarios or challenges are possible. In the first years of primary school, for example, A and C could be constrained to being 5 or under (with $B < A$, $D < C$) and a challenge might be, what are all the possible changes you could generate in one turn? Or, can you generate a change of +2? or -2? Different combinations could be shared and mapped, generating questions about patterns, which might lead to students posing their own questions (condition e).

Older students might be invited into a situation where B and D are proportional to A and C, respectively and similar challenges could be offered. There are of course opportunities to model the situation using technology (e.g., Scratch), and the effect of multiple turns, to explore the longer-term behaviour of the system. Figure 1 can be seen as a general flow system, which can become a climate model if A is energy from the sun, B relates to the albedo (reflectivity) of the Earth, C is the energy emitted by the Earth and D relates to greenhouse gas concentration in the atmosphere.

DISCUSSION

This paper has asked whether an aim of mathematics education could relate to transformative learning. In other words, might mathematics education occasion communities of learners to come to see themselves differently, including their potential and their potential to create different futures? Transformative learning was interpreted to mean a change in the manner of learning (labelled Learning II), or an awareness of the frame (the metacommunicative patterns) in which learners are involved, including a move away from individualism (a possible socio-ecological gesture). There is a need for vigilance about socio-political questions arising from intentions for mathematical learning (Valero, 2023) and a danger that transformative learning (as an aim for mathematics education) becomes one that reinforces the status quo, e.g., by only being available to the already privileged.

Drawing on past work on task design, I introduced the ideas of a symbolically structured environment (SSE), with the suggestion that a SSE can provoke transformative learning. A second key socio-ecological gesture I have alluded to is an ontological commitment to working against separations such as nature and culture. Within such a gesture, a pure mathematics task is itself addressing a socio-ecological issue – since any question of culture (such as within the culture of mathematicians) is not divorced from the living world in which it arose. Any mathematical task contributes to how we frame the living world (e.g., in a more or less extractive manner), and is also framed by such assumptions. In a socio-ecological task design, there might both be attention to the kind of Learning II (or assumptions about learning) occasioned and also attention to including applied content, such as climate change (exemplified above). There are severe limitations in the manner in which climate change is treated, above. A more deeply considered socio-ecological gesture would surely be one where questions of climate justice arise alongside the mathematics of climate change. And, in thinking about how such a curriculum might play out, it seems inevitably that mathematics loses a central place. In Solares-Rojas et al. (2023), there is an image of what such a curriculum could look like, where a motivating idea was the creation of a Memorial Museum to a polluted river.

And, at the same time, I find myself wondering what place there is for developing a deep disciplinary appreciation of mathematics. The tension between inter-disciplinary and disciplinary aims for education is one that perhaps is not to be resolved but rather explored. The notions of a SSE seem to relate to a sense of offering mathematical spaces where students can follow their own lines of inquiry, within a constraining environment that offers feedback. And, in that opportunity to follow your fascinations (Hay, 2024), there is the potential for communal activity and perhaps activism.

As I look to possible futures and what changes in the living world might mean for education, there is surely a need for a reckoning about the mathematics curriculum and its aims, both in terms of content and framing. The UK primary curriculum contains no new mathematics from the last 1000 years, I would guess. Systems theory seems an obvious topic for a renewed curriculum and the study of flow systems is potentially an accessible and powerful starting point. I have offered an account of my own thinking about how the study of flow systems could be introduced, in a manner that might occasion some of the transformative potential I have witnessed in the use of SSEs. My own interpretation of how such work could address Valero's (2023) important challenges around the potential damage done by mathematics education (including education reforms), rest on the move away from individualism and in drawing attention to the metacommunicative work that takes place in mathematics classrooms. I believe it is important to consider, what are the metacommunicative messages relevant to the current

times? In other words, what framings of the endeavour of mathematics education are needed at this time? I suggested some possible aims, in terms of what students might learn about learning, through studying mathematics (e.g., mathematical statements are only true in relation to a set of assumptions; the use of mathematical models has ethical implications). What perhaps seems an important step is that, as an educator, I bring to my own awareness the metacommunicative aims implicated (and ignored) by the tasks I offer in the classroom. For example, a key socio-ecological idea I have only touched on, is the potential need for activism, and aggregating actions, in the learning of mathematics.

References

- Bateson, G. (1972). *Steps to an ecology of mind*. University of Chicago Press, 2000.
- Bateson, G. (1979). *Mind and Nature: A Necessary Unity*. Hampton Press Inc, 2002.
- Bell, C. (1991). *Ritual theory, ritual practice*. Oxford University Press.
- Coles, A. (2023). Teaching in the new climatic regime: Steps to a socio-ecology of mathematics education. In: Ayalon, Michal et al. (Eds.). *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-34). PME.
- Coles, A., & Sinclair, N. (2019). Re-thinking ‘Concrete to Abstract’ in Mathematics Education: Towards the Use of Symbolically Structured Environments. *Canadian Journal of Science, Mathematics and Technology Education*, 19(4), 465–480. <https://doi.org/10.1007/s42330-019-00068-4>
- Coles, A., Solares-Rojas, A., & Le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116(2), 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- De Freitas, E., & Sinclair, N. (2014). *Mathematics and the Body: Material Entanglements in the Classroom* (1st ed.). Cambridge University Press. <https://doi.org/10.1017/CBO9781139600378>
- Friedman, J. (2022). How a New Learning Theory Can Benefit Transformative Learning Research: Empirical Hypotheses. *Frontiers in Education*, 7, 857091. <https://doi.org/10.3389/feduc.2022.857091>
- Haraway, D. (2015). Anthropocene, Capitalocene, Chthulhocene: Donna Haraway in conversation with Martha Keneey. In E. Turpin, & H. Davis (Eds). *Art in the Anthropocene: Encounters Among Aesthetics, Politics, Environments and Epistemologies*. Open Humanities Press (pp.255-270).
- Hay, P. (2024). Creative Pedagogies: School Without Walls and Forest of Imagination. *International Journal of Art & Design Education*, jade.12512. <https://doi.org/10.1111/jade.12512>
- Kirby, V. (2011). *Quantum anthropologies: Life at large*. Duke University Press.
- Skovsmose, O. (2023). *Critical Mathematics Education*. Springer International Publishing. <https://doi.org/10.1007/978-3-031-26242-5>
- Solares-Rojas, A., Arellano-Aguilar, O., García González, M. M., López-Vargas, M. del R., Coles, A., & Méndez Serrano, A. (2022). Mathematics education and social-environmental crises: An interdisciplinary proposal for didactic innovation with rural communities in Mexico. *Research in Mathematics Education*, 24(2), 202–223. <https://doi.org/10.1080/14794802.2022.2062781>
- Stillman, G. (1998). The emperor’s new clothes? Teaching and assessment of mathematical applications at the senior level. In P. Galbraith, W. Blum, G. Booker, & D. Huntley (Eds.), *Mathematical modelling: Teaching and assessment in a technology-rich world* (pp. 243–253). Horwood.
- Valero, P. (2023). Mathematical subjectivation: Death sentence to the planet or chances for a terrestrial life? In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 53-68). PME.
- Wolff, L.-A., Shephard, K., Belluigi, D. Z., Vega-Marcote, P., Rieckmann, M., Skarstein, F., & Cheah, S. L. (2022). Editorial: Transformative learning, teaching, and action in the most challenging times. *Frontiers in Education*, 7, 1041914. <https://doi.org/10.3389/feduc.2022.1041914>

MODELING AND WORLDING: HOW TO IMAGINE MORE-THAN-HUMAN MATHEMATICS

Elizabeth de Freitas

Adelphi University

In this paper I propose a distinction between modeling and worlding, as part of a philosophical reframing of mathematics education in more-than-human ecologies. I discuss examples of topological and stochastic ecological models, and the importance of speculation in imagining alternative mathematical worlds. I advocate for a posthumanist philosophy of mathematics, which allows us to consider the possibility of non-human mathematical practices.

WORLDING

What kind of mathematics education is appropriate for the Anthropocene? Which mathematics curricula might be called for, as the earth convulses under new climatic regimes? Should this be a mathematics well suited to studies of extinction? A mathematics of death and decomposition? A mathematics of entropic systems that track the dissolution of forms and structure? We already have a mathematics of chaos and entropy, and perhaps this is all we need. Perhaps ergodic algebraic models that marshal computational randomness are the best curriculum for future young people? Is this what we should incorporate into our mathematics courses, as we prepare future generations for an increasingly inhospitable and chaotic planet? It seems more urgent now than ever to turn to the potential role of mathematics education in imagining and perhaps pursuing more inclusive future worlds.

The Caribbean philosopher and poet Édouard Glissant (1928-2011) speaks of “world imaginaries” and “world orientation” as well as key terms such as “tout-monde”, “écho-monde” and “chaos monde” and “earth aesthetics”. Glissant was a poet and philosopher situated historically on the island of Martinique, which allowed him to generate a powerful theory of creolization and post-colonial futurity; he resisted the logic of center/periphery or empire/colony in all his work, always stretching nomadically out beyond the local, gathering the world as a whole or *tout-monde*, enlivened through multiplicity and fluid or metamorphic Relationality. This worlding gesture, so dangerous and yet so necessary, formulated from the archipelagic scattering and “diffractive” Caribbean, makes Glissant an extremely important thinker for our times.

How might we foster in children and young people both *inclination* towards worlding gestures, as well as *skill* and *knowledge* for crafting ethical and inclusive worlds? How might mathematics education foster *world orientations* and *more-than-human world imaginaries*? Coles et al. (2024) argue that a socio-ecological framing of mathematics education could help us better attend to the shared ontological nature of human and non-human being. Current emphasis on “real world modeling” within mathematics curriculum often seems to do the very reverse, by misrecognizing and diminishing the agency of the non-human. Barwell (2023) exposes the ways that mathematical ecological models are often destructive of the life worlds of animals, even when such models try to ethically manage ecological change. Like many other scholars in critical mathematics education, he shows how mathematical modeling performs a violent “ethical erasure” (de Freitas, 2008;

Skovsmose, 2005). In other words, he shows how mathematical models misrepresent the real, and often inflict damage on ecologies. But he also notes that mathematics is a powerful worlding activity, which is never an innocent activity, but almost always necessary as one considers possible and perhaps more inclusive futures on a planetary scale.

Different mathematical worlds are possible, and some of them may not look like the mathematics we are familiar or comfortable with. Some mathematical worlds may not look like mathematics at all. What kind of mathematical world does a wolf bring forth? What kind of mathematics does the living planetary ecosystem bring forth? Standard mathematical curricula do not give much space to these kinds of questions. Responding to them requires imagination.” (Barwell, 2023)

How might we develop this mathematical imagination? How can the turn to more-than-human worlding disrupt ongoing delusions of human exceptionalism and cognitive grandeur associated with mathematical expression? Such questions support an image of mathematical practices as diverse, material, conceptual, but also posthuman or more-than-human, and linked across different locations and different forms of life. I turn to the philosopher Rosi Braidotti who has embraced the term posthumanism to characterize a new criticality (Braidotti, 2021), linking it to an ethics of care and cross-species relationality, drawing from Spinozist ontology and Haraway’s cyborg feminism. According to Braidotti (2021), posthumanist feminism rethinks life “as a generative force beneath, below and beyond what we have made of it” (p.240). This kind of vitalism is often associated with Indigenous philosophies and new materialisms, and Braidotti traces the ways in which these ideas have informed “postcolonial ecocriticism” and a “zoe-centered egalitarianism” which “draws productive parallels between the species, while preserving the distinct capacities, propensities, and predispositions of humans” (p.84). Her use of the suffix zoe and geo, rather than bio, is explicitly meant to move the focus towards modes of life that are more vulnerable, monstrous and “deviant” from humans (p. 219).

Following Latour’s (2018) turn to “political ecology”, we need to better understand how human mathematics, in all its diversity, lives and dies in the metamorphic zone of more-than-human activity. Human mathematics is historical, political, and embodied, but it is also a more-than-human achievement – and in that sense, it is never entirely human. The speculative work of the imagination recruits from the earth, and forms all kinds of alliances with non-human agency and process. According to *Inclusive Materialism* (de Freitas & Sinclair, 2014), this approach avoids the ideal universalism of Platonism by exposing the immanence (rather than transcendence) of mathematical concepts. This shift in onto-epistemology will allow us to be even more ethically attuned to the role of mathematics in political ecology, and better able to do mathematics education responsibly.

The difference between *modeling* and *worlding* is helpful here, and might be a way to think through some of the challenges associated with the socio-ecological turn in mathematics education. In the context of applied mathematics, mathematical modeling is that which captures or encodes the material world, typically subtracting or erasing the complexity of the situation, in order to represent it with a simpler model. Through such extractive approaches, mathematical models have played a pivotal role in the destruction of ecologies, but they have also generated models of climate change, the very models that have helped us understand planetary change. Iterative design approaches in mathematical modeling are meant to use feed-back from trials and to thereby develop more responsive models, but

they tend to remain wedded to a mathematics of *representation* rather than a mathematics of *immanence*.

As an alternative, or as a curriculum goal, I propose the concept of mathematical *worlding* which works primarily through speculation (rather than representation), composition (rather than reduction), and attention to generative relationality (rather than fixed order). This is mathematics as an artful practice of imagining alternative worlds. And it is this practice of worlding that I aim to explore in this paper. An emphasis on worlding (as a radically different kind of modeling) could help us build a more robust mathematical pluralism that draws on posthumanist ontologies and engages with the fluidity of matter-meaning mixtures. And an emphasis on worlding can show students how mathematical activity is generative of possible alternative futures, and not just a human language game and inadequate *representation* of the real.

I am contrasting modeling with worlding to help us think about the different ways that *abstraction* occurs in mathematics, so that we might incorporate into our curriculum the speculative power of mathematics in more ethical ways. Abstraction is not only a subtractive act, simplifying the complexity of an ecology, as it is in many models. Abstraction is also a key mode of imagining otherwise and exploring alternative modes of relationality and belonging, and as such it is a key creative mode in both art and science. Mathematician Eugenia Cheng (2023) suggests, in her book, *The joy of abstraction*, that the role of abstraction in mathematics is diverse. It is not only an act of generalization from particulars or a subtraction of features. It is also an important generative force in mathematical activity, bringing forth the new. Cheng’s own mathematical practice uses abstraction as a creative process of composition, an act of opening up to encompass a larger and more diverse set of unknown entities whose *relations* can then be formalized as part of a speculative experiment. This approach to abstraction could bring a certain speculative sensibility into mathematics education, drawing on students’ creative imagination.

TOPOLOGICAL FOLDS OR PROCEDURAL RANDOMNESS?

In this section I consider two contrasting approaches to the role of mathematics in socio-ecology, examining the extent to which they pursue a worlding orientation. Both involve a critique of the anthropocentric habits of mathematical modeling, and offer suggestions for ways forward. The first is a topological approach, and resonates with the work of eco-philosopher Timothy Morton, in his book *Humankind: Solidarity with Nonhuman people* (2019), where he revises Marx’s species-specific and anthropocentric communist agenda so that it might support a solidarity with that which is not human. The urgency and trauma associated with the climate crisis suggests to Morton the need for an overturning of teleological, hierarchical, patriarchal and racist frameworks, and the recognition that humans have never been human insofar as they have always been a cluster of symbiotic relations. Such an approach is always a tad tricky, as it tries to break with conventions of inside/outside, thinking with topological folds of a *being* that is *spread everywhere*. In *Dark Ecology*, Morton (2016) emphasizes the form of the *loop*, when advocating a “knowing that knows itself. Knowing in a loop - a *weird* knowing. Weird from the Old Norse *urth*, meaning twisted, *in a loop*.” (Morton, 2016, p.5). The concept of *weird* is used here as a marker of the ecological loop, a way of engaging the more-than-human. He goes on to claim that “ecological awareness is a loop because human interference has a loop form, and because ecological and biological systems are loops. And ultimately this is because to exist at all is to assume the form of a loop.” (Morton, 2016, p.6). Morton slams “parochial”

Euclidean geometry as “anthropocentric”, and invested in a false image of “smooth homogeneous universality” (p. 12). Pursuing the form or figure of the loop, he suggests we consider the power of strange loops: “A strange loop is one in which two levels that appear utterly separate, flip into one another.” (Morton, 2016, p.7). And then he turns to the Möbius strip, a nonorientable surface, where non-human spatial sensibilities might be explored.

Unnithan Kumar (2023) continues this line of work, and suggests we formulate an “ecological mathematics” in which topological concepts play a key defining role. He wants to recruit topology as an important part of ecological thinking, and to address the impasse he finds between mathematics and ecology in the social sciences. This is an attempt to reclaim the power of abstraction in topological thought, and engage it more carefully in ecology. In the context of the Anthropocene, topological concepts can help us think in nuanced ways about borders, proximities, de/re/territorialized spaces of flow, where risky subjects, citizens of no country, are kept in precarious states. The mathematics of topology lends itself to studying proximal or conjoined territories, as migrating animals and refugees traverse the earth. Some border scholars use the Moebius strip to capture the paradoxical way in which one creature inhabits degrees of internality and externality at such thresholds. Both geographies of exclusion and counter efforts at cross-border solidarity can be studied through the lens of topology. My own use of topological thinking in education research has been in the spirit of a speculative gesture, drawing explicitly on the mathematics of knot theory, as a way to diagram the complex relationality of learning environments (de Freitas, 2012).

The second approach centers the mathematics of stochastic systems. Eglash (2023) notes that previous ethnomathematics research, his own included, had emphasized the power of Indigenous “human knowledge” as it was encapsulated in particular material practices, while neglecting the ways in which nonhuman agency was integrated into those practices. He offers a quotation from D’Ambrosio to make his point:

We see knowledge as emanating from the people, essentially a product of man’s drive towards explaining, understanding and coping with this immediate environment and with reality in general, reality understood in its broadest sense and in permanent change as a result of man’s own action. (D’Ambrosio, 2000)

According to Eglash, these efforts reflected a decolonial reading of non-Western mathematical wisdom that was called for in the latter quarter of the 20th century, but he believes that ethnomathematics must now turn to information theory and become “ethno-bio-mathematics” to attend more adequately to nonhuman design. Eglash (2023) claims that the mathematics of information theory offers a “mutual legibility” between “Native and American math, culture and Nature”, and that Shannon-Weaver entropy metrics are the language of nature. He claims a correlation between Native American agroecology practices that maximize biodiversity and the stochastic concept of entropy as a measure of the “evenness” of a probability distribution: “This basis in probability nicely fits the Native American traditions around coding for maximum information, farming for maximum biodiversity, and viewing the world in terms of probability” (p.293).

Accordingly, he suggests that mathematics curriculum should include the study of automatic self-forming structures and biochemical processes like autocatalytic reactions, by which molecular living things become self-sustaining. Whereas before he kept the cultural separate from the natural world,

he now wants to make more visible the links between cellular automata (CA) and mimetic cultural patterns. For instance, he suggests that fractal patterns in cultural-material practices can be studied as an automatic self-forming process: “It is fractal because human agency and the ecosystem’s agency are in alliance in this CA-like process of morphogenesis, an exemplary case of ethno-biomathematics.” (p. 297).

One of the problems with this approach is how it remains wedded to a reductive form of abstraction, and a computational image of nature. Stochastic models typically study “minimal animals” (Benson, 2014), where animals become data points in an empty space, their movement tracked based on cybernetic circuit models and simulations (McRae et al, 2008). These are models derived in large part from Claude Shannon’s revolutionary information theory (1948) and Von Neumann and Morgenstern’s game theory (1944). Indeed, stochastic systems have come to dominate ecological modeling, channeling spatial ideas about relationality and interaction originally derived from thermodynamics. A socio-ecology based on stochastic systems conceives of agents bumping up against each other like “billiard-ball-inhabitants” in such a manner that larger scale patterns of behavior emerge (Châtelet, 2014, p.72). Rather than a decolonial mathematics, as Eglash hopes, this approach aligns perfectly with capitalist fantasies of self-organizing free markets (Châtelet, 2014). This atomized ontology of self-organizing bits of information has flourished along with what many call our “computational ecology”. Is this the mathematics of nature, or is it just the mathematics of discrete-state machines? I am not disallowing that a mathematic of folds or a mathematics of chance or quantity might also emerge from the life-worlds of wolves or other non-humans, but in order to conceive of these in/human or more-than-human mathematical worlds, we have to do the hard work of decentering human mathematics, using posthumanism and other related speculative approaches.

EMBODIMENT OF HUMANS, ANTS AND SPIDERS

Topology and stochastic system theory offer interesting models for thinking about ecological relationality and the emergence of structured worlds. The extent to which these theories reflect only human embodiment and space-time experience is an open question. In this section, I draw attention to this open question, emphasizing how tacit phenomenological structuring of space sets up spatial concepts about foreground and background and proximity, and overdetermines our notions of twist, loop, even, flat, heterogeneous and entropy – mathematical concepts linked to the fields of topology and stochastics. In other words, our mathematics reflects our embodiment. The human body is implicated in all these concepts from topology and stochastic systems theory. Human depth-perception solicits particular world imaginaries and particular mathematics. With reference to the phenomenology of Husserl, Ahmed (2010) suggests that human orientation marks a “zero-point” or starting point “from which the world unfolds” (p. 236).

But returning to Glissant and his archipelagic spatial thinking, we might push back at this *center-periphery* spatial imaginary and individuation of a bilateral symmetric body, as inappropriate for non-human forms of life. How might we understand alternative body-milieu relationships, and their corresponding different modes of mathematizing? How might we better appreciate the socio-ecological and more-than-human ways in which activity unfolds for lichen or forests? How might we rethink our spatial metaphors for navigation? How might we study the force of bodying, without centering the human body?

The Saharan desert ant – the *Cataglyphis Fortis* - is renowned for its agile navigation skills while traversing a shifting terrain of wind-blown sand. The ant searches under the sun for scraps of insect meals, a journey that may take these tiny insects 100 meters away from their nest. Their return journey home is impressively direct, and not based on chemical trails, relying instead on an internal skylight compass that allows them to discern the correct direction back to the nest. Perhaps even more impressive is that the ant can determine the exact number of steps required to reach its pin-hole nest, travelling over an undulating surface of summits and troughs that shift with the wind. Does this accuracy indicate a cognitive pedometer that measures exact distances or a milieu sensibility?

In a series of experiments, scientists added stilts to lengthen and shorten ant legs, and found that the ants either undershot or overshot their nest, accordingly. Their sense of direction was unchanged, but their “step-count” was off, or rather their skill at “step integration” was compromised (Wittlinger, & Wolf, 2013). They were able to venture out and accurately return if the legs were adjusted in the nest, but not if they were adjusted when out foraging. These ants are probably not counting in some conscious way, but it does seem that their body-sense or sense of their body’s size and syntonicity has changed with the adjusted legs. And why would it not, you might ask. The ant is not a computer or automata, but is a responsive creature that is adaptive to body-milieu relationships. Their geometric ability to recalibrate the number of steps needed as they traverse a complex topological surface is absolutely necessary. In other words, number sense, leg sense, proprioception, and measurement practice are inseparable in foraging excursions and in re-orienting or homing for a nest.

Turning to the spider, Deleuze (2000) suggests that the spider web is a compelling image for an alternative mode of orientation and spatial mobility. What kind of mathematical world does the spider bring forth? Does the spider web offer an alternative to the logical maze? Why do we test animal intelligence in terms of maze navigation? In what ways does the maze, as an exercise in spatial navigation, reveal more about the specific symmetries of human embodiment (right, left) and spatial assumptions about grids? Deleuze (2000) suggests that spider sense is “involuntary” and both preconscious and transindividual: “The spider sees nothing, perceives nothing, remembers nothing. She receives only the slightest vibration at the edge of her web, which propagates itself inside her body as an intensive wave and sends her leaping to the necessary place ... The spider-Narrator, whose web is the Search being spun, being woven by each thread stirred by one sign or another: the web and the spider, the web and the body are one and the same machine (p. 117).

The Argentina artist Tomás Saraceno builds various kinds of large-scale interactive spider-web sculptures, as part of an artistic practice focused on reconfiguring our sensory embodiment under Anthropocene conditions. In 2022, at the SHED in NYC, his soaring ninety-five-foot-wide installation, called *Free the air: How to hear the universe in a spider/web*, involved climbing out onto a hanging mesh net, forty feet above the floor. Once there, the room darkens and one becomes somewhat anxious and disoriented, before undergoing various kinds of sounds and light and vibration while slung mid-air. Also opening in 2022, in the observation deck of the Torre Gloriès, in Barcelona, was a Saraceno installation called *Cloud Cities: A space of communal encounter, discourse and critical speculation*, composed of a 6 km web of tensioned iron cables, connected by 5000 nodes, and forming a complex twisted network of linked resting pods where one could climb, read and recline, as though suspended in a condensed water droplet caught far above the city. Participants enter and dwell for as long as they like, taking their own path across the complex intersecting web. These two

immersive artworks by Saraceno and his team, offer an alternative geometric rendering of communal space, a kind of speculative architecture. Inside and outside are no longer relevant, and the reflected surfaces and symmetry of the pods are disorienting, as one begins to forget what is up and down. To heighten the experience, and diminish gravity's call, participants are asked to empty pockets, and wear a special "web technician" uniform before scampering along the cables. When I nervously climbed into the web, I slowly developed a spider sense of suspension.

CONCLUSION

I believe there is no exodus from the dominant image of the human without a radical reconfiguration of its mathematical imaginaries. There is no climate-escape or cross-species hybrid form that will not carry with it the residue and detritus of past mathematical practices. Modeling and measurement have always been forms of appropriation, of striating and colonizing space. Mathematics is geo-political. Indeed, mathematics has always been geo-political, always that dangerous gesture that is both material in its practice and yet also stretching towards the "tout monde", that worlding gesture that makes alien our own thought, and detaches our perspectives from the here and now. In this way, mathematics has served many problematic idealisms and human desires for transcendental more-than-human truths. But if we claim that mathematics is only an expression of human desire, and predominantly an expression of white/male/European idealism, we actually *reinscribe* human exceptionalism, albeit historically situated, and deny nonhuman others any mathematical nature or propensity. This would be like denying sign-making-facility and worlding capacity to forests, crows, ants, spiders, etc. Why would our extinction spell the end of patterns and persistent structural relations or the conceptual and imaginative thinking of nonhuman animals and nonhuman collective intelligences? I suggest that there is valuable insight to be gained in mathematics education by considering my proposal that the speculative gesture of mathematics is not exceptional or unique to humans. Mathematical abstraction is a practice found in mathematical activity of all kinds, be it human or otherwise.

Neither topology nor the mathematics of stochastic change are sufficient to support a world orientation in mathematics education. Neither of these is in itself adequate for shifting the philosophical assumptions that underlie reductive modeling; neither kind of mathematics will help us move towards more speculative mathematical worlding. But bringing more topology or more stochastic problems into our curriculum may help students develop dynamic spatial sensibilities and perhaps also alternative ways of understanding processes of more-than-human change, as we try to imagine the life worlds of wolves, ants and spiders. Student imagination will also be served if educators turn to artful experiments like that of Saraceno. Artist Natalie Jeremijenko (2017) performs field experiments collaboratively with various aquatic organisms (mussels, fish, algae) seeking new kinds of posthuman models of ecological "mutualism". Together these kinds of efforts can shift mathematics education away from a mathematics of representation and towards a mathematics that is responsive to the metamorphic nature of more-than-human ecology. This *monist* philosophy of immanence does not claim that nature is written in the language of mathematics, but seeks out the material mixtures of mindedness and matter, to show how mathematics can be reconceived as part of more-than-human worlding. Through such philosophical reframing of mathematics, educators might begin to pursue the kind of world orientation and inclusive relationality that is at the heart of ecological thinking. We have to start thinking more creatively about how to bring forth alternative

mathematical worlds. Exposing the earthbound materiality and fallibility of human mathematical practices is the first ‘critical’ step, but then we need to use that grounding to fuel creative experiments in alternative mathematical world-imaginaries.

References

- Ahmed, S. (2010). Orientations matter. In D. Coole & S. Frost (Eds.), *New materialisms: Ontology, agency, and politics* (pp. 234-257). Duke University Press.
- Barwell, R. (2023). Dialogue, responsibility, and mathematics education. Keynote at *12th International Conference of Mathematics Education and Society*. Sao Paulo, Brazil.
- Benson, E. S. (2014). Minimal Animal: Surveillance, Simulation, and Stochasticity in Wildlife Biology. *Antennae* 30: 39.
- Braidotti, R. (2021). *Posthuman Feminism*. Duke University Press.
- Coles, A., Solares-Rojas, A., Le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*. <https://doi.org/10.1007/s10649-024-10318-4>
- Châtelet, G. (2014). *To live and think like pigs: The incitement of envy and boredom in market economies*. MIT Press.
- Cheng, E. (2023). *The joy of abstraction: An exploration of math, category theory, and life*. Cambridge University Press.
- D’Ambrosio, U. (2000). A Historiographical Proposal for Non-Western Mathematics. In H. Selin, (Ed.), *Mathematics Across Cultures. Science Across Cultures: The History of Non-Western Science*, (pp.79-92). Springer, Dordrecht. https://doi.org/10.1007/978-94-011-4301-1_6
- de Freitas, E. (2008). Critical mathematics education: Recognizing the ethical dimension of problem solving. *International Electronic Journal of Mathematics Education* 3(2), 79-95. Available at <http://www.iejme.com/>
- de Freitas, E. (2012). The classroom as rhizome: New strategies for diagramming knotted interaction. *Qualitative Inquiry*, 18(7), 557-570.
- de Freitas, E. & Sinclair, N. (2014). *Mathematics and the body*. Cambridge University Press.
- Eglash, R. (2023). Ethno-biomathematics: A decolonial approach to mathematics at the intersection of human and nonhuman design. M. C. Borba, D. C. Orey (eds.), *Ubiratan D’Ambrosio and Mathematics Education: Advances in Mathematics Education*. Springer.
- Glissant, É (2009). *Poetics of Relation* (B. Wing, Trans.) University of Michigan Press., Ann Arbor
- Jeremijenko, N. & Hannah, D. (2017). Natalie Jeremijenko’s new experimentalism. In (Eds. R. Grusin). *Anthropocene Feminism*. University of Minnesota Press.
- Latour, B. (2018). *Down to earth: Politics in the new climatic regime*. Trans. Catherine Porter. Polity Press.
- McRae, B. H., Dickson, B.G., Keitt, T.H. & Shah, V.B. (2008). Using Circuit Theory to Model Connectivity in Ecology, Evolution, and Conservation. *Ecology* 89(10), 2712–24.
- Skovsmose, O. (2005). *Travelling through education: Uncertainty, mathematics, responsibility*. Sense Publishers.
- Wittlinger, M. & Wolf, H. (2013). Homing distance in desert ants, *Cataglyphis Fortis*, remains unaffected by disturbance of walking behavior and visual input. *Journal of Physiology – Paris*, 107(1-2), 130-136.

RETHINKING MEASUREMENT IN SOCIO-ECOLOGICAL TERMS

Elizabeth de Freitas¹, Nathalie Sinclair²

¹Adelphi University, ²Simon Fraser University

In this paper we show how measurement practices might better resonate with socio-ecological concerns. We argue for a measurement curriculum that would help students understand the relational entanglement of socio-ecological processes. We treat measurement as a mode of forming alliances between different scales of mattering (body and earth), drawing on archeological data. We relate this approach to current scholarship in ecologically oriented philosophy and STS, providing a theoretical foundation for socio-ecological mathematics education.

INTRODUCTION

One of the biggest challenges in developing a socio-ecological sensibility in young people pertains to the vastly different temporal and spatial scales entailed in ecological thinking. At one extreme there are the times and magnitudes associated with astronomic, geologic, and pre-historic processes, while on the other hand there are micro-temporal processes which include the diverse lifespans of bacteria, birds, and humans. Ecological thinking must encompass all these different times and scales into a complex system. But how do we think distinct and heterogeneous spatio-temporalities at the same time? How do we help students develop methods for thinking about complex ecological relationships at multiple scales of relevance? In what way is mathematics helpful to them?

In this paper, we focus on what we take to be a key paradigm of ecological thinking, that being *relationality*. Our emphasis is on *dynamic relationality*, since ecologies are first and foremost *provisional* rather than static (Smith & Young, 2022). We argue that our challenge as mathematics educators is to help students and teachers investigate the *changing relationality* of provisional milieus (see also de Freitas et al., 2022). The current measurement curriculum is almost entirely caught up with static measurements (length, area, volume) and rarely deals with relationships changing over time (de Freitas & Sinclair, 2020). Students learn about linear and exponential growth patterns, which can be used to model the change in an isolated variable, but time and temporality are typically considered too complex or abstract for young learners, whose space-time sensibility is said to be limited by their developmental status. This is an assumption we need to contest and update. It strikes us as significant that “related rate” problems¹ in 1st year calculus courses often stump students, perhaps because we have failed to prepare them to think relationally about *reciprocally informed processes*. Indeed, as the spatial reasoning literature has shown, while mathematicians engage substantially in dynamic, extrinsic thinking (such as co-variation), most of school mathematics involves static and, at best, internal dynamic thinking (how one thing changes) (see Uttal et al., 2013).

We argue for the importance of reframing “measurement” as essentially an act that entails forming dynamic relationships or alliances between differently scaled aspects of a milieu. We believe this will help develop students’ socio-ecological sensibility and mathematical agility for living in this

¹ Problems that ask you to find the rate at which one quantity is changing, if you know the rate at which another is changing. For instance: If you know the rate at which water pours into a cylinder, what is the rate with which the water height rises?

increasingly unpredictable world. Rather than think of measurement as the covering of space with standardized units, a perspective which treats measurement as an act of imposing an “objective” ruler on matter from without, our emphasis better engages the socio-political aspects of dynamic ecologies by revealing how all measures are essentially attempts to create relationships between reciprocally in/formed agencies and processes. Even the most simplistic laying of a ruler must be rethought in these terms.

Within the classical traditions of Western mathematics, we can see this relational ontology at work, although often erased or sidelined by the more dominant images of deductive mathematics. Two of the founding legends of Western mathematics point to the pivotal role of measurement in linking the human day and immediate locale to the earth, the sun, and larger scales of mattering: Erastheneus (200 BCE) worked out the circumference of the earth using only the difference in the angle of the mid-day sun at two locales, while Thales used proportion to discover the height of the Great pyramid of Cheops (600 BCE). These feats of measurement were achieved by enlisting the sun’s rhythms and the starry transits of the night sky.

As we have argued elsewhere (de Freitas & Sinclair, 2012, 2014, 2016, 2017, 2022), mathematical formalisms and material processes are bound together in a metamorphic mixture of mind and matter. Our philosophy of mathematics is a kind of monism, which we call *Inclusive Materialism* – this approach studies mathematical activity as emergent and ecologically distributed.² Unlike dualist ontologies that separate concept from matter, monist onto-epistemologies explore the way that concepts are actualized through material practices, and how material practices are animated by virtual potentiality. It is this complex reciprocating mixture that makes mathematics political and plastic, controlling and liberating, realist and speculative. Our philosophical framework allows us to seek out mathematical activity that *builds alliances* across different kinds of more-than-human agencies and different scales of temporal and spatial mattering. This paper discusses examples of measurement practices that support this socio-ecological sensibility in various ways.

MEASURE, MATTER, EARTH

Kirby (2011) critiques dualisms that separate out mathematics from the material world. She characterizes these as anthropocentric in two different ways: the first imagines that the earth is there to serve humans, whose magical mathematics aims to control it (reductive scientism); the second imagines that the earth is entirely socially constructed by the cultural veil of human mathematics (social constructivism). Kirby suggests instead an approach that dethrones the anthropocentrism of these two traditions and seeks a new empiricism of life, earth, and the more-than-human.

Measure would then not *only* be the anthropocentric habit inscribed in Protagoras’ aphorism ‘man is the measure of all things’ nor reflect a unique human capacity. Instead, measure would be a tendency or potentiality of matter. Geometry, for instance, would be a more material mingling of *geo* and *metric*. For Kirby, too much of socio-cultural theory forecloses this possibility by defining geometry against geology, language against matter, mathematics as a *representation* that codes matter from without. (de Freitas, 2016, p. 656)

We propose here that one way of rethinking measurement as relational rather than representational involves unmasking its dynamic and analogical nature. For instance, the mathematical proportion (a/b

² Inclusive Materialism draws on ideas from Bruno Latour, Gilles Deleuze, Donna Haraway, Karen Barad, Michel Serres, and Gilles Châtelet.

: c/d) should be considered a dynamic variance of two reciprocally informed processes, brought together in a kind of spatio/temporal alliance. Proportions would then be a series of relays of linked processes “in parallel” (in fact, analogy engenders the very notion of parallel) in which temporality would always be presumed (Coles, 2017). In this way, we cast proportions as that which can track the resonance and entanglement, but also divergence, of linked processes (the flow of the river *with* the transit of the sun *with* the migrations of animals). We are so accustomed to thinking that measurement rests fundamentally on a global de-territorialized unit, we forget that cultures across the globe invented their units in response to reciprocally varying processes (Lugli, 2019).

Consider how units have often had to stretch to suit our changing relationships. The sundial, like the gnomon, measures “sun time” (not “clock time”) and a day’s length in “sun time” varies over the year; the unit “hour” thus stretches or shrinks to match the day. The gnomon is a mathematical instrument thoroughly bound to both earthly processes³ and the transit of the sun, forming an alliance between these spatio-temporal scales. Brown (2005) argues that the Babylonian gnomon should be seen as a kind of automatic inscription device that *knows* the earth and the sun; it seems to measure on its own, with no need for human interpretation (unlike a telescope). For the Greeks, the gnomon “discerned, distinguished, intercepted the light from the Sun, left lines on the sand as if it were writing on a blank page and, yes, understood” (Serres, 1995, p. 80). Instruments of measurement must be studied for how they enlist (but also sometimes coerce) the agencies of the non-human world. Indeed, Lemke’s (2000) focus on *heterochrony* raises related questions such as, “On how many different timescales is our social life organized?” (p. 273). Mikulan and Sinclair (2019) advocate for a stratigraphic approach to this question, where the analogical thinking must be made visible, and the different space-time scales conjoined or brought alongside each other, but not collapsed. In the remaining sections, we look at the history of diverse measuring practices because it provides insight into how body-earth-cosmos relationality has always been integral to measurement practices (Alder, 2002).

ARCHEOLOGY OF MEASUREMENT

Most early archaeological evidence for measurement practices is linked to the emergence of sedentism, about 10,000 years ago, when in many parts of the world, humans began to invest in permanent dwellings and agriculture. This marked the onset of what archaeologists Colin Renfrew and Iain Morley call the “tectonic” or constructive phase in the development of human culture. Measurement practices are said to have emerged to structure group relations, distribute resources and commodities, navigate journeys and comprehend time’s passage and cycles (Renfrew & Morley, 2010a). Direct archaeological evidence for measurement prior to this phase is sparse (Renfrew & Morley, 2010b). This sparsity of evidence has implications for us today, since we are heirs to a particular historical account of what and how human measurement practices evolved.

Since nomadic measurement practices have gone largely undetected in the archaeological record, we are left to wonder what these might have been. Farr (2010), for instance, speculates on Neolithic measurements associated with navigation practices in the Adriatic Sea, based on the distribution of obsidian volcanic glass found in different seafaring locations. We know that in other regions, hominids were seafaring as early as 50,000 years ago. Farr suggests that nomadic seafarers might

³ It must be positioned and oriented with respect to the rotational axis of the earth and the local latitude.

have measured time and space in ways very different from those used by sedentary peoples. The measurement practices adopted for navigational purposes by the Marshall Islanders provide some inkling of such measuring practices. Instead of relying on an external, flattened map of the islands and atolls, as western navigators would, they used a physical stick chart. This *mattang* modelled the diffraction of ocean swell both in open sea and in its interaction with the land. A trainee navigator would memorise these diffraction patterns, for different locations, and learn to match them with the swell they felt with their own body, while lying outstretched on the bottom of the boat (Ascher, 2002).

The diversity of measuring habits in the historical and ethnographic record is indexed by a wide range of measuring devices “such as bows, chain links, and goads for driving oxen, as well as spans of the body such as finger-widths, hand-breadths, and arm-lengths” (Cooperrider & Gentner, 2019, p. 1). Many practices include measures that reference the body (the foot, the hand, the thumb, etc.). Medium-scale measures tend to reference events—such as the ‘bow shot’ used in the Andaman Islands and the sonic measures used in Burma, which were based on the distance from which one could still hear a person’s voice. Larger-scale measures, rarer, were often temporal, based on protracted events such as, for the Sámi, the number of coffee stops required on a journey, and for the Mi’kmaq, the number of capes traversed. Early North American Ojibwe used the body to measure how much of a day it would take to travel a certain distance, by superimposing an outstretched hand on the arc of the sun: in this analogical measuring, “one ‘hand-stretch’ was considered one fourth of the arc from sunrise to zenith” (Cooperrider & Gentner, 2019, p. 5).

Cosmology shows how measurement comes to figure prominently in human spirituality as well (Urton & Llanos, 1997). Measurement plays a crucial role in cultural and theocratic regimes of power, as a method for revealing the order that is presumed to lurk within the unruly earth. We invest measurement with the capacity to *uncover* an ordered universe, which serves the human tendency to imagine an underlying stability and explanatory continuum. And yet this delusion of control is not the last word. Proportional and analogical thinking is precisely how mathematics becomes speculative and stretches into the infinite, beyond human perception and human scales. Lugli (2019) recounts how European Medieval measurement standards were built into the churches and public squares, and how the standards became quasi-objects, or “ambivalent entities that exist between physical objects and ideal ratios” (p. 30). Eventually, measurement practices—enforced by friars and statutes whenever possible—were used both to control everyday life and to make the metaphysical palpable: “measuring served as an ideal interpretative channel to recuperate the body of Christ” (p. 146). Measurement always entails this mixture of tangible and speculative dimensions, with one leg in the finite world, and one in an indeterminate virtual realm of potentiality (Deleuze & Guattari, 1987).

THE NON-INNOCENCE OF MEASURE

Measuring practices were also, and continue to be, part of imperialist population control, always implicated in governance, exploitation and subjection. Consider for instance how the Inkan empire used a complex system of knotted colour cords called Khipus to record census data and resources, as well as accounts of conquest and debt. The brilliance of using cords and knots and colours for such a registry is found not simply in its ability to represent place value and difference in kind, but in its reusability—knots can be untied and retied (Urton, 2010). But this brilliance was in the service of Inkan empire building. This account also draws attention to the important role of technology in measurement, where ‘techniques’ serve to regulate populations, monitor community participation,

and control value. Indeed, our emphasis on relationality and plasticity is not meant to eclipse the equally powerful way in which measurement serves imperial interests. Postcolonial theorist Kyla Schuller (2018) suggests that past investments in a sentimental *relationality* have often been deployed to separate the ‘sensitive’ civilized white and able-bodied subject from the abject other who is alleged to lack this sensibility. Analogical thinking and intensive measures may be fundamental in relational ontologies, and may be currently neglected in school mathematics, but they are not innocent gestures without risk. Measurements are always central to all colonial invasions; ‘imperial units’ are typically deployed to *apportion* resources amongst the victors. Deleuze and Guattari (1987) seek “minor” measuring practices that undo the striating and controlling gesture of enclosure, and that resist the ‘covering unit’ that contains and apportions.⁴ Rather than the territorial drive to divide up or distribute space among various claimants, which aligns with extractive capitalism, they seek a mathematics that is better articulated with rhizomatic and ecological images of change (de Freitas, 2016).

Unfortunately, the measuring habit of covering with imperial units is not likely to disappear. For that reason, we are all the more motivated to undermine this image of a “royal science” mathematics, by developing an alternative understanding of measurement. Our geo-historical archaeological approach reveals the temporality of measuring practices, and the metamorphic nature of the mathematics-matter mixture. We affirm the worldly relevance of analogical thinking, and the power of proportionality to link the fluid changing parameters of a milieu. This makes measurement messy, caught up in mucky matters, provisional. And in this way, we also affirm the generative disorder of measurement and celebrate the distinctive and situated minor measuring impulses that emerge in different locales.

We note that the Egyptians, Babylonians, Greeks and Inkas processed their ratios and proportions with *machines*—or the “automatic knowledge”, as Serres (2017) calls it—like knotted ropes, tables of chords, and so on. Compared to classroom instruments such as rulers and protractors, the knotted ropes of the *harpedonaptai* provide a different vision of an analogical approach, where the measuring device itself embodies the varying proportionality of a relational world. Pedagogical approaches that invite students to learn from the early use of machines or tools create opportunities for young people to engage in the analogical aspects of measurement practices. Moreover, this archeological approach helps learners understand the way that measuring tools—be they fingers and feet or rulers and protractors—are not simply transparent recorders of number and magnitude.

PROCESS AND TIME

The relatively recent image of time in Western cultures is that it flows linearly, uni-dimensionally, continuously, sequentially from past to present to future, and is teleological (Mikulan & Sinclair, 2023). While mathematics education discourse often obeys this time-as-arrow imagery, in terms of developmental progressions, for example, or mastery discourse, mathematics itself is often characterized as timeless. Students are said to move from process-based learning (material verbs and embodied enactments) to object-based conceptualisations and proofs, where the latter concern a-temporal concepts and relations. De-temporalisation is a key feature of mathematical proof (Balacheff, 1988), signalling the desire for that which will never change, and has always existed.

⁴ Deleuze and Guattari often point to infinitesimals as examples of elastic measures. This notion was controversial in the 17th century, but used to measure the area and volume of all sorts of unusual shapes.

When time does come up—as it does whenever a process is under study—it is curbed, as with the Cartesian coordinate system or the banishing of curves involving two independent motions (as in Archimedes’ *neuses* constructions or spirals), or the nounification of discourse (Burton & Morgan, 2000).

And yet the socio-ecological is steeped in time, both cultural time and geologic time. We find important alternative temporal tropes across different socio-ecologies, where social structures are meant to be in analogical relation to temporal processes. The mathematics of temporality needs to be unpacked and explored, so that non-linear examples might be discussed. Alternatives to the single arrow might be a web or loop, all structural forms that entail some kind of mathematization. Each of these carries different assumptions about temporal relationality. And each conveys a different socio-ecological perspective, which might be hierarchical or cooperative, extractive or nurturing, imposing subordination or soliciting mutual reciprocity. Time comes in complex forms, but is also lived. The Traditional Tongan social structure is dominated by inequality relations and kin ranking hierarchies, emphasizing structural relations based on status and power across generations and genders (Ascher, 2002). This ordering is arboreal. For traditional Basque communities, “the idea of circularity pervades many of their interactions” (Ascher, 2002, p. 130), shaping their social life and their agricultural rhythms. Village houses are built in circles, in valleys encircled by rising mountains, and “circular exchange” marks their sharing of bread and labour. Ascher (2002) notes that this reliance on circular structures reflects how the traditional Basque community used a notion of “equivalence relation” which relied on a kind of modular or clock arithmetic. Similarly, the Borana in Ethiopia create a circular structure to manage the changing of leadership, from one generation to the next, allowing for sons to replace their elders every forty years. This model recognizes the rivalry between the displaced group and the new group, but also recognizes the alliances formed by two groups who are across the circle (Ascher, 2002).

A proportional study of linked circular processes remains a fascinating mathematical challenge. This challenge with marking time and its repetitive structure relates to how various cycles interrelate, so that closed circles must somehow become open again, as they relate to other circles. Correlating the days, the seasons, and the years, for instance, or the lunar cycle with the solar cycle is a fascinating material-mathematical challenge. And humans have always been challenged to find a way to integrate what are arbitrary counts and units (the week, for instance) with these varying material processes. It is through the cyclical nature of time that humans are said to link with cosmic time and other more-than-human supra-cycles, so that the Mayans are said to have “mathematized time” as a way of linking earthly activity with supernatural forces (Ascher, 2002, p. 63).

CONCLUSION

Measurement practices are both necessary and problematic in ecological research. We have suggested here how measurement might be taught and pursued as part of a means of building ecological alliances across spatio-temporal relations, so that measurement is seen for what it is—an activity thoroughly implicated in complex socio-ecological processes. Rather than treat measurements as abstract, detached and neutral coverings of land and matter, and rather than dismiss all measure as a distorted misconstrual of nature, we seek ways to better understand the imbrication of matter and measurement. This newly instaures Inclusive Materialism and shows how mathematics is not ontologically separate from the generative provisionality of the material world.

We have discussed historical examples where measuring is tuned to dynamic proportionalities. When we look to the school curriculum, we find instead an overemphasis on standardized units and covering objects with those units. This, we feel, ignores the underlying variability and plasticity of the material world, as well as the analogical nature of measuring. This overemphasis on standardized units produces two negative effects: it limits measurement to banal practices that cannot be cultivated as part of a socio-ecological sensibility, and at the same time ensures that measurement will be a destructive erasure of the temporal nature of ecologies.

We believe that students and teachers must become keen observers of reciprocal relational processes. A simple example might be children exploring analogical relations using dynamic grids. In the app called *The Griddler* (Jackiw, 2024), grids become highly plastic and responsive to the environment under study, so that students can create and manipulate grids as they explore the changing terrain or changing climate of an ecology (Figure 1a). Instead of measuring a lake by covering it with standard units, they let the *relation* between the lake and other lakes lead them in creating responsive grid units, so that measurement becomes profoundly provisional and attentive to distinctive differences; the grid becomes a cascade of linked lakes, a meshwork of relationality, merging measurement with materiality—(Figure 1b to 1c). Changing the size/shape of the unit (from square to parallelogram ...) changes what is being measured and how the measurements compare. Fractal measures can be created by subdividing grid units around the unruly border of the lake, so that measurement occurs on two scales simultaneously.



Figure 1(a): Gridding a map; (b) Comparing lake sizes; (c) Changing units to make analogical comparisons

This provides a simple context in which measurement shifts from being a formula-driven, static enterprise resisting matter and time, to one that underscores the relational contingencies of the earth. Could Griddler help students better understand how measuring practices “belong” to matter, to earthly variability, to climatic regimes and fluctuating floods? How can we help students treat grids as malleable and responsive, as pliable and material, as analogical and processual?

References

- Alder, K. (2002). *The measure of all things: The seven-year odyssey and hidden error that transformed the world*. New York: Simon & Schuster.
- Ascher, M. (2002). *Mathematics elsewhere: AN exploration of ideas across cultures*. Princeton University Press.
- Balacheff, N. (1988). Aspects of proof in pupils' practice of school mathematics. In D. Pimm (Ed.), *Mathematics, Teachers and Children: a Reader* (pp. 216-235), London, UK: Hodder and Stoughton.
- Brown, S. (2005). The theatre of measurement: Michel Serres. In C. Jones and R. Munro (Eds.), *Contemporary Organization Theory* (pp. 215-227). Oxford: Blackwell Publishing.

- Burton, L. & Morgan, C. (2000). Mathematicians Writing. *Journal for Research in Mathematics Education*, 31(4), 429–453.
- Coles, A. (2017). A relational view of mathematical concepts. In de Freitas, E., Sinclair, N., Coles, A. (Eds.). *What is a mathematical concept?* Cambridge University Press.
- Cooperrider, K., & Gentner, D. (2019). The career of measurement. *Cognition*, 191, 1-12.
- de Freitas, E. (2016) Deleuze, Ontology, and Mathematics. In Peters, M. (Eds), *Encyclopedia of Educational Philosophy and Theory*. Springer, https://doi.org/10.1007/978-981-287-532-7_374-1
- de Freitas, E., Sinclair, N., le Roux, K., Solares-Rojas, A., Coles, A., & Ng, O. (2022). New Spatial Imaginaries for International Curriculum Projects: Creative Diagrams, Mapping Experiments, and Critical Cartography, *Qualitative Inquiry*, 28(5), 507–521.
- de Freitas, E. & Sinclair, N. (2022). Proportion, Analogy and Mixture: Unearthing Mathematical Measurement practices: In T. Ingold (Ed), *Knowing from the Inside: Cross-Disciplinary Experiments with Matters of Pedagogy* (pp. 63–80). London: Bloomsbury.
- de Freitas, E. & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. New York: Cambridge University Press.
- Deleuze, G. & Guattari, F. (1987). *A thousand plateaus: Capitalism and schizophrenia* (B. Massumi, Trans.). Minneapolis, MN: University of Minnesota Press. (Original work published 1980).
- Farr, R. H. (2010). Measurement in navigation: conceiving distance and time in the Neolithic. In C. Renfrew & I. Morley (Eds.), *The Archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 19-27). Cambridge, UK: Cambridge University Press.
- Jackiw, N. (2024). The Griddler. [software application]. The Tangible Mathematics Project.
- Lugli, E. (2019). *The making of measure and the promise of sameness*. Chicago: University of Chicago Press.
- Mikulan, P. & Sinclair, N. (2023). *Time and Education: Time pedagogies against oppression*. Bloomsbury, London.
- Mikulan, P. & Sinclair, N. (2019). Stratigraphy as a method for studying the different modes of existence arising in the mathematical classroom. *ZDM Mathematics Education*, 51(2), 239-249.
- Renfrew, C. & Morley, I. (2010a). *The archeology of measurement: Comprehending heaven, earth and time in ancient societies*. Cambridge, UK: Cambridge University Press.
- Renfrew, C. & Morley, I. (2010b). Measure: Towards the construction of our world. In C. Renfrew & I. Morley (Eds.), *The Archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 1-6). Cambridge, UK: Cambridge University Press.
- Schuller, K. (2018). *The biopolitics of feeling: Race, sex, and science in the nineteenth century*.
- Serres, M. (2017). *Geometry*. London, UK: Bloomsbury Press.
- Serres, M. (1995). Gnomon: The beginnings of geometry in Greece. In M. Serres (ed), *A History of Scientific Thought* (originally published 1989). Oxford: Blackwell.
- Smith, M. & Young, J. (2022). *Does the earth care? Indifference, Providence, and Provisional Ecology*. University of Minnesota Press.
- Urton, G. (2010). Recording measure(ment)s in the Inka khipu. In C. Renfrew & I. Morley (Eds.), *The Archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 54-68). Cambridge, UK: Cambridge University Press.
- Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C, Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, 139(2), 352-402.

COMPOSITIONS WITH MATHEMATICS EDUCATION, PHILOSOPHIES OF DIFFERENCE AND CULTURALLY DIVERSE GROUPS IN SEARCH OF OTHER POSSIBILITIES FOR A HUMANITY COMMITTED TO SOCIO-ECOLOGICAL CONCERNS

Cronopies+ Collective, Roger Miarka

São Paulo State University (UNESP), Institute of Geosciences and Exact Sciences, Rio Claro

This article presents a discussion about composing Mathematics Education along with philosophies of difference, that has been carried out by the Collective Cronopies+, in Brazil, since 2013, working with diverse cultural groups in situations of social vulnerability, negligence and resistance, in terms of their power to affirm lives, in search for other possibilities for a humanity socio-ecologically committed. Assuming that Mathematics Education can be understood both as a research area and as movements that cross it, which is denoted, respectively, by Mathematics Education and mathematical education, or M[m]athematics [E]ducation when both ideas are brought together, it is asked “In a panorama in which one understands M[m]athematics E[e]ducation as a clash between area and movement, how can we assume other possibilities of humanity in the production of Mathematics Education and mathematical educations?”. As main results, we present ways of operating with difference in methodological terms, what is meant by rigor in this perspective, and how it could help produce other possibilities for a humanity committed to socio-ecological concerns.

INTRODUCTION

In October 2019, at the *Science and Culture Forum of the Federal University of Rio de Janeiro* (UFRJ, 2019), Ailton Krenak – an Indigenous leader, environmentalist, writer, and participant in Brazil's political life – asks: “Why continue reproducing the values of a *vast humanity* that has gone wrong?”. Faced with Krenak's questioning, the audience remains silent. An uneasy silence. A silence absorbed in the absurd: this *vast humanity* that has gone wrong ... What values sustain this *vast humanity*?

One assertion: “*Vast humanity* has gone wrong”. Another assertion: “We continue reproducing the values that sustain this so-called *vast humanity*”. Such *vast humanity*, in its *vastness* – understood as universality, totality – is restricted to a single sign, to a unique mode of participation in humanity. Which humanity? White, Euro-American, male, heterosexual, cisgender, with standardized attributes, both physical and intellectual, employed and contributing to the production of wealth and tax collection ... the same humanity that has been carelessly exploiting the world resources.

What life is affirmed with and in this *vast humanity* with shades of uniqueness? What values sustain it? Uniqueness denies variation and other existences by affirming as life a unique way of existing, a single way of being human, a way that has been destroying any possibility of a future for the human existence.

Having that said, the purpose of this article is to produce ways to investigate other possibilities of humanity within Mathematics Education research, in which it is assumed the power of groups, and the way they live, that have been left out of academia. We understand that bringing other ways of humanity to Mathematics Education can help deal with the socio-ecological concerns established by

the ICMI Study 27.

With this aim, firstly, we are going to establish Mathematics Education as a porous space that can be somehow transformed when it is not taken as reference for transformation, but as a knowledge area that may be changed with contributions of diverse groups whose knowledge has been left out of academia. Then, we are going to present some possible contributions of philosophies of differences to this goal, that have been produced by the Cronopies+ Collective of the São Paulo State University (UNESP) of Rio Claro.

MATHEMATICS EDUCATION AS AREA AND AS MOVEMENT

Clareto and Miarka (2015) discuss how Mathematics Education can be understood as flows of diffuse movements connected to teaching, mathematics, education etc. – what they call mathematical educations – that stabilize themselves in an area – Mathematics Education – and, with this, create a system of belonging and, consequently, of non-belonging. These flows, in turn, operate again in the conformed area, producing cracks and the possibility of its constant updating, what can be denoted as M[m]athematics [E]ducation when both ideas are brought together.

It is important to emphasize that this way of understanding M[m]athematics E[e]ducation – deepened in Silva and Miarka (2017) – does not try to dichotomize Mathematics Education and mathematical education movements, but assumes that the power of creation occurs in “between” one and another in a kind of dialogic game in which area feeds movements that provoke and tear it, and movements gain in possibility of permanence when being received and conformed in an area. Without movements, an area might wane because of the lack of new elements that promote creation; without an area, movements might dissolve by their diffusive nature.

If we use the studies of authors who assume the creative power of difference, commonly called philosophers of difference, such as Deleuze and Guattari (1992, 1995, 1997), we see an area operating amid mechanisms of identity and movements operating within difference.

Since 2013, the Cronopies+ Collective has been seeking to operate within difference by understanding it as a powerful concept for production of inventive knowledge that – not necessarily committed to this idea – has historically been submitted to identity aspects, which can also be seen in research within Mathematics Education, especially in the researches that prioritize categories and generalizing aspects, working through mechanisms of induction or deduction.

To this end, the Collective has been operating with experiments in research involving cultural communities in situations of social negligence, vulnerability and resistance⁵, assumed in their power of life (Gomes, Amaris Ruidiaz & Miarka, 2024), such as quilombola communities⁶ (Gondim, 2021, 2018, Gondim & Miarka, 2020, 2019), Indigenous communities (Orjuela Bernal, 2023, 2018, Orjuela

⁵ *Situational*, to mark that it refers to groups amidst a combination or competition of events or circumstances at a given moment, but with the possibility of change, thus differing from the concept of *condition*. *Negligence*, regarding the indifference of social agents who do not take into account their practices and often do not take the necessary precautions for the maintenance of these groups. *Vulnerability*, to indicate a risky situation, whether for social, economic, environmental, or other reasons. *Resistance*, through their life-sustaining practices despite a socially unfavorable panorama.

⁶ *Quilombola* is the Brazilian term used to designate descendants of Africans who came enslaved to Brazil during its colonial period. It derives from the word “quilombo”, which are communities created by people who had escaped from the farms where they worked enslaved.

Bernal & Miarka, 2018), collectives of black and transgender women (Cortés Suárez, 2020, Cortés Suárez & Miarka, 2023, Cortés Suárez, Amaris Ruidiaz & Miarka, 2020), Brazilian Landless Worker's Movement (Lopes, 2023, 2016) etc. In this movement, attention is paid to researching-with and not about, with immersive fieldwork (Cronopies+, Orjuela Bernal, Lopes & Miarka, 2024, Gondim & Miarka, 2019), initial research questions that can change considering the urgencies of the groups with which one works, dialogues with knowledge from other worldviews, and productions of writings that experiment with other ways of producing knowledge, assuming transdisciplinary possibilities.

In a panorama in which one understands M[athematics] E[ducation] as a clash between area and movement, how can we assume other possibilities of humanity in the production of Mathematics Education and mathematical educations?

In particular, we understand this question as especially important for ICMI Study 27 if we assume that, in order to deal with urgent socio-ecological issues, Mathematics Education needs to be fissured with other possibilities of humanity.

PHILOSOPHIES OF DIFFERENCE AND MATHEMATICS EDUCATION: SOME METHODOLOGICAL ARTICULATIONS

Before we move around this question, it is important to point out that the production of a response to this question is due to the power of its operation. In accordance with philosophies of difference, power is evaluated by the possibility of multiple outputs, that can be evaluated with criteria of *meaning* and *value*, that could take in consideration socio-ecological concerns.

The pieces of work by the Cronopies+ Collective⁷ are presented as compositions of texts, sounds and images, not seeking consistency with a previously given Reference Plan. The productions take place in the “here and now”, in a state of attention to what happens in the environment. It should be realized that even if the Reference Plans already conceived on the studied groups are not taken as primary elements, they are not discarded. They are taken as constituents of a Consistency Plan, to be operated as elements of an Aesthetical Plan, which eventually gets materialized as problematization exercises⁸.

Such exercises of problematization can be understood as movements of fissures of different territories of knowledge (such as Indigenous, quilombola, mathematical education, mathematics, etc) that the works cross with. In particular, with regard to Mathematics Education as an area, one of the purposes of these works is to give way to other connections not yet available, including those ones related to socio-ecological concerns. In this way, the area does not become a basis for discussing the elements of production that occurred next to it, but one more element to compose with. The commitment is with the multiplicity of knowledge and with cultural groups whose knowledge has been historically neglected, in the search for more and more connections, and not primarily with the Mathematics Education area itself. Mathematics Education becomes, in this perspective, an element to be composed with others, not necessarily traditionally used, coming from the most varied places, theories, prays, practices, traces of experiences, books, paintings, music, etc (Miarka, 2019).

⁷ Productions of the Cronopies+ Collective can be found in <cronopiando.org>

⁸ Reference, Consistency and Aesthetical Plans are concepts used by Deleuze and Guattari (1992) to talk about the mechanism of producing knowledge in, respectively, Science, Philosophy and Arts.

Another point to be highlighted is related to the ways of using methodological instruments, which are no longer legitimate by a theoretical framework. If so taken, they would resemble devices in an area, that act as protective elements for the productions that take place in a certain area and which are constructed by already established paths – or striated paths, if we use a Deleuzian terminology – so that other productions could follow. Such logic would approximate that one of Science, in which according to Deleuze and Guattari (1992), the legitimacy of its productions is given by a functional mechanism in coherence with the territory already established for the area.

In the perspective of difference, the use of methodological instruments is given by the intensity they offer to give way to *affects*⁹ of an event. With this, it is not the methodological instruments that legitimize research, but what research produces together with a composition, in which such instruments are only tools to produce intensities, coming from an imaginary box from which we can draw. In conversation with Foucault, Deleuze understands that theories themselves can be taken as toolboxes.

A theory is like a toolbox. It has nothing to do with the signifier [...] It must work, and not for itself. If there are no people to use it, starting with the theoretician who ceases to be a theoretician, then it is not worthy, or the moment to use it has not yet arrived. (Foucault, 1979, p.71, our translation).

With this, the use of different instruments – theoretical or methodological – is not motivated by coherence with what has already been given, experienced and considered successful, with its shades of *vast humanity*. On the contrary, the logic behind that assumes the operation of instruments in composition, as a possibility of invention. It works if it produces multiple outputs. The search is not for the truth, a movement considered arboreal, but for the chaotic, rhizomatic, uncontrollable ramification.

We may wonder how not to get lost amid “so much freedom” of using methodological tools. Most research of the Collective uses the Cartographic Method (Kastrup, Passos & Escossia, 2009), produced with inspiration in the work of Deleuze and Guattari. It is an *ad hoc* path, in which one seeks to follow subjectivation processes.

The word *method* tells one about ways to achieve a goal. In the case of Cartography, the word is used in a very proper sense, which aims to subvert the more traditional sense in which the term says of a path that is expected to be right (or almost certain) to reach an end. In the case of Cartography, the method says more about a set of behaviours that are not legitimized by itself, but by the possibility of maintaining a movement of walking in which the goals are not previously taken but are in the way itself. The researcher, in this perspective, rather than using methodological tools, is concerned with methodological practices, in which elements such as interventionist practice and being in an attitude of attention, composition and production gain importance.

The usual academic writing is no longer enough to show these processes, as in order to work one needs to handle multiple outputs. Writing, then, carries a writing policy, becoming moving, dragging to itself various elements like poetry, music, images etc, in search of the creation of intensity to produce and move researcher, readers and other subjects affected by the research (Gomes & Miarka, 2024, Amaris Ruidiaz & Miarka, 2024, 2018, Miarka, Amaris Ruidiaz, Orjuela Bernal & Gondim,

⁹ Affect is a concept derived from Spinoza that places emphasis on bodily or embodied experience (Deleuze & Guattari, 1995).

2018, Chisté & Miarka, 2022, Cortés Suárez & Miarka, 2023, Gondim & Miarka, 2018, 2017, Casal, 2023).

Moving away from a policy of research and writing hitherto conceived in the Academy does not mean that these works are uncompromising or lacking rigour. On the contrary, there is a constant exercise of commitment to the event, which Suely Rolnik calls ethical-aesthetic-political.

[...] ethical is the rigour with which we listen to the differences that are made in us and affirm the becoming from these differences. The truths that are created with this kind of rigour, as well as the rules that have been adopted to create them, only have value as driven and demanded by brands. Aesthetic because this is not the rigour of the domain of an already given field (field of knowledge), but the creation of a field, a creation that embodies the marks in the body of thought, as in a work of art. Political because this rigour is that of a struggle against the forces in us that obstruct the springs of becoming. (Rolnik, 2003, p.7)

THE WORK OF THE CRONOPIES+ COLLECTIVE WITH MATHEMATICS EDUCATION, PHILOSOPHIES OF DIFFERENCE AND CULTURALLY DIVERSE GROUPS

The Cronopies+ Collective has been operating with philosophies of difference in their investigations in Mathematics Education, understanding that, in this way, they can promote difference as a producer of knowledge committed to culturally diverse groups in search of other possibilities of humanity. From this perspective, Science, Philosophy, and Art, as well as Mathematics Education, are not taken as fields with an end in themselves, but as means to deal with the urgencies of the communities with which they work in interventionist research. Together, these fields are operated with the possibility of, in this composition, creating inventive modes with the power of production alongside elements still little mobilized in research. In this wake, the research of the Cronopies+ Collective has been paying attention to groups whose practices still do not operate assertively in academia, taking them in their power of affirmation of life.

The Collective understands that its research intertwines and operates in the manner of a *rhizome*, with principles of *connection* and *heterogeneity*, where any point can and should be connected to any other; principle of *multiplicity*, where the multiple is treated as a noun – multiplicity – without listing a primordial generating element; principle of *a-signifying rupture*, against overly significant cuts that separate or cross structures; and principle of *cartography* and *decalcomania*, as they are not justified by any structural or generative model (Deleuze & Guattari, 1997).

We can point out some characteristics of this work, such as researching-with and not about; immersive fieldwork; initial research questions that may change in light of the *urgencies* of the groups involved in the research; productions of writings that experiment with other forms of knowledge production; etc (Miarka, 2019).

It is important to highlight that the *urgencies* mentioned above frequently are related to the possibility of the continuation of the existence of the studied groups and of their practices. In particular, the work with Indigenous and quilombola communities has shed light on the importance of bringing to Mathematics Education other ways of understanding the relation with earth (Orjuela Bernal, 2023, 2018, Orjuela Bernal & Miarka, 2018, Gondim, 2021, 2018, Gondim & Miarka, 2020, 2019).

FINAL CONSIDERATIONS INVITATION

In this vein, the Cronopies+ Collective continues to operate, problematizing, and inventing with...

... educations and bands and mathematics and women and black people and ethnomathematics and cultures and nguanguela and hip hop and nasas and children and quilombolas and guaranis and kaiowás and social movements and expressions of gender and sexualities and collectives and peace and writings and empowerment and schools and formations and policies and experimentations and spellings and autisms and fields and ...

With these notes, we conclude this article, which proposes itself to serve as a producer of multiple outputs for the reader who is into its reading, that are invited to look for other possibilities of humanity that may help change Mathematics Education and produce a mathematical education committed to socio-ecological concerns.

Acknowledgements

I thank Cronopies+ for all the support, collaborative and cooperative work. After all, we are green and moist beings, inspired by Julio Cortázar's imagination (Cortázar, 2009), who, distracted, keep on losing what we have in our pockets, and, with them emptied, we learn day after day to produce other worlds with clouds and leaves, seasoned with our own tears and smiles, all in name of a more colorful, diverse, and equitable world

I also thank the São Paulo Research Foundation (FAPESP) for its financial support through grants 2017/23227-1, 2018/24911-6, 2019/01101-1, 2020/14396-7, and 2023/00778-3.

References

- Amaris Ruidiaz, P. J., & Miarka, R. (2024). The Writing Politics of a Body among the Literature [and] the Philosophy [and] the Research... *Folios*, (59), 158-169. (<https://doi.org/10.17227/folios.59-16604>)
- Amaris Ruidiaz, P. J. & Miarka, R. Writing-body-Experience and literature: what can writing (in a [mathematics education] research)? *Alexandria*, Florianópolis, 11 (3), 13-31. (<http://dx.doi.org/10.5007/1982-5153.2018v11n3p13>)
- Casal, J. R. B. (2023). *Escritas em divã: encontros ético-estético-políticos em Educação Matemática*. Rio Claro, Brazil: UNESP. (<https://repositorio.unesp.br/server/api/core/bitstreams/30d19a3b-fe22-4e6b-bd51-c4f72d753eba/content>)
- Chisté, B., & Miarka, R. (2022). um esquizoexercício ensaístico...: fabulações da e na e com a infância. *Alegrear*, 30. (<https://alegrar.com.br/alegrar30-14/>)
- Clareto, S. M., & Miarka, R. (2015). AefFecTivE MATHemAtiCS eDucAtiOn: names and movements in inside outs. *Bolema*, 29 (53), 794-808. (<http://dx.doi.org/10.1590/1980-4415v29n53a01>)
- Cortázar, J. (2009). *História de Cronópios e de Famas*. Rio de Janeiro, Brazil: Civilização Brasileira.
- Cortés Suárez, J. M. (2020). *Denúncias e anúncios sobre camadas de vulnerabilidade social e Educação Matemática junto a um grupo de mulheres pretxs que assumiram empoderar-se por meio da tecnologia*. Rio Claro, Brazil: UNESP. (<https://repositorio.unesp.br/items/6b2279f6-10e1-4fbf-bf57-f77c78bcd320>)
- Cortés Suárez, J. M., & Miarka, R. (2023). Writing, Race, Racism, Education and their Silences. *Alegrear*, 31. (<https://alegrar.com.br/alegrar31-3/>)
- Cortés Suárez, J. M., Amaris Ruidiaz, P. J., & Miarka, R. (2020). Género y raza y sexo y educación matemática: políticas de visibilidad de cuerpos disidentes. In H. J. L. Gonçalves (Ed.). *Educação Matemática & Diversidade(s)*. Porto Alegre, Brazil: Editora Fi, 183-194

- Cronopies+, Orjuela Bernal, J., Lopes, R. M. G., & Miarka, R. research and/in (de)[compo/field]sition. *Bolema*, 38, e230095, 1-21. (<http://dx.doi.org/10.1590/1980-4415v38a230095>)
- Deleuze, G., & Guattari, F. (1997). *Mil Platôs: volume 5*. São Paulo, Brazil: Editora 34.
- Deleuze, G., & Guattari, F. (1995). *Mil Platôs: volume 2*. São Paulo, Brazil: Editora 34.
- Deleuze, G., & Guattari, F. (1992). *O que é a Filosofia?* São Paulo, Brazil: Editora 34.
- Foucault, M. (1979). *A microfísica do poder*. Rio de Janeiro, Brazil: Edições Graal.
- Gomes, D. O., Amaris Ruidiaz, P. J., & Miarka, R. (2024). Affects-experimentations of a MaThemaTics EduCatioN that affirms lives and a correspondence that practices a writing to come. *Hipatia*, 9(1), 144-158. (<https://ojs.ifsp.edu.br/index.php/hipatia/article/view/2482>)
- Gomes, D. O., & Miarka, R. (2024). La Composición de un Rigor sin Órganos a través de una Ópera de Movimientos Aberrantes. *Praxis & Saber*, 15(42), 1–22. (<https://doi.org/10.19053/upct.22160159.v15.n42.2024.16811>)
- Gondim, D. de M. (2021). *Manifestos Quilombolas: desta terra, nesta terra, para esta terra*. Rio Claro, Brazil: UNESP. (<http://hdl.handle.net/11449/215015>)
- Gondim, D. de M. (2018). *Ribeiras de Vales ...e experimentações e grafias e espaços e quilombolas e...* Rio Claro, Brazil: UNESP. (<https://repositorio.unesp.br/handle/11449/152579>)
- Gondim, D. de M., Miarka, R. (2020). The Mandira Quilombola Community and the Production of an Anthropophagic Identity. In M. Rosa & C. C. de Oliveira. (Eds.). *Ethnomathematics in Action*. Springer International Publishing, 41–56. (http://link.springer.com/10.1007/978-3-030-49172-7_3)
- Gondim, D. de M., & Miarka, R. (2019). Uma Comunidade dos Cantos: notas de uma experiência em campo como expressão de uma educação (matemática) dos sentidos. *HISTEMAT – Revista de História da Educação Matemática*, 5(2), 100-111. (<https://histemat.com.br/index.php/HISTEMAT/article/view/273>)
- Gondim, D. de M., & Miarka, R. (2018). To think with the body as well as to think with space: imagetical aforisms that affirm a learning by trails. *Educação Matemática em Revista*, Brasília, 23(60), 169-183. (<https://www.sbemrasil.org.br/periodicos/index.php/emr/article/view/1348>)
- Gondim, D. de M., & Miarka, R. (2017). The Constitution of a Plan of Intensities: learn and maths and difference and avalanche-writing and... *Perspectivas da Educação Matemática*, 10 (22), 115- 131. (<https://periodicos.ufms.br/index.php/pedmat/article/view/3359>)
- Kastrup, V., Passos, E., & Escossia, L. (Ed.). (2009). *Pistas do método da cartografia: pesquisa intervenção e produção de subjetividade*. Porto Alegre, Brazil: Sulina, 2009.
- Lopes, R. M. (2023). *Escola Itinerante Herdeiros da Luta de Porecatu*. Rio Claro, Brazil: UNESP. (<https://repositorio.unesp.br/items/43d84eda-abcc-4290-80bb-38df070ebc24>)
- Lopes, R. M. (2016). *Histórias de uma pesquisa(dora) em uma escola do campo com professores que lecionam Matemática*. Rio Claro, Brazil: UNESP. (<https://repositorio.unesp.br/handle/11449/141480>)
- Miarka, R. (2019) Sobre parafusos, miséria e Educação Matemática: discussões metodológicas na composição de Educação Matemática com filosofia da diferença. In M. A. V. Bicudo & A. P. Costa (Eds.). *Leituras em Pesquisa Qualitativa* (pp.161-176). São Paulo, Brazil: Editora Livraria da Física.
- Miarka, R., Amaris Ruidiaz, P. J., Orjuela Bernal, J. I., & Gondim, D. M. (2018). O que pode a apropriação de elementos artísticos na pesquisa em Educação Matemática? In R. S. R. da Silva (Ed.). *Artes em Educação Matemática* (pp. 123-142). Porto Alegre, Brazil: Editora Fi.
- Orjuela Bernal, J. I. (2023). ... e teko e arandu e produção de subjetividades e Educação Superior e educações outras...: modos de vida criados e afirmados por kaiowás e guaranis. Rio Claro, Brazil: UNESP. (<https://hdl.handle.net/11449/251248>)
- Orjuela Bernal, J. I. (2018). *Indígenas, Cosmovisão e Ensino Superior: [algumas] tensões*. Rio Claro, Brazil: UNESP. (https://flihtml5.com/garcx/xwxi/basic#google_vignette)

- Orjuela Bernal, J. I., & Miarka, R. (2018). [I]ndigenous [E]ducation, [M]athematics [E]ducation and [E]thnomathematics. *Zetetiké*, 26(1), 41-58. (<https://doi.org/10.20396/zet.v26i1.8650820>)
- Rolnik, S. (2003). Pensamento, corpo e devir: uma perspectiva ético/estético/política no trabalho acadêmico. *Cadernos de Subjetividade*, 2(1), 241-251. (<https://www.pucsp.br/nucleodesubjetividade/Textos/SUELY/pensamentocorpodevir.pdf>)
- Silva, M. A. da, & Miarka, R. (2017). Geni, Research in [M]athematics [E]ducation and the Zeppelin. *Perspectivas da Educação Matemática*, 24(10), 752-767. (<https://periodicos.ufms.br/index.php/pedmat/article/view/5331>)
- UFRJ (2019, 11 out.). *Novas subjetividades, novas possibilidades de existir e resistir*. <https://forum.ufrj.br/novas-subjetividades-novas-possibilidades-de-existir-e-resistir/>

RE-VALUING, DE-CENTRING, UN-LEARNING: ENVISAGING A MATHEMATICS EDUCATION FOR SOLIDARITY AND HOPE

Hilary Povey¹, Corinne Angier²

¹Sheffield Hallam University, ²University of Stirling

Neoliberalism and the climate emergency that accompanies it need young people worldwide to believe that they have the power, working together, to re-fashion our relationship with the natural world and with each other. We argue that developing such a conviction requires a mathematics education for solidarity and hope and we consider what that might look like.

INTRODUCTION

As minority world authors engaging with mathematics education's socio-ecological turn (Coles, 2022), we need to recognise that some of its most important insights have been "blindingly obvious" to most of the majority world for centuries. Even within European socialism, we can find an awareness of the interconnectedness of human beings and the living world from the agrarian socialists of the middle of the 17th century who "envisioned an ecological interrelationship between humans and nature, acknowledging the inherent connections between people and their surroundings" (Wikipedia, n.d.) to Marx who argued that we are material beings, belong to nature, exist in its midst and are alienated when the reciprocity between self and nature breaks down as it does under capitalism (Eagleton, 2011).

What is perhaps new is a growing awareness in the minority world of the vulnerability caused by "an illusory sense of our own ontological and ecological independence" (Plumwood, 2012 p.16), fostered by the capitalist, colonial project of the "West", a misnomer that functions to make differentiation from "the other" seem natural (Hall & Gieben, 1992). This hegemonic project "others" people who are deemed unworthy or dispensable, philosophies that are derided as primitive or fairy stories and the planet understood as a resource to be exploited. This community of "others" is ever growing as inequalities deepen and environments are damaged. We shift to a socio-ecological perspective whilst maintaining the critique that the rotten core is the inseparable duo of colonialism and capitalism. Those of us who are privileged are having to un-learn and to acknowledge that survival requires air, oceans, climate systems and species diversity to be held in common. At the very time that we need to collaborate across borders and learn from those who have always been response-able, education in the West and beyond has been saturated with neoliberal values of competitive individualism to prepare for a life of acquisitive consumer capitalism and of the fabrication of the self. Mathematics classrooms can be experienced as hopelessly constrained by the dominant purpose of attaining qualifications with a high exchange value and by a performativity culture.

Limiting ourselves for reasons of space to within-school settings, we address the question: what are the aims, pedagogies and ethical standpoints that lend themselves to the cultivation of solidarity and hope? We understand hope as encompassing an expectation of alternatives and a realisation that they were always already existent and can be creatively re-imagined. Working collaboratively with mathematical activity enriched by plurality can contribute to un-learning the "othering" of people and philosophies and foster hope and, reciprocally, political solidarity (though we are concerned that

access to school may be prevented by war; by regimes hostile to the education of girls ([Why Girls' Education | Malala Fund](#)); by migration due to climate change and conflict; by child labour; or by the inhospitable nature of school causing anxiety and refusal).

The move to socio-ecological perspectives is not to be seen as leading to certainty, resolution, consensus or, even, coherence (Skovsmose, 2014; Povey & Adams, 2021) and we are mindful of the danger of a single story (Adichie, 2009). We need an axiological “shake-up”, challenging the ethics-free thought that mathematics education in neoliberal societies typically provides and resisting the “ethicides [...] that kill ethics and therefore any notion of history and justice” and operate against “the human need for sharing, bequeathing, consoling, mourning and hoping [...]” (Berger, 2016, p.83).

Here, we consider the context within which we write; the nature of mathematics itself; how mathematics education under neoliberalism is understood; socio-ecological approaches; and what they might offer us as we seek to articulate a praxis of solidarity and hope.

CURRENT SOCIAL AND EDUCATIONAL CONTEXT

We write as ex-teachers, teacher educators and grandparents who are aware of the seismic changes that a two generations span of technological change has wrought. We have witnessed and benefitted from hard won, often long forgotten, collective fights and now see reversals and losses that undermine any notion of ongoing improvement in the socio-ecological realm.

For many, a world not based on neoliberalism has become unimaginable. Wendy Brown (2015) refers to it as “a normative order of reason, [...] a widely and deeply disseminated governing rationality [which] transmogrifies every human domain and endeavour along with humans themselves” (p.9). It has colonised language and thinking to such an extent that it has successfully appropriated even social justice within education; teachers are made responsible for “closing the gap” created by inequality and non-recognition, and students are responsible for their own “failure” either through lack of inherent ability or through inadequate mental resilience. Meanwhile a toxic atmosphere of competitive individualism is to be valorised in the mathematics classroom with benevolent pity for those who lack the “wellbeing” to survive such an inhospitable environment.

A new form of education colonisation, the Global Education Reform Movement, the GERM (Sahlberg, 2015), has imposed neoliberal values, thinking and practices on the majority world, seeking to regulate every aspect of what teachers and students do. Enabled through powerful transnational organisations such as the World Bank and OECD, it is spearheaded by Pearson, a latter day East India Company, a giant textbook and testing corporation which claims to work in about 200 countries (approximately the total number of countries in the world) and to have 160 million users worldwide (approximately 1 in 50 of the world’s population) (see [Homepage | Pearson plc](#)). Education is not seen as a right, a (public) good, a moral enterprise, a vehicle for nurturing our links with the natural world and making us fully human; rather, it is as a commodity – a thing, a product, something with an exchange value in the human marketplace of neoliberalism, with national qualification regimes dominating budgets.

CURRENT CONTEXT: MATHEMATICS AND MATHEMATICS EDUCATION

Mathematics formats how we see the social world (Skovsmose, 1994), with political decisions invoking the use of mathematical models presented as neutral, value-free, not subject to question, with unjust policies becoming “unquestionable practical reality, a necessary evil” (Ernest, 2016, p. 8). Implicated in this are the AI systems, underpinned by mathematics, to which decision-making is increasingly being deferred; these can “facilitate housing discrimination, make racist associations, and exclude women from seeing job listings available to men [...] unfairly recommend longer prison sentences for black people, [fail] to accurately recognise faces of black skinned women and [cause] countless additional incidents of harm” (Evan Harris, 2023, p.4). Mathematics provides “a training in ethics-free thought” (Ernest, 2016, p. 6). Mathematical tools are embedded in the cost benefit analysis and statistical models that have privileged intensive destructive land use and the processing of finite resources into throwaway products. Harnessing mathematics to fight back, for example, through climate models exposes what has always been true: that only some results are acceptable and decisions are always ethical acts.

Kollosche (2018) uses a functionalist paradigm to review the possible social benefits of mathematics education. He concludes that there is no evidence that the mathematical content studied has any functional use but rather that mathematics education is appropriated as a mechanism for social selection and as a site for training in rule-following obedience. Biesta (2022) traces a shift from a humanistic to an economic agenda for education, and feminist thinking asks where the narrow canon that has become the almost global school mathematics curriculum came from, whose situated knowledge is it, and who might be damaged when it is mobilised as “a *resource* of western modernity, [...] which operates in the production of sub-humanities, as it upholds idealized subjectivities in school cultures” (Santos Fernandes et al., 2022, p.12) and as essential for the never ending “progress” of our exploitation of the earth (Arendt, 1951/2017; Povey & Angier, 2021). This canon is characterized by inflexibility: there is no space for a student to argue *but this is my mathematics* despite a history of scholars in the subject having done just that.

SOCIO-ECOLOGICAL PERSPECTIVES

This International Commission on Mathematical Instruction (ICMI) Study flags up the need to rethink the nature and purpose(s) of (mathematics) education through socio-ecological perspectives and to problematize not just neoliberalism and “usual” (we owe this phrasing to Mark Boylan) mathematics education but also the relationship between mathematics and our lived experiences, past, present, future, personal, educational, local and global. So, along with Pashby et al. (2020) we ask: “How has the modern/colonial ontology restricted our horizons and what we consider to be possible, desirable, intelligible and imaginable?” (p.159). We add: how can we open ourselves up to being taught by different ways of being in order to experience and sense the world differently, being aware of misinterpretations, idealizations and appropriations that are likely to happen in this process?

One way to do this is to listen to Indigenous scholars who use different imaginaries to move thinking away from binaries and their inevitable oppositional debates. Aguon (2022) uses the notion of desire which resonates with learning as “[i]magination’s heartwork [...] a beckoning of the heart and the imagination, not merely of intellect” (Hogan 2009 p.172) where we are “learning to mind” (Griffiths and Murray, 2017. p.39).

Desire is a thirding of the dichotomized categories of reproduction and resistance. It is neither/both/and reproduction and resistance. This is important because it more closely matches the experiences of people who, at different points in a single day, reproduce, resist, are complicit in, rage against, celebrate, throw up hands/fists/towels, and withdraw and participate in uneven social structures—that is, everybody. Desire fleshes out that which has been hidden or what happens behind our backs. Desire, because it is an assemblage of experiences, ideas, and ideologies, both subversive and dominant, necessarily complicates our understanding of human agency, complicity, and resistance. (Aguon, 2022, p.420-21)

We write in the context of a planet on fire, devastating and increasing inequality with a few corporations “owning” a vastly disproportionate share of the world’s resources, the consequently inevitable migration of peoples across the globe and now, writing in mid-2024, the most dangerous times in terms of nuclear threat since the early 1960s. Yet almost all mathematics, mathematics education and mathematics education research continues as business as normal (Boylan & Coles, 2017).

Drawing on Boylan and Coles (2017), we see socio-ecological approaches including though not restricted to:

- values of cooperation, equity, solidarity and respect (p.4).
- different ways of knowing (p.6) and a nuanced plurality of perspectives and a recognition of the importance of democratization of debates (p.4).
- supporting a collective vision of a desirable future (p.5).
- an ethics of the commons (p.5) and of care (p.7).
- taking into account interconnectedness, relationship and uncertainty (p.5).
- de-centering humans (p.8).

We assert the need for a transformative approach where “teaching and learning mathematics becomes subordinate to a process of students becoming engaged and critical citizens, able to critique the status quo and participate in social action” (Boylan & Coles, 2017, p.3). But the issues of which we speak are, in the West, “hiding in plain sight”. We conjecture that, alongside the hegemonic neoliberal discourse, one reason for this is that “usual” school (mathematics) education produces a sense of powerlessness and despair for many learners amongst the marginalized, the disadvantaged and the “other”. We cannot move forward in the process of transformation without solidarity with each other and with the natural world – and solidarity requires a sense of powerfulness and hope and the conviction that together we can make a difference in the world. It is this solidarity and how to achieve it that is our focus here.

A MATHEMATICS EDUCATION FOR SOLIDARITY AND HOPE

With this focus, we recognize but do not explore here the value of a civil rights approach (see, for example, [Radical Equations: The Algebra Project Drawing on the Past: The Roots of Our Movement — Civil Rights Teaching](#)). Nor do we address the vital role that mathematics has to play in allowing students to interrogate and seek to change the material conditions of their lives and to live in harmony with the planet (for example, Solares et al, 2022; [Data Science - youcubed](#)). We see young people themselves (<https://fridaysforfuture.org/>) demanding we teach the future ([Teach the Future: Campaign for climate education](#)) and “get real” about what mathematics education is needed in the context of the climate emergency and we are confident these ideas, that directly address socio-ecological issues through curriculum content, will be well represented elsewhere in this study. But

our concern here is with a mathematics education that will allow learners to have their agency and voice recognised and supported; to engage in collaborative activity within the community of their classroom; to develop a sense of powerfulness, solidarity and hope in and through themselves with others and with the planet; and to experience hope-in-the-present (Povey & Angier, 2021). We suggest three threads which are all represented in scholarship and practice that might contribute to the axiological re-set for which we are calling.

First is a decolonising ontology that confronts human exceptionalism and Western elitism by placing mathematical knowledge in global and historical contexts. In mathematics, structures are used to contain complexity and contradiction but the structures evolve and change over time. The global history of mathematics points to human decisions and demonstrates that things have been different in the past or in other places and therefore might be different in the future. Becoming aware that mathematics is uncertain and human-made can be significantly empowering (Povey, 2014). There are opportunities in the classroom to consider what happens when people choose to flout the rules and there is sometimes a peculiar power in doing this because it is not expected. Within a broader conception of the nature of mathematics there is room for autotelic exploration of the geometry of art, games and puzzles from around the world, the history of systems of number and measure and so on, themes connected to cultural activity and closer to the mathematics present in Indigenous cultures. The much-contested new curriculum in New Zealand/Aotearoa attempts to do this.

Second is a praxis which seeks, welcomes and values the mathematics within the funds of knowledge of students; recognising the contribution a young person brings reinforces their legitimate participation (Fraser, 2007). This knowledge may relate to the practices of non-dominant groups, practices which are not generally even labelled mathematics. Munir Fasheh, a Palestinian mathematics teacher and scholar, provided a compelling account (1990) of his realization that his mother's dressmaking was also mathematical thus drawing attention to the contrast between the constrained school curriculum and the awareness that the features and foundations of mathematics "like those of art, poetry, religion, and other human activities and cultural productions [...] are as wide and deep as social existence" (Restivo, 2017, p.254).

Third is a praxis that affords space for playfulness and curiosity. Exercising curiosity (Povey, 2020) is a joyful experience and is a powerful and wide-ranging incentive to search for knowledge; such knowledge is imbued with meaning-fullness (Jardine, 2013). Curiosity starts from the world around us and allows the world to speak to us, where we are "caught in its regard not just vice versa" (Jardine, 2012, p.101), addressing us, calling for a response, eliciting something in us and taking our breath away. To allow this to happen, we need a spacious pedagogy (Angier & Povey, 1999), an expansive timescape (Povey, Boylan & Adams, 2019), where we move away from industrial time-consciousness to an ecological time where "things start to regard us and tell us about ourselves in ways we could not have experienced without such whiling" (Jardine, 2008, n.p.). Playfulness (Playful Learning Centre, 2015) includes questioning the authority of authority with everything "up to grabs"; challenging the rules, the conventions of classroom interactions; recognising that learning takes place in an immediate, often emotionally charged, social world; and asserting the agency of players. Playfulness encourages us to engage with others, to create a community, to be flexible, responsive and open. One powerful way to shift from formality to exploratory playfulness is to replace the classroom with outdoor settings.

Respecting funds of knowledge, fostering curiosity, allowing while-ness so that the world can speak to us and responding with agency and openness all provide a space in which students can develop love for the world (Povey & Angier, 2021). Through this we recognise what it is we want to value and protect, helping us to show the next generation that “something is intrinsically good and worth preserving” (Vlieghe & Zamojski, 2019, p.160). Love for the world can also be generated by the awe, wonder and enchantment that mathematics can supply, such experiences encouraging a de-centring of an individualistic sense of self. This prompts a move away from “Are you like us?” towards “What is it like to be you?”, a shift from empathy and identity to solidarity, recognizing and valuing other things without having to identify with them as being like us (Bridle, 2022).

CONCLUSION

We have tried to navigate the paradox of facing up to the “human wrongs” in which we are implicated (Andreotti, 2021, p.498) whilst suggesting ways in which we might support the rewilding of mathematics education, “learning to walk a tightrope between naïve hope and desperate hopelessness” (Andreotti, 2021, p.506) as we try to re-value, de-centre and un-learn. Pessimism is a legitimate response to the global crisis facing us. As Grace Chen (2023) points out:

Even if teachers were to perfectly implement every researched equitable instructional practice, they cannot singlehandedly overturn historical, structural, and symbolic systems of oppression, nor dissolve the material consequences of these systems. (p.28)

Fighting for change is not going to be easy, comfortable or safe and so maybe those of us who have had lives most characterized by ease, comfort and safety are going to find this difficult. However, we seek the cracks where the light gets in. Stuart Hall reminds us that:

No victories are permanent or final. Hegemony has constantly to be ‘worked on’, maintained, renewed, revised. Excluded social forces, whose consent has not been won, whose interests have not been taken into account, form the basis of counter-movements, resistance, alternative strategies and visions ... and the struggle over a hegemonic system starts anew ... history is never closed but maintains an open horizon towards the future. (Hall, 2017, p.334)

There is no reason to suppose that many (most?) teachers do not appreciate wider purposes for mathematics education or the value of pedagogic approaches besides teaching to the test using direct instruction and “teacher proof” material. This despite the fact that in some settings teachers have very limited choice (Carter, 2024) with classrooms regularly monitored for compliance and students required to obey without question “first time, every time” ([REACH Group Strategy Visioning 9th March 2016 \(astreadearne.org\)](#)). In many settings teachers are able to make creative use of the curiously shaped scraps of fabric left after the pattern pieces that fit the requirements have been cut out. It is much harder as a teacher to resist the discourse which labels, stratifies and separates children so that they become estranged from their peers. Teachers need support to do this, support which in the past has come in part from professional networks bringing them into community with other teachers and with teacher educators. We have seen these networks diminish in England through the marketisation of teacher education.

The pinnacle of mathematics education has become an examination hall of silent fearful children desperately hoping they can perform better than enough other people to secure some kind of future. An alternative vision might be a mathematical version of the West-Eastern Divan Orchestra ([Homepage - West-Eastern Divan Orchestra \(west-eastern-divan.org\)](#)), who work with joy and rigor

across deep historical divisions. A community of peers who have collaborated to investigate a mathematical question of their devising then sharing their explorations. Such an experience of solidarity might provide the hope that wicked problems can be tackled and that we can live in partnership with others across the globe and in reciprocity with the planet.

References

- Adichie, C. N. (2009). The danger of a single story. [Chimamanda Ngozi Adichie: The danger of a single story | TED Talk](#)
- Aguon, J. (2022). *No country for eight-spot butterflies: A lyric essay*. Astra Publishing House.
- Andreotti, V. (2021). Depth education and the possibility of GCE otherwise. *Globalisation, societies and education*, 19(4), 496-509.
- Angier, C. & Povey, H. (1999). One teacher and a class of school students: their perceptions of the culture of their mathematics classroom and its construction, *Educational Review*, 51(2), 147-160.
- Arendt, H. (1951/2017). *The origins of totalitarianism*. Penguin Classics.
- Berger, J. (2016). *Hold everything dear: Dispatches on survival and resistance*. Verso.
- Biesta, G. (2022). Reclaiming a future that has not yet been: The Faure report, UNESCO's humanism and the need for the emancipation of education. *International Review of Education*, 68(5), 655-672.
- Boylan, M. & Coles, A. (2017). Is another mathematics education possible? An introduction to a special issue on "Mathematics Education and the Living World: Responses to Ecological Crisis". *Philosophy of Mathematics Education Journal*, 32.
- Bridle, J. (2022). *Ways of being: Animals, plants, machines: The search for a planetary intelligence*. Penguin.
- Brown, W. (2015). *Undoing the demos: Neoliberalism's stealth revolution*. MIT Press.
- Carter, D. (2024). It felt like kids were droids at toxic school. [[Teachers speak of 'fear' at Astrea schools in Cambridgeshire - BBC News](#)]
- Chen, G. A. (2023). Teacherly response-ability: ethical relationality as protest against mathematical violence. *Educational Studies in Mathematics*, 114(2), 277-296.
- Coles, A. (2022). A socio-ecological turn in mathematics education: reflecting on curriculum innovation. *Paradigma*, 43(1) 207-228.
- Eagleton, T. (2011). *Why Marx was Right*. Yale University Press.
- Ernest, P. (2016). The collateral damage of learning mathematics. *Philosophy of Mathematics Education Journal*, 31.
- Evans Harris, D. (2023) What would happen if AI fell into the wrong hands? *Opinion, the Guardian*, 17 June 2023, p.4.
- Fasheh, M. (1990). Community education: To reclaim and transform what has been made invisible. *Harvard Educational Review*, 60(1), 19-36.
- Fraser, N. (2007). Re-framing justice in a globalizing world. In N. Fraser & P. Bourdieu, P. (Eds.) (2007). *(Mis) recognition, social inequality, and social justice*. Routledge.
- Griffiths, M., & Murray, R. (2017). Love and social justice in learning for sustainability. *Ethics and Education*, 12(1), 39-50.
- Hall, S. (2017). The neoliberal revolution 2011. In S. Davison, D. Featherstone, M. Rustin & B. Schwarz (Eds.) *Selected political writings: The great moving right show and other essays*. Duke University Press.

- Hall, S. & Gieben, B. (1992). The West and the rest: discourse and power. In Das Gupta, T., James, C., Andersen, C., Galabuzi, G. & Maaka, R. (Eds.) *Race and racialization: Essential readings*, Second Edition. pp. 85-95. Canadian Scholars.
- Hogan, P. (2009). *The New significance of learning: Imagination's heartwork*. Routledge.
- Jardine, D. (2008). On the while of things. *Journal of the American Association for the Advancement of Curriculum Studies*, 4, n.p.
- Jardine, D. (2012). *Pedagogy left in peace: Cultivating free spaces in teaching and learning*. Continuum.
- Jardine, D. (2013). Time is (not) always running out. *Journal of the American Association for the Advancement of Curriculum Studies*, 9(2), n.p.
- Kollosche, D. (2018). Social functions of mathematics education: A framework for socio-political studies. *Educational Studies in Mathematics*, 98(3), 287-303.
- Pappé, I. & Gopal, P. (2024). Reading the crisis. Online seminar, 7 May 2024, Stuart Hall Foundation.
- Pashby, K., Da Costa, M., Stein, S. & Andreotti, V. (2020). A meta-review of typologies of global citizenship education. *Comparative Education*, 56(2), 144-164.
- Playful Learning Centre (2015) Manifesto: playing with learning. https://helda.helsinki.fi/bitstream/handle/10138/158664/Manifesto_2015.
- Plumwood, V. (2012). Decolonizing relationships with nature. In W. Adams, & M. Mulligan (Eds.) *Decolonizing nature: strategies for conservation in a post-colonial era*. Routledge (pp. 51-78).
- Povey, H. (2014). Walking in a foreign and unknown landscape: studying the history of mathematics in initial teacher education, *Science and Education*, 23(1), 143-157.
- Povey, H. (2020) Thoughts on a playful, curiosity-led curriculum: a walk in Sheffield in May 2020. *Forum*, 62(3), 335-344.
- Povey, H. & Adams, G. (2021). Disordering mathematics, citizenship and socio-political research in mathematics education amongst the “rubble of words”. *Research in Mathematics Education*, 23(2), 306-322.
- Povey, H. & Angier, C. (2021). Against ‘progress’. *Forum*, 63(2), 20-31.
- Povey, H., Boylan, M. and Adams, G., 2021. Regulated time and expansive time in primary school mathematics. *Pedagogy, Culture & Society*, 29(1), 119-136.
- Restivo, S. (2017). *Sociology, science, and the end of philosophy: How society shapes brains, gods, maths, and logics*. Springer.
- Sahlberg, P. (2015). Finnish schools and the global education reform movement. In J. Evers, & R. Kneyber (Eds.) (2015). *Flip the system: Changing education from the ground up*. Routledge. (pp. 162-177).
- Santos Fernandes, F., Giraldo, V., & Matos, D. (2022). The decolonial stance in mathematics education: pointing out actions for the construction of a political agenda. *The Mathematics Enthusiast*, 19(1), 6-27.
- Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Kluwer.
- Skovsmose, O. (2014). *Critique as uncertainty*. Information Age Publishing.
- Solares, A., Arellano-Aguilar, O., García González, M., del Rocío López-Vargas, M., Coles, A. & Méndez Serrano, A. (2022). Mathematics education and social-environmental crises: An interdisciplinary proposal for didactic innovation with rural communities in Mexico. *Research in Mathematics Education*. 24(2), 223-22.
- Vlieghe J. & Zamojski, P. (2019). *Towards an ontology of teaching*. SpringerNature.
- Wikipedia (n.d.) Diggers. [<https://en.wikipedia.org/wiki/Diggers>]

MATHEMATICS EDUCATION IN THE MATERIAL WORLD: WORK, LIVELIHOODS AND ECOLOGICAL JUSTICE

Anita Rampal

Faculty of Education, University of Delhi

Reaffirming the pedagogy of hope and transformative action, to foreground artisanal knowledges of the people living by the earth, we interrogate the aims and discourses of “21st century” mathematics education. We resist hegemonic designs to colonise work tied to “creative” knowledge for the Ethereal, casting routine skills to the Material world. We seek intercultural imaginaries and co-constructions of school mathematics for work-responsive (not “vocational”) education, attentive to livelihoods, places and the planet, with a broader commitment to an ecologically just “pluriverse”.

INTRODUCTION

In recent transnational panel discussions around issues of ecological justice, I was asked to speak on creating national primary mathematics textbooks, with chapters on artisanal knowledges and livelihoods of real-life protagonists - fishworkers, junk sellers, farmers, brick masons and others (NCERT, 2008-2024). Participants from the Global South, working on transformative economies, within local communities or larger networks, have been deliberating on transformative education to address and find ways to resist increasing ecological crises. The term Global South indicates a shared history of subjugation within global capitalism; also to multiple *Souths*, including transnational discourses, practices, and institutions working towards an equal world (Mahler, 2017).

This paper invokes Freire’s (1994) pedagogies of Hope and Indignation, who believed that:

the dream is as necessary to political subjects, transformers of the world as [...] it is fundamental for an artisan, who projects in her or his brain what she or he is going to execute even before the execution thereof (p. 82). Dreams are visions for which one fights (p. 32). We need critical hope the way a fish needs unpolluted water. The idea that hope alone will transform the world, [...] (is) naïveté, [...] an excellent route to hopelessness, pessimism, and fatalism. (p. 2).

Freire presciently addresses the prevailing agenda of “neo-liberal environmentalism”, thriving in a corporatised educational industry peddling naive “hope”, “happiness” and “resilience”, to stifle sociopolitical questioning or collective action. Dismissing the “doom and gloom” of climate change, this agenda individualises responsibility to consumerist behaviour, with trivialised, tokenistic tasks, instead of discussing the structural dynamics of power (Tannock, 2021). Sustainable Development Goals (SDGs) are often cited by curricular research with shallow patronising perspectives. A mathematics study (Haier, 2022) on “sociocritical modeling tasks in sustainable development contexts” asks: “How big would your closet have to be in 10 years if you don't sort out or dispose of anything in between and continue to shop the way you have been?”. However, from a critical mathematics framework there is modeling research on socio-ecological issues (Steffenson, 2023).

Freire asserts that: “To me fighting [...] for the utopia of criticalness, and of hope constitutes [...] a struggle founded in *just rage*, and in effective political-ethical action. I cannot accept, quietly and in “well-behaved” fashion, that the one billion unemployed who will see the end of this century be viewed as a mere fatality of this moment” (Freire, 2004/2016, p102; italics added). To educate for

democracy, freedom, and ethical responsibility, and a sense of history *as possibility*, we need “a knowing that knows itself as a component of a broader universe of knowing” (ibid., p. 111).

We draw upon decolonial frameworks that critique universalism, call for “critical pluriversalism”, by transcending disciplinary divisions, and recrafting of existing paradigms of knowing. In Mbembe’s words: “Decolonization (of knowledge) is [...] about reshaping, turning human beings once again into craftsmen and craftswomen who, in reshaping matters and forms, needed not to look at the pre-existing models and [...] use them as paradigms” (2020, p.62).

Within the broad framework of critical mathematics education, we ask: What kind of a knowing in mathematics education would challenge the hegemonic discourses that invisibilise artisanal knowledges and livelihoods of those who have lived by the earth? How would it pay attention to the scales of “planetary” extraction that endanger them, and the planet? How could mathematics education aim for creative “entanglement” (Coles et al., 2024) with other knowings, to craft a broader universe of knowing - embracing collectivistic aims, not individualistic and competitive?

With a sense of history *as possibility*, this knowing must acknowledge that while mathematics has contributed to the broadening of the universe, it is complicit in the shrinking of the globe; for “nothing can remain immense if it can be measured” (Arendt, 1958/1998, 250). An analytical distinction by the postcolonial historian of subaltern studies, Dipesh Chakrabarty (2024), between the “globe” and the “planet”, lends attention to the “parallax” of looking at “one” planet, through many differentiated unequal worlds. While the “globe” – is a human-centered construct of humans, crafted through empires, capitalism and technology, and a recorded history of only five hundred years; the “planet” – has a deep history of 4.5 billion years, with or without humans to reckon its age. Sustainability is a “global” term, alluding to how humans leave the earth for those coming later, while habitability is a planetary notion that refers to all life. The human-constituted globe is amenable to moral and political questions; but what happens when planetary processes reduce humans to a politics of survival stripped of any sense of morality? (p.6). Gandhi’s famous words have greater import now, than in 1928, written in righteous anti-colonial resistance to the untenable inequality of extractive capitalism: “The economic imperialism of a single tiny island kingdom (England) is today keeping the world in chains. If an entire nation of 300 millions took to similar economic **exploitation**, it would strip the world bare like locusts” (cited in Gandhi, 2008, p.276).

Opposing generic notions of the “we”, the “Anthropos” of the Anthropocene, or even climate “change”, the Global South has asserted a relational notion of ecological “justice”. It encompasses the precarity of livelihoods; conditions of disparity; plurality of knowledges; diverse dreams of freedom and flourishing. After the first IPCC report, environmentalists Agarwal and Narain (1991, p.1) rejected the word “humans”, the “one world-ism” of the West, suggesting computation of per capita emissions. They said “environmental colonialism” intended to perpetuate inequality in the use of the earth’s resources, holding developing countries responsible for the crises. The politics of the universalism of “numbers” - in food insecurity, or “population” science, where women bear the brunt of “population control” - exacerbates the entanglement of the ecological crisis. As we shall see, the disparaging pointing to the “mouths to feed”, makes no reckoning whatsoever of the often uncounted “heads-hearts-bodies *that* feed”. A mathematical knowing of the scale of the earth’s morphing must note that: “championing of the collective in geology” serves to camouflage “how the codification of

geology (as land, mineral, metal, gold, commodity, value, resource) becomes the historical basis of theft” (Yussof, 2018, p.107).

When Conway (2023) delves into the geology of six materials on which the world stands today, he is aghast by the unprecedented planetary stripping. As an economist he debunks delusional myths about dematerialisation, where the Global North flaunts figures of rising GDP, with declining consumption of materials. The truth lies off-shored, with far deeper extractions from where goods are imported. Talking to workers in dusty quarries, sweltering foundries, and ultra sterile fabricating units he is shocked by the “Ethereal” world he has inhabited. A nice place of ideas; trading in management and advice; in the clouds; with utterly shallow perspectives of the “other” – the Material world. Where unsung people and unknown companies dig, raze, and chemically churn mountains, to materialise ideas and goods. Mountains of denuded toxic earth are not counted in the mathematics of this gross domestic product. The “math of minerals” is not revealed by energy budgets; renewable energy infrastructure is highly mineral intensive; replacing 1 natural gas turbine (with 300 tonnes iron, 2000 tonnes concrete and 50 tonnes copper) needs 20 wind turbines (with 30,000 tonnes iron, 50,000 tonnes concrete and 540 tonnes copper).

Questions are seriously asked about where new knowledge or ingenuity can possibly come from - as fewer people attentively engage with the Material world, education abandons and demeans it, while programmes of mining, soil science or metallurgical engineering are fast winding down.

ENGAGING WITH THE MATERIAL WORLD – OF FOOD

The energy scientist Vaclav Smil (2022) worries about a crucial “comprehension deficit”, attributing it to very few people producing the essential materials of the world. Barely 1 percent of Americans (about 3 million) produce food—plowing, sowing, weeding, harvesting, looking after the animals—most have no idea of how it reaches them. Less than 0.25 percent of Chinese (about 3.6 million) produce 1 billion tonnes of steel—smelting, casting and rolling; while the majority stays away. “The proverbial best minds do not go into soil science and do not try their hand at making better cement; [...] disproportionately high rewards are for work completely removed from the material realities of life on earth” (p. 9). His concerns are valid; what about his numbers, and assumptions?

We look at mathematics education and its relation to work within the agri-food system (AFS) - of agriculture, forestry and fishing. For postcolonial India, food (with irrigation and power) was pivotal for development; famines had killed millions under the complicit British empire. AFS is a dominant segment of India’s livelihoods and offers a large canvas for critical curricula through generative themes from the Material world. In some of our school based critical mathematics research we have found that students respond intimately to issues around food within their broader socio-cultural environment. One Grade 8 group volunteered, as participants of a doctoral study on culturally responsive assessment tasks for data handling, to organise the entire school data of the Mid-Day Meal programme, relearning themselves some mathematics they had not understood earlier, and discovered discrepancies in the delivery system not noticed by the school. Clearly what they had learned while undertaking the collectivistic assessment tasks was far deeper and more constructive than the “low performance” they were assigned to through the traditional competitive assessment which focused mostly on memorisation of procedural routines (Yadav, 2018).

In one case, some grade 5 students observed a large amount of water being wasted by the RO (Reverse Osmosis) purifier and nudged their teacher to help examine it. Initially she thought it was a machine defect, but after some reading about the purification process, she conducted some measurements with them. They were deeply concerned to find that the RO was throwing away 3-4 litres of water for each litre it purified. They further estimated that the 4 ROs in the school were together wasting 1000 litres per month during the 15-minute morning break, in a water starved area; computing the amount of precious water wasted in a year was indeed mind boggling, and students wanted to ask the state government to stop installing RO purifiers, which in any case were more expensive and not needed for water that was not brackish (Sethi & Rampal, 2019).

While developing the Grade 5 mathematics chapter (NCERT, 2008-2024) on the connectedness between fish, the sea and fishworkers, the Textbook Committee – of school teachers, academics, and members of field based organisations – sought support from a fishworkers’ collective. The human narratives had to be authentic and engaging for all children from different backgrounds, while responsive to the dignity and knowledge of artisanal communities. The narratives wove around tools, artifacts, types of boats, speeds and distances traversed, their ways of navigation and reckoning of wind currents and weather patterns, quantities of catch, and the vibrant seashore fish market. The chapter showed how the women’s group had fought for a special bus when commuters objected to their “strong smelling” head-load; had collectively bought a boat and took to fishing (against a gendered work taboo); and were budgeting a plan for a fish-drying factory. Detailed information and visuals were shared, but the one question – about the number of people whose lives connected to fishing – was not easy to get an estimate for. Sources suggested 10 million (or one “crore” in India), as a good estimate for young children engaged in fishing – through a visually evocative page on “some big, big numbers” of boats and people.

The numbers are now much bigger, as are the conflicts. Small-scale fishers are among the poorest low-caste communities, increasingly facing climate catastrophes, struggling for their coastal and livelihood rights. They contend with large-scale displacement, from powerful tourism lobbies, aquaculture farming, land reclamation projects, knee-jerk “climate-resilience measures” of embankment and concretisation on several stretches of India’s highly bio-diverse coastline of over 8000 km.

While studying the global contribution of small-scale fisheries to sustainable development (FAO et al., 2023) concerns have emerged about the missing data on small-scale fishers. Globally they are the poorest, most vulnerable to climate crises, but their fishing and aquatic food-gathering activities provide livelihoods to 500 million people; with gender parity (50 percent women) in post-harvest work (as per 2016 data). Importantly, with 40 percent of the global aquatic food catch, they provide “essential nutrition to billions, contribute substantially to household, local and national economies” and “define the livelihoods and culture of a substantial and diverse segment of humankind” (p. 2).

Indeed, people producing food are globally missing in official data. The importance of *counting people, not jobs* is emphasised by a large statistical study (Davis et al, 2023) about all those engaged in agrifood systems. It shows that labour-force enumerations for “employment” data usually undercount people in household farming; subsistence farming is excluded in the term “employment”. The numbers (as per 2019 data) are revealing – the livelihood of 3.83 billion people is reliant on AFS; Asia has the highest with 2.4 billion, followed by Africa with about 1 billion.

A “comprehension deficit” about materials is clearly not because *fewer* people are producing them; it owes more to the few who maintain, even in the 21st century, that education must have little to do with messy work. “21st century mathematics” discourses from the Global North, speak *for* the globe, using a rehashed rhetoric dividing the head from the hand, that goes back centuries, even millennia. Meanwhile, our national textbooks are being rewritten and replaced in phases, regressing from a critical humanist framework to suit a majoritarian ideology; while also pushing for early vocationalisation of education, that further stratifies and segregates. Issues of caste, religion, or social inequality in the existing textbooks have seen major deletions and distortions. Chapters on environmental degradation and people’s movements, in science and social science textbooks for senior grades, have long been dropped.

DOES MATHEMATICS CONTINUE TO DIVIDE THE WORLD?

Dichotomies from the past – of creative versus routine, head versus hand, scholar versus artisan, science versus technology, mathematics versus its applications – have been challenged in the 21st century; connected rationalities of knowledge and material production, are entangled as a complex hybrid process, with inquiry and invention intimately interwoven. It is only two hundred years (Quinn, 2024) since the invention of “Western Civilisation”, when ideas of racial hierarchy and “civilisation” were forged to map historically shared cultures and knowledges onto particular geographies. Grounded in the epistemological shift about what is considered modern, universal, scientific or mathematical knowledge, the 21st century can only respond to such dichotomies as “othering” (Aikenhead, 2017).

Yet, OECD and its partners framing Mathematics 2030, introduced new competencies in PISA2022. The Strategic Mathematics Advisory Group (none of its seven members from the Global South) and other experts, have recommended less explicit teaching of arithmetic, geometry, and algebra, saying that: “Not so long ago waiters in restaurants would multiply and add on paper to calculate the price to be paid. Today they just press buttons on hand-held devices. Not so long ago we were using printed timetables to plan travel [...] Today we just make a direct internet inquiry” (OECD, 2017, p.4). US employment data is used to specify 21st century mathematical skills required for future employment. Agriculture is said to require three areas – Geometry; Numbers & Operations; Statistics & Probability.

The Ethereal world’s education systems are openly being called to prepare students for high paying “Knowledge Age” jobs, for complex and creative tasks; liberated from dull, routine mental tasks which can be automated, or outsourced at lower wages to the “Industrial Age” world, to deal with the grime and environmental hazards (Trilling and Fadel, 2009). The graphical “future of 21st century work” (p.10), is shockingly presented as a hierarchical geo-location of “creative” versus “routine” work, where “creative” occupies the crest of a pyramid (in more developed countries), and “routine” comprises the bulk of its body (in less developed countries).

“21st century competencies” in mathematics, questionably focus on persistence and “resilience”, placing the onus of performance on the “character” of the student, their social background, and attitude towards schooling. Adaptive testing is used to measure both mathematical reasoning, and resilience. PISA uses students’ digital footprints, so that those “taking more time”, perhaps grappling with unfairly difficult questions, are assessed as poor, for their “lack of effort”. Students who do not

attain basic proficiency in PISA2022 (Level 2; of the set of 8 proficiency levels) are labelled “low performers” in mathematics. An average of 31% students are “low performers” across 37 OECD countries. In 35 of the 81 participating countries/economies, *more than half* the students are “low performers”; *in 12 of them more than 80% are “low performers”* (OECD, 2023). *Only 6* participating countries/economies are deemed close to achieving universal basic proficiency in mathematics, because *15% or less* of their students are “low performers”. Prominent OECD countries have significant numbers of “low performers”: approximately 30% in Germany, 29% in France, 28% in Sweden, 27% in Australia, 25% in Finland, 20% in Switzerland, and 16% in Korea. Yet surprisingly, there is no introspection on this “globalisation of low performance” and *no* change suggested in the mathematics content. On the contrary, tremendous effort is invested in placing the responsibility on students; with policy implications that include helping reduce mathematical anxiety, with a “positive” disposition towards failure. This counts for 21st century positivity!

RE-CRAFTING MATHEMATICS AND “VOCATIONAL EDUCATION” THROUGH WORK

New inclusive ways of engaging with the material world will need to be tethered to new sustainable ways of attending to it, shorn of exploitative, hierarchical and extractive forms of work. An empathetic premise comes from an “education of attention”; not looking at the world from a distance, from the outside, not seeing it as a “globe”, but rather from within (Ingold, 2018). Learning, observing, caring, responding, being immersed in a spherical world. Education needs to be reimagined and re-crafted (Rampal, 2021), intrigued and inspired by the material world. We look back, for inspiration from decolonial visions of education *through* productive work – for self-reliance, equity, and rural transformation – which had led to alternative national curricula in countries such as Ghana, Botswana, India, Cuba and Vietnam.

In India the Gandhian system of “*Nayee Taleem*” (also called Basic Education) of “education from life, for life”, was part of its anti-colonial struggle for independence. A productive craft – such as, weaving, carpentry, or collective farming – taken as the *medium* of interdisciplinary learning, was meant to also challenge the stigma of caste-based vocations and “untouchability” deeply entrenched in a hierarchical society. At Gandhi’s Sevagram Ashram in Wardha, the Anand Niketan school was re-started in the 70s on the same humanist principles. Affiliated to a State Board, the school includes productive work – growing food, cooking, pottery, spinning cotton, weaving, sewing of *khadi*, community engagement programmes, and looking after the campus – as part of its extended curriculum. We have included the school as one of the sites of an Intercultural Masters course in Education for Sustainable Development (ESD) for Swiss teachers, where we co-construct our understandings through diverse socio-historical cultural contexts. In an interdisciplinary exploration around “cotton”, for instance, we observe people in creative processes of production – farming, hand-spinning, weaving, and block printing. Looking back at the colonial timeline, of the decimation of a vibrant cotton textile industry, we understand how Gandhi chose spinning and wearing of *khadi* as a collective act of resistance. Working through mind-boggling numbers of the water footprint of a shirt, for instance, when transported from Wardha to the Swiss site, strikes a common revelation of the extractive process of globalised production.

The journey of cotton continues to the Swiss classrooms. Swiss teachers got to study their own forgotten histories of cotton textiles in the 17th-18th centuries. An exhibition on Chintz (Indian cotton

textiles) had been beautifully curated by the Swiss National Museum (2022) and its director conducted an online walk-through for large numbers of our student teachers of Delhi University. She shared new details about the Swiss “triangular trade” – of cotton textiles, slavery, and sugar or coffee – with India, Africa and South America. It enriched our own learnings, about sophisticated Indian artisanal production, acknowledged to have “changed the world”, across various space-time cycles of transnational knowledge sharing.

Indeed tracking the transnational journey of an everyday product used by children, helps conduct critical discussions on the impact of globalisation on material production (Rethinking Schools, 2006). Issues discussed include companies relocating to low-wage countries, child labour, consumerism and questions of ecojustice. Students situate where the object was made, by whom, for what wages, under what conditions, and how the environment could have been impacted – learning to connect with people and places, connected to them through their closest belongings (Rampal & Darragh, in press). In the shrinking space of humanist curricular deliberation, in national or sub-national domains, these transnational curricular conversations allow us to negotiate themes beyond disciplinary boundaries of science, mathematics, history, art, agriculture, economics or geography. We hope to nurture these valuable spaces to move beyond the demoralising Global North projections on the future of education and work; towards more inclusive understandings of lives, livelihoods, places and the planet, for an ecologically just “pluriverse”.

We also need a concerted collective effort to recraft Vocational Education. The head versus hand discrimination continues to be used to ascribe low value and low status to “skill-based” practical work, especially when tied to caste, race or class. This must be abandoned, for a truly creative work-based programme, as we have proposed to a state government. Even in better resourced “well performing” countries like Sweden and Finland, dichotomies between work-place and academic mathematics exist. “Vocational” learners who find the contextual mathematics more engaging are nevertheless viewed as being “in need of support”, using their hands not heads (Boistrup & Selander, 2022). In countries such as India, herded out by school mathematics, vocational education becomes a default “option” for the poor and “low performers”, into “skilling” courses designed for and by the industry, with barely any education. We hope in future to explore and work towards collaborations that can help transform the discourse of school and vocational mathematics, which begins early in the sorting and labelling of children, their communities, cultures, and places.

References

- Agarwal, A. & Narain, S.(1991/2003) *Global warming in an unequal world: A case of environmental colonialism*. Centre for Science and Environment.
- Aikenhead, G.S. (2017). *School mathematics for reconciliation: From a 19th to a 21st century curriculum*. <https://education.usask.ca/documents/profiles/aikenhead/School-Mathematics-forReconciliationvB11.pdf>
- Arendt, H. (1958/1998). *The human condition*. 2nd ed., University of Chicago Press.
- Boistrup, L.B. & Selander, S. (Eds.) (2022). *Designs for research, teaching and learning: A framework for future education*. Routledge.
- Chakrabarty, D. (2024). *One planet, many worlds. The climate parallax*. Primus Books
- Coles, A., Solares-Rojas, A. & Le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics* 116, 165–183. <https://doi.org/10.1007/s10649-024-10318-4>

- Conway, E. (2023). *Material world: A substantial story of our past and future*. Penguin.
- Davis, B., et al (2023). *Estimating global and country-level employment in agrifood systems*. *FAO Statistics Working Paper Series* (pp. 23-34). FAO. <https://doi.org/10.4060/cc4337en>
- FAO, Duke University & WorldFish. (2023). *Illuminating hidden harvests – The contributions of small-scale fisheries to sustainable development*. Rome: Author <https://doi.org/10.4060/cc4576en>.
- Freire, P. (1994/2016). *Pedagogy of hope*. Bloomsbury.
- Freire, P. (2004). *Pedagogy of indignation*. Paradigm Publishers.
- Gandhi, G. (2008). *The Oxford India Gandhi: Essential writings*. Oxford University Press.
- Haier, K., Siller, H. S., & Vorhölter, K. (2022). Criteria for sociocritical modeling tasks in sustainable development contexts. 12th Cong. of the Euro. Soc. of Res. in Math. Edu. CERME12. Hal-03745376
- Ingold, T. (2000). *The perception of the environment*. Routledge.
- Mahler, A. G. (2017). Global south. In E. O'Brien (Ed.), *Oxford bibliographies in literary and critical theory*. Oxford University Press. <https://doi.org/10.1093/obo/9780190221911-0055>
- Mbembe, A. (2020). Decolonizing knowledge and the archive. In M. Smith (Ed.), *Decolonizing: The museum, the curriculum, and the mind*. Vilnius Academy of Arts.
- NCERT [National Council for Educational Research & Training] (2008-2024). The fish tale. In *Math-Magic: Textbook for mathematics, Class V*. (pp.1–15) <https://ncert.nic.in/textbook.php?eemh1=1-14> New Delhi: NCERT.
- OECD (2017). *PISA 2021 mathematics: A broadened perspective*. OECD.
- OECD (2023). *PISA 2022 results* (Vol. I): The state of learning and equity in education. OECD.
- Quinn, J. (2024). *How the world made the west: A 4,000-year history*. Bloomsbury.
- Rampal, A. (2021). Critical mathematics education imaginaries: Culturally situated, situating, transformative, decolonising and more. In R. Barwell & A. Andersson (Eds.), *Applying critical mathematics education* (pp. 240-247). Brill.
- Rampal, A. (2015). Curriculum and critical agency: Negotiating everyday mathematics. In S. Mukhopadhyay & B. Greer (Eds.), *Proc. of the 8th Int. Math. Edu. & Soc. Conf.* (1, pp. 83–110). Ooligan Press.
- Rampal, A., & Darragh, L. (in press). A transnational mathematics curriculum? Imaginaries and implications. In M.A. Clements, et al (Eds.), *Fourth international handbook for mathematics education*. Springer.
- Rethinking Schools (2006). *Rethinking mathematics*. Rethinking Schools.
- Sethi, J. & Rampal, A. (2019). Teaching mathematics for social justice: A dialogic reflection. In J. Subramaniam (Ed.), *Proc. of the 10th Int. Math. Edu. & Soc. Conf.* (pp. 912-16). MES10.
- Smil, V. (2022). *How the world really works: A scientist's guide to our past, present & future*. Penguin.
- Steffenson, L. (2023). Sustainability and mathematical modelling in 5th grade. *Prometeica* <https://doi.org/10.34024/prometeica.2023.27.15290>
- Swiss National Museum. (2022). *Chintz: How a fabric conquered the world*. Author.
- Tannock, S. (2021). *Educating for radical social transformation in the climate crisis*. Palgrave.
- Trilling, B. & Fadel, C. (2009). *21st century skills: Learning for life in our times*. Jossey-Bass.
- Yadav, C. (2018) *Understanding assessment in elementary mathematics classrooms: A sociocultural perspective*. Unpublished doctoral thesis. University of Delhi.
- Yussoff, K. (2018). *A billion black anthropocenes or none*. University of Minnesota Press.

SOCIO-ECOLOGICAL CONCEPTS OF POSITIVE DEVIANCE OF ETHNOMATHEMATICS AS A PROGRAM

Milton Rosa, Daniel Clark Orey

Universidade Federal de Ouro Preto

A search for innovative and alternative pedagogical action such as ethnomathematics is useful for recording historical forms of mathematical ideas, procedures, techniques, and practices, developed in diverse cultural contexts. This program is not an attempt to replace school/academic mathematics, yet it is necessary to acknowledge important contributions of local mathematical knowledge for its inclusion in the mathematics curriculum. The insubordination triggered by ethnomathematical investigations is creative and often evokes a disturbance that causes a review of rules and regulations in many mathematics curriculum contexts. This process enables educators to use positive deviance to develop actions to deal with such norms. Thus, positive deviance involves an intentional act of breaking the rules to serve the greater good of the communities and its natural environment in socio-ecological ways. This process increases the potential for continual growth in the debate about the nature of mathematics as it relates to culture since it proposes a dialogue between local and global approaches to the development of mathematical knowledge.

INITIAL CONSIDERATIONS

Ethnomathematics as a research program focuses on questions related to how members of distinct cultural groups have developed means for surviving and transcending their own reality. It relies on the analyses of mathematical techniques and practices, as well as on the evolution of behaviors and knowledge developed by these members, especially, in relation to the natural, environmental, social, cultural, economic, and political contexts. Thus, these members develop mental strategies (abstract and mental ethnomodels) to critically understand and explain how they cope with natural and sociocultural environments to survive, and to explain reality (cosmic, imaginary, natural, and emotional) through mythology, ontology, aesthetics, and ethics to transcend survival, which is considered as the evolution of knowledge as a human process. Consequently, these members have developed techniques and strategies for coping and dealing with their natural environment, as well as to interact with members of other cultures (D'Ambrosio, 2016).

During their existence, D'Ambrosio (2016) states that members of distinct cultures relate to: a) nature according to principles of physiology, b) members of other groups according to principles of socio-biology as they organize themselves in groups, and c) nature according to principles of ecology. This context enables these members to relate to symbiotic relations with nature and society and, together, they form the essence of their culture. Thus, the needs and interests of these members and their will are internally shared through generations, which help them to develop principles of ecology that are essential for maintaining and improving the primordial triangle: *members of distinct cultural groups ↔ other(s)/societies/communities ↔ nature*. In this regard, to intermeditate the relations between these members and nature, they need to create procedures and strategies based on local daily activities.

We emphasize that these techniques are influenced by the: a) behavior of these members (conflicts with themselves), b) harmony of societies and communities (conflicts with the others), and c)

equilibrium of natural resources (conflicts with nature) (D’Ambrosio, 2016). This context enabled ethnomathematics to arise in opposition to the dominant eurocentric discourse in mathematics education – a discourse which emphasizes a school curricula originally developed by colonizing countries and imposed upon local communities during the process of colonization and conquest. Traditional academic views of mathematics and curriculum are highly valued in educational institutions worldwide. Thus, Rosa and Orey (2016) argue, ethnomathematics has emerged as a perspective that contrasts the overwhelming eurocentric discourse in mathematics, because the ethnomathematics program challenges the view that members of distinct non-dominant cultures develop simplistic or folkloristic techniques for solving problems they face daily.

Eurocentrism has supported a powerful scientific paradigm that, beyond its positive effects has allowed for the development of a destructive military power and created financial chaos and environmental ruin for the planet. Consequently, many procedures and practices found in local traditions, have been lost, many are considered inferior, and therefore judged as unimportant. Yet, there is growing evidence that the current academic paradigm needs to include the moral implications of its uses. Thus, the main objective of mathematics education is to situate educational activity in the real life and experiences of individuals in their own socio-ecological-cultural-economical contexts.

It is important to value how members of distinct cultures develop their own interpretation of the *local* culture (*emic* approach) as opposed to the outsiders’ *global* interpretation (*etic* approach) of that culture, which is one of the aims of ethnomathematics as a program (Rosa & Orey, 2015). In this context, Coles et al. (2022) affirm that “ethnomathematics specifically focuses on the practices of social groups, with the ecological a context in which human activities are developed, for example, land measurement practices” (p. 174). Similarly, D’Ambrosio (2016) affirms that members of culturally identified groups develop deeper relations with nature and their surroundings to cope with their own reality.

This means that nature and culture converge in ways that span values, beliefs, norms, practices, knowledge, and languages. As a result, there exist mutual interactions between cultural systems and the environment by exchanging information. Societies adapt and transform environments their members inhabit (D’Ambrosio, 2016). Yet, these members depend upon the use of natural resources and reduction of its hazards for their survival and material well-being to transcend to other contexts. Societies also assign meanings to the environment that vary over place and time, which help to define their identity and values within the world (Tuhus-Dubrow, 2009).

In this regard, ethnomathematics emerges as a program that can be interpreted as a reaction to this *cultural imperialism*¹⁰, which spread around the world along with the expansion of empires and the great navigations in the fifteenth century (Rosa & Orey, 2015). This can be related to the concept of *positive deviance*¹¹ (Zeitlin, Ghassemi, & Mansour, 1990) that deals with the adaptability of rules and

¹⁰Cultural imperialism can be considered as the economic, technological, and cultural hegemony of developed nations that has come to determine the direction of both economic and social progress, many cultural values and standardizes civilization and cultural environments throughout the world (Sandbacka, 1977).

¹¹The concept of positive deviance first appeared in nutrition research in the 1970s. Investigators observed that despite the poverty in certain communities, some poor families had well-nourished children. Researchers suggested the use of information gathered from these families to plan alternative nutritional programs based on the on the local knowledge that comes from their own natural environment. Thus, positive deviance enables community members to discover the

regulations to achieve the welfare of members of distinct cultures. In mathematics education, it is necessary that educators and teachers are deviant, but with discernment, to oppose prescriptions with no pedagogical sense, which are rooted on educational bureaucracy of public policies.

The positive deviance concept refers to actions that are assumed in relation to many often-unchallenged norms and institutional rules, which aim at a better commitment to the needs of school populations (Rosa & Orey, 2015). Educators, school administrators, and teachers are deviants if they design alternative actions that can achieve better results for the common good of their communities and are constituted by active input from colleagues, students, and parents. This action challenges established authorities that exclude and/or use discriminatory policies.

An important objective of positive deviance is to recognize and include diverse ways of knowing and doing found outside of formal educational environments. This means that members of distinct cultures mindfully develop awareness about when, how, and why to act against established guidelines that are unjust and do not positively serve their communities (D'Ambrosio & Lopes, 2015). As a result, ethnomathematics can be considered as a positive deviance program because it often causes a disruption in the existing order in academic mathematics by encouraging and developing the study of local ideas, procedures, and unique mathematical practices found in specific natural and sociocultural contexts, which are in accordance with the emic (local) perceptions of its members (Rosa & Orey, 2016). In this regard, this program has broken rules and bureaucratic expectations of academic mathematics in order to recognize divergent ways and value the diverse modes of mathematics that is produced in other cultures.

The positive deviance aspect of ethnomathematics initiated a disturbance caused by a review of both traditional and/or Western school and academic mathematical knowledge systems, which increases the potential for growth and the emergence of new opportunities for the discussion of the nature of the mathematics curriculum. In this regard, positive deviance contributes to the confrontation of taboos that suggest mathematics as a universal field of study without traditions, natural, and cultural roots (Rosa & Orey, 2015). According to Gutiérrez (2013), the development of mathematical tools is necessary to set political actions in the school contexts to enable teachers and students to develop their own voices against bureaucratic decisions.

The concept of positive deviance embraces innovative solutions in ethnomathematics research and its pedagogical action because this program also relates to the flexibility of norms in educational institutions, which considers local mathematical knowledge developed in the natural, environmental, social, cultural, political, and economic contexts.

SOCIO-ECOLOGICAL ASPECTS OF POSITIVE DEVIANCE IN ETHNOMATHEMATICS

Since both emic and etic approaches of mathematics are complementary, it is possible to delineate forms of synergy between the local and global aspects of mathematical knowledge. A suggestion is to use a combined local-global approach rather than simply applying one dimension. This combined local-global approach requires educators to first attain local knowledge developed by members of

wisdom they already have, and then to act on it. It is based on the premise that these members self-discover the solutions to problems they face daily (Zeitlin et al., 1990).

distinct cultures. This approach allows us to become familiar with relevant cultural differences in diverse sociocultural contexts, such as natural and environmental ones (Rosa & Orey, 2015).

Accordingly, Alves and Vidal (2024) state that the social construction of nature emphasizes that its concept has multiple meanings, which vary in different sociocultural contexts. This underlines multiple ways in which both structures and members of distinct cultural groups understand, explain, and engage with the natural environment. Consequently, they argue that “nature and cultures/societies are not separate entities but are intertwined in complex and interdependent relationships. Therefore, nature is the result of human perceptions and social practices. The way we interact with, perceive, interpret, and value nature is influenced by a given society’s history and sociocultural factors” (p. 1).

In this perspective, positive deviance is an approach to social change based on observations that in any cultural group there are people whose successful strategies have enabled them to create and find solutions to solve problems. These individuals are referred to as positive deviants (Tuhus-Dubrow, 2009). Thus, Bloch (2001) describes positive deviants as individuals who are focused, persistent, and optimistic in their pursuit. As positive deviants, members of distinct cultures develop common traits such as the sharing of common languages, the generation of diverse knowledge, the development of diverse ways of solving problems, and the creation of myths and symbols, traditions, norms, and wisdoms, as well as ways to organize themselves as hierarchies and social classes. All these categories characterize their culture.

In this perspective, positive deviance becomes an important source for adaptive transformational capacities by members of distinct cultural groups that produce non-conformist actions. Its main goal is to change these norms and rules by applying inclusion, innovation, creativity, and adaptability. It relates to unprescribed techniques and practices developed in the natural environment of members of distinct cultures that produce better outcomes than traditional standard practices (Rosa & Orey, 2015). Thus, positive deviant educators and teachers contribute to the generation of new, and a respect for diverse, forms of mathematical knowledge and aids in resolving ethical dilemmas involved in investigations in this area of study. During investigations seeking to understand and comprehend local mathematical knowledge, educators may be faced with a set of specific characteristics related to ideas, procedures, and mathematical practices that are different from those studied in the academy (Rosa & Orey, 2015). They are individuals who accomplish the aims and goals of the organizations such as schools and are crucial to the success of change efforts.

It is important that ongoing investigations in ethnomathematics describe the ideas and procedures implicit in mathematical practices locally developed by community groups. In this sense, the research on these practices can be regarded as a form of resistance towards the imposition of academic mathematical knowledge as they may suggest actions in search of creative and innovative solutions to these challenges (Lloyd, 2011). Similarly, Coles, Solares-Rojas, and Le Roux (2024) affirm that in a socio-ecological perspective, ethnomathematics focuses on the members of groups marginalized by coloniality and neoliberal globalization and may position itself as a perspective that decolonizes mathematics education.

Fielding, Hogg, and Annandale (2006) affirm that positive deviants are exceptional and high achieving individuals who exceed normal or average levels of performance in a group. They are “extremely resourceful, knowledgeable, and adaptable” (Clancy, 2010, p. 54). As a result, there is a

need for educators to break the greater western-Eurocentric perspective of mathematical knowledge. Hence, mathematical knowledge must be interpreted in the broader sense given that the term *ethno* is associated with members of identifiable cultural groups, such as national and tribal societies, working groups, children of a given age, individuals belonging to distinct professional classes, and marginalized and minority cultural groups (D'Ambrosio & Lopes, 2015).

This perspective aims to reduce the prejudice, inequity, and harm due to ongoing disconnections between knowledge as practiced in school contexts and academies (*etic*) and its practical use in everyday life (*emic*) (Rosa & Orey, 2016). Positive deviance in mathematics curriculum uses theoretical and methodological apparatus of investigations to reveal and combat the privilege and the authority that was granted to the academic mathematical discourse. This approach enables understanding and comprehension of how privilege and authority, stemming from colonization, have influenced the distribution of power in modern society (Fitzsimons, 2003). However, it is necessary that educators and teachers are willing to take the risks associated with their decision-making process, which is understood as a fight against dehumanizing effects of bureaucratic authority (Haynes & Licata, 1995).

Hence, it is important to highlight, once more, that the pedagogical action of this program helps learners to overcome the use of disassociated techniques and formulas often blindly memorized. As well, it allows them to develop strategies to give access to diverse mathematical representations in new formative dimensions of a mathematical nature. And most importantly, it helps them connect to their prior experience and community through the learning of mathematics by valuing and respecting local ways of knowing and doing mathematics. Thus, Coles et al. (2022) state that “As with ethnomathematics, the very nature of relations in these contexts signals the presence of the ecological” (p. 174).

A mathematics curriculum that uses an ethnomathematical perspective offers one way to emphasize the importance of community to schools. It allows educators to construct knowledge and experience based on valuing what and how mathematical thinking occurs in the community first. It is necessary that the school curriculum is designed to value and promote local knowledge and practices (*emic*) developed by members of communities who integrate school contexts and then connect it to the outside environment. This perspective provides a necessary balance to the school curriculum because the integration of these components in the mathematics curriculum enables the conception of ethnomathematics as a program that aims at the humanization of mathematics through contextualized approaches to curriculum development.

This process of positive deviance can be understood as an approach to organizational change and forms a theoretical basis for: a) understanding institutional behaviors (Lindberg & Clancy, 2010); b) as an alternative method of naming best practices (Tarantino, 2005); c) and as a problem-solving technique (Lloyd, 2011). Thus, positive deviance is a valuable tool for teachers, used to identify innovative pedagogical actions for their teaching practices such as inquiry methods and ethnomathematics. Despite the typical top-down hierarchy of many educational systems, there seem to be individuals who figure out solutions to problems and solve them on their own by using local techniques found in the school community.

Following this discussion, Tarantino (2005) suggests that instead of relying on outside parties to determine best practices and solutions, institutions or organizations should identify and use the existing knowledge and procedures that are already practiced in their local context. Similarly, the most efficient way to improve education is to use locally available sustainable and effective approaches (Rosa & Orey, 2016). In this regard, Sternin and Choo (2000) have argued that it is important to identify the relevant positive deviancies within each local community and then adopt that behavior. Hence, Tarantino (2005) also states that an act of positive deviance becomes both an intentional and honorable behavior that differs from the established norms because it contains elements of innovation, creativity, and adaptability.

Because positive deviance values local ways to solve problems by offering advantages over traditional approaches that try to impose solutions from outside of the cultural groups. These advantages are related to: a) the progress that is made rapidly without requiring an outside analysis or resources; b) the resulting benefits that can be sustained since the solution to the problems resides within the community; and c) the approach can be broadly applied because positive deviants exist in every cultural group (Sternin & Choo, 2000). This context shows that “ethnomathematics challenges dominant narratives of the history of mathematics and identifies what are considered culturally and socially embedded ways of thinking and acting mathematically that are decentred by dominant mathematics (education)” (Coles et al., 2002, p. 174).

Thereafter, it is in the school community itself that educators find the didactic and pedagogical elements of the mathematical content necessary in the development of mathematics curriculum (D’Ambrosio & Lopes, 2015). There is a need to diversify teaching strategies used in the mathematics curriculum, in this case by applying an ethnomathematics perspective. It must be acknowledged here that there is no single recipe that will help educators everywhere to acquire all the necessary pedagogical tools for improving the performance of students in mathematics.

FINAL CONSIDERATIONS

In education, positive deviance is exemplified by educators’ and teachers’ intentional bending of rules to accomplish specific goals for students or to promote work efficiency for teachers. In this way, the subversion occurs when experienced, knowledgeable, idealistic teachers feel professional conflict and frustration over certain aspects of students’ performance and consequently *work around* the rules so they can adequately meet their educational needs. The (re)examination of education in *disadvantaged* communities enables the transformative engagement of students in empowering and collaborative experiences that link curriculum, pedagogy and assessment to identity, politics, and social justice (Zyngier, 2009).

Positive deviance can be conceptualized from a behavioral approach that highlights the significance of reference groups and normative standard as the basis for categorizing deviant behavior. The criteria for defining positive deviant behaviors include those that break or depart from reference group rules and norms that are socially or organizationally beneficial (Warren, 2003). In this context, especially in regards with ethnomathematics, positive deviance can be considered as a tool to combat against the dehumanizing effects of curricular bureaucratic authority and as a tool for peace.

According to Zyngier (2009), it is necessary to enable educators to become responsible subversives or positive deviants through enabling a sense of collective effort. If educators are connected and

engaged with students' sociocultural backgrounds while affirming the different strengths that their knowledge adds to the development of pedagogical action in the classrooms, then, student knowledge, history, and their unique experiences are validated and accounted for in the schools and allows them to construct bridges of understanding to academic forms of mathematics and science. Such students' engagement is empowering because it develops a sense of entitlement, belonging, and identification. Finally, positive deviance is a useful decolonial concept. It offers educators a basis for decision making when both normal and unexpected actions collide with conflicting views in the schools for the benefit of their students. This concept is extremely necessary in the development of a mathematics curriculum based on ethnomathematics to assist teachers to serve student learn to address student's cognitive, cultural, and pedagogical needs. Surely, this demonstrates the need for a cultural perspective into the mathematics curriculum for the 21st century.

References

- Alves, F., & Vidal, D. G. (2024). Plural nature(s): an overview of their sociocultural construction. *Encyclopedia*, 4(1), 1-12, 2024.
- Anderson, S. E. (1997). Worldmath curriculum: fighting Eurocentrism in mathematics. Powell, A. B., & Frankenstein. (Eds.). *Ethnomathematics: challenging Eurocentrism in mathematics education* (pp. 291-306). Sunny.
- Bloch, S. (2001). Positive deviants and their power on transformational leadership. *Journal of Change Management*, 1, 273-279.
- Clancy, T. R. (2010). Diamonds in the rough: positive deviance and complexity. *Journal of Nursing Administration*, 40(2), 53-56.
- Coles, A., Le Roux, K., & Solares, A. (2022). Towards a socio-ecological perspective of mathematics education. *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (pp. 171-178). Universidad de Alicante.
- Coles, A., Solares-Rojas, A., & Le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116(2),1-19.
- Crowson, R. L., & Morris, V. C. (1982). The principal's role in organizational goal attainment: discretionary management at the school site level. *Proceedings of the Annual Meeting of the American Educational Research Association* (pp. 19-23). AERA.
- D'Ambrosio, B. S., & Lopes, C. E. (2015). *Creative insubordination in Brazilian mathematics education research*. Lulu Press.
- D'Ambrosio, U. (2016). The ethnomathematics program and a culture of peace. *Journal of Mathematics and Culture*, 10(2), 1-11.
- Fielding, K. S., Hogg, M. A., & Annandale, N. (2006). Reactions to positive deviance: social identity and attribution dimensions. *Group Processes & Intergroup Relations*, 9(2), 199-218.
- Fitzsimons, G. A. (2003). What counts as mathematics? Technologies of power in adult and vocational education. *ZDM – Mathematics Education*, 35(6), 323-324.
- Freire, P. (1993). *Pedagogy of the oppressed*. Continuum Books.
- Gutiérrez, R. (2012). Embracing Nepantla: rethinking 'knowledge' and its use in mathematics teaching. *REDIMAT - Journal of Research in Mathematics Education*, 1(1), 29-56.
- Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, 44(1), 37-68.

- Haynes, A., & Licata, J. W. (1995). Creative insubordination of school principals and the legitimacy of the justifiable. *Journal of Educational Administration*, 33(4), 21-35.
- Hutchinson, S. A. (1990). Responsible subversion: a study of rule-bending among nurses. *Scholarly Inquiry for Nursing Practice*, 4(1), 1-3.
- Lindberg, C., & Clancy, T. (2010). Positive deviance: An elegant solution to a complex problem. *Journal of Nursing Administration*, 40(4), 150–154.
- Lloyd, J. C. (2011). For clues to HAI prevention, seek out positive deviance. *Healthcare Purchasing News*, 35(1), 46-47.
- Lopes, C. E., & D'Ambrosio, B. S. (2016). Professional development shaping teacher agency and creative insubordination. *Ciência & Educação*, 22(4), 1085-1095.
- Rosa, M., & Orey, D. C. (2015). Evidence of creative insubordination in the research of pedagogical action of ethnomathematics program. In: D'Ambrosio, B. S., & Lopes, C. E. (Eds.). *Creative insubordination in Brazilian mathematics education research* (pp. 131-146). Lulu Press.
- Rosa, M., & Orey, D. C. (2016). Ethnomodelling: exploring glocalization in the contexts of local (emic) and global (etic) knowledges. *International Journal for Research in Mathematics Education*, 6(1), 196-218.
- Sandbacka, C. (1977). *Cultural imperialism and cultural identity*. Finnish Anthropological Society.
- Sternin, J., & Choo, R. (2000). The power of positive deviance. *Harvard Business Review*, 1, 14-15.
- Tarantino, D. P. (2005). Positive deviance as a tool for organizational change. *Physician Executive*, 31(5), 62–63.
- Tuhus-Dubrow, R. (2009). *The power of positive deviants: a promising new tactic for changing communities from the inside*. Boston Globe.
- Warren, D. E. (2003). Constructive and destructive deviance in organizations. *Academy of Management Review*, 28(4), 622-632.
- Zeitlin, M., Ghassemi, H., & Mansour, M. (1990). *Positive deviance in child nutrition with emphasis on psychosocial and behavioral aspects and implications for development*. The United Nations University.
- Zyngier, D. (2009). Education through elegant subversion. *Professional Voice*, 6(3), 53-59.

SOCIO-ECOLOGICAL COMPLEXITIES IN ISRAEL? STORIES AND SILENCES IN ISRAELI NATIONAL MATHEMATICS TESTS

Laurie Rubel¹, Nadav Ehrenfeld², Gil Schwarts²

in collaboration with Tommy Dreyfus³, Anna Sfard¹

¹University of Haifa, ²Weizmann Institute of Science, ³Tel Aviv University

In this conceptual paper, we quest for a socio-ecological terminology that interconnects the mathematical, political, and ecological. We suggest such a framing and illustrate it through an examination of Israeli curricular materials. In this time of war and extreme suffering for many, such a study is not merely an academic exercise. Rather, it is a soul-searching effort, part of our attempt to situate our work as Israeli mathematics educators in pressing, interwoven social, political, and environmental crises. We begin with a preliminary attempt to be precise about “socio-ecological” relative to mathematics education, locating it in the literature and framing it as storytelling. We then illustrate one category of stories—about the world in mathematics education—by analyzing contextualized problems in Israeli national tests. We characterized their protagonists, activities, and settings. The analysis reveals emphasis on human protagonists engaged in consumer activities amidst broader silences about socio-political dynamics and local crises, challenges, and opportunities.

INTRODUCTION AND CONTEXT

An Israeli mathematics teacher grading a matriculation examination was surprised to find that the student had penned just the following: “Sorry, I cannot answer all these questions. Believe me, I have many problems in my life, and solving them is much more important than this trigonometric identity.” This moment underscores the potential disconnect between the problems of school mathematics and those of real-life. In a traditional approach to teaching mathematics, mathematics is seen to be a value-free discipline, ostensibly applicable to actual problems. Beyond its crucial lack of responsiveness to learners, an artificially neutral approach falls short in several additional ways. Refraining from teaching about social, political, or environmental injustices in mathematics classrooms signals that mathematics is not about real problems and does not fully realize mathematics education as a space for bettering the world. Moreover, in maintaining silence about the range of social functions of mathematics, a faux neutral approach is complicit with how mathematics or mathematics education are themselves implicated in many inequalities and crises (Skovsmose, 2021; Valero, 2023).

In this paper, we consider aspects of mathematics education in Israel, amidst global crises like climate change, worsening social and economic inequality, and the rise of radical nationalism. At the local level, Israel’s population is marked by considerable diversity, with pronounced ethnic, cultural, and political divides and differing visions of: the state’s democratic nature and civil rights; rising Jewish and Muslim fundamentalism; and above all, the status of the Palestinian people and occupied territories in Gaza and the West Bank. The Israeli context is marked by a longstanding conflict between two peoples, Jewish and Palestinian, who claim historical and present rights to the same land. The land, its airways, and waterways, are never a neutral backdrop. In this case, the non-neutrality is especially heightened since the land is itself the object of contention and a past and present battleground. All these issues, the global and the local, are significant enough to manifest in

mathematics education, whether through relevant curricula or academic research. In this time of war and suffering, we asked whether, and to what extent, Israeli mathematics curricula, as reflected in contextualized problems in annual national examinations, bring real-life problems to the fore.

IN SEARCH OF A SOCIO-ECOLOGICAL DISCOURSE IN THE ABSENCE OF A SOCIO-POLITICAL ONE

Despite narratives of mathematics as objective and universal, its applications in spaces such as industry, public media, and education are fundamentally political. Mathematics instruction can foster critical thinking, awareness, and involvement, or it can create passive, timid, and alienated students (Fasheh, 1982). Today, there are examples of mathematics education research and practice rooted in political sensitivities from seemingly around the globe. However, there is a conspicuous paucity of classroom examples from Israel (and from the Middle East more generally). Outside of studies that compare outcomes of Jewish Israeli students with Palestinian/Arab Israeli students, there are few that deal with socio-political dimensions of mathematics education in Israel. Exceptions include ethnomathematics (e.g., Fouze & Amit, 2023) and explorations of mathematics-related experiences of immigrants from the former Soviet Union (Sfard & Prusak, 2005), ultra-Orthodox men (Ehrenfeld & Heyd-Metzuyanim, 2019), Palestinian/Arab Israeli women (e.g., Rubel & Ehrenfeld, 2020; Sabbah & Heyd-Metzuyanim, 2021), and peer-groups of Palestinian/Arab and Jewish mathematics teachers (Schwartz et al., submitted).

Considering Fasheh's (1982) observation that critical thinking and student awareness threaten established institutions and authorities, these silences in mathematics education in Israel are concerning. We call to question whether and how a focus on the so-called socio-ecological could bring together the mathematical, ecological, and political, in a context like Israel, where the day-to-day is so rife with conflict and yet has received little expression in Israeli mathematics education curriculum, teacher education, or research. In this paper, we argue for ways of thinking about the "socio-ecological" relative to mathematics education, in the context of a protracted silence with respect to socio-political aspects of mathematics education. We suggest one such preliminary conceptualization and then illustrate it through a study that maps the extent to which the Israeli mathematics education community engages with socio-ecological issues.

WHAT DO WE MEAN BY "SOCIO-ECOLOGICAL"?

One way that "socio-ecological" appears in mathematics education research is in reference to Bronfenbrenner's (1979) human-centered ecological framework to foreground, name, and distinguish between different layers of systems in which students, teachers, and researchers participate (e.g., Ehrenfeld, 2022; Louie & Zhan, 2022). A second approach to the socio-ecological in mathematics education is grounded in a "socio-political turn" (Gutiérrez, 2013), but critiques how often, the environment is taken as "a fixed background to social and cultural concerns" (Coles et al., 2024, p. 3). A third body of work is rooted in Indigenous epistemologies and suggests a decentering of humans and a re-imagining of the relations between our discipline, persons, places, and more-than-humans (Barwell et al., 2022; Gutiérrez, 2017; Vossoughi et al., 2023). In total, socio-ecological qualifies an expansion of discourse beyond the "socio-political" to the "socio-ecological" (Coles, 2022) to center the mathematical, social, political, ecological, and their entanglements (Coles et al., 2024).

Valero (2023) ties this third focus to the concept of storymaking as political action. As an example, Valero (2023) presents the longstanding story that mathematics is neutral and universal, and its leading to “the sense of a homeless mind” (p. 59). For Valero (2023), storymaking can be a way to bring closer mathematics education to a “terrestrial” epistemology, in which all beings are interconnected. Valero (2023) prompts: “Which new stories could we make to reimagine and reconfigure the basic assumptions of mathematics education for the Terrestrial?” (p. 65). We take up Valero’s challenge and start by defining the term socio-ecological as follows:

Teaching and learning mathematics (and mathematics) are shaped by the society in which people are embedded and by the non-human material world in which they live. For our purposes, *socio-ecological* describes a paradigm or discourse in which mathematics education researchers tell stories about the teaching and learning of mathematics (or about mathematics) that refer to the interplay between human processes related to mathematics (thinking, learning, teaching) and ecological and political phenomena that shape and are shaped by mathematical thinking and the learning-teaching of mathematics.

One kind of story, and way of thinking about the socio-ecological in mathematics education, refers to how place, politics, and the environment shape mathematical thinking. Some have moved away from the notion of the mind as homeless to study the impact of more-than-human factors (ecology) on mathematical thinking and processes of learning-teaching mathematics. For example, Rubel and Nicol (2020) discuss Polynesian wayfinding and its contrast with conventional mathematics. Other examples are provided by Kelton and Ma (2020) and de Freitas and Sinclair (2013); these studies explore the impact of “more-than-human” factors (ecology) on mathematical thinking and processes of learning-teaching mathematics.

Another type of story explicitly takes up a crisis or challenge (e.g., climate change, inequality, access to resources, and intersections of these crises) through a lens of mathematics as part of mathematics education. That is, mathematics can play a special role in supporting students to better understand crisis or inequality and to act in response. This requires that school curricula make space for and attend to social, political, and environmental crises and challenges. Until recently, in the literature, such attention has been focused on social or political questions, and less on the entanglement of the social, political, and environmental (Coles et al., 2024).

Yet every issue has social, political, and environmental dimensions, even if one dimension is more readily apparent. For example, building systems of clean energy sources with wind farms could appear to be strictly an environmental action. In the case of the Golan mountains, this action carries complexity in social ways (e.g., who is most impacted), with political dimensions (e.g., ownership and property rights of a natural resource). Conversely, local debates over the language of instruction in mathematics courses seem like purely social or political questions (who studies mathematics in their first language and the status of Arabic in Israeli society). Yet language readily connects to a range of environmental dimensions, like air quality or freedom of motion. In these examples and many others, mathematics can be used as a storytelling tool to clarify aspects of a phenomenon, identify relationships among variables, and to communicate ideas for action.

A starting point for our work is to explore the Israeli curriculum in terms of how it makes explicit or connects to crises and challenges in Israel. On one hand, considering Valero’s (2023) critique of a tendency to educationalize social problems, we are not suggesting that mathematics represents in and by itself a better and richer answer to any of these challenges or crises. But inversely, we are hopeful that breaking curricular silences, about the social, the political, the environmental and their

intersections, however challenging, might play out together with other aspects in the ecological system to disrupt the status quo. Below, we take Israeli national tests as a kind of representative text, one that tells stories about protagonists and activities, in settings. We explore whether and how Israel’s many crises and challenges find expression in the storytelling in these texts.

ILLUSTRATIVE EXAMPLE

We focus on Israel’s national mathematics test for 8th-grade. The test is written in Hebrew and translated into Arabic for Israel’s Arabic-language schools. We focused on the Hebrew version of these assessments, using versions from 2014 to 2020. In total, the tests include 236 problems. Most (141, or 60%) did not refer to contexts, but 95, or 40%, of the problems were contextualized problems. For these, we constructed a profile of their protagonists, activities, and settings.

Protagonists

Humans dominated the contextualized problems (85 of 95), either as named protagonists (67), human-implied activities (10), or physical objects (8). Animals (1) and animal habitats (3) appeared infrequently (4). Natural environments (trees or gardens) without mention of people, or physical attributes of place (e.g., temperature, distances, or relative sizes) account for six problems. In almost all cases, the names of the human protagonists were contemporary Hebrew names, portraying Jewish Israelis (Table 1). There were no names that would signal Palestinian/Arab Israeli people, nor were there any names that could signal Russian, Ethiopian, French, or Anglo ethnicities, even though these minority groups currently constitute more than a third of Israeli society. The Arabic version of the test translated the names of protagonists with alternative names and settings. For example, a boy named Bar (a contemporary Hebrew name) leaves Netanya (a majority Jewish city) for cycling training (2019, #17) in the Hebrew-version of the test became a boy named Alaa (an Arabic name), leaving from Nazareth (a majority Palestinian/Arab city) in the Arabic-version.

Zooming in to the homogenous representation of protagonists as Jewish Israelis, 34% of the problems involving human protagonists specifically indicate girls or women. This was more frequent than representations of specific boys or men (24%) and extended to traditionally male-dominated activities, like sports and mathematics. Hebrew, as a gendered language, assigns gender to all nouns and verbs as either masculine or feminine. Although the plural male form has been traditionally used to indicate all people (e.g., students, workers), there are contemporary efforts to make Hebrew more inclusive. However, in all cases of plural nouns, only the male form was used to indicate generic groups of people. Overall, the tendency to normalize masculine as default is unchallenged in these tests and explains the overall skewed gender distribution in Table 1 towards boys and men (61%).

Category	Characteristics (n=67)				
Names	Hebrew	Arabic	Mixed group	n/a	
	39 (56%)	0 (0%)	0 (0%)	28 (44%)	
Gender	Girls or women	Boys or men	Mixed group		
	23 (34%)	41 (61%)	3 (4%)		
Role/Occupation	Consumer	Athlete	Worker	School student	Misc
	19 (28%)	10 (15%)	5 (8%)	14 (21%)	19 (28%)

Table 1. Characteristics of Human Protagonists (n=67)

The exception to a representation of human protagonists as able-bodied came in two questions about designated seats for people with disabilities (2020, #4). Question 4a asked, “How many additional seats need to be reserved for people with disabilities so that at least 40% of the seats in the theater are reserved?” This rare instance explicitly highlights human diversity and values of inclusivity and accessibility. This exception demonstrates the possibility of incorporating inclusive scenarios into the assessments. However, this solitary example underscores the overall absence of diverse representation of humans in these 8th-grade national assessments, alongside very minor representation of more-than-human lifeforms.

That the Hebrew version of the test remains silent about the society outside of Israel’s majority Jewish society reflects the extensive, systematic separation between Jewish and Arab/Palestinian citizens of Israel. Only modest integration occurs in Israel, in a limited number of schools, workplaces (particularly in the healthcare industry), sports leagues, and higher education, but visions of integration or co-existence were entirely absent in these mathematics assessments. On these tests, there were no stories of friendships, joint participation in activities, or references to visiting or living in majority-Arab cities or villages (in the Hebrew version).

Activities and Settings

Eighty-two of the 95 contextualized problems referred to human activities (Table 2a). The dominant trend was money-related activities, in 35% of the items with human protagonists (purchasing and earning). Consumer activities dominated, usually a purchase of objects (e.g., snacks, balls, notebooks) or services (e.g., mailing packages, paying to stow baggage, or paying for a health club), with far fewer activities related to earning money. Besides buying everyday items and pursuing leisure activities, the questions reflected aspects of consumerist decision-making, mostly related to price-comparison. For example, one problem posed price plans for two airline companies that charge for excess baggage at different rates (2017, #21). The question prompts the test taker to consider the price points at the two companies and asks how much money a man could save by choosing the cheaper company. In general, the questions reflect an affluent society in which people dine in restaurants, go on international vacations, and purchase health club memberships. The emphasis is on acquisition, rather than investing, saving, donating, or sharing money.

Other, less frequent, activities were technology or information-related activities, in home settings (e.g., asking how many megabytes Na’ama should erase to free space in her mailbox, 2016, #9a), school settings (e.g., how many children at school are members of social networks, 2015, #4) and community settings (e.g., computer after-school club, 2019, #9), suggesting that these activities are considered by the test authors to be taken as shared. We see infrequent evidence of collective-scale society, counterexamples being questions referring to the recommended amount of water consumption per day for boys and girls (2020, #1a,b) or the recommended distance between a television and viewers (2014, #20a-c). There were no mentions of social service-related activities, nor any mentions of activities that reflect socio-economic diversity. Also notable is a complete absence of activities around mitigating the effects of climate change, addressing economic inequality, caring for the elderly, fairly allocating resources, civil protests, or peace-making.

Activities n=82	Earning/working 4 (5%)	Settings n=95	School 13 (14%)
Purchasing 25 (30%)	Interpreting data 7 (9%)	Stores or businesses 23 (24%)	Home and digispace 13 (14%)
Exercise/leisure 17 (21%)	Motion in vehicles 4 (5%)	On-the-roads or outside 20 (21%)	Newspaper, news, books & public information 7 (7%)
School-related 11 (13%)	Technology-related 4 (5%)	Community spaces 19 (20%)	Misc 0 (0%)
Home design 6 (7%)	Misc 4 (5%)		

Tables 2a,2b. Activities and Settings

One might expect school and home to account for most of the settings, since these are the most basic shared settings for school students. Instead, however, stores or businesses, community spaces (e.g., museum, dance club, fitness center), and outdoor environments (e.g., roads connecting cities) each accounted for more of the settings than either school or home (Table 2b). Residential segregation in Israeli society is strictly reflected in and thus reinforced by the two language versions of these tests. The only hints in these assessments that reflect the ongoing crisis were optimistic names of institutions in the problems, such as the “Hope Community Center” (2019, #9) and the “Peace Restaurant” (2018, #19), which appeared in both the Hebrew- and Arabic-versions.

DISCUSSION

The national assessments are texts with institutional and national legitimacy that likely influence teaching and learning at a national scale. These texts provide a retrospective view of which and whose experiences are considered worthy of investigation and representation, reinforce norms, and act prospectively as gatekeepers around who and what can be represented in mathematics education. What is missing in the storytelling with protagonists, activities, and settings is more conspicuous than what is included. The analysis suggests that certain types of stories are being told and retold, in mathematics education, while many other kinds of stories are not voiced. There is a focus on consumerism, market logic, and leisure, yet no evidence for using mathematics to better understand and challenge inequalities or cope with other pervasive difficulties typical of daily life in Israel. As far as these tests are representative of curriculum, mathematics education in Israel can hardly be considered responsive to calls for socio-ecological relevance.

We can speculate about some of the reasons for this situation. Firstly, addressing contentious issues is risky, for mathematics teachers and others, and avoiding these subjects might be the simplest approach. A second reason relates to a critique of the difficulty of integrating real-world contexts into mathematics curriculum, in terms of fidelity to mathematics. Some view rigorous study of mathematics at the school level as a necessary prelude to future studies of complex social issues. Thirdly, an inclination to background social differences in Israel, rather than address their implications, may be rooted in specific Israeli ideologies and policies, originating from the decisions made in the 1940's by the founders of the state (Zameret, 2002). These choices aimed to downplay cultural divisions within Jewish Israeli society, advocating a “melting pot” approach, focusing on shared survival missions as Jews and as a Jewish nation in the Middle East.

Breaking silences in curriculum or research is not simply a matter of changing the names of people or places appearing in teaching materials, although even the inclusion of Palestinian/Arab Israeli people, places, and concerns on the Hebrew-version of mathematics assessments seems radical in the current moment. The silences identified in our study are indicative of what is valued and considered as important, and thus also what can be troubled or challenged with mathematics. We view the omissions as driven by broader silences, and in particular, indicative of the widely endorsed norm of turning a blind eye to the socio-ecological challenges partly listed in the opening of this paper. Harkening to the dichotomy posed by Fasheh (1982), breaking such silences in mathematics education in Israel would encourage critical awareness and promote the use of mathematics to question and challenge many dimensions of the status quo. For this, mathematics educators should acknowledge that mathematics education is political and, as such, should aim at engaging students in a full range of pressing socio-ecological concerns and crises with mathematical relevance.

We write this paper during a time of war that has produced vast destruction—a war through which humans are taking human life at large scale; burning fields, homes, and forests; polluting waterways; and destroying a full range of human and more-than-human habitats. We recall the Biblical narrative, wherein two women are in conflict over which is the mother of one baby. King Solomon directed the women to share the child by cutting the baby’s body into two parts. One of the women accepted this resolution, but the second woman begged the king not to hurt the baby. In the end, Solomon ruled that this latter woman must indeed be the baby’s mother. In a story of two peoples locked in a violent struggle over ownership and control of the same lands, the lands are living and destructible, much like the baby in the Biblical tale. Furthermore, a metaphor of the lands as one baby highlights the interconnectedness of the ecologies, which do not heed national borders. Unlike the resolution in that ancient drama, the present conflict between two peoples overshadows love and concern for the lands about which they are warring. The disputed land is a casualty of the fighting over it.

Looking forward, beyond urgent questions of human survival in our region, we point to the socio-ecological, and to the land with its potential as teacher, towards breaking the silences we have noted here. Perhaps bringing the health and survival of the contested lands into focus—meaning the lands with their air and waterways, seas and forests, mineral and plant riches, and human and more-than-human inhabitants—could potentially serve as an opening, a kind of crack that lets the light get in.

References

- Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013. <https://doi.org/https://doi.org/10.1016/j.jmathb.2022.101013>
- Bronfenbrenner, U. (1979). *The ecology of human development: Experiments by nature and design*. Harvard University Press.
- Coles, A. (2022). A socio-ecological turn in mathematics education: Reflecting on curriculum innovation. *Paradigma*, 43(1), 207-228. <https://doi.org/10.37618/PARADIGMA.1011-2251.2022.p207-228.id1168>
- Coles, A., Solares-Rojas, A. & le Roux, K. (2024). *Socio-ecological gestures of mathematics education. Educational Studies in Mathematics* 116, 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- de Freitas, E., & Sinclair, N. (2013). New materialist ontologies in mathematics education: The body in/ of mathematics. *Educational Studies in Mathematics*, 83(3), 453–470. <https://doi.org/10.1007/s10649-012-9465-z>

- Ehrenfeld, N. (2022). Framing an ecological perspective on teacher professional development. *Educational Researcher*, 51(7), 489–495. <https://doi.org/10.3102/0013189X221112113>
- Ehrenfeld, N., & Heyd-Metzuyanim, E. (2019). Intellectual identities in the construction of a hybrid discourse: The case of an ultra-orthodox Jewish mathematics classroom. *International Journal of Science and Mathematics Education*, 17(4), 739–757. <https://doi.org/10.1007/s10763-018-9885-z>
- Fasheh, M. (1982). Mathematics, culture, and authority. *For the Learning of Mathematics*, 3(2), 2–8. <https://flm-journal.org/Articles/61C120DB2753AE943E8568787FA72A.pdf>
- Fouze, A. and Amit, M. (2023). The importance of ethnomathematics education. *Creative Education*, 14, 729–740. <https://10.4236/ce.2023.144048>
- Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, 44(1), 37–68. <https://doi.org/10.5951/jresmetheduc.44.1.0037>
- Gutiérrez, R. (2017). Living mathematx: Towards a vision for the future. In Galindo, E., & Newton, J., (Eds.). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Hoosier Association of Mathematics Teacher Educators.
- Kelton, M. L., & Ma, J. Y. (2020). Assembling a torus: Family mobilities in an immersive mathematics exhibition. *Cognition and Instruction*, 38(3), 318–347. <https://doi.org/10.1080/07370008.2020.1725013>
- Louie, N., & Zhan, W.-Y. (2022). A socio-ecological framework for research in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 365–371. <https://doi.org/10.5951/jresmetheduc-2022-0003>
- Rubel, L. H., & Ehrenfeld, N. (2020). Palestinian/Arab Israeli women’s experiences in mathematics education: An intersectional analysis. *International Journal of Educational Research*, 102, 101616. <https://doi.org/10.1016/j.ijer.2020.101616>
- Rubel, L. H., & Nicol, C. (2020). The power of place: spatializing critical mathematics education. *Mathematical Thinking and Learning*, 22(3), 173–194. <https://doi.org/10.1080/10986065.2020.1709938>
- Sabbah, S., & Heyd-Metzuyanim, E. (2021). Integration of Arab female students at a technological university—Narratives of identity in figured worlds. *International Journal of Science and Mathematics Education*, 19(5), 977–996. <https://doi.org/10.1007/s10763-020-10084-8>
- Schwartz, G., Alon, L., & Sabbah, S. (submitted). Bridging the gap between mathematics and real-life: Teachers’ perspectives and imagined possibilities. Paper submitted for presentation at the 2025 Meeting of the American Educational Research Association (AERA), AERA.
- Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, 34(4), 14–22. <https://doi.org/10.3102/0013189X034004014>
- Skovsmose, O. (2021). Mathematics and crises. *Educational Studies of Mathematics*, 108, 369–383. <https://doi.org/10.1007/s10649-021-10037-0>
- Valero, P. (2023). Mathematical subjectivation: death sentence or chances for a terrestrial life. In Michal Ayalon, Boris Koichu, Roza Leikin, Laurie Rubel & Michal Tabach (Eds.). *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education (Vol. 1)*. University of Haifa, Israel: PME (pp. 53–68).
- Vossoughi, S., Marin, A., & Bang, M. (2023). Toward just and sustainable futures: Human learning and relationality within socio-ecological systems. *Review of Research in Education*, 47(1), 218–273. <https://doi.org/10.3102/0091732X231223505>
- Zameret, Z. (2002). *The melting pot in Israel: The Commission of Inquiry concerning the education of immigrant children during the early years of the State*. SUNY Press. <https://doi.org/10.1353/book4500>

TROUBLING THE AIMS OF MATHEMATICS EDUCATION IN THE SOCIO-ECOLOGICAL

Paola Valero¹, Francesco Beccuti²

¹ Stockholm University, ² Università di Cagliari

We propose to engage in a critical examination of the aims of mathematics education in the socio-ecological by challenging some typical standard proposals that may emerge for fostering change and raising awareness concerning current multiple climate crises. The challenge comes not only from the (im)possibilities of connected educational configurations, but foremostly from the investigation of mathematics education as a cultural, political, and economic space of subjectivation in the Modern governing of populations and individuals. We argue that such type of systematic critique is important to understand the limits and promises of mathematics education and well-intentioned proposals for action that may emerge in the field. Without critique, research may risk contributing to a simple “green washing” of the curriculum and the practices of mathematics education.

A TYPICAL WELL-INTENTIONED PROPOSAL FOR CHANGE

As concerned mathematics educators, we are increasingly worried by the rapid progression of environmental degradation and the various alarming symptoms of climate change. In the face of these challenges, mathematics stands out as a crucial ally. Through the application of mathematical modeling, scientists can simulate climate systems and predict future changes with remarkable accuracy. These models help us understand the potential impacts of various environmental policies and human activities, guiding decision-makers towards sustainable practices. Statistical methods allow us to analyze vast amounts of climate data, identify patterns, and assess the effectiveness of mitigation and adaptation strategies. Mathematical optimization techniques can be used to develop efficient resource management plans, minimizing environmental impact while at the same time meeting human needs.

By integrating environmental issues into all types of mathematics curricula, educators can contribute to equip students with the knowledge, critical thinking and problem-solving skills necessary to apply mathematics and address these global challenges. This approach not only enhances students' mathematical proficiency but also fosters a sense of responsibility and empowerment, inspiring them to contribute to the creation of a sustainable future, showing that mathematics is not just an abstract discipline but a vital tool in the urgent fight against climate change because it offers the analytical framework needed to understand its complexities and devise effective solutions.

It is thus imperative that the aims of mathematics education align with the OECD's and UN's Sustainable Development Goals, particularly SDG 4 (Quality Education) and SDG 13 (Climate Action). We must fundamentally rethink and reshape mathematics curricula to incorporate sustainability and environmental literacy. As a consequence, we should integrate real-world problems related to climate change directly into mathematics lessons, making the subject more relevant and engaging for students while simultaneously fostering a deeper understanding of global environmental challenges. For instance, lessons could include analyzing carbon footprints, calculating the impact of various energy sources or modeling population growth and its

environmental consequences or else they could include the development of models for sustainable urban planning, optimize water usage in agriculture or predict the outcomes of different climate policies. Moreover, professional development for teachers should offer training to effectively integrate sustainability into teaching, to bring new technologies and to use new assessment methods. This includes understanding the latest research in climate science and environmental mathematics as well as learning how to foster a classroom environment that encourages socially-responsible thinking and innovation.

By transforming mathematics education in these ways, we can not only equip students with the technical skills needed to address climate change, but also cultivate a generation of informed, responsible, and proactive citizens. This aligns with a vision for education that promotes academic excellence while also developing skills and values necessary for a sustainable and equitable future. Through these changes, mathematics education can become a powerful driver of both personal and planetary well-being, helping to achieve the Sustainable Development Goals and securing a better future for all.

SKEPTICAL EDUCATIONAL OBJECTIONS

Wait, what? Really? From a skeptical educational point of view, a first set of objections to the sketched proposal above can be raised. While integrating climate change themes into mathematics education may seem like a just and promising endeavor, it is essential to critically examine the practical feasibility of this and similar proposals as well as considering whether overall similar initiatives may genuinely contribute to solving the ecological crises or simply serve as possibly empty symbolic gestures with no real environmental gain.

A first difficulty is that proposals such as the above seem to assume a one-size-fits-all approach, aligning with externally mandated guidelines like the UN's Sustainable Development Goals. However, not only do educational contexts vary widely across different regions and countries but also environmental challenges may be different in different areas and geographical situations. Second, a more fundamental difficulty is that the primary role of mathematics education has traditionally been and still is related to the development of students' mathematical knowledge, abilities and competencies. Some mathematics educators —or schools, parents, politicians, employers— would not be willing to divert too much focus from the mathematical core towards environmental themes which might dilute the rigor and depth needed for mathematics instruction of high quality.

Furthermore, mathematics teachers are trained in mathematics, not in environmental or climate science. Hence, a potential restructuring of mathematics education towards environmental goals places a significant and perhaps unfeasible burden on mathematics teachers. Professional development programs can help, but they require substantial investment in time and resources, which may not be feasible for all educational institutions, especially those already facing budget constraints. Indeed, expecting teachers to cultivate not just mathematical proficiency but also environmental topics and communicate a sense of responsibility for the environment may be overwhelming in the current circumstances, possibly leading to increased burnout and reduced effectiveness and well-being.

Moreover, there could be a risk that the overemphasis on mathematical modeling is more detrimental than beneficial in treating environmental issues. Indeed, mathematical models, while powerful tools,

are not infallible but come with assumptions, approximations and simplifications. Over-reliance on models without understanding their constraints could mislead students about the predictive power of mathematics. For example, climate models often involve simplifications of complex natural systems and are influenced by the quality and quantity of data available as well as by the assumptions made by scientists. Misunderstanding these factors can lead students to overestimate the precision and reliability of model predictions.

Hence, there is a risk that typical well-intentioned proposals to align mathematics education with environmental concerns could become superficial, focusing more on the appearance of addressing climate change rather than achieving meaningful outcomes.

CULTURAL-POLITICAL OBJECTIONS

A further critical stance can help us problematize the proposal above which exemplifies the usual way of responding to societal challenges in education, in general, and mathematics education, in particular. The proposal can be seen as an articulation of the existing “order of discourse” — in Foucault’s terms — which frames what is perceived as familiar, research-based responses to new problems of practice. Indeed, resonances can be found between the proposal and reports with suggestions by international organizations such as the OECD (e.g., Nusche et al., 2024) which suggests rethinking STEM education to “get the foundations right”. UNESCO (2024) emphasizes how climate change affects education — worsening results in mathematics achievement of affected populations — and how a shift towards social and emotional, and action-oriented learning should help mitigating climate change. The response from educators and agencies governing education appeal to a sense of urgency and to the necessity of action, and provide expert-based avenues to steer education in new directions.

As well intentioned and — most of the time — well-supported the responses may be, they mobilize a logic of educationalization of social problems (e.g., Tröhler, 2017): the characteristic of Modern education to be thought of as the most effective mechanism to solve different types of social challenges by regulating changes in the knowledge, cognitive capacities, moralities and behavior of populations and individuals. Educationalization leads to an overestimation of the role of education in solving complex, non-educational problems such as, for instance, global climate change. While education indeed may help in raising awareness, it is not a panacea. Comprehensive solutions to the ecological crises — an any other social problem, for that matter — require large coordinated efforts across multiple sectors, including policy, industry and communities. Expecting mathematics education alone to drive significant change may set unrealistic expectations and divert attention from other essential actions. In other words, orienting the formulation of the aims of mathematics education in what is perceived as a new time of socio-ecological crises with the logic of the proposal above may simply continue following the same Modern impulse. But if that logic has worked before — think for example the time of the Sputnik crises —, why not now?

We would argue that a stance that critically challenges the Modern impulse is worth mobilizing, given that what is identified as socio-ecological crises are a manifestation of how the cultural, economic and political project of Modernity with its impulse for the transformation, appropriation and exploitation of humans and not humans has overpassed what the planet can bear (e.g., Latour, 2017). Therefore, the stance that we adopt to trouble the aims of mathematics education in the socio-

ecological considers mathematics education as an important cultural-historical network of subjectivation practices in education. As argued in studies on the *cultural politics of mathematics education* (e.g., Ziols & Kirschgasler, 2024), mathematics as a subject of the school curriculum in Modern mass education has served not only the purposes of qualifications through the expansion of elements of mathematical knowledge, skills and competences, but also and inseparably the fabrication of types of subjectivities that embody central values, moralities, epistemic virtues and ways of conceiving of the world and the self (cf. Beccuti, 2024). Investigating how school mathematics knowledge and being are articulated is central in understanding the aims of mathematics education as a space to make culture, to create certain notions of nature, and to envision which functions people’s mathematical qualifications are to play.

Also, the fact that mathematics education is studied in relation to the making of subjectivities renders both mathematics education and research as political (e.g., Kollosche, 2016). That is, power is being effected in and through these practices, as populations and individuals are governed, classified and ordered with respect to their coming closer (or not) to desired forms of knowing and being. In this sense, mathematics education instantiates power, resulting in the (re)production of multiple in(ex)clusions.

Thus, an exploration of the aims of mathematics education adopting this stance directs our attention to the ways in which mathematics education is materially and discursively articulated in different sites: not only in schools and classrooms, but also in the many other sites that make part of the distributed network of institutions and practices that support its cultural importance for producing mathematical subjectivation. Theoretically/methodologically, the exploration brings us to cultural studies, sociology and philosophy, to look for ways of unpacking how and in which conditions such subjectivation is effected, and to challenge its consequences.

CULTURAL-ECONOMIC OBJECTIONS

Thinking with Latour (2017), Valero (2023) has argued that the forms of subjectivity fostered in/through school mathematics education practices have historically been tied to Modern narratives of progress and development which now show to be an unsustainable project for the planet. Furthermore, the project of Modernity has also been associated with the endless expansion of capitalist forms of exploitation of both human and non-human resources. As Fraser (2023) pointed out, not only the mode of production of current neoliberal Capitalism concerns the mere act of endless material exploitation of nature, but also, since its Modern inception, tends to reproduce forms of subjectivation connected to a peculiar form of representation of nature itself. This fundamentally serves to sustain the perpetuation of its exploitation: nature is represented as an inert mass of exploitable and disposable objects, the ultimately inessential and inert “other” of humankind (*ibid.*).

If the cultural link through history between mathematics education and the project of Modern technoscientific progress and capitalism are inseparably linked in governing and producing types of people who are functional to the Modern capitalist order — even in its recent manifestations —, then mathematics education cannot really serve a function of safeguarding the planet or deterring its brutal continuous exploitation, as has been the case until now. Mathematics education in its current form — articulated aims and curricular and pedagogical organization — from the outset cannot be used to promote different forms of subjectivation that are more “socio-ecological aware”, “planet-friendly”

or even less “planet-rescuing” force. And proposing more of the same type of mathematics education that has been contributing to the situation that we are at now may lead nowhere than to the worsening of the situation. We seem to be facing a type of “Catch-22” situation, confronting a central impasse for mathematics education: the desire of mathematics education to “maintaining and promoting Modern forms of mathematical subjectivation risks proclaiming a death sentence on the planet, since the planet cannot bear any further project of modernization” (Valero, 2023, p. 56) that mathematics education is designed to perform. Furthermore, and making clear Valero’s argument on subjectivation, the cultural-historical entanglement of the Modern, technoscientific enterprise and Capitalism are also an element at the core of the type of subjectivities that mathematics education has stived to produce. Not in vain is mathematics education in recent times closely governed by agencies such as the OECD and its PISA (see e.g., Andrade-Molina, 2022).

If this is the case, what can be said about the proposal for change sketched above? What if efforts to align the aims of mathematics education with the desire to mitigate the socio-ecological crises are merely ritualistic gestures that give the illusion of a solution? Here we need to engage in a new form of critique. According to Žižek (2011), many contemporary institutional practices are characterized by the creation of a semblance of change while allowing the underlying structures of power and exploitation to remain unchallenged. This is one of the mechanisms of the ideological functioning of Capitalism, and mathematics education is deeply entangled in it (Straehler-Pohl & Pais, 2014). In this context, initiatives that appear progressive but lack substantive impact might function as ideological rituals by providing a sense of participation and moral satisfaction without addressing the deeper, structural causes of ecological degradation. Popkewitz (1982, 1988) emphasized the performative nature of many educational reforms and efforts to strive for change, arguing that they often end up reinforcing existing power structures rather than disrupting them. He highlighted how reforms or changes frequently aim to adjust practices within the existing framework of education, thus failing to challenge the deeper socio-economic and political contexts that shape educational goals and methods.

Efforts that only superficially integrate socio-ecological themes into the curriculum may thus *simulate* solutions rather than enact real change. For example, incorporating a few environmental examples in mathematics problems or holding occasional workshops on sustainability can create the appearance of addressing the crises, but without a fundamental shift in the educational paradigm and societal values, these actions might be little more than performative gestures. Following Žižek, then these ritualistic activities can serve to maintain the *status quo* by channeling potential radical energy into harmless activities. In this sense, incremental and superficial changes in mathematics education could serve to “pacify” calls for more radical transformation, thereby preventing more significant and necessary systemic changes from occurring. Žižek speaks about symbolic acts that satisfy our need to “do something” while avoiding real sacrifice or change. Educational reforms that do not challenge the underlying capitalist and technoscientific paradigms may fall into this category, allowing educators and policymakers to claim they are addressing the ecological crises without fundamentally altering the systems that contribute to the problem. These reforms thus run the risk of amounting to little more than lip service to environmentalist discourse rather than achieving systemic change, a potential type of “green-washing” of mathematics education practices and, overall, the school mathematics curriculum. Such “green-washing” would not be more than the educationalization of the risks of the multiple socio-ecological crises that mathematics education is called to solve.

The concept of the “ideology of certainty” elaborated by Borba and Skovsmose (2004) can be particularly illuminating here also with reference to the aforementioned problems connected to the (mis)use of mathematical modeling in the classroom. As understood by these authors, this is an ideology arising primarily in mathematical classrooms which fosters a belief in the absolute accuracy and objectivity of mathematical knowledge and models, which can create a false sense of security and control over complex issues like climate change (cf. also Skovsmose, 2023; Mbembe, 2021). By relying heavily on mathematical models to address climate change, educators and policymakers might unwittingly perpetuate this ideology, giving the impression that mathematical solutions and approaches are infallible and/or unquestionable. Borba and Skovsmose warned that this ideology can mask the inherent uncertainties and limitations of mathematical models, leading to an over-reliance on these tools and potentially overlooking other crucial dimensions of the ecological crises. This without even mentioning that new forms of mathematical models, such as those underlying current Artificial Intelligence technologies, create new forms of certainty that are not only problematic in themselves, but are also a new force contributing to the socio-ecological crises (e.g., see Andersson & Valero, in this volume).

Furthermore, illusions arising in the classroom connected to the mathematization of environmental problems with the purpose to sensitize towards them can ultimately go in the direction of nurturing “green capitalist” solutions with doubtful or detrimental environmental impact. Think for instance of the current carbon credit offset market and its connection with the idea that we can quantify the carbon emission of each and every human activity. While this idea is frequently used in the classroom to raise awareness about individuals’ environmental impact, these activities can also be seen as actively connected to the push for adoption of carbon offsetting mechanisms as a major institutional solution to the environmental crises, ultimately masking the reality that “carbon offsets are primarily used to justify ongoing emissions, rather than reduce them” (Cullenward et. al., 2023, p. 1085).

Overall, the ideology of certainty reflects what Jablonka and Gellert (2007) described as the “myth of the infallibility of technology” (p. 8; cf. Strahler-Pohl, 2017). Applied to environmental crises, this myth implies that there is no doubt that a solution will ultimately come from advanced technological interventions, possibly including, say, forms of extreme geoengineering. Latour (2017), criticized this *hubris* as a particular type of frenzy of technoscientific domination which serves to acquiesce in many the anxiety related to the upcoming environmental disaster. The key, in this perspective, is to accelerate with even more technoscientific advancements — surely the invention of a new device for zero-emission energy is just around the corner! Of course, mathematization is fundamental for this pursuit. And for mathematization to generate a sense of certainty, mathematics education has to promote a subjectivation that inserts in individuals and populations the epistemic virtues of mathematics, all of which are Modern in character (Valero, 2023). This is why the typical proposals of change in mathematics education may not break the dominant educational-political-cultural-economic logic that governs the very same functioning of the different institutions implicated in producing the current socio-ecological crises.

ALTERNATIVE ORIENTATIONS?

Some mathematics educators might argue that the most important task of mathematics education is to provide universal access to a high-quality education that equips students with both strong mathematical knowledge and skills and the ability to transfer and apply the former to real-world

problems. From this perspective, the integration within mathematics education of the modelling of climate environmental phenomena would appear solely in the positive light of offering a means of fostering mathematical knowledge, developing transfer and application skills, and also raising environmental consciousness. To these mathematics educators the critical approach adopted in this paper could perhaps appear to be counterproductively pessimistic and ultimately leading to the overlooking of the significant real-world impact that mathematics can have — leaving aside a potentially divisive and paralyzing effect in the field itself that critical approaches are often condemned for. As Coles, Solares-Rojas and le Roux (2024) pointed out, the usual practices of mathematics education, if they remain unchallenged, run the risk of becoming, in the words of Latour (2004), like those “mechanical toys that endlessly make the same gesture when everything else has changed around them” (p. 225). Therefore, they argued that for mathematics education to produce significant new actions, “[t]here is an urgency to (re)evaluate the “reach” of these practices of mathematics education and consider both what ecological collapse means for mathematics education and whether mathematics education might have a meaningful response” (Coles et al., 2024, p. 166).

To truly go in the direction of addressing the ecological crises by means — or perhaps despite — mathematics education, a radical rethinking thus seems to be required: one that moves beyond the usual, learned responses to reform mathematics curricula and practices, which may easily turn into symbolic gestures not challenging the core principles and structures of the current educational paradigm. Drawing on the analytical moves of studies on the cultural politics of mathematics, it is possible to engage in a critique of the types of typical proposals by troubling mathematics education as part of the network of practices and power that make it a central element in the school curriculum. It is also possible to focus on the economic critique that posits mathematics education as central for the function of mathematical subjectivation in capitalist economies and their organization. These problematizations are important to disturb the belief that better mathematical curricula and practices aligned to internationally proclaimed sustainability goals can potentially make a fundamental difference. This critical task requires us to go beyond the established limits and possibilities of mathematics education.

An objection to the critical stances in mathematics education that we propose here could be that no concrete, actionable proposals of change in practice are offered here. To such objection we first respond that the ethical responsibility of research as a serious intellectual activity engaged with the world and its problems is to strange the familiar, to provide understanding, and to theorize. Research is a very political act with the potential to make new imaginations possible. Without critique, it is unrealistic to disrupt the strength of reproduction built in many well-intentioned proposals for change in mathematics education, also in times of socio-ecological crises. A second response is that critique is at the base of the possibility of articulating new imaginations not only conceptually but also in what is actionable. This is indeed a collective challenge as we move—hopefully—towards new unexplored directions.

Acknowledgement

We thank the members of SOCAME, RelMaS (VR 2019-03679) and IMMPACT (VR 2021-05235) for the collaboration and discussions that support our work.

References

- Andersson, C. & Valero, P. (this volume). Mathematical AI-modeling in the socio-ecological: futuring critical mathematics education. In Le Roux, K., Coles, A., Solares-Rojas, A., Bose, A., Vistro-Yu, C., Valero, P., Sinclair, N., et al. (Eds.), *Proceedings of the 27th ICMI Study Conference (Mathematics Education and the Socio-Ecological)* (pp. 416-423). ICMI.
- Andrade-Molina, M. (2022). When numbers dictate common sense. Transnational aspirations of a global curriculum. In W. Zhao et al. (Eds.) *Epistemic colonialism and the transfer of curriculum knowledge across borders* (pp. 103–117). Routledge.
- Beccuti, F. (2024). Meaning and subjectivity in the PISA mathematics frameworks: a sociological approach. *Educational Studies in Mathematics*, 116(1), 49–65.
- Borba, M. C., & Skovsmose, O. (1997). The ideology of certainty in mathematics education. *For the learning of Mathematics*, 17(3), 17–23.
- Coles, A., Solares-Rojas, A. & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116(2), 165–183.
- Fraser, N. (2023). *Cannibal capitalism: How our system is devouring democracy, care, and the Planet – and what we can do about it*. Verso Books.
- Latour, B. (2004). Why has critique run out of steam? From matters of fact to matters of concern. *Critical Inquiry*, 30(2), 225–248.
- Latour, B. (2017). *Facing Gaia: Eight lectures on the new climatic regime*. Polity.
- Mbembe, A. (2021). *Out of the dark night: Essays on decolonization*. Columbia University Press.
- Nusche, D., Rabella, M. F., & Lauterbach, S. (2024). *Rethinking education in the context of climate change*. OECD education working paper nr. 307. OECD.
- Popkewitz, T. S. (1982). Educational reform as the organization of ritual: Stability as change. *Journal of Education*, 164(1), 5–29.
- Popkewitz, T. S. (1988). Educational reform: Rhetoric, ritual, and social interest. *Educational Theory*, 38(1), 77–93.
- Skovsmose, O. (2023). *Critical mathematics education*. Springer.
- Straehler-Pohl, H. & Pais, A. (2014). Learning to fail and learning from failure – ideology at work in a mathematics classroom. *Pedagogy, Culture & Society*, 22(1), 79–96.
- UNESCO. (2024). *Education and climate change. Learning to act for people and planet*. UNESCO.
- Valero, P. (2023). Mathematical subjectivation: death sentence or chances for a terrestrial life? In M. Ayalon et al. (Eds.) *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol 1, pp. 53–68). PME.
- Ziols, R., & Kirchgasser, K. L. (2024). Slow emergencies of racism in mathematics education. *Educational Studies in Mathematics*, 116(3), 371–389.
- Žižek, S. (2011). *Living in the end times*. Verso Books

IMPLICATIONS OF ANTHROPOCENE AND GAIA IN MATHEMATICS EDUCATION

João Viola¹, Júlio Faria Corrêa²

¹Federal University of Mato Grosso do Sul, ²Federal University of Santa Catarina

This essay aims to produce implications of Anthropocene and Gaia in Mathematics Education, specifically on socio-ecological issues. To do so, we briefly disclose these two concepts and produce some effects in Mathematics Education. We do not discuss a mathematics education of improvement, development, and progress. Not even a mathematics education with socio-ecological issues as a context for teaching and learning mathematics. Gaia and the Anthropocene imply redesigns of political-economic-ecological assumptions of Mathematics Education. For example, we present concepts of scale and belonging in the scenario of the River Doce in a mathematics education entanglement with Gaia and the Anthropocene.

INTRODUCTION

white people sleep a lot, but they can only dream about themselves

Davi Kopenawa Yanomami, Indigenous leader

Throughout this paper, we will engage with the following question – a powerful invitation to produce other ideas – presented in the Discussion Document of the 27th ICMI Study: “What does the socio-ecological condition of the world mean for mathematics (education), and what might mathematics (education) mean for this condition?”. We begin by listing some elements of the current socio-ecological condition worldwide and some future projections.

By 2050, we will have more plastic in the oceans than fish. In Brazil, around 27 million tons of recyclable waste, not garbage, are produced annually, of which only 4% is recycled. Kevin Anderson, a British scientist, stated that by the turn of the millennium, it would be possible to think of an evolutionary solution, within the political and economic system, in the face of climate emergencies. However, after two decades of denialism and strategies that have little impact on the actions of the human species towards the Earth, only a revolutionary action, from an economic and political point of view, is capable of reversing the situation in which we live and not exceeding an increase of 1.5 degrees. German climatologist Johann Thalhuber, in response to the question, “What is the difference between an increase of 2 Celsius degrees or 4 degrees in the Earth System?” stated: simply our civilization.

These statements make explicit not only a necessity but urgency for discussions and problematizations regarding *climate emergencies* in our society, as well as in mathematics teacher education and mathematics classrooms. Not just a climate change or a crisis, but an emergency. And not climate emergencies just as a possibility of contextualizing mathematical content, rather calling into question how school and academic mathematics are also constituents of this scenario. Paraphrasing ideas from anthropologist Bruno Latour (2020), what other relationalities can the human species build between terrans (humans and non-humans), who connect, relate and become entangled with Gaia, a political agent? Other relationalities not just as a function of a more harmonious way of living with other species, but for the sake of our existence.

In this meander of ideas, emergencies, mathematics education, and the potent question of the 27th ICMI Study, this essay aims to produce implications of the Anthropocene and Gaia in Mathematics Education, specifically on socio-ecological issues. To do so, we briefly engage with these two concepts and unfold some effects in mathematics education. We drift apart from a mathematics education of improvement, development, and progress, as well from a mathematics education with socio-ecological issues as a context for teaching and learning mathematics. Gaia and the Anthropocene imply redesigns of political-economic-ecological assumptions of mathematics education. For example, we present concepts of scale and belonging in an entanglement with mathematics education and Gaia and the Anthropocene and ...

ANTHROPOCENE

Since the second half of the 20th century, the Anthropocene, a political-economic-ecological concept, has crossed several areas of research and used in different intensities, ethical, philosophical and epistemological assumptions. From the biologist Eugene Stoermer's first insights, until 2019, when the Anthropocene Working Group – an interdisciplinary research group established as a part of the International Commission on Stratigraphy, chaired at that time by paleobiologist Jan Zalasiewicz – advocates for the official recognition of the Anthropocene as a new geological era by the International Union of Geological Sciences. The Anthropocene, in its problematics and potentialities, offers an opportunity to redesign our practices in Mathematics Education.

In the Anthropocene, the geological era in which the Earth System finds itself, the human species carries out actions of global magnitude. On a planetary scale, those actions affect and modify systems, biomes, glaciers, the entry and exit of energy flows, sea currents, and an equilibrium between species (Costa, 2022). Therefore, there is an urgent need for political-economic-ecological projects on a global scale, which are constituted in the crossroads of the Anthropocene emergencies, including refugee crises, the habitability of the planet with other species, and climate emergencies. An Anthropocene-implicated Mathematics Education is urgent. As the Brazilians anthropologists Renzo Taddei and Stello Marras (2020) state,

In 2020, the amount of artificial materials present on the planet's surface, in mass, exceeded that of the entire biosphere. This means that there is more concrete, bricks, asphalt, metals, and aggregates used in construction than everything living on the planet. Civil construction moves more sediment than all the river basins in the world combined. The amount of energy used in human activities has reached the scale of geological processes, such as the movement of tectonic plates (p. 10 – our translation)

The Anthropocene as a concept is both powerful and inappropriate, mistaken, inconvenient, elusive, and slippery. It operates a narrative of the effects of human actions at geological levels with the Earth System. However, it still places humans at the center (Anthropos). Facing this supposed contradiction, researchers from many different areas propose other characterizations in to deal with the “built-in problem” of the Anthropocene, to cite some: Plantationocene, Capitalocene, Chthulucene. This polysemy of theoretical productions explains the power of this concept towards a planetary thinking in the 21st century, with humans, rivers, plants, mountains; bacteria, fungi, stories; stars, dreams, smells, ghosts ... the Anthropocene implies an invention of other ideas of temporality, ancestry, and relationalities between different species. Of course, it also implies a distrust of the still-present Eurocentric colonizing thought, which inhabits our grammar, desires, and ways of life. Make kin, not babies! – proclaims Donna Haraway in her famous slogan. It is urgent and necessary to

produce a practice of planetary relationalities in inventions with multiplicities, always operating with(in) the difference.

Therefore, we believe that it is necessary to explore this concept as an opportunity for a Mathematics Education in which scales, relationalities, belongings, and alliances between species with Gaia (our second concept to be explored next), can produce other ways of sociability, and especially other ways of economic, ethical, political, and ecological productions entangled with schools. Facing the Anthropocene, Mathematics Education practices need to be problematized. How can a mathematics classroom be constituted with and by the concept of the Anthropocene? Does the big and powerful narrative around the necessity of mathematics learning in schools still hold? Should one of the schools' pivotal aims be still based on human learning? On human mathematical learning? Why always us, humans?

GAIA

Sometimes the Earth, or the Earth System, is conceived as a place in which humans and non-humans are born, grow, and die. A dwelling, a source of resources. A space where species inhabit more or less harmoniously, being prey at times, predator at others; some with more power and influence, others with less. There was a time when the Earth was the center of the Universe, and humans were the center of the species on Earth. At another time, Earth was just another planet co-inhabiting a system, along with many others. However, the human was still at the center. In these narratives, the Earth is always on one side and humans on the other. Earth belongs to the realm of the immaterial, without a soul. The human belongs to the realm of rationality and culture, without a body (or at least, in a Eurocentric standpoint, having the body being purposely erased from what constitute the human).

In the 1970s, James Lovelock and Lynn Margulis proposed the Gaia hypothesis, an idea of Earth as living entity, as if the planet behaves like a single living organism, from the earliest forms of life to the agents that produce the atmosphere. The Gaia hypothesis was produced in a scenario in which the human was no longer seen as the center of all species, at a time of several upheavals in the ways of producing scientific knowledge, both in the so-called human and natural sciences. Taking this hypothesis seriously is a call to question our very theoretical, ethical, ontological, and epistemological assumptions: If the Earth is no longer just a place we inhabit, how can we operate with humans relating to the Earth? If Earth is not on one side and humans on the other, what other narratives are possible to be produced? What about if we replace human in the above questions for other beings and entities?

With Gaia – a living organism capable of producing energy for its functioning, which regulates its climate and remains in balance in maintaining, preserving, and sustaining the lives of species –, our grammar can invent itself in words (ideas/concepts) such as agency, relationalities, entanglements, effects in time-space-matter. The nature/culture binary, as well as subject/object, among many others that unfold from these, are pushed away, in an attempt to invent other alliances. In this sense, other possibilities for territories named until now as schools are necessary and urgent to be constituted.

The Gaia hypothesis stands up for another relationality with Earth. Earth as no longer an address or a resource, but an organism that regulates itself and affects us. Gaia establishes herself as a political agent. We are with Gaia and we do not live on Gaia. What concepts, ideas, and ways of seeing ourselves change when we consider Gaia as a political (and also ethical, economic, and ecological)

agent? Who is this human facing Gaia? What can a teacher, a student, or even mathematics do facing Gaia?

In 2013, following a series of lectures given in Edinburgh, Bruno Latour published his book *Facing Gaia*, which we took as inspiration to think about the implications of the Anthropocene and Gaia in Mathematics Education. According to Latour (2020), in the preface of the Brazilian version:

Gaia is not virgin nature. She is not the mother goddess. She is not the mother of anything. It is not even a whole, a global existent. It is simply the consequence of the successive inventions of living beings that ended up completely transforming the physical-chemical conditions of the initial geological earth. Today, each element of the soil, air, sea and rivers results, to a large extent, from modifications, creations and inventions of living organisms. Gaia is all living beings and the material transformations that they have subjected to geology, diverting the sun's energy for their own benefit. (p. 10-11 – our translation)

Gaia imposes upon us other agreements, other socio-abilities, other *socio*. It is no longer about subjects (teachers and students), and objects (a way of performing mathematics) inhabiting a place (mathematics classroom). With Gaia, other ideas/concepts of subject, object, and mathematics must be on the political agenda of Mathematics Education to be produced. What if trees had political representation in our schools? What if rivers, air, rain, and animals could be constitutive of and constituent in our values, assumptions, intents, and existence of territories called schools?

The concepts of the Anthropocene and Gaia, briefly explained, imply another Mathematics Education. This is not about other demands arising from our contemporary times and, with them, our efforts to deal with these problems. It is about inventing other grammar, forms of life, and modes of existence. The implications of these concepts push our bodies in other directions and, in this tension, some possibilities emerge. How to produce Mathematics Education facing the Anthropocene and Gaia? Why a (produced by and for) human Mathematics Education? Why Mathematics Education?

MATHEMATICS EDUCATION: IN BETWEEN SCALES AND BELONGINGS

In another essay, in a speculative fable with the idea of Gaia and Us, we stated:

The idea of We, terrans, moving away from We, humans, expands the potentialities of relationalities, imaginable and speculative. It pushes us towards other political regimes of coexistence and social organization. Colonize and dominate are verbs to be put into disuse, for use of other verbs such as inhabit and happen in gaps and crevices. We, terrans, as opportunities in other relationalities and entanglements, in a becoming-terrain, in contagions, reverberations, and genuine interests in the potential of others. Terrans in, terrans with. Terrans and Gaia (Viola dos Santos & Correa, 2024, in print, p. 9).

In the crossroads of the concepts Anthropocene and Gaia, we are entangled with two other concepts that, perhaps, can be useful to operate in a scenario of binaries, of causes and consequences, of relations of colonization, hierarchization, and disposability of some in the function of others. We operate the concepts of scales and belongings in a scenario of Rio Doce (Sweet River), a river of more than 853 kilometers, *who* runs through two Brazilian states, Minas Gerais and Espírito Santo, and *who* has its mouth in the Atlantic Ocean. These concepts offer other modes of sociality between species in the Anthropocena and with Gaia.

Scale is a concept commonly seen in public policies as a major challenge. Thinking about a microcosm in a given region may be, in a way, easy and possible. However, thinking about a macrocosm, a huge territory with many agencies, people and ways of life is complex and complicated. But those are not the only two possibilities of operationalize the concept of scale. Moving away from

this linear idea of the scale going from micro to macro, or vice versa, the scale can be characterized as:

a way to understand and unfold the implications of a chosen scale opens to a particular situation. When we do a characterization of the concept of scale in such way, there is no possibility to determine a comparison ratio or a measurement unity as a political strategy to measure or denote a phenomena. It is not about build, test and calibrate a scale and then produce an effect, but instead, to do a composition with variations and readings of implications and possibilities that happens when we use such scale (Correa & Viola dos Santos, 2023, p. 333).

How can we produce sociability, forms of life, and political regimes between species operating with the aforementioned concept of scale? An example would be an event with *Rio Doce* (Sweet River). In 2015, the *Barragem do Fundão* (Fundão dam) collapsed in the city of Mariana, in the state of Minas Gerais, in Southeast Brazil. This dam was built to store waste from iron ore production. This type of dam is relatively cheap for companies, as they use cheap resources such as earth and rock. Unfortunately, the Fundão dam broke and created a catastrophic chain event, with floods of tailing mud in communities and small towns.

On one scale, iron ore extraction in Brazil is the largest mineral extractive segment of the Brazilian economy. This segment corresponds to 4% of Brazilian gross domestic product. This sector generates many jobs for Brazil and contributes significantly to the production of steel that is used in the construction of buildings, bridges, machines, cars, household appliances, etc. Along this scale, the extraction of iron ore can be considered an important activity for sociability, specifically among humans, in maintaining and intensifying their ways of life.

On another scale, the construction of this dam violently transforms the lives of animals, humans, river, and their ways of relating. As a result of the construction of large reservoirs to store water from dams, sometimes an entire ecosystem is altered, with consequences for many species in a given context. What is the price of energy (produced by a hydroelectric plant with a huge reservoir), if families who had to leave their homes for the construction of a dam are included in this equation? What is the price of energy, if we include in this equation the extermination of an entire chain of animals by a flood or a change in the river course?

On a third scale, still in the context of the collapse of the Fundão dam, we had the death of a river *who is* the ancestor of indigenous Krenak peoples – an Indigenous community of the region of the dam.

The Doce River, which we, the Krenak Nation, call Watu – our grandfather – is a person, not a resource, as the economists like to call him. He is not something you can own or appropriate; he is part of our construction as a collective society that dwells in a specific place into which we have been gradually corralled by the government, forcing us to live and breed in bubbles subject to increasingly crippling external pressure (Krenak, 2020, p. 27).

How to create agency in terms of representing the Doce River in economic, political, and ecological decisions, from the construction of dams to the effects produced on and by humans and non-humans? In between these three scales, it is not just a scaling from the micro to the macro or vice versa. In variations and together with the concepts of the Anthropocene and Gaia, a Mathematics Education of terrans can be produced in a collectivity among species. Scale can also be a tool to be operated, and problematized, and not just a concept to be learned.

In the *school of moderns*, for instance, humans decided — based on their values, ethics, and interests — to brutally modify the riverbed and to commit violence against the region's ecosystem. The units of measurement and scales used are produced based on exclusively human interests. In a *school of terrans*, the use of scales, its levels of complexity, its varying compositions, and the readings and effects offered in these variant compositions need to take into account representations of non-humans. In the case of Doce River, the ancestors, animals, humans, and all other entities that inhabit this space, as well as a global scalability of the necessity, urgency, or pertinence of building the dam to extract iron ore (or even produce energy) needs to be addressed in collective productions.

When we change the scale, we change the problem! When we operate with scale in terms of composing variations and reading implications of what happens and would happen and what relates to these variations, other ways of life and other political regimes are produced. How to operate scales in Mathematics Education entangled with the Anthropocene and Gaia? Maybe, one idea is *staying with the trouble*, as proposed by Haraway (2016).

Another tool, that can be operated in school territories, in implications of the Anthropocene and Gaia in Mathematics Education, is the concept of belonging. A classic and mathematical way of operating the idea of belonging is from the point of view of *an element of a set*. An element that does or does not belong to a set. Belonging throughout identity, in a binary of being inside or outside.

Implicated by the concepts of the Anthropocene and Gaia, we try to operate the concept of belonging in other grammars, affects, and policies. Belonging as a strategy to relate terrans with Gaia. Relating without the chance of excluding, or being excluded, from and with Gaia. Belonging as a willingness to relate across differences. As Krenak (2020) states, we belong because we are capable of attracting each other through our differences. Belonging as a political-ethical-economic-pedagogical-ecological verb that entangles terrans not because of their traditions, cultures, identities, nor even by knowledge, but rather as a flow, a crossing between terrans in multiple territories. As we argued in another essay:

The idea of belonging to a country or to a cultural group is related to a bond with values, principles, believes which can traced in the traditions of such a group. The interactions between different groups has as a limit the identity process which maintain and differs these, principles, values and believes, its form of life. However, a concept of belonging could be built not by identity process but by relations in which identity is not build a priori. A concept of belonging could be oriented by a politics of indifference (Correa & Viola dos Santos, 2023, p. 333)

Belonging happens between different species when an enterprise decides to extract iron ore in a region and, at the same time, produces wealth for some and destroys an entire ecosystem and the lives of uncountable many others. It is paradoxical that an iron enterprise that contributes positively to the GDP of a country and supports humans in living with their privileges in their homes with their cars and electronics made from steel (a metal produced from iron ore), is the same iron enterprise that develops Environmental, Social, and Governance (ESG) Programs (sometimes limited, sometimes interesting) to contribute to the balance of an ecosystem and repairing damage caused by their very extractive activity. In all these actions that take place interconnected in variations and potentialities, the criteria that provide conditions for decision-making are still concentrated in the hands of (certain) humans and for (certain) humans. Again, and always, humans are in the center. How to think about belongings in which the center is not human? How can Gaia have an agency, beyond being a supporting character?

With scales, compositions in variations offer different possibilities of readings, and belongings happen with different species and also with Gaia. Humans, rivers, dreams, bacteria, and mathematics are entangled and related in inventions of forms of life, modes of existence, and political regimes of sociability. What would a classroom be like in a school in which the invention, maintenance, and deconstruction of belongings were central? Would it be plausible for us to still focus on human (mathematics) learning? What if schools were constituted as territories to produce belongings with different species? Once again with statements from an Indigenous thinker, we corroborate the following excerpt:

In Brazil, there's a rocky mountain near the stretch of the Rio Doce affected by the burst mining tailings dam. The Krenak village is on the left bank of this river, about thirteen kilometres from Resplendor, in Minas Gerais, but not far from the border with Espírito Santo. This part of the river is flanked on the right by a range of mountains. I learned that those mountains have a name — Takukrak — and a personality. Early in the morning, down in the village, the people look up at the mountain to see if it's going to be a good day or whether it would be best to stay inside. They can read this from the mountain's mien. If the mountain looks grumpy, they take extra care. When the mountain wakes up splendid, beautiful, and all done up with fluffy white clouds drifting around its peak, the villagers say: "Today we can throw a party, dance, fish, do whatever we like. It's all good." (Krenak, 2020, p. 16)

A school and a classroom can also be a territory in which different species are entangled in different ways of belonging to each other and also to Gaia, like the mountain is for the Krenak's community. A territory in which links, alliances, compositions, and other relationalities can be invented. A Gaia School can also operate with belonging in terms of supposed maintenance and stability of certain ideas, narratives, values, and ethics. Always temporary assumptions, because a desire pulses in inventing, inventing oneself, in fables, stories, mathematics, among other narratives. Historicities and ancestries, as well as futurisms, are also part of entanglements of belonging that always operate with what happens and distance themselves from an idea of replacement.

In the discussions with the Anthropocene and Gaia, operating scales and belongings with the Doce River scenario, we have some indicators of what socio-ecological conditions of the world could mean for Mathematics Education, and what might Mathematics Education mean for these conditions. The River Doce is a potential scenario for our theorizations. Other scenarios, such as dreams, deaths, and myths are necessary and urgent to operate implications of the Anthropocene and Gaia in Mathematics Education. Entangled with Donna Haraway (2016), "it matters which stories tell stories, which concepts think concepts. Mathematically, visually, and narratively, it matters which figures figure figures, which systems systematize systems (p. 101).

WHAT CAN MATHEMATICS EDUCATION MEAN FACING THE ANTHROPOCENE AND GAIA

In this essay, we attempted to problematize old meanings and aims that constitutes the foundation of Mathematics Education. We consider the modern school to be that of humans, of (mathematical) subject, of teaching and learning aimed at progress and human development, that is, the triumph of humans over nature. We seek to shake up the values and aims of Mathematics Education that circulate in modern schools, bringing Gaia and the Anthropocene into dialogue.

On one hand, the Anthropocene, as a political-pedagogical concept, allowed us to push the centrality of the human-centered project of modernity, which portrays nature as an infinite resource to be explored. The Mathematics Education implicated by the Anthropocene could push the school of

moderns and give rise to the school of Terrans — those focused on Gaia and who no longer share the values of progress, but seek to create other relationalities with humans and non-humans.

On the other hand, Gaia emerges as a political agent who can no longer be silenced. Gaia imposes other types of agreements, other socio-ability, other *socio*. It is no longer about thinking about subjects (teachers and students), or objects (a way of constituting mathematics) inhabiting a place (mathematics classroom). With Gaia, other ideas/concepts of subject, object, place, and mathematics must be on the political agenda of Mathematics Education, to be produced/invented.

For instance, we mobilized scale and belonging, not as content to be applied, but as a way of operating and producing other relationalities and agencies between human and non-human actors. In this sense, the three different River Doce scales and belongings give rise to a much richer and more complex political-pedagogical game that could inspire other mathematics education.

Finally, we would like to reinforce that we do not discuss a mathematics education of improvement, development, and progress. Not even a mathematics education with socio-ecological issues as a context for teaching and learning mathematics. The Western human, who only dreams about *himself*, and the Earth, an (in)exhaustible source of resources, are elements of an anthropocentric mathematics education. The multispecies entangled with the Anthropocene and Gaia can operate concepts such as scale and belongings, and many others to be invented by necessity or even contemplation, in a Mathematics Education of Terrans. Inspired Davi Kopenawa Yanomami's epigraph that opened this essay and provoked us to think-sense in new directions, we proclaim: *Mathematics Education sleeps a lot, but it (yet) can only dream about itself*.

Acknowledgements

We thank the Fundect-MS for the financial support to the authors.

References

- Costa, A. A. (2022). Antropoceno: desmandamentos gravados em rocha. In D. Danowski, E. Viveiros de Castro, & R. Saldanha (Eds.), *Os Mil Nomes de Gaia: do Antropoceno à Idade da Terra* (Vol. 1, pp. 106-186). Rio de Janeiro: Machado.
- Corrêa, J., & Viola dos Santos, J. R. (2023). Mathematics Education of Helplessness. *Prometeica - Revista De Filosofia Y Ciencias*, (27), 327–335. <https://doi.org/10.34024/prometeica.2023.27.15306>
- Haraway, D. J. (2016). *Staying with the trouble*. Duke University Press.
- Krenak, A. (2020). *Ideas to postpone the end of the world*. House of Anansi Press Inc.
- Latour, B. ([2015]2020). *Diante de Gaia: oito conferências sobre a natureza no Antropoceno*. Ed. Ubu.
- Taddei, R., & Marras, S. (2022). Introdução: do Antropos aos mundos confluídos. In S. Marras & R. Taddei (Eds.), *O Antropoceno: sobre modos de compor mundos* (Vol. 1, pp. 9-30). Belo Horizonte, Brazil: Fino Traço.
- Viola, J. & Corrêa, J. (2024) Nós e Gaia: entre educações matemáticas dos terrestres. *REVEMAT*, in print.
-

THEME B – SCALES OF MATHEMATICS EDUCATION

STUDENTS' SOCIO-ECOLOGICAL GESTURES IN DRAWINGS

Aehee Ahn

Massey University

This discussion aims to illustrate certain students' socio-ecological gestures in mathematics which emerged through their drawings on fraction concepts, and to offer a potential direction to entangle socio-ecology and mathematics. I conducted a drawing task, Fraction Land, with sixth-grade Korean students, asking them to create an image representing a fraction land. Viewing drawing as an embodied act of doing mathematics, I focus on two students' drawings that brought socio-ecological issues – specifically flood and pollution – in relation to fractions. These observations exemplify the students' socio-ecological gestures that recontextualise ecological issues within mathematics, creating meaningful connections between socio-ecology and mathematics. Their engagement highlights how learning can extend beyond the traditional mathematics school curriculum, demonstrating the potential for socio-ecological awareness to enrich sustainable mathematics education within and outside the classroom.

INTRODUCTION

A socio-ecological perspective examines the interconnections and entanglements between living/non-living beings and socio-ecological systems. In addressing local and global ecological concerns, mathematics can play a role in predicting environmental uncertainties and supporting ecological sustainability. Renert (2011) identified characteristics of sustainable mathematics education, arguing that mathematics education needs to connect mathematical concepts with the environment (e.g., interpreting large numbers as ecological quantities). This approach enables mathematics to provide sustainable solutions to ecological challenges, such as predicting food production and consumption, or anticipating the development of new materials through mathematical models. In this context, Coles and Helliwell (2023) emphasised the importance of teaching ecological sustainability in mathematics classrooms, encouraging educators to pay attention to mathematical situations that relate to ecological issues. For a sustainable mathematics education, Solares-Rojas et al. (2022) designed mathematical activities for a curriculum innovation to foster a conceptual shift in students when engaging with ecological issues. By reminding students of the historical importance and beauty of a river (now polluted) in Mexico, schools and local communities designed educational activities to address the issue of pollution. One such activity involved the creation of a museum dedicated to the polluted river, which links times (past, present, and future) and connects community members (students, teachers, and local residents) with their environment. These activities in ecological mathematics research seek to create space for students and teachers to re-view and reflect on socio-ecological issues in mathematics. This led to my interest in students' enactment of and perspectives on socio-ecology: How do students bring socio-ecology into their understanding of mathematics?

The aim of this discussion is to illustrate certain student's socio-ecological meaning-making in enacting with socio-ecological issues in mathematics, based on Coles et al. (2024), who work on socio-ecological entanglement. Coles et al. (2024) view the social and ecological as inseparable and entangled. They perceive the aspects of the social and ecology as existing in inseparable relations, beyond mutual influence on each other. With this perspective on socio-ecology, Coles et al. (2024)

define socio-ecological gestures as “practices that produce research, recontextualized knowledge, curriculum organizations, learning materials, professional development opportunities, and anything that becomes visible in the context of teaching and learning” (p. 166). From their example, recognising, expanding, or considering diverse innovative research on local and global socio-ecological issues can be a socio-ecological gesture that listens for the entanglement of socio-ecology.

Following their perspective, I interpret students’ socio-ecological gestures as the recontextualisation of their knowledge and learning of their entangled socio-ecological world in their context. As part of my own socio-ecological gestures, I listen for students’ socio-ecological gestures observed in their drawings, which reflect their knowing of socio-ecology and mathematics. In particular, I investigate how individual students bring forth their ecological views as visual images when depicting a mathematical concept, specifically the concept of fractions. I bring two students’ drawings to illustrate how the students shape ecological issues in a mathematical way, as their socio-ecological gestures, and I then discuss their socio-ecological ideas within and beyond the mathematics curriculum. Before examining the students’ ecological gestures in their drawings, I outline my view of a student’s drawing as embodied representation and my stance on how I analyse drawings.

EMBODIEMENT AND DRAWING

To address how individual students bring together mathematics and socio-ecology, I approach the scope of ecological mathematics through the lens of human differences and embodiment. Embodied activities enact a world, and bodily-grounded conceptions bring forth a world (Varela & Poerksen, 2006). Embodied cognition puts a focus on the importance of bodily actions in knowing, and bodily actions cannot be considered separately from knowing. Nemirovsky and Ferrara (2009)’s exemplar cases involving students’ bodily gestures and utterances illustrated how bodily performing enacted with thinking, showing that thinking is bodily actions themselves. Lakoff and Núñez (2000) explained that mathematical concepts are grounded in embodied everyday experiences, which they refer to as conceptual metaphors. They demonstrated how mathematics is made sense of by relating to material worlds, such as numbers tied to counting objects. While their work on embodied mathematics, particularly conceptual metaphors, highlights sensory-motor experiences as the foundation for conceptualising abstract mathematical ideas, my focus is toward how students link these embodied experiences to ecological contexts. Kumar (2024) discussed the ecological implications of Lakoff and Núñez’s work in mathematics, suggesting that they might ask “this ecological phenomenon *provides the conceptual ground* for which mathematical ideas?” rather than “which mathematical ideas *apply* to this ecological phenomenon?” (p. 10, originally italic). This embodied mathematics perspective offers valuable insights into students’ learning experiences as potential conceptual resources in bringing their understanding of socio-ecological dynamics into a mathematics context.

Drawing as an embodied act brings thinking into being. Visualisation is a powerful tool for making the invisible visible and for re-enacting on them (Arcavi, 2003). Drawing enables students to revisit and revise their images, providing a way to capture the emergence of students’ understanding of concepts. Visual images act as written descriptions of mathematical ideas and offer visual captures of real-time thinking. As embodied representations, these images can play a crucial role in representing students’ interpretations of mathematical concepts. De Freitas and Sinclair (2011) proposed that drawing – or “diagramming” as they refer to it – is intricately entwined with thinking. Rather than separating materials from thought, they explore how diagramming moves beyond static

representation, emphasising its dynamic and performative nature. They argued that diagramming is not about distinguishing between internal and external representations, or the virtual and the actual. Instead, they regard student's tactile actions, such as diagramming, as performative acts of knowledge creation, akin to bodily gestures. I view students' drawings as embodied acts that bring forth their understanding of concepts. In the next section, I describe my methodological approach and touch on how I prompted students to draw images.

METHODS

The drawing task was part of a broader investigation into students' conceptions of fractions, and the primary aim of the whole data collection was not specifically to explore students' socio-ecological understanding in mathematics. The drawing task, titled "Fraction Land", involved creating visual images related to the concept of fractions. I reached out to 27 sixth-grade Korean students (aged 11-12) through their schoolteacher. Students' participation was voluntary, and consent for using students' responses was obtained from both the students and their caretakers. As both a researcher and a teacher, I prompted the students to draw any images that came to mind when they thought about fraction land. The specific prompt, given in Korean, was: "please draw any images of FRACTION LAND". The Korean term for "land" denotes a part of the surface without water or a country/state, suggesting that fraction land could symbolize a country/state where fractions exist. The fraction land task was individual, with no specific principles or guidelines provided for the drawing. At the time, the student participants had learned most mathematical concepts related to fractions at a primary school level, except for the concept of ratios. Following the drawing activity, to listen for their drawings, one-to-one online interviews were conducted. The key question asked was: "Can you tell me about your fraction land drawing?". For the safety of both the students and myself, the schoolteacher accompanied me during the drawing task and the interviews.

In analysing the dataset, I initially focused my attention on students' use of metaphors as their understanding of fractions, drawing on Lakoff and Núñez (2000) work on conceptual metaphors. As a researcher, I recursively visited students' drawings to find their conceptual sources they used to make sense of fractions and identified generative metaphoric themes as a particular form of fraction understanding. My interpretations were also shaped by my shared cultural background with the students as a Korean, as I sought to understand the cultural contexts embedded in their drawings and the interpretive nature of knowing situated in contexts. Recognising the diversity of the students, I placed an emphasis on their individual agency. While analysing the drawings, I noticed that two students introduced ecological issues into their fraction land images. This connection to ecology was unexpected, as the task had no explicit socio-ecological component. Since the interviews were primarily focused on discussing fractions, they did not delve deeply into the socio-ecological aspects of the drawings. Despite this limitation, I found the works of two students, Dami and Bora (pseudonyms), particularly noteworthy because they incorporated ecological concerns into their fraction drawings. Through these two cases as exceptional and evoking works, I aim to exemplify the students' socio-ecological gestures, indicating how their ecological thinking became entangled with their understanding of mathematics.

SOCIO-ECOLOGY IN DRAWINGS

Dami illustrated her fraction land as a kingdom where a king and a queen exist. Her fraction land is a space equipped with social infrastructure for fraction residences, such as apartments, a square, and a department store. The king and queen are natural numbers with a fraction number form: $1/1$ and $3/1$, respectively, while the proper fraction resident $3/7$ is in a square. I noticed that Dami's fraction land has several spaces with water, including a pond and indoor and outdoor swimming pools. She also drew a river that runs across the land, dividing it into upper and lower regions. Bridges span the river, connecting the two regions and leading to roads. Figure 1 provides a close-up of the river in her fraction land. In her drawing, Dami included written texts in Korean, which I translated into English in the image. The translated texts are positioned in the same places and with similar font sizes as her original Korean text. The students' images (Figures 1 and 2) are presented with the translated English text.

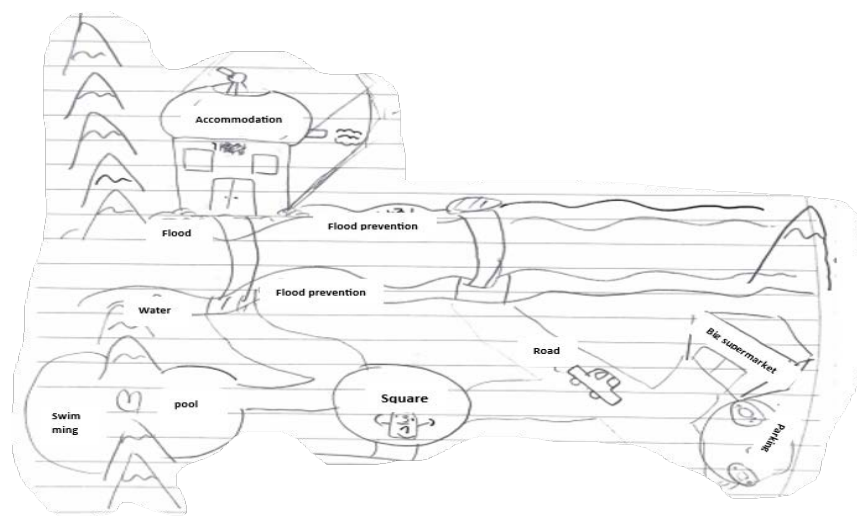


Figure 1: Socio-ecology in Dami's fraction land

As shown in Figure 1, several mountain peaks are drawn on both the right and left sides of the river, with bridges leading to a swimming pool, a square, and a supermarket with a parking space. The border of the river is surrounded by a layer labeled "flood prevention." In the interview, Dami mentioned the river-related details as follows:

- Dami: When I watch historical dramas or things like that, it seems like kings or people like that go for walks in pavilions or around ponds and use river water. A river has bridges that connect villages and castles, but they also need to divide them, so I drew a river.
- I: What does "flood prevention" mean here?
- Dami: Because it's river... and there is an accommodation right in front of it. Rivers are at risk of overflowing during floods.

I first noticed the entanglement of water within the fraction land society. According to the interview, the river plays a dual role in both separating and connecting the upper and the lower regions. The accommodation building is located near the river, which could suggest a residential area with scenic river views or a tourist attraction. On the left side of the drawing, water appears to flow from the mountains, connecting to the river and the swimming pool, which the fraction land residents could use in their daily lives. This connection indicates the essential role of water for practical and aesthetic

purposes. Another key observation is the layer drawn along the riverbank. The layer drawing seems to denote its role as an embankment designed to prevent flooding. This detail addresses a flood-related concern. Heavy rain, a global issue, often causes significant social damage, such as property loss. In Korea, on a local level, it also complicates parenting when schools close for student safety, during heavy rains. Climate change has led to unexpected downpours in certain areas. Urban areas are particularly affected by floods each year due to large impermeable surfaces, such as asphalt roads, as well as the presence of Korean-style semi-basement households, typically inhabited by economically vulnerable people (Eum & Kim, 2024). This shows how flood issues in Korea are entangled in social, economic, ecological, and educational aspects. The primary school students who participated in the fraction land drawing task lived in areas generally safe from flood damage and with low economic vulnerability, but they were familiar with water issues as their homes were located near seaside beaches. For Dami, the river appears to be linked to the risk of flooding.

Dami's drawing illustrates the relationship between nature elements (river water and mountains) and artificial elements (e.g., swimming pool, bridge, roads, embankment), showing how these components are intertwined with water, like a socio-ecological narrative. This story implies a water-related ecological perspective within her fraction land society, highlighting the practical, aesthetical, and problematic aspects of water. It can be seen as an example of the externality, dynamics, and creativity in thinking in drawing (de Freitas & Sinclair, 2011). The inspiration for Dami's drawing seems to stem from her life experiences and prior learning outside of mathematics class. In her interviews, Dami mentioned that her ideas about water (such as the pavilion on a pond and the river) came from a historical TV series she had watched. She appeared to connect her experiences outside of school to a mathematical context, linking aspects of everyday life with mathematical thinking.

On the other hand, Bora's fraction land introduces another socio-ecological issue, related to the fraction $\frac{1}{2}$. Her fraction land explores the concept of halves, such as a bisected table, land divided into half sea and half ground, and the equivalence between $\frac{4}{8}$ and $\frac{1}{2}$ through an image of a leftover pizza. In her drawing, I observed that Bora addresses a pollution issue by illustrating objects – a phone and earphones – that are halved. Figure 2 shows the part of her whole drawing relevant to this ecological issue.



Figure 2: Socio-ecology in Bora's fraction land

Figure 2 portrays the fraction being $\frac{8}{11}$ using a mobile phone and a connected earphone. Both the mobile phone and the earphone are designed around the concept of halves. The mobile phone is shown at half its conventional size, addressing an environmental pollution issue. Similarly, the earphone is drawn as being halved with only one earpiece, rather than the usual two. Bora went on to explain this imagery further during the interview:

- Bora: This is a phone split in half, and it's made for preventing environmental pollution. The earphone is also split in half, so there's only one side.
- I: Can you tell me more about the idea of preventing environmental pollution?
- Bora: Because of the severe environmental pollution and global warming in our country right now, I drew it this way.
- I: Do you think having just half like this would help reduce environmental pollution?
- Bora: I heard that making phone parts releases greenhouse gases and stuff, so I thought maybe reducing the size of the phone might make it somewhat better than before.
- I: What about the earphone?
- Bora: Speaking of phones, I also thought of earphones, so drew them in half as well.
- I: I see. How did you come up with this?
- Bora: I learned it in my essay writing class.

For Bora, the concept of half seems to be the idea of “less” in the context of sustainable use. She believed that reducing the size of items can lead to sustainable benefits by consuming fewer resources and less energy, which aligns with principles of sustainable product design. In the interview, she articulated her vision for a better future, indicating a focus on sustainability. In this context, the concept of half emerges as a symbol of conservation and sustainable use. Her drawing demonstrates how she brings an ecological concern into mathematics to move toward a more sustainable future. For Bora, understanding the concept of half may support and propose sustainable practices in everyday life, establishing a connection between environmental issues and mathematics, specifically fractions. Similar to Dami, Bora's conceptual resources in her drawing did not stem solely from her mathematics education. She connected her understanding of pollution, gained through essay writing, to her fraction drawing. When doing mathematics, she draws on knowledge from outside of the mathematics classroom as everyday conceptual sources and links it to her mathematics.

STUDENTS' SOCIO-ECOLOGICAL GESTURES

In exploring the fraction land drawings of Bora and Dami, I observed how they address socio-ecological issues within mathematics. In thinking of fraction concepts, Dami depicted the entanglement of nature and society connecting with water, including potential flooding concerns. Bora connected the concept of half with reduced consumption and emissions, illustrating the relationship between a sustainable future and mathematics. This exploration shows how students bring socio-ecological issues into mathematics, demonstrating the entanglement of socio-ecology with mathematical thinking. Dami and Bora move back and forth between mathematics and socio-ecological issues and recontextualise these socio-ecological issues in a mathematical framework.

I interpret these ecological reflections enacted in mathematics as their socio-ecological gestures. Lakoff and Núñez (2000) regarded *understanding* as the process of creating and structuring relations between concepts. Building on their perspective, the socio-ecological gestures observed in the two students can be seen as acts of understanding socio-ecological issues through mathematics, thereby shaping their socio-ecological thinking. Students can use socio-ecological issues as resources for mathematical thinking while simultaneously shaping their socio-ecological understanding through their mathematical insights. Establishing relations among students, socio-ecology, and mathematics can be viewed as their socio-ecological gestures.

A significant insight gained from this exploration is that when discussing mathematical concepts, what Dami and Bora bring and generate is not confined solely within the realm of mathematics. In the Korean primary mathematics curriculum (Ministry of Education, 2015), socio-ecological issues are not explicitly addressed. Although some textbook activities regarding fractions incorporate everyday life contexts including ecological aspects, such as fractional proportions in gardening, fraction concepts are typically presented through contexts of object sharing or length and area measuring. Consequently, mathematical concepts and socio-ecology are treated separately within the curriculum. However, Dami and Bora did not confine fraction concepts to the boundaries between mathematics and non-mathematics. They acted as agents in enacting their everyday resources related to socio-ecological aspects. This approach shows an entanglement of time (their previous experiences and present knowledge) and the relationships between mathematics and socio-ecology.

Dami and Bora seem to create mathematics spaces that involve the ecological conditions of their society, linking their past experiences to their present context. The resources they use to form socio-ecological relationships with mathematics could stem from what they learned and thought. These resources can help students establish connections and are not limited to the mathematics curriculum. Students can think mathematical concepts and ecological issues through experiences inside and outside of the classroom, as illustrated in the two fraction land drawings, such as writing essays, watching nature documentaries at home, and discussing these topics with friends. The time and space derived from a student's everyday life can be significant gestures for enacting socio-ecology in mathematics. This observation shows students' listening for recontextualising their perspectives on ecology and fraction concepts and for connecting ideas across mathematics and beyond the classroom. In shaping socio-ecology with mathematics, students appear to play roles as actors in bringing mathematics to socio-ecological issues, or vice versa. Socio-ecological resources can serve as learning spaces where students gain valuable experiences both inside and outside school. Interpreting the socio-ecological project by Solares-Rojas et al. (2022) through this lens suggests that their work may provide students with outside-classroom resources to entangle their experiences, social contexts, and mathematics, thereby supporting their socio-ecological gestures. Engaging with socio-ecology in mathematics could enable students to form relationships with the socio-ecology surrounding them.

LOOKING FORWARD

I explored how students recontextualised social-ecological issues within their mathematical contexts, as their socio-ecological gestures, to investigate the entanglement of socio-ecology and mathematics. Although I identified two instances of socio-ecological gestures in their drawings, the fraction land task did not seem to encourage many students to engage with socio-ecology issues in mathematics. To prompt more students to connect with socio-ecological thinking in their fraction drawings, the task could be revised to evoke the socio-ecological aspects of fraction land. For example, the drawing prompt could explicitly encourage consideration of the environment or socio-ecology by stating, "Draw any image considering the environment of FRACTION LAND" or "Draw any image imagining the ecology of FRACTION LAND society".

My forward question is: How can we (e.g., I, researchers, teachers, and teacher educators) enhance or foster students' socio-ecological gestures? How can mathematics promote the enactment of socio-ecology? My initial idea regarding these questions is to provide more time and space as conceptual resources that address socio-ecological issues. The fraction land drawing task created a space for the

two students to recontextualise socio-ecological issues in a mathematical way. It may suggest that mathematics learning should not always focus on mathematical concepts. The typical contexts used to introduce fraction concepts in the curriculum – such as sharing and measuring objects – may be limited for addressing environmental uncertainties. Teaching mathematics might need to encourage students to connect their mathematical understanding with their pasts, other subjects, and socio-ecological issues, rather than confining them to traditional mathematics curricula. This approach may provide a rationale for developing innovative curricula aimed at reaching sustainable education in a rapidly changing world.

References

- Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52(3), 215–241. <https://doi.org/10.1023/A:1024312321077>
- Coles, A., & Helliwell, T. (2023) The role of mathematics teacher educators in preparing teachers of mathematics to respond to global challenges within their classrooms. *London Review of Education*, 21(1), 1–13. <https://doi.org/10.14324/LRE.21.1.02>
- Coles, A., Solares-Rojas, A., & le Roux, K (2024) Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- de Freitas, E., & Sinclair, N. (2012) Diagram, gesture, agency: Theorizing embodiment in the mathematics classroom. *Educational Studies in Mathematics*, 80, 133–152. <https://doi.org/10.1007/s10649-011-9364-8>
- Eum, J.G., & Kim, H.K. (2004) 서울특별시 반지하 가구를 고려한 홍수 취약성 평가 [Assessment of flood vulnerability of semi-basement households in Seoul]. *Journal of Climate Change Research*, 15(1), 35–46. <https://doi.org/10.15531/KSCCR.2024.15.1.035>
- Kumar, S. U (2024). Towards an ecological mathematics. *Interdisciplinary Science Reviews*, 0(0), 1–22. <https://doi.org/10.1177/030801882412327>
- Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being* (1st edn). Basic Books.
- Ministry of Education. (2015). *Mathematics Curriculum for Republic of Korea*. Notification No. 2015-74 of the Ministry of Education [Supplement No. 8]
- Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics*, 70, 159–174. <https://doi.org/10.1007/s10649-008-9150-4>
- Renert, M. (2011) Mathematics for life: Sustainable mathematics education. *For the Learning of Mathematics*, 31(1), 20–26. <https://www.jstor.org/stable/41319547>
- Solares-Rojas, A., Arellano-Aguilar, O., García González, M. M., López-Vargas, M. D. R., Coles, A., & Méndez Serrano, A. (2022). Mathematics education and social-environmental crises: An interdisciplinary proposal for didactic innovation with rural communities in Mexico. *Research in Mathematics Education*, 24(2), 202–223. <https://doi.org/10.1080/14794802.2022.2062781>
- Varela, F. J., & Poerksen, B. (2006). Truth is what works: Francisco J. Varela on cognitive science, Buddhism, the inseparability of subject and object, and the exaggerations of constructivism – a conversation. *The Journal of Aesthetic Education*, 40(1), 35–53. <https://doi.org/10.1353/jae.2006.0012>

LIFE IN A DROP OF WATER: A PARTICIPATORY DESIGN PROJECT TO TACKLING EDUCATIONAL POVERTY

Giancarlo Artiano^{1,3}, Emilio Balzano^{2,3}, Maria Mellone^{2,3}

¹Università degli Studi della Campania Luigi Vanvitelli, ²Università degli Studi di Napoli Federico II, ³Università degli Studi Suor Orsola Benincasa

Educational poverty, a significant and far-reaching phenomenon, demands attention due to its profound impact on the economy and society. Extensive research, including cross-sectional studies and time series analyses, reveals that these disparities are closely linked to gender, ethnicity, disability, geographic location, and economic status. The situation becomes more complicated if we consider that students from challenging social environments face a high risk of not acquiring essential skills such as mathematics and their mother tongue. They are also barred from scientific careers and have great difficulties with scientific literacy. Triggering processes in society to allow everyone access to public, quality education is a significant challenge for mathematics and science education research. The current contribution primarily discusses an action research approach to promote a participatory design project to create a pond for communities living in underprivileged areas. The goal is to promote a scientific culture in education, with a specific focus on the role of adults in reducing educational poverty.

PREFACE

We begin our contribution to ICMI Study 27 with a framework of the socio-political context that shapes our daily operations. Educational poverty stands out as one of the most pressing issues characterising our metropolitan area. This issue assumes a complex form within our territory, marked by a myriad of variables such as public childcare services not available to all, poor basic skills, school dropout rates, and a dearth of youth opportunities, all of which collectively represent the dire state of educational poverty within our region. The urban area in which we operate is characterised by a significant number of students with low proficiency in their mother tongue and mathematics as shown by conclamated national and international survey tests (OECD, 2023; Mullis et al., 2021). Notably, cases of school dropouts have seen a dramatic surge following the lockdown imposed by the pandemic emergency, a situation that has deeply affected students across all grades. Furthermore, the prevalence of unemployment and youth involvement in criminal activities in our area ranks among the highest in Europe, compounding the challenges faced by the community. These circumstances are further exacerbated by the insufficient and ineffective allocation of public resources, perpetuating the lack of equitable and accessible education, particularly in socioeconomically disadvantaged territories. Schools catering to the most marginalised communities often lack essential educational provisions, such as full-time programs, updated study materials and opportunities for educational engagement throughout the day, tailored to meet the unique needs of the local context. These institutional inefficiencies and teaching challenges, particularly in the realm of mathematics education, contribute to a lack of appeal for formal schooling, prompting families in urban areas to seek alternative activities for their children.

Our mathematics and physics education research group has been working for years to develop strategies and intervention models to address the educational challenges in our metropolitan area

(Artiano & Balzano, 2023; Mellone et al., 2019). In this project, we will discuss our unique experience with a participatory design project to create a pond in an abandoned school area in a disadvantaged neighbourhood of our city. The project involved designing and constructing a pond that has become the focal point for many mathematics and science educational activities for the school community. We engaged students, adults (especially parents of at-risk children), educators, teachers, and volunteers from social associations in several outdoor scientific activities, particularly the analysis of the geometry and scientific characteristics of the pond and its surrounding environment. This work was made possible thanks to the immense effort and collaboration of a group that promotes environmental awareness and the school's headmistress. We will describe the activity in more depth in the following sections.

In conclusion, one of the goals of this contribution is to examine it within the socio-ecological framework for teaching mathematics as indicated by Coles et al. (2024). We see the present action-research process and its relative participatory design project as one of the many possible socio-ecological gestures to address educational poverty. We think the pond example is a research activity that integrates multiple scales of socio-ecological entanglements. We try to explain this through our statements in the conclusion section. Therefore, the ICMI 27 represents an opportunity to share our experience with the community of researchers and educators and deepen our skills to become researchers in mathematics and physics education with a greater focus on socio-ecological issues.

THE POND

The EduCare Project

The pond was built in the area of the complex belonging to the 83rd "Porchiano Bordiga" Comprehensive Institute in Ponticelli, a popular district on the eastern outskirts of Naples. The property located at Via Argine 917b, from which it takes its name, has experienced years of neglect due to a substantial decrease in enrollment, leading to the cessation of educational activities within the building. Starting from January 2020, thanks to the firm will of the current headmaster of the school and a synergistic effort of numerous volunteers from various associations in the area, numerous initiatives have been promoted aimed at the recovery of the structure and at designing new spaces and new uses of the complex. These include the EduCare project, which lasted three years and aims to implement an integrated strategy of interventions involving the local educational community, which comprises schools, teachers, families, public institutions, and associations in the eastern suburbs of Naples. As research groups, we have been collaborators on the project. Our role has focused on recognising the impact that scientific, mathematical, and technological education can have on the cultural education of the public. This entails promoting a non-traditional approach to teaching mathematics and physics within the education community. The research methodology we adopt in the project is based on a mixed-method approach. Specifically, we use an up-to-date Design Experiment framework to design experimental activities (Cobb et al., 2003; diSessa & Cobb, 2004). That approach implies that we design and implement interventions in authentic classroom contexts, whereas our regular presence over an extended time frame allowed for a seamless integration of the intervention in the students' everyday school experience. The iterative nature of the design approach involves a process of micro-cyclic design (on the scale of classroom activities), evaluation and revision in which interventions are planned according to a provisional assessment of learning outcomes. This implies that learning and design can be studied in their mutual interdependence.

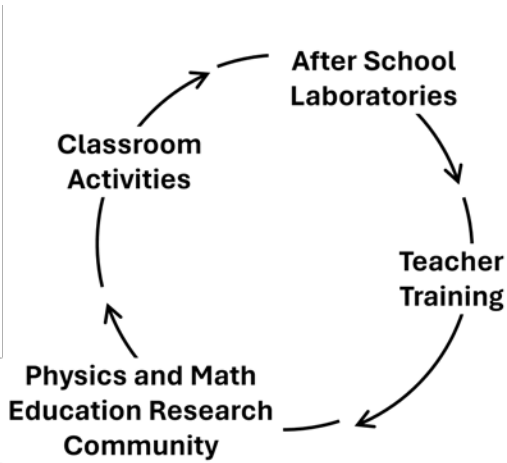


Figure 1: The macro-cycle design research

Research activities in real teaching-learning contexts are crucial (Guidoni, 2005b). This allows us to use the collected data to provide valuable suggestions for teacher training and scientific workshops for parents and to prompt deep reflection on revisiting the fundamentals of disciplines. This iterative and macro-cyclic design (on the scale of research) is useful for bridging the gap between research and practice. That approach can lead to didactic proposals that align with “what happens” in the classroom, favouring a bottom-up rather than a top-down approach. Figure 1 shows the macro-cyclic design. The macro-area of interventions for the EduCare project were: a) Providing teacher training courses, b) conducting

experimental classroom activities with students of all grade levels from kindergarten to middle school; c) developing and evaluating extracurricular science education programs and scientific dissemination initiatives targeting students, parents, and the social community. To synergise our interventions and enhance the territory's resources, we have started a participatory design process to create an immersive scientific environment in the Via Argine structure. Initially, we set up hands-on laboratories in the internal areas. Afterwards, we started the construction of the pond in the external area. The images used in the present work are from the documentation of the EduCare project. We have permission to share these materials based on the guidelines in the consent form, which has been signed by all participants, including the legal guardians of minors.

Design and Implementation



Figure 2: Pond construction timelapse

The school assigned the design and implementation of the pond to the Verde Speranza Association. They are one of the many groups who care about the place, and their mission is to carry out actions concerning the care of greenery to raise awareness among citizens on ecological and environmental issues. Therefore, the educational pond project was a great example of activities aligned with their policy. They designed the pond in collaboration with our research group. We make it essential. The pond has an oval shape and an area of about 30 square meters (Figure 2). It has two levels of depth, one

part of 40 cm and the other of 60 cm. We created these two different areas to make the pond an ecosystem open to a plurality of flora and fauna. We planted water plants such as Lotus, Elodea canadensis, Nymphaea, and Papyrus. We choose to have only plants on the perimeter and in the water and not to have consumer carnivores. This choice, a part of biological equilibrium, was made to

discuss ethical aspects regarding the legitimacy or otherwise of putting living organisms in an artificial environment. Tap water was used to fill the pond, which is rich in chlorine and evaporates shortly afterwards thanks to its high volatility. The water was, therefore, ready to host life. Right from the start, the pond was populated with mosquitoes and dragonflies ready to give life to the ecosystem.

A complex ecosystem

The pond is a complex ecosystem with reciprocal interaction between abiotic, biotic and anthropic components. In educational and training discussions, educators usually forget the anthropocentric vision of nature as a critical point of discussion. Instead, with our artificial pond, we begin by sharing that every living being represents a point of view because it lives in its ecosystem and is accessible to its modes of perception, movement, feeding and reproduction needs. The pond comprises an expanse of stagnant fresh water fed by rain or an aquifer, in which vegetation and aquatic organisms depend on each other to feed and survive. The organisms produced in the pond are microscopic plants, mostly algae, which comprise the so-called phytoplankton. Despite its invisibility, phytoplankton produces more oxygen than macroscopic plants (seagrasses rooted in the bottom or floating algae), which are only important in shallow waters. There are two types of consumers: herbivores and carnivores. Herbivores (or primary consumers) feed on plants or parts of plants. Among them, the herbivorous zooplankton is essential in the water column, but there can also be herbivorous fish such as carp. Other consumers, such as predatory insects, fish, or carnivorous zooplankton, feed on primary consumers and thus become secondary consumers (carnivores). The chain continues with carnivorous fish-eating carnivores (tertiary consumers) and so on. Also very important are the detritivores present, especially in the benthos (a term that indicates the fauna present on the bottom). Their shredding activity makes it easier for decomposers to work. Finally, decomposers are mainly present in the bottom mud, where dead organic matter accumulates, and in the water column rich in particulates. These are bacteria, fungi and flagellates, which can release the nutrients contained in dead organisms so that they are recycled and reused. All these generic structures can be observed appropriately by planning educational activities. Bearing in mind, therefore, the complexity of the ecosystem constituted, we organised five-month educational activities involving students from kindergarten to lower secondary school, teachers and families from all class groups. The design of the activities involved the choice of a common macro-theme modulated in terms of complexity based on the age of the participants entitled “Studying a complex ecosystem across different scales.” This macro-theme is divided into three sub-topics, which are:

- water and soil
- shape and dimension of a drop
- life in a drop of water

STUDYING A COMPLEX ECOSYSTEM ACROSS DIFFERENT SCALES

Water and soil

The activities begin with children and adults taking elements of the terrain near the pond (Figure 3). What students harvest is placed on the worktables, and through direct observation with magnifying glasses and measuring meters, they define the characteristics of fruits, flowers, leaves, branches and pieces of bark, giving them names, sizes, colours, compositions, sensation to the touch, the smell

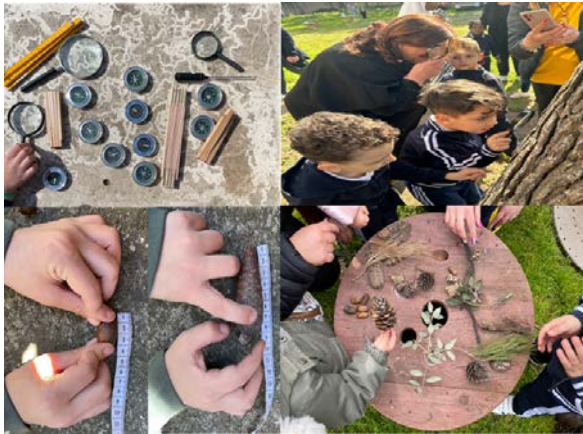


Figure 3: Observations of the soil elements

when dry and wet. We then move on to characterise the soil, which, after being collected, is sifted to define the granules' size and observe the organic material contained. Students, with the help of sieves, determine the granules' size. They compare and analyse several soil types (gravel, sand and clay) and discuss the differences present. Then, they classify soils according to the size of the granules (gravel granule diameter greater than 2 mm, sand granule diameter between 2 mm and 0.1 mm, clay diameter less than 0.002 mm). We discuss with students that the different types of soil used during the activity result from natural phenomena capable of

transforming the rock in place into incoherent material (erosion, transport, deposit), of which the primary architect is water. We propose to study the power of water transport and storage (Figure 4).

By placing small quantities of soil samples in a jar, adding water and stirring, and using the power of water to drag, students observe, mix and deposit the different soils according to exact rules, which we discuss collectively. As the speed of water transport decreases, the coarser material settles on the bottom and the finer. The last particles to settle will be those in suspension. At the end of the experiment, a stratigraphic succession will be observed in the jar that starts from the coarser material (gravel) and ends with clay. Clay is essential in forming natural ponds as it determines the stagnation of rainwater and is used for ours, too.



Figure 4: Studying the power of water transport

Shape and Volume of a Drop

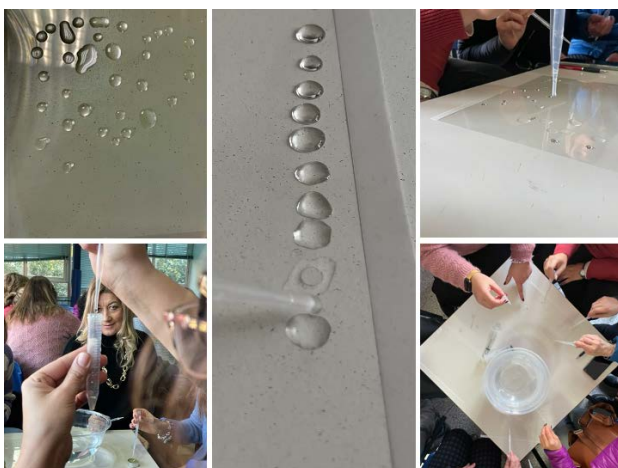


Figure 5: Shaping and measuring volume drops

Surface tension is a fundamental force that governs the behaviour of liquid droplets, such as water. This force arises from the cohesive nature of water molecules, which strive to minimize surface energy. Consequently, water droplets assume a spherical shape, representing the configuration with the minimum surface area for a given volume, thereby minimizing the total surface energy. The spherical shape also offers stability, as the surface tension distribution around the perimeter is uniform, preventing distortion or division into smaller droplets (Figure 5).

Using a pipette provides an insightful experimental approach to understanding the formation and detachment of water droplets. The meticulous manipulation of the dropper pipette's distance from the substrate and the applied pressure enables the observation and analysis of the resultant variations in droplet shapes. Subsequently, the droplet volume can be quantified, albeit with specific challenges. An approach involving the enumeration of droplets required to fill a volume, followed by a division of the total volume by the number of droplets, yields an estimate of the average volume of a single droplet. Notably, these estimations are subject to experimental nuances and inherent limitations. Throughout the activity, the adults worked with entire groups of children. Our responsibility was to facilitate free exploration activities by providing examples of creating various droplets and openly discussing the space occupied by them. The activity was captivating, and we recommended extending the laboratory experience at home.

Life in a Drop of Water

In this educational activity, we gather adults and children near the pond to explore various aspects of the ecosystems. The focus is on understanding the concept of a pond as an ecosystem, the organisms that inhabit it, and the challenges they face. By understanding these challenges, we can foster a sense of environmental responsibility. We also compare the water of a pond with everyday water sources, such as tap or bottled water, and conduct measurements of temperature, dissolved oxygen, and turbidity in pond water samples using sensors. The activity also involves collecting water samples from different areas and depths of the pond and observing the microorganisms present in the samples using magnifying glasses. However, there is a whole life in a drop of water that is not visible to the naked eye. To make the learning experience more interactive, we guide the participants in transitioning from observing the macro scale to a thousand times smaller by asking them to show us a specific part of their body (fingertip, nails, a hat) while looking at it under a microscope. We connect a microscope to a large screen to allow everyone to see what the microscope is magnifying (Figure



Figure 6: Observations with the microscope

6). We then analyse the water samples in detail, giving us a vision of the microscopic world enclosed in a drop of water. The focus is on discussing the ecosystem's complexity and the organisms' adaptations to their environment. We observe specific organisms, such as crustaceans, including *Daphnia* (water flea), copepods, decapods, and ostracods, to understand their internal systems, organs, and reproductive mechanisms. We also present the participants with additional organisms, such as hydra, dipteran larvae (chironomids and culicids), oligochaetes, and molluscs. Throughout the activity, we actively encourage participant engagement

through observations, drawings, and descriptions of the organisms. This promotes an interactive learning experience, piquing interest and creating an engaging atmosphere. We avoid overwhelming participants with excessive biological details, instead emphasising intriguing aspects such as live embryos within the body of *Daphnia* and predator-prey interactions involving the hydra.

CONCLUSION

Results

The mathematics and science education research project for designing, building, and conducting pond activities commenced in September 2022. It involved a community of nearly 500 individuals, including adults and children, from an urban area of Naples with high levels of educational poverty. Even though the EduCare Project concluded a year ago, the Verde Speranza cooperative, our Research Group, and the school community are collaborating to develop and implement practices to ensure the sustainable management of the pond. Green spaces are scarce in this neighbourhood, and those that exist are often neglected. As a result, the pond stands as one of the few natural ecosystems within several kilometres. The school's proximity to the pond enables frequent class visits, fostering ongoing engagement and the development of long-term educational trails. This, in turn, encourages the documentation of observations through diverse mediums and sparks questions that lead to a better understanding of fundamental concepts such as variables and interrelationships among components. By nurturing the survival and well-being of the ecosystem over time, a sense of community and emotional attachment is cultivated, promoting respect for the environment. Children and their families are provided with a place to explore and analyse other similar ecosystems, fostering environmental awareness and responsibility. The impact on the community is significant, and we are proud to be part of this collective effort.

Discussion

We report some ideas by Banks (2007) and Gutiérrez and Rogoff (2003), who point out that “individual development and disposition must be understood in (not separate from) cultural and historical context”. All people engage in sophisticated learning shaped by the cultural and contextual conditions in which they live. In this sense, all people learn, but a group may learn different knowledge and practices and may organise its learning differently (NRC, 2009). Children’s learning is a social and cultural process, and they develop mathematics and science skills starting in childhood through family relationships and play. In this sense the nature of the environments to which individuals are exposed influences their conceptions of scientific principles and ways of knowing. It is critical to consider diversity issues and the science learning of vulnerable communities for several reasons: to ensure equitable treatment of all individuals; to continue to develop a well-trained workforce; to create a well-informed, scientifically literate citizenry; and to increase diversity in the pool of scientists and science educators who can bring new perspectives to science and the understanding of science (NRC, 2009). Adults, family members, and teachers play crucial roles in supporting children’s learning in science, making sense of their experiences and supporting reasoning about interpreting natural phenomena. The involvement of citizens, especially adults, in science education is a general objective often addressed with disappointing results. Most proposals are limited to dissemination and could be more effective. From our commitment to the field, we have learned that planning stimulating activities in a welcoming environment that leverage the enhancement of the knowledge that the adult already possesses helps him to recognise that he can carry out a scientific activity together with the child independently and to deepen the topic addressed by continuing the experimental activities at home. We want to conclude the present contribution with the school's headmistress in her own words:

Usually, the school sets up a top-down conversation with the respective school, and the parents perceive it. However, when the discussion plane is shifted and set on an equal basis, the discussion becomes more authentic, and parents become more available [...] The two main social activities involved in this work (the pond), namely the practice of science and the dissemination of scientific knowledge, were pursued as collective efforts with significant implications for individuals and the communities in which the research was conducted.

Acknowledgements

We want to express our gratitude to L.E. and all the members of the Verde Speranza cooperative, as well as to the principal of the Porchiano Bordiga Comprehensive Institute and the entire school community.

References

- Artiano, G., & Balzano, E. (2023). Teaching and learning physics in formal and informal contexts. *Il Nuovo Cimento*, 100(182), 46.
- Banks, J.A. (2007). *Educating citizens in multicultural society* (2nd ed.). New York: Teachers College Press.
- Cini, M. (1991). Science and sustainable society. *Capitalism Nature Socialism*, 2(2), 39–54.
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, 32(1), 9–13.
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024) Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *Journal of the Learning Sciences*, 13(1), 77–103.
- Guidoni, P., Iannece, D., Tortora, R., et al. (2005b). Multimodal language strategies activated by students in understanding and communicating mathematics. *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education*, Sant Feliu de Guixols, 841–851.
- Gutiérrez, K.D., and Rogoff, B. (2003). Cultural ways of knowing: Individual traits or repertoires of practice. *Educational Researcher*, 32(5), 19-25.
- Mellone, M., Sabena, C., Carotenuto, G., Gagliano, M., Bisogni, C., & Sapio, R. L. (2019). Informal Mathematical education contexts to prevent the school drop-out. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.). *Annual Proceedings of the Forty-Third Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 163). PME.
- National Research Council (NRC). (2009). *Learning science in informal environments: People, places, and pursuits*.
- Mullis, I. V., Martin, M. O., & von Davier, M. (2021). *TIMSS 2023 Assessment Frameworks. International Association for the Evaluation of Educational Achievement*.
- OECD. (2023). *Pisa 2022 results (Volume I): The state of learning and equity in education*. PISA, OECD Publishing. Paris.

BUILDING CONNECTIONS ACROSS SCALES TO STIMULATE AND REALIZE TRANSFORMATION IN MATHEMATICS EDUCATION

Carrie Bala, Brynja Kohler

Utah State University

This paper reports on collaborative efforts to reimagine mathematics education within the complexities of a socio-ecological system. Mathematics is often taught in isolation from meaning, community, and culture, through and within a Western academic system. Alternatively, authentic problem-solving through mathematical modeling and data science invites and values ways of knowing and doing that are deeply connected to community, culture, and place. The authors are engaged in two projects aimed at building connections across scales to stimulate and realize transformation. One initiative brings together mathematicians, educators, and policy makers to reimagine meaningful mathematics courses and implement pathways of opportunity for secondary and university students. Another project pairs secondary teachers in rural and Native American communities with university faculty to co-develop mathematical modeling and data science curriculum meaningful to place. We present and analyze project components with a socio-ecological framework and reflect on the entanglement of entities in these projects.

INTRODUCTION

Mathematics education today is embedded in a complex web of social, ecological, economic, and technological change. The need exists to support the development of students able to address these very real challenges with authentic problem-solving skills (Wolfram, 2020; Yanez et al., 2019). At issue is the gap between the deeply contextualized challenges of today and the procedural skills taught in traditional, Western school mathematics. Mathematics through a Western way of knowing is regularly taught in isolation from meaning, community, and culture, and is indeed often assumed to transcend culture (Luecke & Sanders, 2023; O'Neill et al., 2023). While a classical way of knowing is well-established and formidable, it is incomplete and insufficient to engage and prepare students to address the complexities of today. As all knowledge is connected to specific communities, a uniform or monocultural approach to problem-solving has limitations and should necessarily stimulate questions of what mathematics is meaningful to the contemporary world (Coles et al., 2024; Kincheloe & Steinberg, 2014). Mathematics education requires a shift, a transformation from an emphasis on low-level skill such as classical algebraic manipulations towards engaging, meaningful, and sustainable problem-solving. This shift must center the complex challenges of specific communities, invite and elevate different ways of knowing, and utilize the relevant technologies of today (Luecke & Sanders, 2023; O'Neill et al., 2023; Wolfram, 2020).

Transformational change in mathematics education may occur through newly imagined communities across layers of social and ecological activity. Louie and Zhan (2022) analyzed approaches to equity through a socio-ecological framework, with layers detailed in Table 1. These authors provided a mechanism to describe both the flow of power between social layers and the need for transformational efforts at every level. Further, Coles et al. (2024) call to recognize the entanglements of culture, community, and place at all layers of social activity. This article incorporates the entangled socio-ecological framework to detail collaborative change efforts across scales for two projects.

Layers of a Socio-Ecological Framework
Macrostructures & Policies
Interactions Between Local Settings
Meanings & Relationships in Local Settings
Moment-to-Moment Interactions
Individual Sense-Making

Table 1: Layers of a socio-ecological framework (Louie & Zhan, 2022)

MATHEMATICAL MODELING AND DATA SCIENCE CONNECTED TO COMMUNITY, CULTURE, AND PLACE

The projects described in this paper envision mathematical modeling (MM) and data science (DS) practices as means for centering education on authentic problem solving that makes meaningful impact in communities. MM and DS are both professional practices that lend themselves naturally to project-based learning connected to community, culture, and place (Anhalt et al., 2018; Boaler et al., 2024).

The exploration process in DS and MM is very similar for both students and professionals. Data scientists begin by asking questions, just as mathematical modelers attempt to make sense of a situation or problem relevant to the community. This leads to a need for researching the problem by gathering and organizing data and local knowledge of the situation. Next, modeling requires simplifying the complexity, precisely identifying contextual variables and constants, making assumptions and decisions about relationships, and deciding on a mathematical/statistical approach. The decisions stem from personal knowledge and experiences, and multiple approaches add value. Once a model and approach are selected, it is possible to analyze and synthesize, make calculations, and subsequently interpret results back in the context of the situation. Investigators communicate the process and interpretation to others which can lead to social action in the community. Results also lead to new questions or a need to refine and improve models beginning the cycle again.

Investigations provide teachers and students opportunities with moment-to-moment interactions that promote critical thinking and elevate personal and community assets. Anhalt et al. (2018) outlined how mathematical modeling naturally connects with culturally responsive pedagogy:

Since mathematical modeling draws on students' mathematical knowledge while offering opportunities for new mathematical content to be developed, teachers can support students in critical thinking about their approach to mathematical modeling. For this reason, modeling tasks have the potential for teachers to leverage diverse students' everyday lived experiences for meaningful engagement with challenging mathematics. The way the students maneuver around the modeling process is informed by their culture and 'ways of thinking' which are formed by their everyday lived experiences (p.321).

Similar in process, both mathematical modeling and data science carry these connections to community, culture, and place.

The research on mathematical modeling and data science in education provides evidence of these subjects to be effective means for transformation in mathematics education. Kramer and colleagues used randomized student allocation in a study showing that mathematical modeling enacted with

evidence-based teaching strategies improves outcomes for calculus students, regardless of race, gender, or academic pathway (2023). Modeling practices in calculus promotes “inclusion and increases access for students with different mathematical backgrounds, cultural identities, and life experiences by allowing them to use their mathematics skills in a supportive, nonthreatening environment” (Kramer et al., 2023). Results, synthesized as a new standard of care, suggest needed transformation at the policy level. Another recent study examined student outcomes at the secondary level and revealed similar benefits for students (Boaler et al., 2024). Data science classes included a higher percentage of students from underrepresented groups compared to other advanced mathematics courses, and these students had a more positive outlook regarding their future in STEM. Students in a high school data science course implemented with similar pedagogical approaches scored higher on an assessment of data and functions than counterparts who took Algebra II. Results from these exemplars suggest that transformation efforts incorporating mathematical modeling and data science empower students at the secondary and university levels.

A great challenge lies in updating curriculum with DS and MM approaches in ways that honor diverse individuals, communities, cultures and spaces. The projects described in the next section are attempts to re-imagine communities for collaboration and coordination of transformation in mathematics education across the state of Utah in the Western United States.

BUILDING CONNECTIONS ACROSS SCALES TO STIMULATE CHANGE

Using the socio-ecological framework, the authors describe and analyze Utah’s Launch Years Initiative (LYI) and Utah State University’s Master Teacher Fellowship Program of Data Science in Rural Utah with Mathematical Modeling (DRUMM), highlighting the connections to community, culture, and place. Roles and experiences of these projects are entangled amongst layers of this framework, creating a strong systemic web in support of transformation.

LYI and DRUMM Project Framework Layers
Macrostructures & Policies: USHE, USBE, USU policies, procedures for change, curriculum, course options, credentials
Interactions Between Local Settings: Committee meetings, partnerships and collaborations, cultural exchange and community-building activities
Meanings & Relationships in Local Settings: Developing patterns of practice, ways of knowing, sense-making, problem-solving in local settings, and leveraging local assets
Moment-to-Moment Interactions: Developing and reflecting on learning experiences within classrooms, community, culture, place
Individual Sense-Making: Elevating funds of knowledge -- students, teachers, faculty, policymakers share the role of learner/developer

Table 2: Project layers within the socio-ecological framework

As shown in Table 2, characteristics of both projects include intentional efforts at the macrolevel, including work within the National Science Foundation, the Utah State Board of Education (USBE), the Utah System of Higher Education (USHE), and Utah State University (USU). The LYI and DRUMM projects aim to build and create communities across commonly disconnected groups,

elevating cultural and community ways of knowing and doing, and ultimately promoting student engagement in authentic problem-solving of complex and relevant challenges.

The Utah Launch Years Initiative (LYI)

Led by the Charles A. Dana Center at the University of Texas at Austin, the LYI is a collaboration of 22 U.S. states and mathematics organizations with a mission to create and support opportunities for students to develop the skills and dispositions to tackle the complex challenges of today. With the participation of a wide assortment of stakeholders, the leadership team from the state of Utah coordinates the efforts of the High School Pathways committee, the Early College committee, and the Support Structures committee.

The High School Pathways committee's charges include identifying the mathematics needed for realized and engaged citizens, providing students with choice based on interest and aspiration, and establishing a modernized content progression to set students up for success. Gathering feedback from students, teachers, parents, mathematics faculty, and community and industry leaders, this committee has compiled and recommended specific mathematical knowledge, skills, and habits of mind for Utah's students of today. These recommendations serve as a guiding framework for the core standards revision process outlined by the Utah State Board of Education. During this revision process, USBE has also provided individual school districts the option to teach a secondary Data Science course in place of a third-year course in the traditional pathway. The High School Pathways committee works closely with the Early College committee to ensure continued access to relevant and engaging mathematics through concurrent enrollment coursework, including statistics, quantitative reasoning, and college algebra.

The Early College committee's responsibilities include rethinking quantitative reasoning requirements and removing barriers to student engagement with meaningful mathematics. To those aims, the Early College committee is creating a new quantitative reasoning course, mathematical modeling with spreadsheets, building upon the recommended knowledge, skills, and habits of mind and elevating local contexts and individual sense-making. With purposeful inclusion of secondary teachers, mathematics faculty, and business leaders, this committee is working within the Utah System of Higher Education to pilot this new college mathematics course at various institutions and secondary schools across the state and to research the effectiveness of its implementation.

The Support Structures committee's charges include compiling and evaluating best practices of student support at the high school and college level. This committee continues to gauge the effectiveness of corequisite or supported mathematics coursework and just-in-time supports across the state through survey efforts and program analysis. Because course offerings differ at individual institutions and just-in-time supports are moment and site specific, the Support Structures committee seeks to coordinate best practices to connect macro-level policies with moment-to-moment interactions in the mathematics classroom. Recognizing the complexities of student learning, members of this committee will compile resources to support students within their local communities, both within and beyond the mathematics classroom.

The Data Science in Rural Utah with Mathematical Modeling (DRUMM) Project

With new funding from the National Science Foundation's Robert F. Noyce Master Teacher Fellowship program, Utah State University's DRUMM project will expand math modeling and data

science opportunities for secondary students in rural and Native American communities in Utah. The project will recruit and support 24 Master Teaching Fellows (MTFs) who will earn a Data Science and Mathematical Modeling Teaching Leadership credential through courses and professional development in data science, mathematical modeling, culturally responsive pedagogy, curriculum development, computer coding proficiency, and a scholarly project. These MTFs will also collaborate with USU mathematics/statistics/education faculty to co-create and teach meaningful investigations.

Preparation for the project has been years in the making through connections developed between our Mathematics and Statistics Department and rural communities. USU is a public Land-Grant research-intensive institution with a mission to serve students across the state. Approximately 20,000 students attend the main campus in northern, suburban Utah; 6,500 students attend one of 31 campuses or education centers. In particular, the Uintah Basin and Blanding campuses offer key facilities for building connections to rural and Indigenous communities. Our team has partnered with school districts in Duchesne, Uintah, and San Juan County as well as the Ute Tribe Department of Education.

USU Blanding hosts regular outreach celebrations of Science Technology Engineering Art and Mathematics at their STEAM Expo events, and our department has added teacher workshops, cultural talks, and excursions in the same weekend. Reflections and artifacts from these events show that incorporating cultural exchange opportunities with relevant modeling contexts corresponds with a strong sense of belonging and connection to the learning community and high motivation to incorporate mathematical modeling into classrooms (Bala et al., 2023; Tidwell & Kohler, 2019). The DRUMM project will begin to host similar events in the Uintah Basin in the fall to complement the events in Blanding in the spring.

The main feature of the DRUMM project is the centering of MTF and faculty collaborations. Their charge is to co-create and implement modeling and data science curriculum resources in the MTF classrooms. Master teaching fellows have local knowledge and connections to identify opportunities for community-engaged learning and to provide contexts for lessons. They work and live in these communities with enhanced awareness of culture, familiarity with the natural world, families, and students. Funding will allow multi-day faculty site visits to remote areas to promote co-teaching and observation of the moment-to-moment interaction between students and teachers. Secondary lessons will be revised and shared within and beyond the community as a resource for others. We expect these collaborations to result in positive student outcomes, empowered secondary teacher leaders, and faculty equipped to better serve all students at the early college level.

Collaboration Across Scales

Currently, several faculty and teachers are involved in both the LYI and DRUMM projects, supporting interconnected transformation efforts across scales. As part of the DRUMM program orientation in August 2024, faculty and MTFs collaborated in sessions focused on team building, mathematical modeling, culturally relevant curriculum, and local community connection events. Through shared experiences hiking along the Green River and in the Blue Mountains, building curriculum based on play of the Navajo Stick Game, and story-telling personal teaching highlights and challenges, those present developed strong connections to empower future work across the state.

As a specific example, orientation participants engaged in a math modeling task from a draft of the LYI quantitative reasoning course. Given basic population data for their county in the years 2000 and

2023, MTFs first drew on knowledge of their local context and shared possible explanations for any changes in the population. In the Uintah Basin, MTFs highlighted an influx of people to support the oil industry, while MTFs in Carbon County described population loss due to the closing of the coal mine. Teachers then looked for data that helped to quantify these change factors and worked to build a model to estimate the total number of people who have resided in their county so far this century. As a whole group, participants shared assumptions, strategies, and findings, and informally discussed population predictions for their local community.

This experience illustrates efforts to build faculty and teacher capacity for mathematical modeling and to highlight its value for engaged, authentic problem-solving. Additionally, faculty and teachers involved in the LYI will use the anecdotes and models from this modeling experience to enhance community and cultural connections in the quantitative reasoning course and inform task facilitation as the course is piloted in their own classrooms in the coming semesters.

Interdependent Relationships

Teachers are certainly the lynchpin to change efforts in mathematics education, responsible for orchestrating authentic problem-solving experiences for students and subject to policy restrictions and limited opportunities at the local level. However, only by attending to all layers and interdependencies of the socio-ecological framework can sustainable transformation occur (Louie & Zhan, 2022). Including secondary school teachers and college instructors in policy committees serves to leverage their insight and check the feasibility of reforms.

As detailed in the project descriptions, individuals from many different communities can take on roles of leadership to leverage the power of macro-level institutions in rewriting opportunities for students in mathematics education. For example, the LYI committees require active participation from secondary educators, university faculty, community members, and state-level policymakers to critique and update state mathematics course standards for today's challenges and to provide opportunities to design and implement new MM and DS courses in high schools and universities across the state. The DRUMM project places teachers side by side with faculty members who have various expertise in DS, MM, and education, which also levels the playing field and aims to elevate student and community assets in this transformation.

Simultaneously, individuals can take on the roles of listener and learner to elevate ways of knowing and doing mathematics that have traditionally been absent from educational experiences. Within the DRUMM project, master teacher fellows from rural areas of the state assume the roles of local knowledge holder and pedagogical expert, sharing their experiences and craft with faculty and fellow educators. Through this collaboration, new curricular materials highlight the challenges specific to culture and place, and local teachers showcase innovative ways of seeing, solving, and orchestrating student experiences with mathematics.

Bringing together mathematicians, mathematics educators, cultural leaders, and policy makers in prolonged interactions enhances these projects. The LYI project makes use of teleconference meetings to connect people in different localities, and the DRUMM project enables interaction of all parties in rural areas to bolster the incorporation of Indigenous knowledge in transformation efforts.

The authors recognize the multiple roles and experiences of individuals within these projects and support the notion that the interdependence of these roles and experiences form a broad and sustainable effort towards transformation in mathematics education.

REFLECTIONS AND CONCLUSIONS

The case for change in mathematics education seems hindered by powerful forces sustaining the status quo. Our roles and experiences within the public education system in the state of Utah reveal challenges impeding the pace of reform. Processes to obtain approval, experiment and study results, and implement new standards and courses require time, energy, and funding. Many hurdles can and do disrupt momentum, including leadership change, academic timelines, assessment requirements, and program pathways. At the national level, Schoenfeld and Daro (2024) argue that a false “equity versus excellence” dichotomy has harmed students. These scholars offer principles in curriculum design that preserve future options for students, favor flexibility, and align with students’ various aspirations, suggesting that modularity in course design could provide a mechanism for the early introduction of data science and mathematical modeling for the benefit of all students. They further acknowledge that even as we know more about equitable and ambitious learning environments, there is more to learn regarding student identities, how they are impacted by schooling, and the social environment of the classroom.

The LYI and DRUMM projects offer opportunities to further the understanding of the socio-ecological entanglements which influence the transformation of mathematical learning environments for students. Such learning environments, centered on the flexible processes of mathematical modeling and data science, empower students to respond to the changing needs of a complex and uncertain world. Collaborations among listeners and learners in secondary schools, rural areas, and universities will author new stories of mathematics education in Utah. Through a web of efforts at all levels of social activity, these two projects provide hopeful images of mathematics education rooted in community, culture, and place.

Acknowledgements

The authors gratefully thank funding and support from the Utah State University Department of Mathematics and Statistics and Statewide Campuses, the Charles A. Dana Center at The University of Texas at Austin, Education Strategy Group, and Student Achievement Partners, for the Launch Years support, and the National Science Foundation Robert F. Noyce Program for Award #2345194 “Community Impact with Teacher Leaders in Data Science and Mathematical Modeling.”

References

- Anhalt, C.O., Staats, S., Cortez, R. & Civil, M. (2018). Mathematical modeling and culturally responsive teaching. In Y. J. Dori, Z. Mevareach, & D. Baker (Eds.). *Cognition, Metacognition, and Culture in STEM Education* (pp. 307-330). Springer International.
- Bala, C., Anhalt, C., & Kohler, B. (2023, August 2-5). *Coupling Mathematical Modeling Professional Development with Community Connection Events*. Paper presented at the Mathematical Association of America MathFest, Tampa, FL, United States.
- Boaler, J., Conte, K., Cor, K., Diekman, J.A., LaMar, T., Ramirez, J., & Selbach-Allen, M. (2024). Studying the Opportunities Provided by an Applied High School Mathematics Course: Explorations in Data Science. *Journal of Statistics and Data Science Education*, 1-20.

- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- Kramer, L., Fuller, E., Watson, C., Castillo, A., Oliva, P.D., & Potvin, G. (2023). Establishing a new standard of care for calculus using trials with randomized student allocation. *Science*, 381, 995-998.
- Kincheloe, J. L. & Steinberg, S. R. (2014). Indigenous Knowledges in Education: Complexities, Dangers, and Profound Benefits. In N.K. Denzin, Y.S. Lincoln, & L.T. Smith (Eds.), *Handbook of Critical and Indigenous Methodologies* (pp.135-156). Sage Publications, Inc.
- Louie, N., & Zhan, W. Y. (2022). A socio-ecological framework for research in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 365-371.
- Luecke, D., & Sanders, D. (2023, July). Dakota/Lakota Math Connections: an epistemological framework for teaching and learning mathematics with Indigenous communities and students. *Frontiers in Education*, 8(1151376), 1755-1785.
- O’Neill, T., Finau-Faumuina, B. M., & Ford, T. U. L. (2023). Toward decolonizing STEM: Centering place and sense of place for community-based problem-solving. *Journal of Research in Science Teaching*, 60(8), 1-31.
- Schoenfeld, A. H., & Daro, P. D. (2024). More Math, Less “Math War.” *Science*, 383(6689), 1297-1299.
- Tidwell, W., & Kohler, B. R. (2019). Mathematical modeling lesson adaptation for increasing local relevance. In W. M. Smith, K. M. Callahan, J. F. Strayer, R. S. Jones, & L. C. Augustyn (Eds.), *Proceedings of the eighth annual Mathematics Teacher Education Partnership conference* (pp. 45-55). Association of Public and Land-grant Universities.
- Utah Division of Indian Affairs. (2023). *Tribal nations*. <https://indian.utah.gov/tribal-nations/>
- Wolfram, C. (2020). *The Math(s) Fix: An education blueprint for the AI age*. Wolfram Media, Inc.
- Yanez, G. A., Thumlert, K., De Castell, S., & Jenson, J. (2019). Pathways to sustainable futures: A “production pedagogy” model for STEM education. *Futures*, 108, 27-36.

POLICY ENGAGEMENT: INTERNATIONAL CURRICULA, THE SOCIO-ECOLOGICAL AND MATHEMATICS

Mark Boylan

Sheffield Hallam University

If mathematics education is to have a role in addressing socio-ecological issues at scale, then engagement with policy is essential. To support policy engagement, I consider the fluid meanings of terms used for referring to socio-ecological issues in educational policy, particularly when focused on national curricula and pedagogy. I then identify how mathematics is largely at the margins of curriculum policy through an analysis of case studies of 80 countries curriculum policies in relation to climate change, sustainability, and the environment, as well as the gap between the concerns of mathematics educators and the curriculum policy. I reflect on the challenges mathematics educators face in influencing policy. This underscores the importance of understanding how generic policy models inform policy development and applying these principles to the development of mathematics education policies. Considering the difficulty in implementing policy changes, I propose some possibilities for mathematics educators to enhance their impact on policy, thereby promoting the adoption of socio-ecological considerations.

INTRODUCTION

The socio-ecological crises call for global and national action, including in education. The gap between the necessary action to address the interconnected socio-ecological crises and the ongoing activity is a recurring theme in the global response. Considering climate change, the accepted goal of keeping the global temperature increase below 2°C and aiming for 1.5°C will still lead to further social and ecological challenges, such as to biodiversity (Nunez et al., 2019). However, the current National Determined Contributions to mitigation appear unlikely to meet this goal, as they “fall far short of what is required” (den Elzen et al., 2019, p. 238). We need changes to education at the national scale as an aspect of the necessary societal action.

If mathematics educators are to contribute to addressing socio-ecological issues at scale, then engagement with curriculum policy is essential, as is the development of a socio-ecological programme of mathematics education research and practice. One way of understanding policy is as narrative (Van Eeten, 2017), and the policy stories told and untold speak directly to how attending to scale highlights whose voices and stories are heard and those which are not. However, policy engagement to amplify socio-ecological concerns is likely to be challenging, given that, generally, mathematics educators have had limited success in influencing policy in many education systems.

Furthermore, generally, national curriculum frameworks have not yet integrated issues like climate change or other socio-ecological concerns. In a review of curricula from 100 countries, 53% did not mention climate change, and 40% only included minimal content (UNESCO, 2021). When subjects and areas of study address other socio-ecological concerns, such as the environment and sustainability rather than addressing them as separate, standalone topics, the framing often fails to embrace socio-ecological perspectives. Subjects such as geography, science, social studies, and even environmental education, if they address the socio-ecological, do so in restricted and limited ways. For instance,

they may include the science of climate change but not the political economy of climate change or the concept of climate justice.

To contribute to exploring these issues, I offer some reference points for mathematics educators to engage with socio-ecological policy narratives. Then, informed by this, I provide a summary of how these narratives appear or not in national curriculum frameworks and education plans. In doing this, I aim to make connections to discussions by mathematics educators of socio-ecological issues. I review UNCESO case studies of curriculum policies in 80 countries that address climate change, sustainability, and the environment. One reason for considering these country reports is that they help to introduce, even in a limited way, how mathematics education and the socio-ecological may be relevant in countries without mathematics education research infrastructure and so otherwise might not be represented in the ICMI study conference. Following this, I consider the challenges for mathematics educators to influence policy and point to the value of considering models of how policy change may be initiated from outside policy networks. This leads to suggestions for future activities by mathematics educators.

THE SOCIO-ECOLOGICAL IN POLICY DISCOURSE

In this section, I consider key terms found in national policy, and among transnational actors, that are relevant to socio-ecological concerns. The ICMI Study 27 focuses on the socio-ecological. This term brings together the social and ecological, and the rationale for this is articulated and argued for in the study call (and in Coles, 2022, 2023; Coles et al., 2024). However, there is limited global usage of the term “socio-ecological” in the field of education. Global education policy actors, like UNESCO, employ various terms in their attempts to influence national policies. Such terms are used by both national and transnational actors that seek to influence policy and initiate curriculum and pedagogical change. National curricula and related policy documents, like UNESCO’s 2021 report, commonly include:

- environment,
- sustainable development and sustainability
- climate change
- and climate justice.

These different terms have different rhetorical purposes, and the use of them in curriculum policy signals different positions or ideologies; they tell different stories. These terms describe a range of potentially less or more critical ways to think about society and the environment. From “environment” in policy discussions, which may focus on individual relationships to a separate “natural world” through to “climate justice,” which points to the interrelationship of ecological, societal, political, and ethical concerns. Thus, these terms describe a range of potentially more radical ways to think about these issues.

However, this continuum is not simple or straightforward. Considering “environment”, this can point to different types of relationships, from individualized relationships of domination to interdependent relationships of ecology (Barwell et al., 2022). A variant of domination is the idea of humans as protectors of the environment (Kyburz-Graber et al., 2006), but who are separate and outside an environment as “other”. Alternatively, some currents in environmental education thinking embrace

critical perspectives that include critical race theory, decolonization theory and social justice (Riggs Stapleton, 2019). Conversely, environmental education may be framed in neo-liberal terms of natural services and the importance of markets (Hursh et al., 2015).

Similarly, “sustainability” has multiple interpretations. It was conceptually centered on early engagement with socio-ecological issues in mathematics education (Renert 2011), proposing different understandings of sustainability:

- accommodation – education about sustainability
- reformation – education for sustainability
- transformation – education as sustainability

The last of these refers to teaching and learning mathematics subordinated to a process of students becoming engaged and critical citizens able to participate in social action.

Sustainability as used by transnational actors can carry other meanings, particularly when linked, as it often is to ideas of sustainable development (Kopina, 2020). The concept of sustainable development has considerable policy currency through advocacy for the UN Sustainable Development Goals. The goals connect social and ecological issues with economic and social development, yet, like other terminologies, their interpretations are subject to debate. One of the goals is sustainable economic growth, but policy can also focus on economic priorities.

Addressing climate change is one of the Sustainable Development Goals. In the introduction, I noted that most national curricula either do not address it or do so minimally. Thus, explicitly addressing climate change in depth could be a sign of acknowledging the importance of the socio-ecological perspective, despite its singularity as a lens for examining socio-ecological crises.

At the same time, climate change may only be considered in narrow terms, such as atmospheric science, rather than the social and political causes and implications. The use of ‘climate justice’ is relatively less ambiguous. The concept of justice draws attention to the disparities that exist between those who contribute to climate change and those who experience its effects and highlights the connection between human activity’s impact on the climate and broader global systems of injustice. However, the concept of ‘climate justice’ is subject to contested meanings, ranging from those advocating for climate mitigation policies that involve redistribution to those advocating for more fundamental power shifts and challenges to the social and economic systems that contribute to climate change and are fundamentally unjust (Scandrett, 2016).

CURRICULA GLOBALLY: SUSTAINABILITY, ENVIRONMENT, CLIMATE CHANGE AND MATHEMATICS EDUCATION

This section draws on a summary analysis of 100 countries climate change and environmental education policy (UNESCO, 2021; UNESCO, 2023) and further analysis of 80 country case studies that describe curriculum policies in relation to climate change, sustainability, and environmental education.

Through this further analysis, I sought to establish whether mathematics is explicitly identified as relevant or, if not explicit, if there are opportunities or openings to include mathematics, for example through cross-curricula activity, and potential relationships to themes and concepts identified as

important in the socio-ecological in mathematics education (e.g., Coles et al., 2024; Boylan & Coles, 2017; Barwell et al., 2022). The climate change vulnerability index is based on consideration of likely impacts of climate change for a country but also resources to respond to such changes. There are some clear limitations to this analysis, including the use of these secondary sources rather than primary sources and the focus on policy documents that may express intention rather than enactment. The issue of enactment is particularly acute in countries where the provision of basic education is challenging due to lack of resources and impacts of colonialism and neo-colonialism. The 80 countries include countries of different population and economic size, with different levels of contribution to climate change through emissions and different degrees of vulnerability to its effects as identified in relevant indices.

Climate change at the margins of National Curriculum Frameworks

UNESCO's (2021) analysis of the relative frequency of references in 100 countries national curriculum frameworks indicates that climate change is marginalized in curricula. Only just over half (53%) of national curriculum frameworks include climate change, and in those that do, the depth of inclusion is usually minimal.

Further my review of 80 country cases¹² indicates that for many of those that do mention climate change, this is in relatively recent documents about plans for curriculum change rather than current practice. Each of the case studies includes an analysis of the type of discourse used and how far documents variously refer to environment, sustainability, biodiversity, and climate change.

Countries more vulnerable to climate change are more likely to include climate change in the curriculum (UNESCO, 2021). Considering the case studies, there is a distinction between a focus on mitigation and adaptation. Countries that have greater responsibility tending to focus on mitigation, and within that on individual responses and actions. Those that are more vulnerable tend to either include both mitigation and adaptation or make adaptation more prominent. The following is a list of countries that have adaptation of equal importance or more importance than mitigation. For each a climate change vulnerability index is given as a rank (1 is the least vulnerable, 185 is most vulnerable)¹³:

Argentina (71), Bangladesh (156), Brazil (58), Cabo Verde (98), Chile (27), Colombia (88), Dominican Republic (92), Fiji (114), Haiti (146), Israel (16), Japan (62), Jordan (49), Maldives (155), Saint Lucia (79), Tuvalu (not determined), Vanuatu (164).

Adaptation speaks to Coles' (2017) contention that the Anthropocene means living with crises and problems is our new and permanent condition. This challenges what an appropriate mathematics education should be. However, as I now consider, how mathematics education may support a curriculum for adaptation is largely absent from these policy documents.

Climate change and environmental education in curriculum: the place of mathematics

Countries curriculums adopt one or more of the following, sometimes with differences across educational phases:

¹² <https://education-profiles.org/themes/~climate-change-communication-and-education>

¹³ <https://gain-new.crc.nd.edu/ranking/vulnerability>

- specific environmental, climate change or sustainability education, sometimes in the context of a broader generic skills curriculum
- inclusion in science and geography curricula, and in some case social studies
- cross-curricular approaches that extend to more subjects.

Eighty cases encourage cross-curricular approaches. Out of these eighty cases, only four explicitly mention or use mathematics as an example. These are Azerbaijan, France, India, and Nigeria. However, the reference to mathematics is minimal in all four of these cases. India, for example, has federated governance in relation to climate change education within the National Curriculum Framework. The Framework recommends integrating environmental education across different subjects and gives one mathematical example of tracking declining rainfall patterns over time (MECCE, 2022).

In summary, only four of the hundred cases explicitly mention mathematics. Of those that do recommend cross-curricular activities, only four specifically mention mathematics. So, it is clear from these four cases that mathematics is marginal to environmental, climate change, and sustainability education. It is also notable that climate change has minimal references in those four countries. Therefore, when mathematics is mentioned, it typically appears in curriculum frameworks that are not fully developed in terms of socio-ecological concerns.

Curriculum opportunities for socio-ecological mathematics

Despite the infrequent mention of mathematics in climate change education, certain curriculum policies offer opportunities for mathematics educators, including more critical mathematics education perspectives. There are some countries where there are links to recent mathematics educators' attempts to rethink human-nature relationships (Barwell et al., 2022; Boylan, 2017; Coles 2023) and embrace Indigenous perspectives (Coles et al., 2024; Guitierrez, 2017).

The Dominican Republic embraces an eco-pedagogy approach to environmental education. This approach emphasises the interrelationships between natural and social ecology, with the aim of evolving current human, social, and environmental relationships. Costa Rica, similarly, bases climate education on ethical principles of interaction, sustainability, and interrelationship. Ecuador acknowledges socio-environmental crises and frames action to address them within the concept of 'the plentiful life' or 'good living', aiming to foster a democratic, equitable, peaceful, and nature-respecting society. In Bolivia, an equivalent concept of "living well" informs environmental education as critical social practice for citizens committed to Mother Earth's rights. Other countries that draw on Indigenous ancestral concepts are New Zealand and the Cook Islands.

These are a relatively small number of countries, and it remains unclear whether such policy intentions are enacted, nor how mathematics education contributes to and is informed by such principles. Nevertheless, these countries provide examples of national policies that align with the philosophy of mathematics education, which adopts a relational approach to the socio-ecological.

MATHEMATICS EDUCATION, POLICY AND POSSIBILITIES

Before offering some possibilities, I note that in general, the relationship between mathematics educators and educational policy is complex, context-dependent, and often problematic. Even when mathematics educators seek to influence policy in line with policymakers' technical concerns for

traditional learning outcomes (Valero & Knijnik, 2016), influence may not be straightforward. Mathematics educators' goals and initiatives may start with different concerns than central and often top-down policy directions and priorities (Geiger et al., 2023).

In this situation, it is helpful to look into more complex models of policy development rather than the more straightforward model of policy cycles. The policy cycle view posits that a need or problem is identified, then a solution is proposed, and finally the policy is put into place. We identified two alternative policy models that are relevant to the development of mathematics education policies in both England and internationally (Boylan et al., 2023; Adams & Boylan, 2023): the multiple streams analysis and the advocacy coalition framework (see Cairney, 2012).

An aspect of multiple streams analysis identifies that the political "stream" of policy decision-making may happen in parallel with the creation and development of activities that can be taken up in policy. Therefore, instead of policymakers first identifying or recognizing a problem and then seeking a solution, the presence of a 'solution' may aid in the identification and framing of the problem. When mathematics educators make innovative contributions to socio-ecological issues, they should consider how these innovations align with policy concerns to enhance their chances of inclusion in future policies.

A focus on advocacy coalitions proposes that the boundary between policy influences, intermediaries, and makers is fuzzy, and policy making is a process of negotiation by shifting and changing coalitions inside and outside government. Reflecting on the national cases, it is noteworthy that globally a diverse range of civil society actors engage in advocacy and innovation in climate change education. These actors play a crucial role in bolstering environmental and climate change education through teacher training, the development of curriculum materials, and advocating for inclusion of climate change and environmental education in schools and educational settings. As a result, mathematics educators can engage with potential partners in existing advocacy coalitions. This might be more fruitful than focusing on habitual mathematics education channels to influence policy.

CONCLUSION

If mathematics education is to contribute substantially to a change in socio-ecological relationships, then this will include changing national curricula. Considering the various terms used in policy discourse highlights how the adoption and argument for a socio-ecological perspective can diversify the narratives and voices reflected in current policies. However, given the relative marginalisation of such concerns in national curricula, adopting a socio-ecological discourse can potentially create friction in seeking to influence policy. My summary review of national curriculum frameworks further underlines the gap between social action needed and current educational practice, including in mathematics education. Boylan and Coles (2017) proposed a roadmap for the progressive development of educational practices to address this gap across the following areas of activity:

- curriculum, pedagogy, and practice,
- teacher education and professional development,
- the establishment and promotion of networks,
- theoretical and empirical research.

As part of this roadmap, they suggested that a significant moment would be when “one country changes its national curriculum to put ecological awareness and stewardship of the planet at the core of all teaching and learning” (p. 13). Arguably, the goal was too modest at that time because, in principle, at least two countries had already done this (Bolivia and Ecuador). However, curriculum policies in these countries did not explicitly integrate mathematics. Thus, the review of national curriculum frameworks highlights the complex and multiple challenges of mathematics educators working for policy change and the importance of addressing the specific educational and policy contexts. In all countries, an important policy goal is demonstrating the importance of mathematics as part of developing environmental, sustainability, climate change and climate justice education. Insights from mathematics education about the importance of integrating ecological and social justice concerns also suggest that mathematics educators need to ally themselves with others seeking to embed more critical and transformative perspectives in policy, ones that are found in a socio-ecological perspective.

Acknowledgements

There was no external funding for this research.

References

- Adams, G., & Boylan, M. (2023). *Landscaping Mathematics Education Policy: Horizon scanning of international policy initiatives*. <https://royalsociety.org/-/media/policy/projects/maths-futures/landscaping-international-mathematics-education-policy.pdf>
- Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1-16.
- Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013.
- Boylan, M., Adams, G., & Birkhead, A. (2023). *Landscaping mathematics education policy: landscaping national mathematics education policy*. <https://royalsociety.org/-/media/policy/projects/maths-futures/landscaping-national-mathematics-education-policy.pdf>
- Boylan, M., & Coles, A. (2017). Is another mathematics education possible? An introduction to a Special Issue on “Mathematics education and the living world: Responses to ecological crisis”. *Philosophy of Mathematics Education Journal*, 32. <https://education.exeter.ac.uk/research/centres/stem/publications/pmej/pome32/index.html>
- Cairney, P. (2012). *Understanding public policy: theories and issues*. Basingstoke: Palgrave Macmillan.
- Coles, A. (2022). A socio-ecological turn in mathematics education. *Reflecting on curriculum innovation*, 1, 207-228.
- Coles, A. (2023). Teaching in the new climatic regime: Steps to a socio-ecology of mathematics education. *In Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-33). PME.
- Coles, A. T. (2017). Habits and Binds of Mathematics Education in the Anthropocene. *Philosophy of Mathematics Education Journal*, 32. <http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome32/index.html>
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024) Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- Den Elzen, M., Kuramochi, T., Höhne, N., Cantzler, J., Esmeijer, K., Fekete, H., ... & Vandyck, T. (2019). Are the G20 economies making enough progress to meet their NDC targets? *Energy policy*, 126, 238-250.

- Geiger, V., Gal, I., & Graven, M. (2023). The connections between citizenship education and mathematics education. *ZDM–Mathematics education*, 55(5), 923-940.
- Gutiérrez, R. (2017). Living Mathematx: Towards a vision for the future. *Philosophy of Mathematics Education Journal*, 32. <http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome32/index.html>
- Hursh, D., Henderson, J., & Greenwood, D. (2015). Environmental education in a neo-liberal climate. *Environmental Education Research*, 21(3), 299-318.
- Kopnina, H. (2020). Education for the future? Critical evaluation of education for sustainable development goals. *The Journal of Environmental Education*, 51(4), 280-291.
- Kyburz-Graber, R., Hofer, K., & Wolfensberger, B. (2006). Studies on a socio-ecological approach to environmental education: a contribution to a critical position in the education for sustainable development discourse. *Environmental Education Research*, 12(1), 101-114.
- MECCE (2022). *CCE country profile: India*. https://mecce.ca/country_profiles/cce-country-profile-india/
- Nunez, S., Arets, E., Alkemade, R., Verwer, C., & Leemans, R. (2019). Assessing the impacts of climate change on biodiversity: is below 2° C enough?. *Climatic Change*, 154, 351-365.
- Renert, M. (2011) Mathematics for life: Sustainable mathematics education, *For the Learning of Mathematics*, 31(1), 20-26.
- Scandrett, E. (2016). Climate justice: contested discourse and social transformation. *International Journal of Climate Change Strategies and Management*, 8(4), 477-487.
- Stapleton, S. R. (2020). Toward critical environmental education: A standpoint analysis of race in the American environmental context. *Environmental Education Research*, 26(2), 155-170.
- UNESCO (2021). *Getting every school climate-ready: How countries are integrating climate change issues in education*. <https://unesdoc.unesco.org/ark:/48223/pf0000379591>
- UNESCO, (2023). *Education and climate change: learning to act for people and planet*. <https://unesdoc.unesco.org/ark:/48223/pf0000389801>
- Valero, P., & Knijnik, G. (2016). Mathematics education as a matter of policy. *Encyclopedia of Educational Philosophy and Theory*. Springer Singapore, 1-6.
- Van Eeten, M. M. (2017). Narrative policy analysis. In Fischer, F., & Miller, G. J. (Eds.). *Handbook of public policy analysis* (pp. 277-296). Routledge.

CROSS-CULTURAL EXPERIENCES IN CURRICULUM INNOVATIONS: VOICES FROM ASIA

Mary Jane A. Castilla¹, Heidi Lynn P. Faustino², Melodee T. Pacio³, Ram Krishna Panthi⁴,
Veronika Fitri Rianasari⁵, Catherine P. Vistro-Yu⁶

¹University of Santo Tomas, ²Hoke County High School, ³Philippine Science High School,
⁴Tribhuvan University, ⁵Sanata Dharma University, ⁶Ateneo de Manila University

In this paper, scholars from three Asian countries share their collective experiences in curriculum innovation. The common point being highlighted is the need for countries to focus on the very heart of mathematics education – its people. Mathematics in schools and other institutions of learning should draw from Indigenous knowledges and unique contexts of its people, paying special attention to the rich cultural heritage of the communities. We argue that mathematics curriculum is part of a socio-ecological system and, thus, cannot be innovated piecemeal. Instead, by considering all factors that make up the ecosystem, curriculum innovations can truly be more responsive to the learners' needs in each culture. We identify and analyze the tensions that all three countries have experienced.

INTRODUCTION

Every country hopes to provide the best education they could offer to their pupils. But clearly not all countries are the same; many struggle to reach the desired achievement levels for a variety of reasons – low budget, marginalizing policies, ineffective teaching methods, among others. Poor achievement levels are often attributed primarily to curriculum; especially in mathematics, often the first thing that a country scrutinizes is its curriculum. Education leaders scramble to “fix” the curriculum but fail to get to the heart of the problem or fall short in culling all interventions.

In this paper, we, scholars, from three countries in Asia, come together to share our ideas and experiences in addressing curricular issues in mathematics. Because we are scholars who are not in national leadership positions, our contributions to “fixing” the mathematics curriculum may be seen as small-scale innovations in mathematics. Yet, as we shall point out at the end of the paper, these small-scale curricular innovations must be given more attention as they address more the most essential parts that coherently make a mathematics curriculum truly responsive to the needs of their pupils. This is where the socio-ecological perspective becomes valuable. Curriculum, specifically, the mathematics curriculum is at the heart of an ecological system in mathematics education and as such can only be fixed by attending, synchronously and coherently, to the variables of the system.

CURRICULUM INNOVATIONS IN INDONESIA

Pendidikan Matematika Realistik Indonesia (PMRI) is an Indonesian adaptation of Realistic Mathematics Education (RME), a domain-specific instruction theory for mathematics founded by Freudenthal in the Netherlands (Sembiring et al., 2008; Zulkardi et al., 2020). The adaptation of RME into the Indonesian context was initiated as there was a growing awareness among Indonesian scholars of the need to improve mathematics teaching in Indonesian classrooms (Sembiring et al., 2008). The development of PMRI from 1994–2015, conducted using a bottom-up approach, included numerous programs aimed at introducing a new way of teaching and learning that emphasized students' activity in their reconstruction of mathematical knowledge under the guidance of the

teachers (Zulkardi et al., 2020). Some of these programs included initiating the International Master's Program on Mathematics Education (IMPoME), conducting international conferences on design research, and organizing mathematical literacy contests (Zulkardi et al., 2020). These efforts resulted in research studies focusing on designing and testing innovative ways for students to learn mathematics through guided reinvention, which were published either in national or international journals. Additionally, the fifth author has also conducted studies on the implementation of PMRI in Indonesian schools (see Larasati & Rianasari, 2017; Rianasari et al., 2012; Rianasari & Julie, 2018). The research results from Indonesian scholars on the implementation of PMRI at Indonesian schools indicated that mathematics instructions designed based on the principles of PMRI make positive contributions to the quality of mathematics teaching and learning.

Reflection on the impact of the reformation movement

Despite the positive contributions of PMRI to mathematics teaching and learning in Indonesian classrooms, the mathematics performance of Indonesian students in solving mathematics problems situated in real-life contexts was poor, as evidenced by the PISA results released by the OECD. From 2009 to 2022, Indonesia ranked among the lowest-performing countries in PISA assessments. In the PISA 2022 results, 82% of Indonesian students did not attain level 2 proficiency, indicating that they could only solve simple problems requiring routine mathematical procedures (OECD, 2023). The stagnancy in students' mathematics performances in PISA assessments seems contradictory to the positive contributions of PMRI in schools.

Analyzing the main factors that caused this stagnancy is not an easy task since education, especially mathematics education, is a complex field. In addition to this, there have been very few studies criticizing the adaptation of RME in Indonesia. A study by Revina and Leung (2019) revealed that the RME adaptation in Indonesia has deviated from its original form. First, there were inconsistencies between the Indonesian curriculum and RME. While the Indonesian curriculum emphasizes the mastery of mathematical content, the Dutch curriculum emphasizes learners' cognitive and personal development, which aligns with RME (Revina & Leung, 2019). Second, the Indonesian curriculum implements a centralized decision-making process for the contents to be taught, which might hinder the realization of some characteristics of RME (Revina & Leung, 2019). These findings indicate that much of the research on the development and implementation of PMRI has not provided suggestions to orient Indonesian education policymakers toward finding a proper balance between RME and the Indonesian educational context. Third, the implementation of RME in Indonesia was somewhat inconsistent with the theory of RME, possibly due to cultural differences between Indonesia and the Netherlands. Indonesian culture, rooted in the Javanese culture, emphasizes compliance, obedience, and harmony, which differ significantly from Dutch culture (Revina & Leung, 2019). In conclusion, the findings of Revina and Leung's (2019) study indicate that much of the research on the development and implementation of PMRI has not provided a critical explanation to Indonesian mathematics educators on how to interpret the borrowed approach (RME) through their own culture and to identify what is best for their local conditions.

In conclusion, the analysis of factors influencing the mathematics curriculum innovation in Indonesia reveals a complex interplay of cultural, educational, and policy-related challenges. This discrepancy highlights the need for a deeper analysis of the factors influencing Indonesian mathematics education.

Integrating Ethnomathematics into Socio-Ecologically Responsive Education

The challenges faced by mathematics education in Indonesia extend beyond the classroom, calling for further research and critical reflection to ensure that curriculum innovations align with the socio-ecological realities of the country. Indonesia's rich cultural diversity, particularly among Indigenous communities, offers a unique opportunity to develop curriculum materials that are not only contextually relevant but also deeply rooted in these communities' cultural practices, knowledge systems, and lived experiences. The integration of ethnomathematics into the curriculum, particularly for Indigenous people in Indonesia, is illustrated by various culturally rooted practices. For instance, the Kampung Naga Indigenous community in West Java demonstrates the practical use of geometric concepts in their daily lives without formal mathematical education. The community's craftsmanship, architectural design, and traditional measurement systems embody mathematical principles that can be used to create culturally relevant mathematics education. This approach preserves cultural heritage and provides students with relatable and meaningful mathematical learning experiences (Hermanto and Nurlaelah, 2019). In another example, the Korowai people of Papua live in a unique socio-cultural environment where their traditional treehouse architecture and forest-related activities form the basis of their learning needs. Integrating these cultural elements into the school curriculum can help bridge the gap between formal education and the student's *lives*. The school environment can be adapted to reflect these cultural aspects by incorporating traditional building designs and maintaining a connection to the surrounding forest. This culturally responsive approach helps ensure that education is inclusive, engaging, and relevant to the Indigenous students' cultural context (Fiharsono et al., 2024). These examples show how ethnomathematics can be a powerful tool to connect students' cultural backgrounds with their mathematics learning, making education accessible and meaningful.

CURRICULUM INNOVATIONS IN NEPAL

Nepal's current mathematics curriculum suffers from a one-size-fits-all approach, prioritizing rote memorization and theoretical content at the expense of real-world application and cultural relevance. This fails to cater to the diverse needs of students in a multicultural country like Nepal. Nepal boasts a rich cultural landscape. As per the 2011 population census, there are 125 ethnic groups and 123 linguistic groups in the country. Indigenous peoples (Adivasi Janajati) comprise 36% of the total population of 26.5 million (Upadhaya, 2023). However, the current curriculum ignores this diversity, focusing on standardized content delivery (top-down approach) instead of engaging their students through their lived experiences (bottom-up approach). This approach leads to disengaged learners and fails to cultivate a deep understanding of mathematical concepts.

The wealth of Indigenous knowledge systems within Nepali communities offers a valuable resource for curriculum reform. By interweaving math concepts with Indigenous knowledge and their lived experiences, the students develop a sense of ownership and a deeper understanding of math concepts (Panthi, 2021). However, incorporating cultural elements risks perpetuating a colonial narrative if it does not address underlying power dynamics (Battiste, 2013). True integration requires a decolonized approach that acknowledges colonialism's legacy and social justice's importance. This involves critically reflecting on the current curriculum while seeking ways to integrate relevant Indigenous knowledge. For instance, the teachers can use "nanglo" (a traditional round woven bamboo tray) or "sikka (coins)" in the discussion of circles and "dhungro" (a bamboo barrel used for cooking) in the

discussion of cylinders, drawing connections between these artifacts and formal mathematics. There is a need to construct a curriculum from a multicultural perspective.

Building a curriculum from a multicultural perspective also requires the use of critical theory as a lens, recognizing how social and cultural forces influence education (Yang & Li, 2019). This approach allows for studying curriculum practices within a sociocultural context, highlighting the challenges faced by educators during innovation. The country's current top-down curriculum development approach, which relies solely on subject experts, should be expanded to include a wider range of stakeholders, including students themselves (Sullivan, 2011). Cross-cultural learning research emphasizes understanding how students from diverse backgrounds approach learning. Connecting mathematics with students' communities' experiences and aspirations makes learning meaningful and can address issues like poverty, climate change, and social justice.

Connection between Mathematics Education and Socio-Ecological Environment

As discussed, Nepal's current mathematics curriculum suffers from a disconnect between rote memorization and real-world application. This gap extends to critical socio-ecological challenges like climate change, pollution, and resource management. We need a curriculum that bridges this gap and equips students to grapple with these pressing issues, especially since these are what their communities are deeply concerned about.

In other words, mathematics education must become more socio-ecological. This doesn't mean abandoning core mathematical concepts but finding ways to connect them to the environment and the social issues impacting it. Is mathematics useful to reduce pollution? Is mathematics helpful to address climate change issues? Is mathematics useful to reduce socio-ecological issues? The urgency is clear – the very systems sustaining life on earth are threatened. As Coles et al. (2024) argue, mathematics education needs to re-evaluate its role in addressing the broad socio-ecological crisis.

This requires a shift in perspective. Traditionally, we view ecological and social relations as separate entities. A more progressive approach recognizes them as entangled and interdependent. This "entanglement" offers a framework for reforming mathematics education. Coles et al. (2024) propose three key practices: listening for socio-ecological entanglement; attending to the scales of socio-ecological entanglements; and living entanglement as mathematics educators. There are several barriers to curriculum innovation through socio-ecological perspective including curriculum policy, bureaucracy, teachers' and students' interest, politics, geographical diversity, multicultural and multiethnic society. We can address the issues by engaging relevant stakeholders.

By embracing a socio-ecological approach through these practices, we can equip students not only with mathematical skills but also with the critical thinking necessary to address the environmental and social challenges of our time. This aligns perfectly with the call for a curriculum that incorporates cultural relevance and empowers students from diverse backgrounds.

CURRICULUM INNOVATIONS IN THE PHILIPPINES

The Philippines is home to over 9.84 million indigenous people, comprising 9.1% of the total household population (Philippine Statistics Authority, 2020). These communities live across the islands, have deep connections to their ancestral lands, and face significant challenges. The education system historically marginalized indigenous communities, but in 2011, the Department of Education

(DepEd) released a policy framework (DepEd Order No. 62 s. 2011) aiming to empower them in shaping their educational experience (Philippine Department of Education, 2011). The shift in the education curriculum was influenced by international agreements such as Education for All (1990) and the Millennium Development Goals (2000), which pressured national governments to provide inclusive education for marginalized groups (Victor & Yano, 2015). Following the release of the policy framework, DepEd implemented the K to 12 curriculum in 2013, focusing on addressing the diverse needs of learners, including indigenous communities. The Indigenous Peoples Education (IPEd) Curriculum Framework (DepEd Order No. 32, s. 2015) was released in 2015, providing strategies for tailoring the curriculum to the specific cultural contexts of indigenous learners.

Indigenization through Collaboration

learners by incorporating their rich cultural heritage and ecological context into the curriculum. This curriculum innovation has resulted in significant changes in classroom practices, as lesson preparations become collaborative endeavors, with teachers no longer being the sole source of knowledge. Learning extends beyond the classroom walls, taking place within the community's ancestral domain.

A cornerstone of this approach is "learning with" indigenous communities. This recognizes the invaluable knowledge and practices that Indigenous Peoples (IPs) possess. Indigenous elders and culture bearers are given significant roles in deciding curriculum content and delivery.

The collaborative project of the DepEd Indigenous Peoples Education Office and the Department of Science and Technology Science Education Institute (DOST-SEI) on indigenization in STEM education, where one of the Filipino authors serves as a technical expert, exemplifies this approach. In this project, a team of technical experts, teachers, local leaders, elders, and cultural knowledge bearers collaborate to indigenize science and mathematics lessons. In the Ilocos Region (situated in northern Philippines) leg of the project, the team focused on indigenizing mathematics lessons, which are arguably the most difficult to indigenize, for the Bago and Kankana-ey indigenous groups. One lesson plan developed under the study integrated the Bago's "panagrama" fishing method into probability lessons, later expanding to include geometry concepts.

The lesson plan underwent a rigorous development process, ensuring accurate representation of indigenous practices and values alongside sound mathematical content. Importantly, cultural practices were discussed in the context of change and sustainability. Using panagrama, the dangers of dynamite fishing were compared to the eco-friendly process, which involves natural resources.

The teacher who was developing the lesson plan was in constant consultation with elders and culture bearers to ensure that the indigenous practices and values were accurately covered in the lesson, and with the technical experts to make sure the math content remained accurate and challenging, maximizing learning opportunities. The project's ultimate goal is to empower teachers and create adaptable lesson plans for various indigenous communities. To ensure best practices are shared, the team communicates their approach through various speaking platforms, benefiting future efforts.

There are also independent efforts of helping in the contextualization of mathematics lessons for IPs. The second author, for instance, collaborated with teachers and elders of the Dumagat Indigenous group in developing lessons on addition and subtraction of fractions for Dumagat primary school students. The lessons leveraged on natural resources available in the ancestral domain of the Dumagat

group, using these as springboards and learning aids. The lesson included a discussion of the Dumagat's practices of farming, betel nut chewing, and their core value of camaraderie. Doing so made the abstract concepts of fractions relatable and engaging, demonstrating the far-reaching positive impact that indigenization can have in these communities.

Indigenizing education in the Philippines faces challenges on multiple fronts. Standardized curricula clash with the unique needs of Indigenous learners, with rigid testing leaving little room for incorporating valuable local knowledge and cultural learning approaches. Gaining community support is also complex, as students often prioritize family and traditional activities over regular school attendance. This necessitates a more flexible academic calendar. Bureaucracy further hinders progress, with lengthy approval processes from various government departments stifling research and innovation in indigenization initiatives. Finally, logistical challenges inherent to remote locations can make research efforts difficult. Increased funding, improved infrastructure, and making Indigenous communities and schools more accessible are all essential. Despite these challenges, the potential benefits of indigenization are undeniable. Continued efforts are needed to address the challenges faced by indigenization initiatives and invest in culturally responsive education.

Socio-Ecological Issues and Practices that Influence Curriculum Innovations

Traditionally viewed as a theoretical subject, indigenization transforms mathematics into a tool for understanding the interconnectedness of social and ecological systems. By incorporating elements of indigenous knowledge, such as the Bago's "panagrama" fishing method or Dumagat practices, students grasp mathematical concepts while gaining a deeper appreciation for sustainable resource management and the relevance of mathematics in their cultural context. Thus, indigenization builds a holistic understanding of mathematics and its role in environmental stewardship. Furthermore, indigenization promotes inclusivity and equity. Collaborative curriculum development with indigenous communities ensures lessons resonate with their values and needs. It empowers learners by fostering a sense of cultural identity and environmental responsibility. Learning about traditional practices alongside mathematics creates a connection between cultural heritage and environmental sustainability. Indigenization addresses socio-ecological issues by enriching mathematical understanding and cultivating responsible and culturally aware future citizens.

SYNTHESIS

Looking across the curriculum innovations in three Asian countries, our focus centers on culturally responsive approaches considering socio-ecological factors, including learners' unique needs and context. A recurring theme is the significance of integrating cultural heritage and ecological context into the mathematics curriculum and addressing students' needs, as exemplified by Ethno-Realistic Mathematics Education in Indonesia and the process of indigenization in Nepal and the Philippines. The scholars' endeavors to create an inclusive, culturally relevant, learner-centered curriculum may be perceived as small-scale innovations. Still, they are pivotal in addressing the fundamental aspects of a mathematics curriculum. Listening to the different voices, we witness the dynamic interactions among curriculum innovations, socio-ecological factors, educational and cultural practices, and the evolving demands of society. Indeed, the mathematics curriculum lies at the heart of a socio-ecological system in mathematics education. The shared experiences shed light on various educational decision-making approaches, from the top-down model in Nepal and the Philippines to

the bottom-up approach in Indonesia. While the top-down method limits diverse ideas and the bottom-up strategy struggles to catalyze large-scale improvements, the potential of collaborative efforts to drive innovation within a centralized curriculum is a beacon of hope. It is important to note that challenges in curriculum innovation are intertwined with complex socio-ecological issues.

Using the lens of critical theory, the researchers identified tensions in developing and implementing an inclusive and culturally responsive curriculum in mathematics classrooms. These tensions are not just challenges, but they are urgent issues that need to be addressed. First, there are tensions among teachers as they try to align their teaching tasks with indigenous perspectives or use resources provided by the country's leading government agency in charge of education. Changing the typical class structure is different from the usual mathematics teaching methods. The conformity demanded by the departments or bureaus sometimes limits or hinders the teachers' desire to innovate. It is hoped that the government or school leaders will allow for *reinvention* rather than merely *following* prescribed formats and structures for curriculum innovations. To Freire, reinvention "demands the historical, cultural, political, social, and economic comprehension of the practice and proposals to be reinvented" (Freire & Macedo, 1987, p. 133, as cited in Gutstein, 2012, p. 23). Another tension exists between the curriculum offered and the needed curriculum in mathematics classrooms. With the model of top-down education power, the national curriculum is prescribed, and the top government agency centralizes the management with some appointed government officials. Gutstein (2012) states that these authorities influence curricula and textbooks with little or no responsiveness to local needs or desires. Because of this, it is not easy to apply any innovation in curriculum or textbooks targeted at a specific area or group in all government-supervised schools. Lastly, drawing from Bishop's (1994) assumption that "all formal mathematics education is a process of cultural interaction, and every child experiences some degree of cultural conflict in the process" (Bishop, 1994, p. 16), the schools are different from social institutions (home or community). Tension exists between the school (formal mathematics curriculum) and the community (non-formal curriculum). With the unclear educational tasks, the learners encounter dissonance between out-of-school and in-school cultural norms.

References

- Battiste, M. (2013). *Decolonizing education: Nourishing the learning spirit*. Purich Publishing Limited.
- Bishop, A. J. (1994). Cultural conflicts in mathematics education: Developing a research agenda. *For the learning of mathematics*, 14(2), 15-18.
- Coles, A., Solares-Rojas, A., & le Roux, K (2024) Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- Fiharsono, A., Carey, M., Hyde, M., Beazley, H., & Yektiningtyas-Modouw, W. (2024). Culturally based learning needs of Korowai students in a lowland-remote area of Indonesian Papua: school physical environment and building design. *The Australian Educational Researcher*, 51(2), 611-629.
- Freire, P., & Macedo, D. (1987). *Literacy: Reading the word and the world*. Bergin & Garvey.
- Gutstein, E. R. (2012). Mathematics as a weapon in the struggle. In O. Skovsmose, O. Skovsmose, B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education. New directions in mathematics and science education* (pp.23-48). Sense Publishers.
- Hermanto, R., & Nurlaelah, E. (2019). Exploration of ethnomathematics on the kampung naga Indigenous peoples. *Journal of Physics: Conference Series*, 1315(1), 012072.

- Larasati, S. R., & Rianasari, V. F. (2017). Mathematical literacy profile of grade VIII students of SMP Pangudi Luhur 1 Yogyakarta using Pendidikan Matematika Realistik Indonesia Approach. *International Journal of Indonesian Education and Teaching*, 1(1), 62–72.
- OECD. (2023). *PISA 2022 results: Factsheets Indonesia*. <https://oecdch.art/a40de1dbaf/C108>
- Panthi, R. K. (2021). Indigenization, decolonization, and reconciliation in mathematics education. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 3, pp. 737–745). Tredition. <https://doi.org/10.5281/zenodo.541593>
- Philippine Department of Education. (2011). *Department Order No. 62 s. 2011. Adoption of the National Indigenous Peoples Education Policy Framework*. <https://www.deped.gov.ph/2011/08/08/do-62-s-2011-adopting-the-national-Indigenous-peoples-ip-education-policy-framework/>
- Philippine Department of Education. (2015). *Department Order No. 32 s. 2015. Adopting the Indigenous Peoples Education Curriculum Framework*. <https://www.deped.gov.ph/2015/07/29/do-32-s-2015-adopting-the-Indigenous-peoples-education-curriculum-framework/>
- Philippine Statistics Authority. (2020). *Household Population, Number of Households, and Average Household Size of the Philippines (2020 Census of Population and Housing)*. <https://psa.gov.ph/content/household-population-number-households-and-average-household-size-philippines-2020-census>
- Revina, S., & Leung, F. K. S. (2019). How the same flowers grow in different soils? The implementation of realistic mathematics education in Utrecht and Jakarta classrooms. *International Journal of Science and Mathematics Education*, 17(3), 565–589.
- Rianasari, V. F., Budayasa, I. K., & Patahuddin, S. M. (2012). Supporting students' understanding of percentage. *Journal on Mathematics Education*, 3(1), 29-40.
- Rianasari, V. F., & Julie, H. (2018). Secondary school students' construction of knowledge: The case of fractions division. *International Journal of Indonesian Education and Teaching*, 2(2), 137–146.
- Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM – International Journal on Mathematics Education*, 40(6), 927–939.
- Sullivan, P. (2011). *Teaching mathematics: using research –informed strategies*. Australian Council for Educational Research.
- Upadhaya, H. P. (2023). Gender quota in the civil service and female participation: Evidence from Nepal. *Pacific Economic Review*, 28(2). 191-205.
- Victor, M. L., & Yano, B. (2015). Actualizing the inclusion of Indigenous peoples' rights in education: A policy initiative in the Philippines. *Human Rights Education in Asia-Pacific*, 6(1), 133-161.
- Yang, W. & Hui Li, H. (2019). Changing culture, changing curriculum: a case study of early childhood curriculum innovations in two Chinese kindergartens. *The Curriculum Journal*, 30(3), 279-297.
- Zulkardi, Z., Putri, R.I.I., Wijaya, A. (2020). Two Decades of Realistic Mathematics Education in Indonesia. In: van den Heuvel-Panhuizen, M. (Ed.), *International Reflections on the Netherlands Didactics of Mathematics. ICME-13 Monographs* (pp.325-340). Springer.

SCHOOL [MATH] CURRICULUM: TOWARDS AN ETHICAL-POLITICAL PERSPECTIVE SITUATED IN THE 'TERRITORY'

Oscar Charry¹, Carolina Tamayo², Diana Jaramillo³

¹ Universidad de Antioquia, ²Universidad Federal de Minas Gerais, ³Universidad de Antioquia

This paper presents a “problematization” of the disciplinary structure of the School [Math] Curriculum (S[M]C) from an Indigenous “territory” called “Sabaleta”. This problematization is inspired by different perspectives articulated by “family resemblances”: Wittgensteinian, Derridean, Foucauldian and Decolonial; the latter, in turn, inspired by the “traces” of the struggles and resistance of historically colonized Indigenous peoples. It is argued that the disciplinary structure has been operating as a process of normalization (homogenization) of the “other” (in this case, the Indigenous) in different historical-cultural contexts. This disciplinary process has left in its wake a trail of socio-ecological (environmental) imbalances in Indigenous “territories”. As a form of resistance to the disciplinary structure, Indigenous peoples, like Sabaleta, have been moving towards an ethical-political perspective of the S[M]C, which does not focus on disciplines, but is situated in the “territory”.

A 'PROBLEMATIZATION' SITUATED IN THE SABALETA 'TERRITORY'

This paper emerges from the doctoral research titled “Following the *traces* of Sabaleta: *walking the word* towards the healing of *territory*” (Charry, 2023). This research was developed by one of the authors of this document, in collaboration with some Indigenous Embera Chamí teachers from the “Educational Institution (EI) Katío Chamí – Sabaleta”: José, Alberto, Efraín, Jhon, and Nelly (Embera Chamí woman), among some of them¹⁴. This EI is in the Embera Chamí community of Sabaleta in the municipality of “El Carmen de Atrato”, in the department of Chocó (Colombia). The records and data of this doctoral research were produced, collectively, through different intertwined activities: walking through the territory in the company of the “Elders” (Wise), teachers and students; and telling stories about the community, among other activities. The voices (personal communications) of the protagonists of this research emerged from the stories told by them about the community of Sabaleta.

Thus, this paper presents a *problematization* of the disciplinary structure of the School [Math] Curriculum (S[M]C) from this Indigenous territory called Sabaleta. This problematization is inspired by different perspectives articulated by *family resemblances*: Foucauldian, Wittgensteinian, Derridean, and decolonial; the latter, in turn, inspired by the *traces* of the struggles and resistance of historically colonized Indigenous peoples. It is argued that the disciplinary structure has been functioning as a process of normalization (homogenization) of the *other* (in this case, of the Indigenous) in different historical-cultural contexts; this disciplinary process has left in its wake a trail of socio-ecological (environmental) imbalances in Indigenous territories such as Sabaleta.

According to Charry (2023), the S[M]C can be seen as a political territory populated by countless hierarchical oppositions that constitute the disciplinary structure. This disciplinary structure is

¹⁴ José, Alberto, Efraín, Jhon and Nelly are also Wise Embera Chamí Indigenous people and leaders (authorities) of the community, that is, members of the Indigenous Council of the Sabaleta Reservation.

supported, in the first instance, by a strategy of marking the other (the Indigenous person) through a logic of dualistic metaphysical thought, which forces the Indigenous to occupy the subordinate place of a binary opposition. This marking strategy would be followed by a purification process (perhaps Platonic) that would elevate her (or him) to the privileged position. To do so, she (or he) would have to be subjected to disciplinary power. And power, in a Foucauldian perspective, does not have a single center; power is not possessed, but rather exercised, and anyone can do it. As Foucault (1993) says, “power functions and is exercised through a reticular organization. And in it meshes, individuals not only circulate, but are placed in the condition of suffering and exercising it” (p. 32). In that sense, the process of normalization (homogenization) through disciplinary structure has not occurred under the passive gaze of Indigenous peoples; on the contrary, it has been in permanent tension with their movements of struggle and resistance. Therefore, it could be said that the S[M]C of Sabaleta bears the traces of colonization, but it also bears the traces of resistance.

Without a doubt, a Derridean perspective would adhere to the struggle and resistance movements of historically colonized peoples. It would be like an act of justice with otherness itself, with that other who has been historically subjected or condemned to always occupy a subordinate position. From a Derridean perspective, one would seek to *deconstruct* the hierarchical oppositions that sustain the disciplinary structure. To do so, Derrida (1981) would invite us to assume a double gesture. One of them consists of *inverting the hierarchy* at some point in the problematization; this gesture seeks to assume a change of position, that is, to try to look at the disciplinary structure from the perspective of the other. But we cannot stay there (in that position), we must try to advance towards another perspective that goes beyond hierarchical oppositions, that is, to make a *positive shift*.

Similarly, from a Wittgensteinian perspective, one would seek to problematize the *pictures* (or linguistic expressions) that naturalize the hierarchical oppositions of the disciplinary structure. For Wittgenstein (2009), a *picture* keeps us prisoners (*enchanted*) and does not let us out, because it resides in our language, which seems to inevitably repeat it to us. The call of this philosopher is to *disenchant* ourselves with these *pictures*, trying to broaden our view, that is, to have a *panoramic view* of the disciplinary structure that constitutes the S[M]C. Here it is important to clarify that the use of the square bracket seeks to indicate that what is being problematized in this article is the disciplinary picture of math.

In short, it could be said that there is no essence between the Wittgensteinian, Derridean and Foucauldian perspectives, but rather similarities that overlap and intersect as happens between the different members of a family; as Wittgenstein (2009) would say: “I cannot think of a better expression to characterize these similarities than ‘family resemblances’” (§ 67). In that sense, it could be said that these three perspectives present family resemblances, since, finally, they are situated on the margins of philosophy in its struggle with the hierarchical oppositions (typical of Western thought). These forms of struggle are those that could be articulated to those of historically colonized Indigenous peoples, that is, to a decolonial perspective. According to Walsh (2017), assuming a decolonial perspective implies understanding that it is not possible to erase the traces of the colonial past, but rather to permanently fight against the hierarchical oppositions that naturalized the subordination, marginalization and submission of historically colonized Indigenous peoples.

In this way, this perspective implies assuming an ethical-political attitude, which invites us to situate ourselves (as teachers and researchers) in the territory of the other, in this case, in the Sabaleta territory. For Sabaleta,

The territory constitutes the vital space, it is our mother that welcomes us, protects us and gives us everything, life and existence to all beings both visible and invisible, and within it we find warmth, love, tenderness, harmony, food, shelter and roof (Sabaleta, 2019, p. 13).

In an Embera¹⁵ perspective, everything is woven; the body is not separated from the mind or the heart, thought (knowledge) is not separated from practice, and nature is not something external to the human being. As Mecha¹⁶ (2015) says, “there is a true relationship and union of fibers or intertwined veins between the Indigenous and the existence of everything in the universe” (p. 16). For the Embera, all beings that exist on earth and in the universe (humans and non-humans) have life (*spirit*); therefore, each of them would be necessary to maintain balance and guarantee the survival of all (Yagari¹⁷, 2017). Thus, this is also an invitation to follow the traces of Sabaleta, to resort to the oral memory of the ancestors, where the memories that gave direction to the forms of struggle and resistance for the defense of the territory live. As Professor José says:

We must continue moving forward and fighting, resisting for our territory, and defending it. Today I advise all my Indigenous colleagues and young people, to be followers of our leaders, and wise, because we have a great library here within the community. The library we have is our elders, our ancestors, because if we do not open that book, we will not have knowledge for the future. (Personal communication, March 15, 2021).

In this sense, following the traces of Sabaleta implies learning to *walk the word*. For Professor Jhon, walking the word “is to dialogue, to have a deep conversation with the wise, (...) with all those people who handle knowledge. Walking the word is when we communicate with everything, and we also know how to read nature” (Personal communication, March 4, 2022). Thus, walking the word following the traces of Sabaleta takes us to a multiplicity of *references*, that is, traces that *refer* to other traces and to traces of others (Derrida, 1981). These references cross multiple space-time scales that allow us to understand how the disciplinary structure of the S[M]C was constituted in their territory, but, at the same time, also recognize the forms of struggle and resistance of their ancestors to the process of normalization (homogenization) of their people.

Following the traces of Sabaleta: towards an ethical-political perspective

Professor Alberto says that in 1979 education was born in Sabaleta “because children were taken to a boarding school in “Lloró” (Chocó). Parents found it difficult to spend a year without their children. They did not like this and instead wanted to bring education to the community” (Personal communication, March 15, 2021). In this sense, with the arrival of boarding schools to Indigenous territories, the relationships of children with their families began to break down. For example, Cáisamo¹⁸ (2012) says that in his community children over eight years old were locked up in these boarding schools, passing into the care of priests and nuns. For him, in these boarding schools “the

¹⁵ Embera Indigenous people had differences depending on the place where they lived: there were the Embera Dóbida (who were river people), the Embera Eyábida (mountain people), and the Embera Chamí (jungle people).

¹⁶ Baltazar Mecha is an Embera Dóbida Indigenous leader from the Catrú community of Chocó.

¹⁷ Dora Yagari is an Embera Chamí Indigenous woman from the Karmata Rua community in the department of Antioquia. Currently, Embera Chamí people are distributed in different parts of Colombia.

¹⁸ Guzmán Cáisamo was an Embera Dóbida Indigenous leader from the Catrú community of Chocó.

discipline was very rigid” (p. 36), where the sound of a bell indicated the call for the development of different routine activities, or rather, the bell oversaw organizing the children’s lives.

For his part, Yagarí (2017) relates that in the courtyards of the boarding schools, children had to line up to pray the Catholic prayers and sing the Colombian national anthem, while in the classrooms they had to sit still and remain silent almost all the time; when they did not obey the orders imposed on them, they were punished. In addition, the teachers treated them as if they were brutes, uncivilized and, therefore, wanted to strip them of all signs of their savagery: their typical clothing, their customs, their languages, their culture (Cáisamo, 2012; Yagarí, 2017). Regarding this, Yagarí (2017) narrates the following: “the teachers watched us so that we did not speak in our language, the ‘capunía’ [non-Indigenous] children told the teachers who was speaking in ‘Embera’ [Embera language], and they punished us” (p. 26).

Thus, in these boarding schools, one can glimpse some characteristic features of the disciplinary structure in a Foucauldian sense: the distribution of individuals in space and the control of their activities, among some of them. For Foucault (2002), the disciplinary structure “sometimes demands closure, the specification of a place that is heterogeneous to all the others and closed in on itself. (...) the boarding school appears as the educational regime, if not the most frequent, at least the most perfect” (p. 130). In boarding schools, permanent surveillance could be exercised over the individuals that were sought to be subjugated, to punish them (sanction them) every time they deviated from the norm (Foucault, 2002).

At that time, some believed that this disciplinary educational model would help the Indigenous people to achieve a more civilized way of life. This belief can be seen in the description that the Comptroller General of the Republic¹⁹ made of one of the Chocó districts in 1943:

Its territory is inhabited by Indigenous people with a semi-wild and therefore precarious life. The government, concerned about the fate of these almost helpless human beings, has advocated the establishment of a boarding school or colony to reunite and civilize them, on the Catrú River. (p. 676).

In this way, boarding schools became the social institution in charge of disciplining the bodies of Indigenous children, that is, converting their animality (savagery) into humanity. This was undoubtedly a Kantian picture of education, in which the main function of disciplinary power would be to straighten out the behavior of children, that is, to create docile bodies (easy to lead) (Foucault, 2002). However, this would not be the only task of boarding schools; they would also oversee disciplining the thinking of Indigenous children, through a structure of disciplinary thought that fragments, hierarchizes and reduces knowledge to disciplines, and that privileges knowledge called scientific (Foucault, 1993).

In this sense, some believed that the Indigenous people could adopt a human way of life, that is, achieve civilization, if they adopted the way of thinking (reasoning) of Western culture. As Guzmán Cáisamo pointed out:

(...) one view of being civilization was that we had to learn the way they think, the way they reason, their logic of thought, their logic of solving problems, because that is also how Western math is oriented. (Cáisamo, 2020, 9:10)²⁰.

¹⁹ This entity is the highest fiscal control body of the Colombian State.

²⁰ This conference is part of the audio recording of the Katío Chamí Educational Institution – Sabaleta, 2020.

In other words, for the Indigenous people to achieve civilization, they had to learn, among other disciplines, Western math. From a Platonic perspective, it is as if math made possible that path of ascension (of purification) towards a civilized way of life, where the Indigenous people could complete the truth. In this way, the Indigenous people would become virtuous human beings and submissive to the laws. Only in this way would it be possible for them to recover the lands that were one day taken from them by the colonizers.

In Colombia, the return of lands to Indigenous peoples began towards the end of the 19th century with Law 89 of 1890, “which determines the manner in which savages who are gradually reduced to civilized life should be governed”. Under the protection of this legal framework, Indigenous peoples began to have their territory returned, but in the form of reservations (pieces of land), which were to be administered by them according to what Law 89 dictated. These lands were to be divided and distributed among the Indigenous peoples in an equitable manner. To do so, these portions of land had to be measured and well delimited. Additionally, the reservation leaders (authorities) would fulfill the role of delivering the population census at the end of each year, in which a count of the people who lived in the community was made, as well as the registration of births and deaths.

In this way, the territory begins to take the form of a Cartesian space, which can be defined in geometric terms of area and perimeter; a space that can be divided and even sold. Likewise, the population would be represented in statistical terms as a group of people who inhabit the territory. Without a doubt, this Cartesian conception of space continued to fracture the relationships of the Indigenous people with their territory, not only by dividing the space into different dichotomies (public/private, rural/urban, interior/exterior, etc.), but also by feeding the picture of a human being superior to nature, who could use math, science and technology to subdue it at will.

In short, in this historical-cultural context, the evangelizing (disciplinary) educational model was charged with fulfilling the mission of purifying differences, that is, stripping the Indigenous of their savagery, which was manifested, supposedly, in their traditions, their language, their ancestral knowledge, their culture, or rather, their form of life. In addition, with the statistical techniques of counting people, births and deaths, which were introduced into Indigenous territories, mainly with Law 89, a form of power began to emerge that was no longer centered on the body (on the individual), but on the population, something that Foucault (1993) called *bio-power*; it is a set of techniques of regulation and control of the population, which seek to invade life completely.

Thus, as a form of resistance to the evangelizing educational model, Indigenous peoples began to think of an education that respected and valued their way of life. Thus, because of the struggles of these peoples, Ethno-education was born in Colombia in 1994. However, for more than a decade, Indigenous peoples like Sabaleta have assumed a distance from this policy, as it is a proposal that recognizes diversity in Colombia, but without transforming the disciplinary structure of the standardized national educational model; an educational model that was born towards the end of the 90s, related to the ideals of *progress* and *development*. In this regard, the Minister of Education at the time said the following: “on the eve of the new millennium and in the context of the new knowledge society, education is recognized as the main cause of progress and the advances that we know as development” (MEN²¹, 1998, p. 2). The Minister was referring to a new vision of development

²¹ Ministry of National Education of Colombia.

(human and sustainable), which would make progress lasting both for us and for new generations. In this way, the S[M]C would end up opening the doors to development in Colombia, by taking charge of creating citizens with forms of reasoning that would grant them a certain emancipatory universal capacity to dominate, control and transform the world and society (Valero & García, 2014).

In this historical-cultural context, math would acquire great relevance, being considered as one of the essential skills for the full participation of citizens in modern economic activities. As Professor Efraín says: “for us, math is important because numbers are essential for doing the math; with addition, subtraction, multiplication, division. In life, it is important to manage one’s own economy, and it must be learned” (Personal communication, October 24, 2019). In this context, a set of standardized tests (national and international) begin to proliferate, which would be responsible for correlating, in statistical terms, performance in math with indicators of human and sustainable development (Charry, 2023). In that sense, even if Indigenous peoples had the freedom to develop an Ethno-educational proposal, they would ultimately end up being evaluated with a standardized model that privileges Western knowledge. As Professor Efraín points out, regarding the “Saber Tests 11”:

The truth is, we accept the Saber Tests 11 i.e. it is what the State mandates. It is stipulated in national education law, and we respect it, that they [our students] know that world too. However, I would also like that they [ICFES] get to know our world, to collect our history, and that story comes out in that ICFES test. Because those ICFES tests only bring things from the Western world. (Personal communication, October 24, 2019).

Efraín refers to a standardized test applied by the Colombian Institute for the Evaluation of Education (ICFES) to students in their final year of school, which is a requirement for access to higher education. Thus, in a disciplinary structure, the *exam* (in this case, the SABER Test 11) plays a normalizing (homogenizing) role. As Foucault (2002) states: “the success of disciplinary power is undoubtedly due to the use of simple instruments: hierarchical inspection, normalizing sanction and their combination in a procedure that is specific to it: the exam” (p. 158). In this case, we can see how the techniques of control and regulation of the population have been strengthened by having greater power to process and circulate information. Thus, standardized tests can be a good example of bio-power, something that shows us how old disciplinary institutions have been clothed in new techniques of management over life (Foucault, 1993). In this context, Indigenous people have been paying a very high price for social inclusion (in economic terms). To access the labor market or higher education in search of better opportunities, Indigenous people would have to abandon their language, their culture, their territory, or rather, they would have to stop being Embera Chamí Indigenous people (in the case of Sabaleta). All this in the name of a new project that has been called development.

According to Escobar (2004), development is an economic, political and cultural project that demands the endless conquest of new territories, as well as their socio-ecological transformation in accordance with Western rationality; this project has historically promoted the exploitation of natural resources, putting the interests of economic growth before the care and protection of nature. Along the same lines, this author argues that forced displacement has been closely related to development. The community of Sabaleta has not been immune to this scourge of forced displacement. In 1998, during disputes between guerrilla and paramilitary groups for control of the area, the community was forcibly displaced to the municipal capital of El Carmen de Atrato, which caused some socio-ecological (environmental) imbalances in the territory. In this regard, Professor Alberto says the following: “in the displacement we lost some of our culture” (Personal communication, March 15,

2021); in addition, he also mentions in his diary²² some *spiritual* disharmonies caused during the forced displacement: “from so many bombings and rifle shots our mother earth is sterilized, it is no longer as it was before, crops and animals in the forest were abundant”.

As a way of healing the disharmonies that this standardized national educational model has left in their territories, Indigenous peoples have been working on the construction of their Own Indigenous Educational System²³, which will allow them to have greater territorial autonomy. This is an educational proposal that does not aim at the interests of the development project, but at the care and protection of Indigenous territories. The people of Sabaleta have been moving in the same direction, that is, in the construction of their own education. From the Embera perspective, their own education is experienced beyond the school walls; it is found in each of the spaces where ancestral knowledge is shared through oral tradition: traditional rituals, dance, music, stories, etc. As Efraín says: “Our own education is in our ancestors” (Personal communication, October 24, 2019).

In short, from this perspective, the S[M]C is the territory. It is an ethical-political perspective, which does not focus on disciplines, but is situated in the territory; a perspective that seeks to heal the socio-ecological disharmonies (imbalances) caused by civilization and development. Thus, math would no longer be in an abstract world (Platonic), nor would it be an instrument of domination of nature, but rather it would be intertwined with the social practices that human beings carry out in interaction with other beings of nature. This vision of math is like that of Wittgenstein (2009), for whom math can be seen as *language games* that are guided by a set of rules that point to the socio-cultural purposes of the way of life in which they are developed. Wittgenstein (2009) calls language games “the whole formed by language and the actions with which it is interwoven” (§7). In that sense, math constitutes the actions and interactions that humans establish among themselves and with other natural, artificial and even spiritual beings in different social practices (Miguel & Vianna, 2019).

This view of math implies that human beings can also learn from nature, if they know how to read it. The language of nature can be seen in a Derridean perspective as writing, that is, trace (*inscription*). According to Derrida (2008), by understanding the writing as trace it is possible to “cross the borders of anthropocentrism, the limit of a language confined to human discourse and words” (p. 125). For this philosopher, the trace concerns “differentially all living beings, all relations of the living with the non-living” (p. 125). In this way, other forms of writing manifested through weaving can be recognized. For example, Professor Nelly says that the women of the community wove their own dresses with a material that they took from the bark of a tree; these dresses were adorned with different figures “with meanings of nature. For example, (...) figures [that] represent mountains and roads” (Personal communication, September 9, 2021). In sum, from an ethical-political perspective, the S[M]C can be seen as a tissue of social practices that intertwine different forms of life (human and non-human).

In conclusion, by following traces of Sabaleta it was possible to cross multiple space-time scales to recognize both historically silenced voices and amplified voices in the constitution of the S[M]C. Likewise, viewing the S[M]C from an ethical-political perspective situated in the Sabaleta territory

²² This journal is part of the digital record of the aforementioned doctoral research.

²³ According to the MEN, Colombia is the first Latin American country to have its Own Indigenous Education System. Information available at: <https://www.mineducacion.gov.co/portal/salaprensa/Comunicados/421823:Colombia-es-el-primer-pais-en-latinoamericana-en-tener-un-Sistema-Educativo-Indigena-Propio>

not only sought to make silenced voices visible, but also to contribute to building a different alterity with nature, that is, to recognize nature as that other on which our own existence depends. This view coincides with the purposes and priorities pointed out by a socio-ecological perspective of mathematics education, aimed at caring for life in all its forms (Coles et al. 2022). In this way, it is possible to continue weaving the sacred tissue of life in our territories, in interaction with mathematics.

Acknowledgements

To the Excellent Young People and Leaders of New Chocó Project and the Bicentennial Doctoral Excellence Program of Minciencias for funding the doctoral thesis that gave rise to this paper.

References

- Cáisamo, G. (2012). *Pensar bien el camino de la sabiduría* [Unpublished doctoral dissertation]. Universidad de Antioquia.
- Charry, O. (2023). *Tras los rastros de Sabaleta: caminando la palabra hacia la sanación del territorio*. [Unpublished doctoral dissertation]. Universidad de Antioquia.
- Coles, A., le Roux, K., Solares, A. (2022). Towards a socio-ecological perspective of mathematics education. In: C. Fernández, S. Llinares Ciscar, A. Gutiérrez Rodríguez & Núria Planas I Raig (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 171-178). PME.
- Contraloría General de la República (1943). *Chocó, Geografía Económica de Colombia, Tomo VI*. Bogotá.
- Derrida, J. (1981). *Positions* (A. Bass, Trans.). Athlone.
- Derrida, J. (2008) *El animal que luego estoy si(gui)endo* (C. de Peretti & C. Rodríguez Marciel, Trans.). Trotta.
- Escobar, A (2004). Desplazamientos, desarrollo y modernidad en el pacífico colombiano. In S. Feldman, C. Geisler, & L. Silberling (Eds.) *Restrepo y Rojas, Conflicto e invisibilidad. Retos de los estudios de la gente negra en Colombia*. (pp.190-205), Universidad del Cauca.
- Foucault, M. (1993). *Genealogía del Racismo* (A. Tzveibel, Trans). Altamira.
- Foucault, M. (2002). *Vigilar y castigar; nacimiento de la prisión* (1st ed.) Siglo XXI Editores, Argentina.
- Mecha, B (2007). Una mirada de emberá sobre el conocimiento y la investigación. *Revista Educación y Pedagogía*, 49, 103-118.
- Mecha, B. (2015). *La unidad entre la diversidad natural y cultural como herramienta metodológica*. (Unpublished document). Quibdó.
- Miguel, A. & Vianna, C. R. (2019). Tigres, talos e deuses ex machina. *Wittgenstein na Educação*, 21-46.
- Ministerio de Educación Nacional. (1998). *Lineamientos Curriculares de Matemáticas*. Bogotá: MEN.
- Sabaleta (2019). *Fortalecimiento de las prácticas ancestrales en el Resguardo indígena de Sabaleta*. (Unpublished document). El Carmen de Atrato.
- Valero, P. & García, G. (2014). El Currículo de las Matemáticas Escolares y el Gobierno del Sujeto Moderno. *Boletim de Educação Matemática*, 28(49), 491-515.
- Walsh, C. (2017). *Entretejiendo lo pedagógico y lo decolonial: luchas, caminos y siembras de reflexión-acción para resistir, (re)existir y (re)vivir*. Alternativas.
- Wittgenstein, L. (2009). *Philosophical investigations* (GEM Anscombe, Trans.) Blackwell.
- Yagarí, D. M. (2017). *Ēbērã Sõ Bía (Embera de Buen Corazón), Referente de la Educación Propia Ēbērã Sõ Bía Kavabidru: Dachi Evarimiká nurēadaita* [Unpublished master dissertation]. Universidad de Antioquia.

MATHEMATICS LANGUAGE-KNOWLEDGE DISPARITIES, DE/COLONIALITY AND THE SOCIO-ECOLOGICAL IN THE PHILIPPINES

Ivy Estrella, Paola Valero, Hendrik Van Steenbrugge

Stockholm University

Based on an ethnographic study conducted in the Philippines, we identify some tensions between school mathematics and mathematical practices used in local communities, and how these tensions are connected to issues of ecological precarity of local communities. By using vignettes from mathematics classrooms and interviews from local communities, the study revealed that particular kinds of school mathematics languages and knowledges are privileged over the mathematics and languages of the communities. Through this, we have worked toward questioning the foundation of the Western code, its effects on Indigenous culture, knowledges, and being; and opening up for other options of thinking and doing mathematics that can contribute to epistemic, ontological, and ecological justice to local communities. This study sheds light on how mathematics education can unpack socio-ecological problems in the concrete context of the Philippines.

COLONIALITY AND DECOLONIALITY

When the concept of time entered history, it put societies in a linear and chronological order starting from nature to culture, barbarism to colonization, going through a progressive path towards a point of arrival (Mignolo, 2011). This point has been termed the Eurocentric Modernity, the universal epistemic code, or the “Western code” which assumes that Europe or the West is the culmination of modern civilization. As an end to which aspire, the Western code is to colonize other people by denying humanity, intelligence and knowledge (Quijano, 2007). In a world ruled by the Western code, codified power relations determine who includes and who is included (Mignolo, 2011). This type of codified power relations known as *coloniality of power* or *coloniality* pertains to the control of economy, authority, gender and sexuality, as well as of subjectivities and knowledge (Ndlovu-Gatsheni, 2012; Mignolo, 2011). In such order, the locus of enunciation — “the geo-political and body-political location of the subject that speaks” (Grosfoguel, 2007, p. 213) including the historical, cultural, bodily, ideological and other possible dimensions from which one speaks and produces knowledge (de Figueiredo & Martinez, 2019)— is the basis of inclusion and exclusion. It holds the control of knowledge and power of decision on gender, race, political orientations, and economic relations. Although the Western code and its coloniality of power only serve a small portion of the population, it has been deemed as the only option and model for the entire planet.

Coloniality, however, is resisted. There is a continuous fight in the form of what Mignolo (2011) calls “options”, over control of knowledge and authority from the state, business entities, and religious groups along with the fight over the freedom from the control of the coloniality of power. One possibility to shed light on and rethink the Western code is *decoloniality*. The decolonial “option” or movement invites us to rethink and critique Eurocentrism from the perspectives of the subaltern side and to decolonize Eurocentric epistemology (Grosfoguel, 2007). It aims to delink from the Western code and to participate in the production of knowledge from the perspective of those who were left out of history, dialogue, and knowledge-making (Mignolo, 2011; Walsh, 2013). The task of

decoloniality is two-fold: 1) to liberate the knowledge and becoming that the coloniality of power (knowledge and being) had prevented by questioning the foundation of the Western code or Eurocentric epistemology and ontology (Mignolo, 2018); and 2) to change the locus of enunciation through the development of decolonial narratives that will legitimize decolonial ways of doing and living, by opening up “other” options of logic of thinking, doing, and living, aside from Eurocentric perspectives (Mignolo, 2011; Mignolo, 2018; Veronelli, 2015).

The decolonial option is also concerned with the socio-ecological sphere. Coloniality is not only a cultural and political phenomenon, it also manifests and materializes through human and environment deterioration due to rising of what Williams et. al. (2017) called “economics of extraction” caused by corporate power and Western scientific materialism. This includes the dislocation of people and other species from their own lands, as well as the oppression of Indigenous knowledges, cultures, and their being, which causes the degradation of humans and their environment, and the sense that such degrading exploitation is necessary for “development”. Modernity has caused the colonization of perception and focus on materialism that heavily depended on repression of Indigenous ways of knowing and being and on the extraction of value from people and nature alike (Williams et. al., 2017). Putting it this way, the focus on materialism and valuing of Eurocentric epistemology has resulted in devaluing of Indigenous knowledge and culture, and on the land and nature. Therefore, questioning the foundation of Eurocentric epistemology and ontology by revealing the other side of Modernity, that is, coloniality, is a crucial step in the process of decoloniality and in advancing towards understanding and resisting the role that it plays in current socio-ecological crises.

In the Philippines, coloniality is maintained through the organization of the educational system (Casambre, 1982), which translates and puts into operation through the curriculum the prevailing idea in the Philippine society that advancement is based on Western standards of progress and development. Since English, the language of the American colonizers, is the language of power and prestige, and thus the language of the government, the elites, and the educated (Bernardo, 2004), mastery of English determines success. In 1974, through the Bilingual Education Policy, English was decided to be the main language of instruction of school mathematics, while mother tongue languages of the students, Filipino and others, are only used as auxiliary languages of instruction. These languages get replaced by English as children progress in school. This means that students need to be able to speak the dominant (colonizing) language to be able to access and produce mathematical knowledge. And with the replacement of students’ mother tongue languages by English comes the replacement of the local mathematics which are part of work and everyday practices in communities by school mathematics. In this sense, language and school mathematics contribute to inequitable access to mathematical knowledge and marginalization of Indigenous knowledge and culture as Indigenous knowledge are in the local language and culture and in contrasts with school mathematics. Indigenous knowledges are kept through language and by disregarding and devaluing the language, Indigenous knowledges and cultures are also devalued (Gutierrez, 2019). To learn school mathematics in the Philippines is to speak the dominant languages, thereby learning a particular perspective and becoming a particular person, and thus a form of colonization of the mind (Barwell & Radford, 2016).

The language(s) and forms of knowledge(s) in Philippine mathematics classrooms show the tensions between school mathematics and mathematical practices used in local communities. These tensions

extend to a larger scale, that is the Philippine society, as these also connect to issues of socio-ecological precarity of communities, and what is being done about these issues. Coles et al. (2022) argue that humans and non-humans are entangled in complex interrelationships, and knowledge about this is significant in understanding those relationships. As previously stated, modernity has shifted the emphasis on materialism and Eurocentric epistemology which resulted in suppression of Indigenous culture, language, and knowledge. Hence, this study asks the question: *How does privileging certain types of language(s) and knowledge(s) in Philippine mathematics classrooms affect the socio-ecological systems in local communities?*

MATHEMATICS EDUCATION IN THE PHILIPPINES AND COLONIALITY

We draw from an ethnographic study in which data were collected from classrooms and communities in a province in the Philippines. Data includes classroom observations and field notes gathered from two primary schools that include four classes and five teachers. The interviews are conducted in the local communities in the same town where the schools are located. The students, teachers, and people from the communities belong to non-Indigenous, major ethnic/ethnolinguistic group called “Tagalog”. This group constitutes 30% of the country’s total population (World Bank Group, 2024). The Tagalogs are located in Tagalog region and their livelihood consists primarily of agriculture and fishing (Defense Language Institute Foreign Language Center, 2011). The main crops include rice, coffee, sugarcane, corn, tobacco, coconuts, mangoes, and bananas. Fishing is also a main source of livelihood as ocean surrounds the region. Other industries include manufacturing of chemicals, textiles, and metal products; processing of lumbers, harvesting of bamboos, milling of plywoods, and mining of copper, gold, nickel, chromite, and iron.

Based on the gathered data, disparities between school mathematics practices and community mathematics practices were identified. These community practices are part of Indigenous knowledge or local knowledge that is distinct to a particular ethnic group or culture and obtained by the members of the group through experiences, informal experiments, and close understanding of their environment (Chikaire et.al, 2012). These practices are kept by local people such as farmers, fishermen, cattle-rearers, and others. Inspired by Indigenous mathematics (Sternberg, 2011), we describe community mathematics as the mathematics of a certain culture that conveys a collective worldview. It is the mathematics that a community is using and created by living in a particular place for a long time.

In the schools, lessons on telling time, and identification of solid figures were observed, while people in the community were interviewed about their jobs. From this, it was found that students are taught to read and estimate time in English but outside the classrooms, time is usually expressed in Spanish and Filipino. The interviews showed that people in the community or outside the school, use body and nature as basis of measurements as shown with measuring the size of fishes through the number of fingers or length of the arms, and measuring height of the water by using body parts and trees as basis, and are expressed using Spanish or Filipino language.

One episode that encapsulates the tension between school and community mathematics knowledge and practices is a vignette on the lesson about solid figures. This is a 5th-grade lesson in which Euclidian geometry on solid figures are taught to the students. In the Philippines, the curriculum mandates teaching one mathematics lesson for four days in which 50 minutes is designated for each

day. The teacher stretches the lesson from introduction to doing activities and exercises throughout the week. This vignette happened on the third day wherein the students were given a task to identify the solid figures outside their classroom but within the boundaries of the school. For this activity, the students were divided into five groups and were given an activity sheet in which they have to fill out by looking for 10 objects, identifying what solid figure these objects are, and identifying the type of solid figure whether polyhedron or non-polyhedron. The groups were tasked to go to five designated stations and identify two objects in each station. The stations are: station 1 is classroom, station 2 is basketball court, station 3 is canteen, station 4 is the flagpole field and station 5 is the garden. When the students were in the garden, one member of a particular group kept on asking the other members about a mango that he saw on the ground. He picked it up, kept looking at it while going around the garden and asked his members using their mother tongue about it: “What about the mango? What figure is this?” The teacher was with them in the garden but this student did not ask the teacher about it. Even when they were back in the classroom, the student did not ask the teacher. In the end, the students did not include the mango as one of the objects they saw in the garden because they could not identify what solid figure it is. Ivy (the observer) asked the teacher about the mango. According to her, a mango is an irregular solid figure and this lesson is not part of the curriculum. They mention it to students but since it is not part of the elementary mathematics curriculum, it is usually ignored. From this vignette, it can be seen that students have experiences that are not part of the curriculum and these experiences are obliterated, and thus this knowledge is not recognized inside the classroom. In this sense, not asking the question to the teacher and the teacher not discussing it in class because it is not part of the curriculum can be seen as making abstract the connections to the local environment. To the student, and arguably, the community, the shape of the mango is a central aspect. However, since it is not part of the curriculum, and the student did not ask the teacher in English, the shape of the mango is not taken up as particularly relevant.

In contrast, in the community, knowledge of solid figures is important for the livelihood of the community. An interview with a fruit vendor in the market shows this. This particular fruit vendor sells fruits depending on the season. Since the interview was conducted in March, summer fruits are abundant and this includes mangoes and melons. For that day, she was selling different types of mangoes which are only distinguishable if the buyer or seller knows the shape. There are very small differences in the shapes so it was difficult for untrained eyes. For the day of the interview, she was selling three types of mangoes: *piko*, *sapadera*, and *kalabaw*. Comparing the three types, some very subtle differences can be seen. *Sapadera* and *kalabaw* mangoes look the same, except that *kalabaw* mangoes are a bit rounder on the bottom part; *piko* mangoes have are more pointed at the bottom part compared to the other two varieties. With the knowledge of shapes of mangoes, one can tell the quality, taste, and kind of mango they are buying. Thus, the shape of mangos is important as selling fruits is one of the main livelihoods in the region. However, as shown in the previous vignette, shapes of mangoes are not taught as part of school mathematics because these are not part of the curriculum. From here, it can be said that there is a disconnection between school mathematics and the mathematics used in the communities.

TRACING SCALES IN SCHOOL MATHEMATICS AND COMMUNITY MATHEMATICS

The disparities between school mathematics and community mathematics show tensions that extend to the Philippine society as these also connect to issues involving local communities. Examining these

tensions with the lenses of decoloniality allows identifying the multiple scales at which mathematics education practices are perpetuating coloniality. Looking at the society in terms of the differential between occupations and jobs that are categorized as elementary occupation and high-end occupations, we can further see the role of language(s) and mathematical knowledge(s) in the Philippines. Elementary occupations include laborers which do not require economically-valued qualifications or skills, while high-end occupations include professional workers which require economically-valued qualifications and skills. From this, occupations and industries which belong to the high-end of an economic scale require higher qualification and skills which include school mathematics skills, while elementary occupations require lower qualifications and skills, which, most often do not include school mathematics and rely on local knowledge. Although school mathematics opens up more opportunities for students and aids in the development of the country, school mathematics also participate in perpetuation and re(production) of coloniality through the idea of progress, in the sense of Eurocentric modernity, that disregards and ignores the issues regarding ecological situations of communities. This means that school mathematics opens opportunities for students to acquire high-end occupations and paves the way for industrialization, urbanization, and economic development of the Philippine society. However, the idea of progress and development through modernity, endangers the existence and continuity of Indigenous or local culture, knowledges, and their environment. The vignettes and interviews evidence the disconnection between school mathematics and community mathematics, and the different types of practices and ideals they are connected with.

Coloniality is also reproduced in mathematics education and in classrooms through the creation of disparities in language(s) and forms of knowledge(s). Acquiring school mathematics and science knowledges is connected to personal advancement and national economic growth. This is reflected in mathematics education as school mathematics is taught and learned mainly in English; and school mathematics knowledge is different from mathematical knowledge and practices outside the school. In turn, local mathematics or mathematics used in the communities are in local languages while mathematical knowledge and practices in the communities are not economically valued and only deemed useful for bare survival. Inevitably, local mathematics languages and knowledges are placed in a subaltern position compared to school mathematics languages and knowledges. This privileging and putting of school mathematics in a superior position creates types of learners who are well versed in English and school mathematics; but who become alienated from their locality. In this regard, coloniality permeates mathematics education by determining who is at the centre or who controls mathematics education; which knowledge and thinking are privileged inside mathematics classrooms; and what type of people are being created through mathematics education.

Thus, it can be said that English language and school mathematics curriculum in the Philippines which is dominated by Eurocentric epistemology has a large impact on the neglect and ignorance of Indigenous knowledge (Chikaire et.al, 2012). In this sense, it can be said that language and school mathematics curriculum are significant factors in the maintenance and perpetuation of coloniality in the Philippines. Indigenous knowledge, including community mathematics, cannot be disregarded as it is people's connection to their environment, land, culture, and identity (Chikaire et.al, 2012). It includes spiritual relationships, environment relationship, relationships among people, and is shown on people's language, social organization, culture, and laws. Indigenous knowledge enables local communities to survive.

RESISTING THROUGH DECOLONIALITY

Our classroom observations and interviews with people from the community have revealed that particular kinds of school mathematics languages and knowledges are privileged over the mathematics and languages of the communities. There are student experiences that are not part of the curriculum, and therefore not recognized inside the classrooms even though these experiences are significant part of their communities. The disconnection between school and community mathematics in the Philippines showed that English language and school mathematics curriculum which is dominated by Eurocentric epistemology has a large impact on the neglect and ignorance of Indigenous knowledge and thus have an impact on issues of ecological precarity of local communities. We understand this privileging and disconnections as traces or signs of coloniality in primary school mathematics classes. In this sense, the vignettes show how the locus of enunciation is enacted inside mathematics classrooms. It shows who and what are included and privileged inside mathematics classrooms. Consequently, the locus of enunciation is placed to those who represent school mathematics and is kept from the people of the community. Through this, in the context of the Philippines, mathematics education centers Eurocentric modernity through the ideas of industrialization, urbanization, and economic development of the Philippine society; Eurocentric epistemology is privileged inside mathematics classrooms as shown by the disparities between school and community mathematics; and mathematics education creates people who are knowledgeable in English and can participate in occupations that are economically-valued. In other words, it is clear that school mathematics practices devalue informal and culturally-rooted ways of doing mathematics outside school settings (Gutierrez, 2019). It displaces the locus of enunciation from local people, their language and knowledge to those who represent (or appropriate) school mathematical knowledge. Following Valero (2023), school mathematics actively works to make abstract the connections between communities and their environment, generating forms of being that are modern and Western, thus prone to be aligned with coloniality.

Maintaining coloniality through mathematics education practices in turn has consequences on socio-ecological crises of local communities in the Philippines. School mathematics promises advancement and growth by creating particular types of people, and through changes in the communities in the form of urbanization, industrialization, and economic growth of communities. However, the reality is that these advancement and development displace resources, livelihoods, and development of local lands and environment. In this vein, progress and development, that is coloniality concealed by modernity, removes the knowledge and value of the communities by privileging capitalistic enterprises. With that, there is a displacement of mind, being, and power. School mathematical knowledge and language change the perception of people, and devalues the knowledge, culture, and language of their communities, thereby taking away their power over their communities. Challenging the locus of enunciation back to those with local knowledge will invoke a sense of solidarity among people of the communities to reestablish and retain the value of their knowledge, culture, and language that will otherwise be taken away if the current system of power relations is sustained.

We understand the privileging of school mathematics and disconnections between forms of knowledge as traces or signs of coloniality in primary school mathematics classes. In this sense, the vignettes show how the locus of enunciation is enacted inside mathematics classrooms. It shows who and what are included and privileged inside mathematics classrooms. Western modernity, it seems,

operates/colonizes through an entanglement of a variety of scales, ranging from globalization and English language, over school mathematics and the Philippine society to the communities' local languages, practices, knowledges, and environment. Thus, we have worked toward questioning the foundation of the Western code, its effects on Indigenous cultures and knowledges. We are not alone in this endeavor, as recent research work raises serious critique on how the organization of school mathematics reproduces coloniality in African (e.g., Osibodu, 2024) and Latin American countries (e.g., Parra & Valero, 2021) in relation to different forms of Indigenous and local knowledge. From here, discussion can start opening up for other options of thinking, and doing mathematics that can contribute in the epistemic, ontological, and ecological justice to local communities, two central aspects of resistance through decoloniality. We are left with the challenge of finding and systematically practicing other forms of school mathematics that support more sustainable lifeworlds and that attend to the balance between humans and non-humans, where other values and forms of living different from an idealized Western standard of “progress” are privileged.

Acknowledgements

We thank the members of the SOCAME research group, and the ReMaS Graduate School (VR-2021-00537) and the IMMPACT project for the collaboration and discussions that support our work.

References

- Casambre, N. J. (1982). The impact of American education in the Philippines. *Educational Perspectives*, 21(4), 7–14.
- Chikaire, J., Osuagwu, C. O., Ihenacho, R. A., Oguebuchulam, M. N., Ejiogu-Okereke, N., & Obi, K. U. (2012). Indigenous knowledge system: The need for reform and the way forward. *Global Advanced Research Journal of Agricultural Science*, 1(8), 201–209.
- Coles, A., le Roux, K., & Solares, A. (2022). Towards a socio-ecological perspective of mathematics education. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas, (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education, Spain* (Vol.2, pp.171-178). PME-Universidad de Alicante.
- De Figueiredo, E.H.D. & J. Martinez (2019). The locus of enunciation as a way to confront epistemological racism and decolonize scholarly knowledge. *Applied Linguistics*, 42(2), 355-359 doi:10.1093/applin/amz061
- Defence Language Institute Foreign Language Center. (2011). *Tagalog culture orientation*.
- Escobar, A. (2007). Worlds and knowledges otherwise: The Latin American modernity/coloniality research program. *Cultural Studies*, 21(2-3), 179-210.
- Grosfoguel, R. (2007). The epistemic decolonial turn: Beyond political-economy paradigms. *Cultural Studies*, 21(2-3), 211–223.
- Gutiérrez, R. (2019). Mathematix: Towards a way of being. In J. Subramanian (Ed.), *Proceedings of the 10th International Mathematics Education and Society Conference* (Vol. 1). Hyderabad India.
- Mignolo, W. (2011). *The darker side of western modernity: Global futures, decolonial options*. Duke University Press.
- Mignolo, W. (2018). The Conceptual Triad: Modernity/Coloniality/Decoloniality. In W. Mignolo & C. Walsh (Eds.), *On Decoloniality Concepts, Analysis, Praxis* (pp.135-152). Duke University Press.
- Ndlovu-Gatsheni, S. J. (2012). Coloniality of power in development studies and the impact of global imperial designs on Africa. *The Australasian Review of African Studies*, 33(2), 48-73.

- Odal, G.P. Lowland Cultural Group of the Tagalogs. Retrieved from <https://ncca.gov.ph/about-ncca-3/subcommissions/subcommission-on-cultural-communities-and-traditional-arts-sccta/northern-cultural-communities/lowland-cultural-group-of-the-tagalogs/>
- Osibodu, O. (2024). Racialization through coloniality in mathematics curricula: Problematizing the Cambridge Assessment International Examination in Sub-Saharan Africa. *Educational Studies in Mathematics*, 116, 445–461.
- Parra, A., & Valero, P. (2021). Propio as a decolonising tool for mathematics education. In A. Andersson & R. Barwell (Eds.), *Applying critical mathematics education* (pp. 71–99). Brill.
- Quijano, A. (2007). Coloniality and modernity/rationality. *Cultural Studies*, 21(2-3), 168-178.
- Sterenberg, G. (2013). Learning Indigenous and Western mathematics from place. *Mathematics Education Research Journal*, 25, 91-108.
- Valero, P. (2023). Mathematical subjectivation: Death sentence to the planet or chances for a terrestrial life. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th PME Conference* (Vol. 1, pp. 53-68). PME – University of Haifa.
- Veronelli, G. (2015). The coloniality of language: Race, expressivity, power, and the darker side of modernity. *Wagadu: A Journal of Transnational Women's & Gender Studies*, 13(1), 108-134.
- Walsh, C. (2013). Shifting the Geopolitics of Critical Knowledge: Decolonial thought and cultural studies 'others' in the Andes. In W. Mignolo, & A. Escobar (Eds.), *Globalization and the decolonial option* (pp. 78-93). Routledge.
- Williams, L., Bunda, T., Claxton, N., & MacKinnon, I. (2018). A global de-colonial praxis of sustainability—undoing epistemic violences between Indigenous peoples and those no longer Indigenous to place. *The Australian Journal of Indigenous Education*, 47(1), 41-53.
- World Bank Group. (2024). *No Data, No Story: Indigenous peoples in the Philippines*. The World Bank.

RESTOR(Y)ING MATHEMATICS: AN EXPLORATION IN KANAKA MAOLI TARO FARMING

Rochelle Gutiérrez

University of Illinois Urbana-Champaign

Mathematics is often used to control, dehumanize, and categorize. Restor(y)ing mathematics centers a version that reconnects us to each other and the natural world, extending the concept of Indigenous relationality and expanding how we think of mathematics to create new futures. As a theoretical framework, it builds upon living mathematx and intervenes by defining mathematics as patterns, structures, and logics between living beings and centers Indigenous storytelling. Restor(y)ing mathematics expands current interpretations and applications of ethnomathematics, with the goal of reconceptualizing mathematics and the patterns we live out with each other as humans and in relation to our more-than-human relatives (lands/waters). In this exploratory study, I explore the theory of restor(y)ing mathematics by drawing upon examples from my work with Kanaka Maoli taro farmers in Hawai'i. More specifically, I highlight some of the patterns, structures, and logics that farmers navigate as they enact local innovations to draw waters back to their lands, organize each other, share interdisciplinary knowledge, and reinvigorate Native foodways.

INTRODUCTION

We are in a moment of great awakening; people everywhere are concerned with a climate crisis, extreme violence in the world, and the importance of justice. Touched by the grief of diaspora, we are all trying to find our ways home. What role should mathematics play in this moment? Should it: describe the nature of global problems; offer excellent prediction models; or provide data analysis and statistics for calculating risk? This project engages how we might think differently about our relationship between mathematics, humans, and the planet. Mathematics is traditionally used to control, dehumanize, categorize, oppress, and serve as a gatekeeper. In response, *restor(y)ing mathematics* (Gutierrez, 2023; 2021) centers a version of mathematics that reconnects us with each other and the natural world, building upon the concept of relationality (Simpson, 2017) and expanding how we think of mathematics (e.g., incorporating various stories/histories) to create new futures. The project intervenes by defining mathematics as patterns, structures, and logics between living beings and by centering Indigenous storytelling.

ETHNOMATHEMATICS & SOCIAL JUSTICE MATHEMATICS

Although purported to be universal, there are many mathematics. Mathematics are localized and arise from the needs of people in relation to their environments (Turnbull, 2000), as evidenced by the field of ethnomathematics established in 1977 by Ubi D'Ambrosio in Brazil. Anthropologists have chronicled fractal patterns in textiles and baskets woven by various Mesoamerican and African cultures (Ascher, 2002); vectors and triangulation used during Polynesian sea voyaging (Furuto, 2019), and fuzzy logic to gauge the health of seal colonies (Berkes & Berkes, 2009). Although researchers have developed ethnomodelling practices (Rosa et al., 2023) and created design tools to highlight ethnomathematics (Eglash et al., 2006), in general, ethnomathematics is not well known by many mathematics teachers or the public at large. Instead, Western mathematics prevails and has tended to support domination and destruction of our planet, through an agenda of national security

and warfare (BooB-Bavnbeek & Høyrup, 2003; O’Neil, 2016). With its emphasis on quantifying, categorizing, and reducing complex and multi-layered relationships between persons to mere abstractions, Western mathematics often supports a fallacy that modeling, big data, and software can solve anything. The introduction of recent artificial intelligence tools (e.g., ChatGPT, Wolfram Alpha, AIR MATH) further threatens our intimate relations with each other through mathematics.

Another form of mathematics, grounded in statistics and quantitative reasoning, seeks to support social justice and centers anti-racism and sustainability (Coles et al., 2013), partly through forms of mathematics education that support community members to engage in important dialogues about our future (Barwell, 2013). Students are taught to use classical mathematics as a tool to read and write the world (Gutstein, 2006), to develop their sociopolitical consciousness and mathematical proficiencies. But, in general, the tool itself is not questioned. Knowing we cannot use the master’s tools to dismantle the master’s house (Lorde 1984), we must also reconceptualize mathematics (and mathematical practices). That is, the mathematics that got us to this moment is not the mathematics that will get us out. This paper extends the concept of Indigenous relationality to mathematics and draws upon storytelling by local peoples to take up issues of epistemology and ontology in moving this work forward.

RESTOR(Y)ING MATHEMATX

Restor(y)ing mathematics is situated at the nexus of mathematics, philosophy, Indigenous studies, and decolonial theory and honors many mathematics coming into relation to consider the human condition. Relational theory highlights that we are always in relation to one another as humans, a form of co-becoming (López & Coello, 2021); this aspect of relationality connects us to each other and to the natural world. Relationality helps us recognize that plants and animals, winds and skies, waters and mountains, humans and more-than-humans, are always in relation – a form of ecology to which we all belong (Little Bear, 2009). When we extend the concept of relationality to mathematics, we see that mathematics are also forms of relation: patterns, structures, and logics. In nature, these forms of relation are produced, embodied, and practiced and can restore some of the ancestral wisdoms that have been muted/erased. Shorelines, ice and snow, as well as galaxies, expand through fractal patterns similar to tree branchings and human vein systems, showing that we are all related. Moreover, the precarious and uncertain world in which we live—where, among other things, a climate crisis, racial capitalism, and ongoing state-sanctioned violence, are occurring—also shows we are all related and part of a larger sociopolitical ecology. How we choose to uplift certain forms of mathematics can both restore us (e.g., bring back ways when we have been in good relations with each other and the natural world) and restory us (e.g., allow us to change) so we can spiral into more liberatory and otherwise futures (Gutiérrez, 2021; 2022; 2023).

Current efforts to reconsider mathematics and their role in our global society tend to rely upon a utilitarian version of mathematics to better survive on this planet. In contrast, *restor(y)ing mathematics* suggests that a form of mathematics relating to all living beings, embracing a different philosophical stance, including metaphysics (Deloria, 1979), is more likely to change our relationships with each other in this universe. Elsewhere (Gutiérrez, 2017; 2019), I have argued that as we begin to reimagine mathematics, we can reimagine the mathematician, as well as how mathematicians are influenced by the mathematics they do. In this early work, I laid out a vision of living mathematx that highlights the ways our more-than-human relatives embody (i.e., live) pattern,

structure, and logic and where we can learn additional forms of mathematics from them. I underscored three concepts to serve as guiding principles: In Lak’ech, nepantla, and reciprocity. In Lak’ech captures the notion that we seek to recognize and affirm “others” in “us” and “us” in “others.” By focusing on ways that we are similar and different, In Lak’ech supports us to learn from others, including lands/waters. That is, lands/waters can teach us new things and expand what we currently accept as patterns, structures, and logics (i.e., mathematx). Nepantla acknowledges that we do not need to work within binaries but can operate within a space of tensions and multiple realities to birth new knowledge, leading to many mathematics. Embracing nepantla allows us to see many forms of patterns, structures, and logics that can contradict one another and create tensions. Moreover, the concept of nepantla helps illuminate the ways living beings navigate those tensions, rather than collapsing them under one umbrella or under a single way of viewing the world. Reciprocity highlights actions necessary when acknowledging that different living beings have unique strengths/needs and must rely on others. That is, acknowledging and reciprocating the patterns, structures, and logics of another can bring us into closer relation. Together, these three principles—the concept of mathematx as a “living practice” and “way of being,” along with what I have referred to as “a spiritual turn in mathematics” (Gutiérrez, 2022)—offer the field a language and structure of restor(y)ing mathematics that can support us to create new mathematical futures that reflect Indigenous futurity.

THE CASE OF KANAKA MAOLI TARO FARMERS

Kānaka Maoli farmers in Hawai’i are leaders in conceptualizing and maintaining water rights, especially around growing *kalo* (taro). But, since the 1800s, colonizers arrived and began applying extractive agricultural practices that reshaped the territories. Portions of the island that originally received copious amounts of water from streams higher up in the mountain now had their sources diverted to support colonizers and the sugar cane farms they had established in other areas. By 1920, tunnels and ditches were diverting upwards of 800 million gallons of water from mountain streams to settler sugar cane farms, thereby remaking the lands and territories and the ways Kānaka Maoli could engage with and relate to lands and waters. Over the years, many Kānaka Maoli lost access to lands and sufficient water to grow wetland *kalo* and some were left with little other option than to abandon their traditional foodways for products they could purchase in stores. Today, ninety percent of the foods consumed by Hawai’ians are imported. The socioecological problem Kānaka Maoli face is not having enough locally produced foods to support an Indigenous diet and losing touch with the cultural practice of making of poi, among other things, in everyday households. Even if they were successful at repurchasing lands that were taken from them after the illegal overthrow of the Kingdom of Hawai’i, those who want to grow *kalo* must also fight for rights to waters.

Kalo, considered the first Kānaka Maoli, the original ancestor, is important for many reasons. The root is pounded into poi as a starch staple; the large heart-shaped green leaves are used to wrap and steam fish and meat as well as a cooked vegetable on its own (lū’au stew or hāhā stew). Out of deep respect for plant as ancestor, when someone is preparing poi, they must refrain from uttering bad words or having bad thoughts. Cultivation of *kalo* is not just for consumption but for spiritual well-being and sovereignty. More recently, and in response to developers’ encroachment of lands and waters, Kānaka Maoli farmers have organized to support each other to restore Indigenous foodways (Fujikane, 2021; Goodyear-Kaopua et al., 2014). Prominent farmers from Oahu, one of the main

islands of Hawai'i, are integral to this movement of reading the lands and laws to realize their goals. Their work offers powerful lessons about the patterns, structures, and logics needed to read the world and interact with others to restore and restory specific Native wisdoms.

Networks of collaboration among *kalo* farmers have supported them to build upon their knowledge of patterns, structures, and logics in several ways. First, by learning the colonial logics of the state of Hawai'i legal system and becoming expert witnesses in trials about water rights, farmers have been able to divert large amounts of water back to their lands in order to cultivate *kalo*. Doing so required they temporarily set aside Native ways people are connected to lands and waters. Brayboy and Chin (2020) capture this relationship to lands well when they state, "Relationships between lands and peoples represent how communities coalesce and connect to different lands, forming connections that are reverent and sacred, and rarely recognized by colonial laws like property or individual ownership" (p. 23).

Having reverent and sacred connections to lands should be the logics employable by Kānaka Maoli farmers to be able to argue for and ensure they have sufficient water to cultivate an abundance of *kalo*, as had occurred before the Hawai'ian kingdom was overthrown. However, this is no longer the case. Instead, they must learn and apply colonial logics. For example, local farmers have learned to appeal to the fact that although they, as Kānaka Maoli, do not have rights to lands/waters through the courts, *kalo* does have rights. Therefore, rather than employing a logic that, as farmers, they need water to grow *kalo* to continue their Native foodways and contribute to a system that is less dependent on outside sources of produce (a socioecological problem artificially created by settlers who developed sugar cane farming and developers who seek to build on areas of the island that do not have sufficient waters for industry and housing), they put forth the argument that *kalo* has rights to water. So, by virtue of planting *kalo* on their lands, the state has a responsibility to supply them with water for *kalo* to thrive. Teaching this colonial (state) logic to other farmers, they were able to support a growing number of Kānaka Maoli choosing to clear a part of their land to plant *kalo*. Moreover, as activists, they created collectives of community members (usually about 100 people, each bringing a shovel) who would clear the land of anyone who wanted to grow *kalo*, thereby restoring an older practice that had served people well. Then, they would teach the farmers the patterns, structures, and logics that would be needed to grow *kalo* in a way that created dense, high quality *kalo* to sell to local people and that would also be protected by state laws. The stories they told both restored the kinds of mathematics that were practiced and restored the lands/waters through new forms of mathematics.

Secondly, the transdisciplinary knowledge that was constructed through such collaborations included sharing how to properly measure *kalo*'s water requirements. The state of Hawai'i legal courts used a simple subtraction model (i.e., input – output) to measure the amount of water that taro needs to grow. That is, they placed measuring instruments on the entry point of stream water for farmers' *lo'i kalo* (taro patches) to track the amount of water flowing into the fields and a measuring instrument at the point of water flowing out of the fields. This form of logic assumed that *kalo* simply consumed the amount of water needed to grow and left in the fields the water not needed, which flowed out of the *lo'i*. However, experienced Kānaka Maoli farmers who understood both the length of time that taro needs to grow (9-12 months) as well as the way *kalo* grows (paying close attention to patterns), could articulate to other farmers, activists, community members, and the courts that *kalo* uses a process that takes in water and gives water back. It begins growing by first taking in water and creating the leaves.

As it grows, it takes in additional water to create the corm. Eventually, the leaves turn yellow and die back, allowing taro to release back into the *lo'i* the water it no longer needs to support the leaves. However, if taro is not provided enough water to support both the corm and the leaves, it will not flourish. Moreover, *kalo* needs rapidly flowing water to keep the roots cool, prevent fungus, and create a dense root. Finally, many *kalo* farmers need to consider their responsibilities to 'alae 'ula (mud hens) that nest in *lo'i kalo* (taro patches) and are considered deities. Disturbing their nests would go against proper ways of being in relations to sacred relatives, thus requiring additional time for some *kalo* to remain in the *lo'i*, unharvested until the mud hens moved on. So, a simple input-output function is not the correct form of mathematics to measure *kalo's* water consumption. Furthermore, every crop of *kalo* has its own needs depending upon the amount of sun and incline of the fields. In sharing all of this knowledge, Kānaka Maoli farmers are able to sustain Native foodways and produce high quality (i.e., dense) taro that can stay in a field for up to 2 years before being harvested, making local people less dependent upon any given season/year/climate. Kānaka Maoli farmers were able to build upon that knowledge to articulate the need for an abundance of flowing water (a kind of restor(y)ing of mathematical practice), not simply a minimum gallon requirement stipulated by the courts. Moreover, from the perspective of living mathematx, we see how *kalo* performs an important pattern and “teaching” for Kānaka Maoli: always give back to the system that which has sustained you temporarily but which you now no longer need.

A third way in which patterns, structures, and logics arise in mathematics is when developers outlined “plots of land” to purchase, they developed boundaries and argued that purchasing such land did not disrupt Kanaka Maoli access to sacred lands which were determined (by earlier courts) to be located elsewhere. However, activists and Elders were able to highlight that such divisions went against Native traditions. In a cultural impact assessment (Souza & Hammattt, cited in Fujikane, 2021) they articulate,

Community members were adamant that one piece of the cultural landscape cannot be cut off from another. Each part of the landscape exists in relationship to each other, just as the 'ohana of Maui as landforms are in relationship to each other: the mother is in the cave above, the son born below near the ocean, the sisters as surrounding valleys. (p. 43).

The concept of boundaries as a structure is important to consider here. For Kānaka Maoli, boundaries are considered points of connection that allow for both specificity (e.g., locality, part of a whole) and integrity (e.g., wholeness). Yet, for settlers, such boundaries are lines of disconnection/separation that open the doors for “exceptions” to state laws that were developed to originally protect Kānaka Maoli. In this sense, a colonial mathematical construct like “boundaries” and “plot lines” used in the making of maps by developers has been important for Kānaka Maoli farmers to learn. In doing so, they have been able to flip the script (to restore mathematical practices while restoring them). In navigating state laws, they can use this very colonial structure and demonstrate through historical maps (which documented original pathways of streams before the 1900s) to show that they are not, in fact, diverting water away from other lands (e.g., developers); *kalo* farmers are simply restoring the original pathways of streams that had once flowed from the mountain above. In terms of living mathematx, we also see how water connects past, present, and future. That is, because it continuously flows above and below land, it performs an important structure and “teaching” for Kānaka Maoli: everything is connected and in constant motion.

This case study has highlighted some of the ways that Kānaka Maoli farmers who cultivate taro have been able to read the patterns, structures, and logics of lands/waters to navigate the politics of the state, address a socioecological problem of overreliance on imported foods and a dispossession of Native foodways and sovereignty, and share that knowledge for community well-being and more liberatory futures. These ways highlight both a restored and a restoried mathematics local to the island of Oahu. This shared knowledge of living mathematx and the enactment of restor(y)ing mathematics, where relationality is central, contributed to the defense of their stories/histories and territories. Their respect for and cultivation of *kalo*, as well as forms of activism, demonstrate that they are not simply restoring “traditional” mathematical practices. Rather, being able to hold intact several logics, patterns, and structures—and for multiple purposes—highlights their political clarity and deep wisdom, the ability to restory for Native futures. In addition, these somewhat competing patterns, structures, and logics as part of a larger system also challenges the underlying axiomatic characteristic of Western mathematics that suggests only one set of “givens” can be drawn upon. Learning from this case can support other communities to consider the ways that local patterns, structures, and logics play out at different scales in their spaces to defend stories and territories.

References

- Archibald, J.A. (2008). *Indigenous storywork: Educating the heart, mind, body, and spirit*. UBC Press.
- Ascher, M. (2002). *Mathematics elsewhere: An exploration of ideas across cultures*. Princeton Uni’ Press.
- Barwell, R. (2013). The mathematical formatting of climate change: Critical Mathematics Education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16.
- Berkes, F., & Berkes, M. K. (2009). Ecological complexity, fuzzy logic, and holism in Indigenous knowledge. *Futures*, 41(1), 6-12.
- BooB-BavnbeK, B. and Høyrup, J. (2003) *Mathematics and war*. Birkhauser Verlag.
- Brayboy, B. M. J., and Chin, J. (2020). On the development of terrortory. *Contexts*, 19(3), 22-27.
- Coles, A., Barwell, R., Cotton, T., Winter, J., and Brown, L. (2013) *Teaching secondary mathematics as if the planet matters*. Routledge.
- Closs, M. P. (2001). Mesoamerican mathematics. In Selin, H. (Ed.), *Mathematics across cultures: The history of non-Western mathematics*, pp. 205-238. Kluwer Academic.
- Deloria, V. (1979). *The metaphysics of modern existence*. Harper and Row.
- Eglash, R.; Bennett, A., O’Donnell, C., Jennings, S., Cintonino, M. (2006). Culturally situated design tools: Ethnocomputing from field site to classroom. *Anthropology and Education*, 108(2), 347-362.
- Fujikane, C. (2021). *Mapping abundance for a planetary future: Kanaka Maoli and critical settler cartographies in Hawai’i*. Duke University Press.
- Furuto, L. (2019). We never voyage alone. In Aikau, K. A. and González, V. V. (eds), *Detours: A decolonial guide to Hawai’i*, pp. 362-272. Duke University Press.
- Goodyear-Kaopua, N., Hussey, I., and Kahunawaika’ala Wright, E. (Eds.), (2014). *A nation rising: Hawaiian movements for life, land, and sovereignty*. Duke University Press.
- Gutiérrez, R. (2017a). Living mathematx: Towards a vision for the future. *Philosophy of Mathematics Education Journal*, 32. <http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome32/index.html>

- Gutiérrez, R. (2019). *Mathematx: Towards a way of being*. In J. Subramanian (Ed.), *Proceedings of the tenth international Mathematics Education and Society conference* (pp. 67–110). MES10. <https://www.mescommunity.info/proceedings/MES10.pdf>
- Gutiérrez, R. (2022). A Spiritual Turn: Towards Desire-Based Research and Indigenous Futurity in Mathematics Education. *Journal for Research in Mathematics Education*, 53(5), 379-388.
- Gutiérrez, R. (2023, June). Restor(y)ing Mathematics, Restor(y)ing Ourselves: A Spiritual Turn in Mathematics Education. In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, K. & A. Hughes (Eds), *Weaving mathematics education from all perspectives: 45th Annual Meeting of the Mathematics Education Research Group of Australasia*, pp. 15-16. New Castle, New South Wales. Australia. <https://merga.net.au/Public/Annual-Conference/Programme.aspx>
- Gutiérrez, R. (2021, April). *Restor(y)ing mathematics teacher education*. A Division K Invited Session at the Annual Meeting of the American Educational Research Association. San Diego, CA.
- Gutstein, E. (2006). *Reading and writing the world with mathematics*. Routledge.
- Harding, S. (2008) *Sciences from below: Feminisms, postcoloniality, and modernities*. Duke University Press.
- Harjo, L. (2019). *Spiral to the stars: Mvskoke tools of futurity*. University of Arizona Press.
- Jacob, M. M. (2014). *Yakama rising: Indigenous cultural revitalization, activism, and healing*. University of Arizona Press.
- King, T. L., Navarro, J., & Smith, A. (Eds.). (2020). *Otherwise worlds: Against settler colonialism and anti-Blackness*. Duke University Press.
- López, L. L. & Coello, G. (2021). *Indigenous futures and learnings taking place*. Routledge.
- Little Bear, L. (2009). *Naturalizing indigenous knowledge. Synthesis Paper*. Prepared for the Canadian Council on Learning's Aboriginal Learning Knowledge Center.
- Lorde, A. (1984). *Sister outsider: Essays and speeches*. Crossing Press.
- Karaali, G. & Khadjavi, L. (2021). *Mathematics for social justice: Resources for the college classroom*. Mathematics Association of America Press.
- Million, D. (2009). Felt theory: An Indigenous feminist approach to affect and history. *Wicazo Sa Review*, 24(2), 53–76.
- O'Neil, C. (2016) *Weapons of math destruction: How big data increases inequalities and threatens democracy*. Crown Publishing.
- Pipes, C. (2020). *Growing resistance: Taro as the symbol of postcolonial Hawai'ian identity*. A thesis presented to the faculty of the Department of Sociology and Anthropology, University of Canterbury.
- Powell, A. and Frankenstein, M. Eds., (1997). *Ethnomathematics: Challenging eurocentrism in mathematics education*, (pp. 331-371), SUNY.
- Rosa, M., Orey, D. C., and de Sousa Mesquita, A. P. S. (2023). An ethnomodelling perspective for the development of citizenship education. *ZDM*, 55(3), 953-965.
- Simpson, L. B. (2017). *As we have always done: Indigenous freedom through radical resistance*. University of Minnesota Press.
- Su, F. (2021). *Mathematics for human flourishing*. Yale University Press.
- Tachine, A. R., & Nicolazzo, Z. (Eds.). (2022). *Weaving an otherwise: In-relations methodological practice*. Stylus Publishing.
- Turnbull, D. (2000). *Masons, Tricksters, and Cartographers: Comparative Studies in the Sociology of Scientific and Indigenous Knowledge*. Taylor and Francis

CONNECTING TO INDIGENOUS WAYS OF KNOWING AND BEING IN MATHEMATICS BY LISTENING TO THE VOICES OF STUDENTS

Jodie Hunter¹, Julia Hill²

¹Massey University, ²University of Melbourne

A goal across many nations is improving equity in mathematics education for all students. This goal intersects with a focus on wellbeing, social justice concerns, and ecological crises impacting Indigenous and minority communities. An important aspect of developing equity is first listening to the voices of Indigenous community members and then considering the connections that can be made within the mathematics classroom. This research paper draws on data collected from two research studies drawing on the voices of young Pacific people to examine the opportunities for mathematics education to connect to social justice and socio-ecological issues by building on both family and community experiences and values related to mathematical wellbeing (MWB). The findings highlight the possibilities of connecting to knowledge systems related to growing, collecting, and cultivating food in mathematics classrooms through modelling tasks and mathematics for social justice-oriented tasks. Also evident is the importance of examining the intersection between cultural values and MWB and considering how values can be leveraged to develop student agency.

INTRODUCTION

Internationally, there has been a growing awareness of the need to work towards equity in mathematics education for all students. For example, the United Nations goals of sustainable development include the development of quality education systems that encompass equitable lifelong learning opportunities and inclusivity for all, along with another key goal of promoting wellbeing (United Nations, n.d.). Intertwined with the need for more equitable education and a wellbeing focus are other concerns related to social justice and ecological crises, which significantly (and unevenly) impact Indigenous, marginalized, and minoritized communities. In Aotearoa New Zealand and Moana-nui-a-Kiwa (Pacific Ocean region), there is a need to address goals of equity and wellbeing for Pacific peoples who are also being disproportionately impacted by climate change (Kerr et al., 2018). In relation to mathematics education, this requires educators of students of Pacific heritage to enable these students to access opportunities in mathematics across all levels of education and to perceive mathematics as a useful, relevant subject rather than being disconnected from their everyday experiences, cultural heritage, and values (Hunter, 2022; Kidman et al., 2013; McAllister et al., 2022). An important aspect of this is first listening to Pacific students' voices and then considering how educators can connect to Indigenous ways of knowing and being in the mathematics classroom. We also consider that by building on student voice, there are opportunities for educators to reposition mathematics education as a powerful tool to both develop a more socially just world and to position learners to become activists for social justice and the environment for their own communities. In this paper, we use exemplary case studies from two research projects, both focused on the voices of young Pacific people, to illustrate the opportunities for mathematics education in relation to a focus on the socio-ecological by attending to the family and community experiences related to mathematics of marginalized groups and values related to mathematical wellbeing. Specifically, we ask: what are the

experiences and values of young Indigenous Pacific people that mathematics educators could connect to when centering socio-ecological aspects while teaching mathematics?

Moana-nui-a-Kiwa and Pacific peoples

Moana-nui-a-Kiwa refers to the Pacific Ocean region, which links the Pacific with Aotearoa New Zealand. Historically, Pacific people were navigators who voyaged the Pacific moana (ocean/sea), settling in different Pacific nations. In Aotearoa New Zealand, Pacific people have ancestral links with Māori and are a group composed of both multiple generations and diverse groups of Indigenous people from nations including Tokelau, Cook Islands, Tonga, Fiji, Niue, Samoa, Tuvalu, Tahiti, and beyond. These groups each have their own cultural ways of being and knowing along with different languages and experiences (Coxon et al., 2002). However, there is commonality across Pacific people and nations in relation to cultural values, which are shaped by a collectivist way of life (Podsiadlowski & Fox, 2011; Uehara et al., 2018). Building from collectivist values, *kopu tangata*, an extended view of the family as a wide grouping rather than a small nuclear group, is often central to everything in the lives of Pacific people (Kerr et al., 2018). Related to being part of *kopu tangata* is a set of values interpreted through a collectivist lens, including belonging, love, service, spirituality, reciprocal relationships, respect, inclusion, and leadership (Kerr et al., 2018; MoE, 2013, 2018; Rimoni et al., 2022).

Indigenous ways of knowing and being related to mathematics

Indigenous and local forms of knowledge have been fundamental to the flourishing of communities of people and the wider society. This knowledge includes cultural activities, values, beliefs, and ways of being and relates to the holistic, multi-disciplinary systems underpinning practices, including education, sustainability, food production, and environmental conservation (Bianchi, 2018; Gutiérrez, 2017). Despite the richness of these sources of information, in many education systems, including both Pacific nations and Aotearoa New Zealand, this local and Indigenous knowledge has often been ignored, neglected, and devalued. For example, Kidman and colleagues (2021) examined Indigenous student experiences in science education in New Zealand and Taiwan. The study found that situating Indigenous knowledge on the periphery of the science curriculum contributed to both disengagement of students and inequitable outcomes. A key disconnect was between Indigenous knowledge of the natural world and what was positioned as high-status scientific knowledge. Similarly, Kerr and colleagues (2018) argue that a lack of connection between Western science and Indigenous ways of knowing, along with teaching that is disconnected from Pacific values, are key contributors to the broken educational pathway in science, technology, engineering, and mathematics (STEM) for Pacific people.

The disconnect in educational systems between Pacific Indigenous ways of knowing and being and mathematics means that educators are often unaware of both the rich knowledge of Pacific people related to the environment and the potential of building on values of collectivism, service, and reciprocity. Another result of this is the missed opportunity to both position members of Indigenous communities as knowledgeable and as working in scientific and mathematical ways but also to provide potential solutions centered on ways of living sustainably and addressing the current ecological crises. It appears that few studies have explored the potential of connecting Pacific Indigenous and local knowledge and ways of being and socio-ecological contexts with mathematics.

This research paper aims to make an initial contribution by addressing the gap in the field with a focus on sharing examples from young Pacific people from research studies and interrogating how these connect to socio-ecological and social justice aspects and mathematics.

Connecting Pacific values, wellbeing, and the socio-ecological in Mathematics

According to the Value Fulfillment Theory of Wellbeing (Tiberius, 2018), an individual's wellbeing is enhanced when they pursue, realize, or fulfill their core values. Values act as a blueprint that guide actions, thoughts, and behaviors to promote both individual and collective flourishing (Hill et al., 2021; 2024). In the mathematics classroom, educators have an opportunity to support student wellbeing by helping learners connect mathematics learning to their personal and cultural values. By listening to student voices and designing learning experiences that facilitate Pacific learners to view mathematics in meaningful ways – such as serving their communities or demonstrating respect for the environment – educators can create a values-aligned mathematics education promoting wellbeing for the individual student, their school community, wider society, and the environment. Centering Indigenous Pacific values and voices in this way opens up possibilities for re-envisioning mathematics education as a vehicle for positive social and ecological impact.

Wellbeing extends beyond the individual to include others and the environment. Cultivating Pacific students' interpersonal and collectivist values can lead to behaviors that consider factors external to the students themselves, aligning with the aims of environmental education. Environmental education seeks to develop learners' knowledge, attitudes, values, and behaviors to address ecological challenges and promote sustainable living (Ardoin et al., 2020). As identified by Radovich et al. (2018) "Indigenous knowledges and eco-justice have much in common, as they both have at their center the notions of relationality, dependence, and inter-sectionality. They also stress the relationship between the natural world, values, meaning, and the politics of existence" (p. 111). While often explored through science and STEM disciplines, mathematics also has an important role to play in environmental education (Barwell, 2018). However, unlike science, which is often valued as more humanistic (Bishop et al., 2006), the relationship between mathematics and environmental education can often be less explicit (Radakovic et al., 2018).

For Pacific communities facing disproportionate impacts of climate change, the wellbeing of people is inherently linked to the wellbeing of their land and sea environments (Andrew et al., 2022; Savage et al., 2019). Pacific knowledge offers deep insight into living in a sustainable balance with nature. By making space for Pacific students to draw upon their out of school experiences along with Indigenous knowledge and values, mathematics educators could support learners in developing creative, culturally grounded mathematical approaches to understanding and addressing socio-ecological challenges facing their communities. Engaging with Pacific student's collectivist values opens up ways to re-imagine mathematics education as more holistic, relational, and in greater harmony with the environment we are embedded within and dependent upon. However, this also necessitates understanding the grounding of student values from their perspective.

METHODOLOGY

The research reported on in this paper is from two research projects undertaken in Aotearoa New Zealand and Niue (a small remote Pacific nation). Both projects focused on different aspects of documenting Pacific ways of knowing and being. The first project (see Hunter 2022) documented the

mathematical funds of knowledge of Pacific learners. Data was collected from 36 children aged between five years old and 15 years old from 34 families of Pacific descent who attended a primary and secondary school in Niue and a primary school in New Zealand. Visual data was collected in the form of photographs taken by student and family participants to capture activities and artefacts that they viewed as connecting to mathematics. Photo-elicitation interviews using Talanoa (Cammock et al., 2021; Vaioleti, 2006), a form of storytelling conversation, was used with students to explore the context of the photographs. For this paper, the data collected from photo-elicitation interviews was re-analyzed with an initial focus on identifying examples from the interview data that were connected to socio-ecological and social justice contexts. A second layer of data analysis and coding identified the themes related to the context of the photographs and interview data. These were broadly identified as growing, collecting, and cultivating food; undertaking service in the community; using natural resources for craftwork; and Indigenous and local knowledge.

The second project (see Hill & Hunter, 2024) was a survey study examining students' mathematical wellbeing (MWB) across New Zealand. For this paper, we focus on the responses from 2109 Pacific students (1125 females, 960 males, 24 unspecified) who completed the survey in 2023. This included students across Year 3 ($n = 149$), Year 4 (296), Year 5 (381), Year 6 (416), Year 7 (339), Year 8 (381), Year 9 (84) and Year 10 (63). Students completed the survey online, responding freely to the questions: *What makes you feel really good and/or do really good in your maths class? Why does this make you feel really good and do really good in maths?* The survey responses were analyzed using a deductive thematic approach (Braun & Clarke, 2006) and guided by the MWB framework developed by Hill et al., (2021). MWB encompasses seven core values (accomplishment, cognitions, engagement, meaning, relationship, positive emotions and perseverance). We focus only on the relationship values here, which align with collectivist cultural values.

RESULTS AND DISCUSSION

We begin the findings by providing an overview of the first project, which focused on the experiences of young Pacific people both in Aotearoa, New Zealand, and Niue, which mathematics educators could connect to when centring socio-ecological issues while teaching mathematics. We present this and discuss both how the context could be built upon in mathematics classrooms while also framing the connections to values embedded within the experiences. In this paper, due to space constraints, we provide a snapshot of the data with a focus on growing, collecting, and cultivating food.

Across both countries, students shared examples of their experiences in growing and cultivating food for their wider family and community to eat. The participants from Aotearoa, New Zealand, commonly referred to growing and collecting fruit such as oranges, passionfruit, lemons, and vegetables such as taro and beans and using the produce in meals. For example, one of the young participants who was six years old said: "I am picking up leaves and we're gonna [sic] eat those leaves, the big ones... We eat it with coconut cream, Dad helps me to cook it and we call it la'au, it's in Samoan, la'au". Similarly, in Niue participants referred to growing and harvesting food including coconuts, taro, and bananas, however, in this context, there was also careful attention given to the space needed and how different parts of the crops would be reused as mulch or for other purposes such as feeding the pigs:

I had to find a plantation [space] that consists of all banana leaves so it was a square plantation [space] and there were banana leaves so you need to know how much basically it needs to grow. If they're all crowded they won't grow easily or they won't get bananas on them...around the outside they have a square of taro.

Additionally, participants from Niue referred to their care for domestic livestock (chickens and pigs) as well as hunting uga (coconut crabs), collecting food such as seafood called alili (cat's eye) and fishing both with nets or from vaka (traditional canoes) and then sharing the catch around the village community:

We go out and fish for tuna and wahoo. Sometimes a shark comes in and eat our bait, I have caught two before but I threw them away, I cut the line... If you go in the morning, then you fish for small fish and then use the small one for bait and catch the big ones, the tuna...Sometimes we go at night-time, the different fish comes at night when you go fishing in the canoe. We catch the fish and we keep some and give them away to people around us.

The importance of *kopu tangata* (family) was embedded throughout the examples, with the participants referencing the care for the family through cultivating and collecting food. This extended further to the wider community in some of the examples, with reciprocity being central along with care for the environment in regard to the sustainability of cultivation practices.

Local knowledge and experiences of growing, collecting, and cultivating food represented in the examples from the participants could be connected in mathematics classrooms to both social justice and socio-ecological contexts. In the Pacific Island nations, traditionally, local produce has been the key components of Indigenous food systems, including *kai moana* (seafood) and Indigenous plant materials (taro, yam, cassava, coconut, and breadfruit (Naika et al., 2024)). In more recent times, the importation of processed food has both impacted food systems in the Pacific and led to increases in non-communicable diseases (Naika et al., 2024; Savage et al., 2020). Additionally, for both Pacific people in Aotearoa New Zealand, and a growing number of Pacific nations, food insecurity is an issue caused by poverty and climate variability and change (Savage et al., 2020). We argue that mathematics educators could build on young Indigenous Pacific students' experiences in growing, collecting, and cultivating food through the development of mathematical modelling tasks connected to the complex systems (English, 2007) related to food systems in the Pacific. Tasks could also connect to mathematics and social justice (Gutstein & Peterson, 2005) by examining the role of climate change on disruption to Indigenous food systems and the impact of the importation of highly processed food both to people and the environment.

The second project drew on the voices of young Pacific people In Aotearoa, New Zealand to examine both their MWB and the connection to values that influenced this. We argue that surfacing student voice to highlight the impact of cultural values on MWB offers educators an opportunity to consider how to teach in meaningful ways where students can see how both their ways of being, and mathematics can be used to develop social and ecological change. From the study, we present a subset of student responses that indicated relationships as important for their MWB. We argue that this is significant to social justice and socio-ecological contexts as relationships are embedded within the wider cultural value of collectivism. Working collectively is one means to address issues related to social justice and the environment. However, to fully optimize understanding of the value of relationships in mathematics classrooms, it is also necessary to understand the types of relationships that students view as important for their MWB and why these are valued.

Data analysis showed that almost half (41%, $n = 834/2019$) of the Pacific students noted the importance of relationships for their MWB. Three types of relationships were signaled as important for MWB, peer, teacher, and family. We summarize the types and frequency of the relationship values in Table 1.

Value	Description	Count
Peer collaboration	Working together with classmates or friends on math problems or tasks	259
Peer support	Receiving help, explanations, or encouragement from friends or classmates	211
Teacher support	The teacher is supporting, helping, or encouraging with mathematics	182
Group discussions	Participating in group discussions about math concepts or problems	89
Sharing ideas	Feeling good about sharing mathematical ideas with others	72
Helping others	Feeling good about helping classmates understand math or solve problems	58
Inclusive environment	Feeling included, comfortable, or safe with others in the math class	41
Family support	Receiving encouragement or help from family members with math	25

Table 1: Relationships themes and frequencies. The total frequency (937) exceeds the number of students due to multiple coding across themes.

Notably, students mostly valued peer support and collaboration to foster their MWB. For example, one student noted “When I have a good group of people around me that support me when I need help or don’t understand the question that I have been assigned to” (Year 8 student). Students also pointed to the importance of reciprocity, helping, and caring for others, including: “I try helping people in maths. They’ll help me back like buddies.” (Year 6 student); and “What makes me feel really good in maths is helping others and working together as a group and making sure we finish our maths work.” (Year 5 student); and “When I help my friends with math. Because it makes me feel good and reinforces my own understanding.” (Year 9 student).

Pacific students’ collective valuing, that is, “us” and “we” as opposed to “me”, could be leveraged to extend valuing the broader community and environment. The emphasis on collaboration, helping others, sharing ideas, and creating inclusive environments aligns well with the collective action and community-mindedness required for addressing ecological challenges. Ronen and Kerret (2020) proposed an integrative “sustainable wellbeing” approach. They argued that “sustainable wellbeing is achieved when improving individual wellbeing is correlated with improving the wellbeing of other members of society and the natural environment” (p.3). This concept aligns closely with the collectivist values observed in Pacific students’ responses, suggesting that these values could also be extended to encompass care for the environment. By fostering these collectivist values in mathematics education, educators can nurture students’ “socio-ecological mathematical wellbeing”, where individual, community, and environmental flourishing are interconnected and mutually reinforcing in the context of mathematics education.

CONCLUSION

This study explored the experiences and values of young Indigenous Pacific people that mathematics educators could connect to when centering socio-ecological aspects while teaching mathematics. We highlight that local knowledge and experiences of students could be used as platforms by mathematics educators to connect to social justice and socio-ecological contexts. Our findings also indicate the influence of cultural values and ways of being on MWB. We argue that by leveraging both student experiences and values, mathematics educators can build a sense of community agency and social interaction skills, which are crucial for addressing broader societal and ecological challenges. By viewing the mathematics classroom as a microcosm of the community, educators can leverage ways of being such as collectivist values to nurture a consciousness that extends care beyond immediate individual needs to encompass societal and environmental wellbeing. This work underscores the importance of centering Indigenous voices and knowledge in mathematics education. By doing so, we can create more equitable, culturally responsive, and environmentally conscious learning experiences that prepare students to address the complex socio-ecological challenges of our time.

Acknowledgements

This work was supported by a Royal Society of New Zealand Rutherford Discovery Fellowship research grant: RDF-MAU1903; Marsden Fast-start research grant 19-MAU-060/MAU1910 and a Teaching and Learning Research Initiative fund administered by the New Zealand Council of Educational Research.

References

- Andrew, N. L., Allison, E. H., Brewer, T., Connell, J., Eriksson, H., Eurich, J. G., ... & Tutuo, J. (2022). Continuity and change in the contemporary Pacific food system. *Global Food Security*, 32, 100608.
- Ardoin, N. M., Bowers, A. W., & Gaillard, E. (2020). Environmental education outcomes for conservation: A systematic review. *Biological conservation*, 241, 108224.
- Barwell, R. (2018). Some thoughts on a mathematics education for environmental sustainability. In P. Ernest (Ed.), *The Philosophy of Mathematics Education Today* (pp. 145-160). Springer.
- Bianche, C. (2018). Reflections on the implications of Western theories on Indigenous populations: Decolonizing and indigenizing the classroom. In G. J. Sefa Dei, & C. S. Jaimunga (Eds). *Indigeneity and decolonial resistance : alternatives to colonial thinking and practice* (pp. 51 – 76). Myers Education Press.
- Bishop, A. J., Clarke, B., Corrigan, D., & Gunstone, D. (2006). Values in mathematics and science education: Researchers' and teachers' views on the similarities and differences. *For the Learning of Mathematics*, 26(1), 7–11.
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101.
- Cammock, R., Conn, C., & Nayar, S. (2021). Strengthening Pacific voices through Talanoa participatory action research. *AlterNative: An International Journal of Indigenous Peoples*, 17(1), 356-368.
- Coxon, E., Anae, M., Mara, D., Wendt-Samu, T., & Finau, C. (2002). *Literature review on Pacific education issues*. Ministry of Education.
- English, L. D. (2007). Complex systems in the elementary and middle school mathematics curriculum: A focus on modeling. Festschrift in Honor of Gunter Torner. *The Montana Mathematics Enthusiast*, 139-156.
- Gutiérrez, R. (2017). Living mathematx: towards a vision for the future. *Philosophy of Mathematics Education Journal*, 32. <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/index.html>

- Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers*. Rethinking Schools.
- Hill, J. L., & Hunter, J. (2024). Diverse Students' Mathematical Wellbeing. *New Zealand Journal of Educational Studies*, 59, 211-234.
- Hill, J. L., Kern, M. L., Seah, W. T., & van Driel, J. (2021). Feeling good and functioning well in mathematics education: Exploring students' conceptions of mathematical wellbeing and values. *ECNU Review of Education*, 4(2), 349–375.
- Hunter, J. (2022). Challenging and disrupting deficit discourses in mathematics education: positioning young diverse learners to document and share their mathematical funds of knowledge, *Research in Mathematics Education*, 24(2), 187-201.
- Kerr, J. Q., Hess, D. J., Smith, C. M., & Hadfield, M. G. (2018). Recognizing and reducing barriers to science and math education and STEM careers for Native Hawaiians and Pacific Islanders. *CBE—Life Sciences Education*, 17(4), mr1.
- Kidman, J., Yen, CF. & Abrams, E. (2013). Indigenous students' experiences of the hidden curriculum in science education: A cross-national study in New Zealand and Taiwan. *International Journal of Science and Mathematics Education*, 11, 43–64.
- Larson, E. C., Luloff, A. E., Bridger, J. C., & Brennan, M. A. (2015). Community as a mechanism for transcending wellbeing at the individual, social, and ecological levels. *Community Development*, 46(4), 407–419.
- McAllister, T., Naepi, S., Walker, L., Gillon, A., Clark, P., Lambert, E., ... & Alipia, T. (2022). Seen but unheard: navigating turbulent waters as Māori and Pacific postgraduate students in STEM. *Journal of the Royal Society of New Zealand*, 52(sup1), 116-134.
- Ministry of Education. (2013). *Pasifika education plan 2013–2017*. Ministry of Education.
- Ministry of Education. (2018). *Tapasa: Cultural competencies framework for teachers of Pacific learners*. Ministry of Education.
- Naika, A., Pillay, A., & Paliwal, A. (2024). Indigenous food system for sustainability: South Pacific study. In *Sustainable food systems (Volume I): Framework, sustainable diets, traditional food culture and food production* (pp. 35-53). Springer.
- Podsiadlowski & Fox, S. (2011). Collectivist value orientations among four ethnic groups: Collectivism in the New Zealand context. *New Zealand Journal of Psychology*, 40(1), 5 – 18.
- Radakovic, N., Weiland, T., & Bazzul, J. (2010). Transdisciplinary, critical mathematics education, eco-justice, and the politic to come. In L. Jao, & N. Radakovic (Eds.), *Transdisciplinary in mathematics Education* (pp. 109–124). DOI 10.1007/978-3-319-63624-5_6
- Rimoni, F., Glasgow, A., & Averill, R. (2022). *Pacific educators speak: Valuing our values*. NZCER Press.
- Ronen, T., & Kerret, D. (2020). Promoting Sustainable Wellbeing: Integrating Positive Psychology and Environmental Sustainability in Education. *International Journal of Environmental Research and Public Health*, 17(19), 6968.
- Savage, A., Bambrick, H., & Gallegos, D. (2020). From garden to store: local perspectives of changing food and nutrition security in a Pacific Island country. *Food Security*, 12(6), 1331-1348.
- Tiberius, V. (2018). *Wellbeing as value fulfillment: How we can help each other to live well*. OUP.
- Uehara, D. L., Chugen, J., Staley Raatior, V. (2018). Perceptions of Pacific Islander students in higher education. *Journal of Diversity in Higher Education*, 11(2), 182-191.
- United Nations. (n.d.). *Sustainable Development Goals*, from www.un.org/sustainabledevelopment/
- Vaiolleti, T. (2006). Talanoa research methodology: A developing position on Pacific research. *Waikato Journal of Education*, 12, 21–34.

CARTOGRAPHIES FOR (RE)STORYING SCALES OF SOCIO-ECOLOGICAL INTERACTIONS IN MATHEMATICS EDUCATION

Kate le Roux, Siddique Motala

University of Cape Town

Social and ecological change is experienced across multiple scales of time, space, and ways of knowing, being, and communicating, that cross traditional boundaries. The work of mathematics education in an uncertain, changing, precarious world demands a turn to focusing on complex territories of socio-ecological interactions. In this theoretical paper we think with two cartographic practices: the mathematical scientific cartographic method, and a critical posthuman cartography. We use situated, in the present instances of activism in our context of South Africa to explore how these practices may be put to work to (re)story scales in mathematics education's legacy in socio-ecological territories, and yet to be imagined role towards socio-ecological justices.

INTRODUCTION

...ecojustice has no allowable one-variable approach to the cascading exterminations, immiserations, and extinctions on today's earth. But blaming [...] some or other "not us" for ongoing destruction webbed with human numbers will not work either. These issues demand difficult unrelenting work; but they also demand joy, play, and response-ability to engage with unexpected others. (Haraway, 2016, p. 208)

...we need stories (and theories) that are just big enough to gather up the complexities and keep the edges open and greedy for surprising new and old connections. (Haraway, 2016, p. 101).

Thinking about the work of mathematics education in an uncertain, changing world of social and ecological precarity, as described in these opening quotes, requires a turn to focusing on complex territories of socio-ecological interactions. These involve, each at multiple scales, and in all their (dis)similarities: individuals and communities (locally, nationally, and globally; from elementary particles to ecosystems); human and non-human entities, both living and inanimate; knowledges and ways of knowing (physical, cognitive, spiritual, emotive); ways of being and of communicating; and times (pasts, presents, and futures) (Coles et al., 2024). As opposed to the noun "territory" as physical space, territories of interactions reference an active "'metamorphic zone' that engenders, actuates, dramatizes, dies" (de Freitas, 2023, para. 4), by nature of the interactions themselves.

Pursuing this turn presents challenges for dominant narratives of/about mathematics education, which value knowledge structures that prompt stable, hierarchical categorisation and vertical abstraction, and have an intrinsic logic of truth, while offering a transferable, universalizing gaze on the world. They centre (some, individual) human knowers, who, having followed a linear learning development, are capacitated as "global" mathematical citizens, with a gaze for solving social, economic and environmental concerns. And the field is absorbed by questions of "scaling up" curricula and pedagogies, and of the "boundaries" of mathematical knowledge, the knower, language, and so on.

The challenges for mathematics (education) of interacting, at varied scales, in these territories, are recognised. Rubel and Nicol (2020) discuss how maps can/have been used in interactions between school mathematics and space, for example, to teach mathematical knowledge, such as measurement, scale, and sampling, and also to critically reason about socio-ecological interactions involving human mobility and relations to land. Yet using maps to teach for spatial justice may, inadvertently,

(re)produce hierarchised scales of knowledges and knowers (Rubel & Nicol, 2020; Rubel et al., 2022). Coles et al. (2024b) discuss an interdisciplinary curriculum project focusing on a polluted river in Mexico. While temporal scales of past, present and future were given meaning in a community Memorial Museum, questions of how the curriculum might travel across spatial scales of local, regional and global, required grappling with “connections and entanglements, as well as disconnections and singularities.” (p. 10). And, even in the immediate “local” context of the project, not all the socio-ecological entanglements were calculable mathematically, but engaged other scales of ways of knowing, such as memory, imagination, and even fear. In mathematics teacher education, Helliwell and Ng (2022) use dialogue and futures thinking to engage their respective locals, that is in pre-service courses in the United Kingdom and Hong Kong, with transnational and global spatial scales. Using Indigenous knowledges, Gutiérrez (2022) proposes a practice of futurity that intertwines timescales of pasts, presents, and futures, and engages ways of knowing such as ethics and values towards remembering and healing. Indeed, it is argued that attention to socio-ecological sustainability in mathematics education requires of the field itself practices of humility, reflexivity, and reciprocity (e.g., Coles et al., 2024b; Gutiérrez, 2017; Swanson & le Roux, in press).

These works give substance to de Freitas’ (2023, para. 4) argument that “spatial and metric imaginaries are both mathematical and material, abstract and situated, speculative and embodied.” She argues for imaginings in mathematics education that “reterritorialize the earth as a multiple earth shared across differences”, her argument reverberating with the prompts of others, for example, for “restor(y)ing” (Gutiérrez, 2022) which involves interactions across the bounded, hierarchical binaries that traditionally characterise mathematics (education) and its relations to the socio-ecological world.

In this theoretical paper we explore how two conceptualisations of cartographic practice – the mathematical, “scientific” cartographic method, and a critical posthuman cartography, and their use with two- and three-dimensional maps – may be put to work to (re)story scales in mathematics education’s legacy and yet to be imagined role in complex territories of socio-ecological interactions. We write from the socio-ecological context of South Africa, where tightly bounded, hierarchically-ordered physical and conceptual categories of physical space, society, peoples, knowledges, and languages (re)produce the logics of racial capitalism, hindering efforts to deeply and systematically pursue equity in society (Green, 2020) and in education [e.g., professional education in cartography science (Motala et al., 2023); and mathematics education (le Roux & Swanson, 2021)]. Our contribution is grounded in our histories and experiences as scholars in this context. Kate le Roux was born in South Africa, into ways of being and knowing with a strong Western tradition, which has engendered for her a strong commitment to equity and socio-ecological justice in her scholarship and practice. Siddique Motala was born into, and spent all of his childhood under, apartheid in South Africa. Being differentially marked as “Indian South African” and subject to absurd apartheid logic has had an impact on his pedagogies and research. We proceed first to describe the two cartographies (Section 2), before thinking with examples of practice in our context to explore their potential for (re)storying scales in territories of socio-ecological interactions (Section 3).

CARTOGRAPHIC PRACTICES FOR (RE)STORYING SCALES

Cartography 1: Cartography as objective mathematical science

During an approximately century-long “cartographic reformation” starting in the mid-1600s, cartography went from being regarded as an “art” to a “science” (Edney, 2011), with map creators acting in imperialist projects (Motala, 2022). Founded in the Cartesian system of the Western Enlightenment (Motala, 2018), mathematics based human action (Yasukawa et al., 2012) was central to the visualisation practices of this science. This system links numeric calculations with “data” (such as measurements of land and time) with visual images, by relating planar Euclidean geometry to a system of multivariable algebraic formulas variables (Anania & Stiglitz, 2023). Thus, “raw material” about the socio-ecological world becomes “information” as it is extracted, processed, and narrated into “readable” models, such as maps. It is this essence, and a striving for accuracy thereof, that continues to be the normative view of contemporary cartography, and its tools and practices such as photogrammetry, geographic information systems (GIS), geodesy, remote sensing, and land surveying. This view assumes a stable, singular, discrete ground “out there” that can be observed by the professional, measured mathematically, and represented unambiguously, in a map, for a user (Edney, 2011; Motala & Bozalek, 2022). Not only is this tradition the focus of professional education in a range of engineering and built environment disciplines (Motala et al., 2023), but it is maps produced in this tradition that are predominantly used in school mathematics (Rubel & Nicol, 2020).

Cartography 2: Cartography as critical posthuman practice

The fact that Cartography 1 is based on a mathematical, scientific method makes map creators and users believe that maps are neutral. Yet thinking in the traditions of critical cartography and critical mathematics education argues that the method is onto-epistemic, ideological, historical and political, with its discursive realisations in visual images having material effects. Crucially, as noted by Motala (2018), these are humanist and anthropocentric ideologies of Western philosophy, prompting boundedness and hierarchies of scales, between: humans/“others”; individuals/community; culture/nature; hard/soft sciences; and social/material. Data visualisations such as maps become objects with epistemic “gatekeeping” properties, and are put to work “objectively” to model pasts and presents, *and* to think about futures (Anania & Stiglitz, 2023). Indeed, Motala (2018) notes that, over 400 years of colonial and apartheid relations in South Africa, Cartography 1 (re)produced enduring logics of a bounded, hierarchized racial capitalism, invisibilising and fixing historically subjugated beings, knowings, and knowledges, and rendering (some) people and nature as exploitable resources. Motala’s critical analysis is one example that points to the profound implications of how human based mathematics in action in Cartography 1 shapes thinking about and materialises complex territories of socio-ecological interactions (see also Rubel et al. (2022) on maps used in school mathematics). These are territories in which anthropogenic climate change is rapidly materializing across the multiple scales (Anania & Stiglitz, 2023) that traverse traditional boundaries of time, space, and ways of being both human and non-human, living and inanimate (Haraway, 2016; Motala, 2018).

This demands of mathematics education “new forms of cartographies” (Rubel & Nicol, 2022, p. 182). We present, as one such line of thinking, critical posthuman cartography (Cartography 2), which draws broadly on critical posthumanism, feminist, new materialist, non-representational, and decolonial theories (e.g., Barad, Braidotti, Deleuze, Haraway). We describe key features using Motala’s (2018; 2022) and Motala and Bozalek’s (2022) syntheses. Rooted in relational ontologies, this practice thinks with assemblages (Deleuze & Guattari, 1987), with becoming “in between”, in relations (Haraway, 2016), rather than in discrete objects such as peoples or things. This thinking

strives against universalising, closedness, but sees boundaries as temporary, fluid and permeable. It strives against generalisations, and is comfortable with “micropolitical, situated instances of activism” (Braidotti, 2011, p. 269). A non-representational stance emphasises the usefulness of description and re-description, tracing complex and sometimes contradictory entanglements at different scales. Cartography 2 provides a reading of the present at various scales of space (geopolitical and ecological), and time, all of which are imbued with power relations (Braidotti, 2002). Power is both restrictive and productive, residing in the individual and in relations (Braidotti, 2002). It is comfortable with complexity, multiple scales, non-linearity, situatedness, movement, difference, and incompleteness, and is attuned to presences and silences. It embraces these conditions as generative for analysing and navigating socio-ecological interactions. Indeed, assemblages (including currently dominant mathematics education assemblages) contain potential in their ability to change, with “deterritorialisation” (Deleuze & Guattari, 1987), a movement that produces change.

Cartography 2 views stories as process/practice/performance and product. Stories are not objective, neutral representations of an “out there”, but are analytic and creative. Motala (2022) sees potential in *storying* as a reterritorializing practice: it constitutes disciplinary knowledges (Cartography 1 being one such story), but also transcends bounded hierarchical scales in terms of what counts as knowledge and the knower, visibilising knowledges and subjectivities of subjugated “others” (racialised, naturalised, gendered), and geographical boundaries and linear temporal scales. Countering universalising narratives, all stories emerge in situated conceptual and material processes. Crucially, this way of thinking about stories offers the possibility of *(re)storying*, for example, through participatory, counter-mapping practices (which may or may not use Cartography 1 maps). These practices use maps to challenge interactions that *(re)produce* injustices, by evoking the interests and voices of historically marginalised peoples and possibly other living organisms (Peluso, 1995).

POTENTIALS FOR (RE)STORYING SCALES IN TERRITORIES OF SOCIO-ECOLOGICAL INTERACTIONS IN MATHEMATICS EDUCATION

Motivating his *storying* practice in professional education in the cartographic sciences at a South African university, Motala (2018, p. 29) argues: “It has the potential to facilitate pedagogies that are socially just by opening up new territories through transdisciplinarity, relationality, affect theory, the use of emerging technologies and a focus on non-anthropocentric critical pedagogy.” In this section we draw inspiration from the situated instances of activism using both Cartography 1 and 2 by Siddique Motala (and others) to think their potentials – together – for *(re)storying* scales in complex territories of socio-ecological interactions in mathematics education. Recognising the limits of what can be presented in this paper, we alert readers to the referenced texts of the thinkers we cite.

Motala (2020, 2022) uses maps, produced in South Africa since the mid-1600s, as central characters to *story* conceptualisations and materialisations of socio-ecological interactions across scales: between humans, and between humans and non-human entities, both living and inanimate, within and across local, national and global scales, and across timescales. He starts with Cartography 1 maps produced by colonial surveyors just prior to the cartographic reformation, during initial entanglements of Indigenous and Anglo-European peoples in what is now Cape Town. These maps marked the land as bounded property, backgrounding Indigenous peoples and natural resources on which settlers were dependent, thus shaping and changing peoples’ relations to the land. For Indigenous peoples, Table Mountain was a “prevailing weather, a settlement, and a mountain where

rock, cloud, the creator, rains, streams, and fresh growth are one” (Green, 2020, p. 27), and their land use changed as they moved seasonally (Motala, 2022). Motala then changes spatial and temporal scales, including using a contemporary dot density map, on which colonial boundaries are superimposed, to show how, despite post-apartheid attention to changing relations between peoples and between peoples and the land, the boundaries continue to live in racialised, city spatial planning. These inequities in socio-ecological relations come into stark reality in maps of a local informal settlement in contemporary Cape Town, a settlement produced in the aforementioned entanglements (Musungu et al., 2012). These maps are produced using multi-criteria assessment of vulnerabilities of shack housing, using indicators such as fire, flooding, sanitation, disease and income. Motala (2018) also zooms out to national and global spatial scales in Cartography 1 maps to story multinational companies as actors in socio-ecological interactions, for example, fossil fuels.

As ways of knowing, mathematical practices of interpolation, formatted in contemporary GIS softwares, and which students of cartographic science are required to understand, are central to Cartography 1 maps. In interpolation, a range of algorithms can be used to create a continuous three-dimensional surface from “observed” three-dimensional co-ordinates, with each algorithm giving a spatial weight to each co-ordinate. Motala also gives voice to different scales of knowers. For example, students use Cartography 1 to produce various maps for the storying of socio-ecological interactions (Motala, 2017, 2018), and the above-mentioned informal settlement maps involved community members in participatory GIS, a form of counter-mapping (Musungu et al., 2012).

The examples discussed thus far involve instances of storying territories of socio-ecological interactions across scales of space, time, and human and non-human interactions. We now turn to what we consider restorying these territories. Firstly, Motala’s pedagogy includes a critical posthuman Cartography 2 of Cartography 1 itself, that is explicit engagement with how human and non-human actors in Cartography 1 hierarchise complex, historical, social, political, economic, and ecological interactions as “civilised/uncivilized, Black/White and nature/culture” (Motala, 2018, p. 83) using the dualistic characteristics of backgrounding (or denial), radical exclusion (hyperseparation), instrumentalism (objectification) and homogenisation (stereotyping) (Plumwood, 1993). For example, mathematical interpolation algorithms weight near co-ordinates as more related than distance points, rendering mobility as a “problem”, and create smooth, continuous surfaces from discrete points, naturalizing notions of vulnerability and property value in ways that mask underlying interactions. Motala (2018) notes that, despite the counter-mapping intentions of the above-mentioned vulnerability maps, these objectify the “knowing” community members as participants, and promote normativity of classed, raced, and spatial experiences of socio-ecological vulnerabilities.

Secondly, Motala’s restorying practice incorporates additional scales of ways of knowing and being, and of knowers. For example, he zooms in on the contemporary space of the university of his practice: in District Six, an area on the slopes of Table Mountain from which peoples not legally classified as “white” were forcibly removed during apartheid. He narrates how, when he looked around the campus, he wondered “what was here before? What ghosts haunt this place?” (Zembylas et al., p. 11). To explore, he and the students turn to the GIS technologies of Cartography 1, to map the changes to District Six over time. He also uses these maps in physical walking tours of the area, the goals of which are to afford scales of knowers (e.g., academics, students, former residents) “a deeply affective and transformative experience” (Wallace & Motala, 2022). While left undeveloped for decades, the

full ecological diversity of Table Mountain has not yet returned to the rubble-strewn site of District Six, with flora and fauna haunting the site in the stories of ex-residents (Motala & Bozalek, 2022).

Thirdly, students as knowers are invited to narrate stories, across spatial and temporal scales and across ways of knowing, being and communicating (e.g., various language modes and language codes). For example, Motala (2018, p. 182) uses choropleth maps of Cape Town suburbs, by population and socio-economics, produced using census data, to prompt students to story their lives in these spaces: “Do any of you live in one of these areas? What is life like there? [...] What are some of the issues that you face?” Another example is a digital storytelling project in which students as knowers work across these multiple scales to restory personal, historical, social and ecological narratives of migration, animal poaching, cultural initiation practices, and so on (Motala, 2017).

While early surveying practices of colonial South Africa “silenced” Indigenous peoples’ relations to the land, scales of knowing and being – such as shamanic trance dance, rock art paintings, and oral storytelling – of these peoples narrates different entanglements of peoples, other species, and times. To think further with this idea, we turn briefly to scholarship in the discipline of archaeology, specifically that of Wessels et al. (2023) in our context. Like cartographic science, the dominant archeology assemblage has strong origins in a Western Cartesian system: practices of cleaning, correcting and optimizing strives for accurate models, commonly erase the lived natural environment and people (Wessels et al., 2023). These scholars restory these practices in narratives of archaeological sites as living in time and space, and ontological in constituting lifeworlds, using the case of the Ga-Mohana Hill-site in South Africa. This site, which includes painted and engraved rock images, has been imbued with a range of cultural meanings, across scales of pasts and presents. The restorying practice engages multiple scales of knowers and ways of knowing: an interdisciplinary team (a geometrician, archeologists, an architect and community members, some with Indigenous heritage); digital tools such as GPS, laser scanning and photogrammetric mapping; and ethical data capture processes. This is realized in dynamic, three-dimensional maps that transcend multiple scales of space, time, and beings (humans and non-humans, both living and inanimate).

We end our thinking about restorying practices section by briefly highlighting recent scholarship by Anania and Stiglitz (2023) on ecological data visualisations. Noting that data science has traditionally used Cartesian and Euclidean geometries, they offer a powerful demonstration of the use of Indigenous knowledges in visualisations to understand ecological change and to think about futures.

CONCLUSIONS

In this paper we have thought with situated instances of activism to explore the use of mathematical, cartographic science and critical posthuman cartography, together, to (re)story mathematics educations’ interactions, its becoming in between, at varied scales, in complex socio-ecological territories. We have begun to explore the “difficult unrelenting work” (Haraway, 2016, p. 208), but also the creative, relational work that our current socio-ecological precarity demands. This thinking prompts questions of mathematics education, such as: What happened here? What is happening here? Where are we heading? Where? Who is involved? Why? How? And with what implications? What should happen? (Rubel & Nicol, 2020, p. 174, following Greenwood, 2013). Thus, guiding our understandings of the deep, “not us” (Haraway, 2016, p. 208) hierarchical boundedness of our discursive and material experiences of territories of socio-ecological interactions. But also,

reterritorializing these experiences by visibilising “unexpected others” (p. 208), that is, historically subjugated beings, knowings, and voices, and their (dis)connections. (Re)storying offers promise, not only as a more ethical way of mathematics education’s being in the world, but also as an analytical and navigational practice (Haraway, 2016; Motala, 2018) for interacting, across spatial scales (Rubel & Nicol, 2020), *and* scales of time, knowledges, and ways of knowing, being and communicating. The thinking and practices we share may offer mathematics education “stories (and theories) that are just big enough to gather up the complexities and keep the edges open and greedy for surprising new and old connections” (Haraway, 2016, p. 101) in its entanglements with the socio-ecological.

References

- Anania, K., & Stiglitz, C. (2023). Touching variables: Decolonial approaches and new tools for ecological data visualization. *Leonardo*, 56(6), 579-585. <https://www.muse.jhu.edu/article/916036>.
- Barad, K. (2010). Quantum entanglements and hauntological relations of inheritance: Dis/continuities, spacetime enfoldings, and justice-to-come. *Derrida Today*, 3(2), 240-268. <https://doi.org/10.3366/E1754850010000813>
- Braidotti, R. (2002). *Metamorphoses: Towards a materialist theory of becoming*. Polity Press.
- Braidotti, R. (2011). *Nomadic theory: The portable Rosi Braidotti*. Columbia University Press.
- Coles, A., le Roux, K., Borba, M., Bose, A., Geiger, V., Gutiérrez, R., Makramalla, M., Solares-Rojas, A., Sinclair, N., Valero, P., Vistro-Yu, C., Leung, F.K.S., & Dorier, J-L. (2024a). *ICMI Study 27 Discussion Document. Mathematics Education and the Socio-Ecological*. ICMI.
- Coles, A., Solares-Rojas, A., & le Roux, K (2024b) Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- de Frietas, E. (2023). Metamorphic mathematics and down-to-earth abstraction. In: *Proceedings of the ICMI Symposium on Mathematics Education and the Socio-Ecological*. <https://www.mathunion.org/fileadmin/ICMI/Conferences/Socio%20Ecological/Proceedings-ICMI-symposium-20th-march-2023.pdf>
- Deleuze, G., & Guattari, F. (1987). *A thousand plateaus: Capitalism and schizophrenia*. University of Minnesota Press.
- Edney, M.H. (2011). Cartography without “progress”: Reinterpreting the nature and historical development of map making. In M. Dodge (Ed.), *Classics in cartography: Reflections on influential articles from Cartographica* (pp. 305-329). Wiley-Blackwell.
- Green, L. (2020). *Rock/water/life: Ecology and humanities for a decolonial South Africa*. Wits University Press.
- Greenwood, D. (2013). A critical theory of place-conscious education. In R. Stevenson, M. Brody, J. Dillon, & A. Wals (Eds.), *International handbook of research on environmental education* (pp. 93–100). Routledge.
- Gutiérrez, R. (2017). Living mathematx: Towards a vision for the future. *Philosophy of Mathematics Education*, 32(1). <https://www.exeter.ac.uk/research/groups/education/pmej/pome32/index.html>
- Gutiérrez, R. (2022). A spiritual turn: Toward desire-based research and Indigenous futurity in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 379-388. <https://doi.org/10.5951/jresmetheduc-2022-0005>
- Haraway, D. J. (2016). *Staying with the trouble: Making kin in the Chthulucene*. Duke University Press.

- Helliwell, T., & Ng, O. L. (2022). Imagining possibilities: Innovating mathematics (teacher) education for sustainable futures. *Research in Mathematics Education*, 24(2), 128-149. <https://doi.org/10.1080/14794802.2022.2079553>.
- Le Roux, K., & Swanson, D. (2021). (Re)Imagining spatialities for equity in mathematics education. In D. Kollosche (Ed.), *Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 2, pp. 603-612). MES.
- Motala, S. (2017). In/between science and art: Posthumanist ruminations on Geomatics education. *Proceedings of the 4th Biennial Conference of the South African Society for Engineering Education* (pp. 194-204), Cape Town.
- Motala, S. (2018). *Critical posthumanism in geomatics education: A storytelling intervention* [Unpublished doctoral dissertation]. University of Western Cape.
- Motala, S. (2020). A place-based response to Fikile Nxumalo. *Critical Studies in Teaching and Learning*, 8, 50-59. <https://doi.org/10.14426/cristal.v8iSI.273>
- Motala, S. (2022). The two cartographies: A posthumanist approach to geomatics education. In R. Dopplhijn & R. Braidotti (Eds.), *Deleuze and Guattari and fascism* (pp. 105-124). Edinburgh University Press.
- Motala, S. & Bozalek, V. (2022). Haunted walks of District Six: Propositions for counter-surveying. *Qualitative Inquiry*, 28(2), 244-256. <https://doi.org/10.1177/10778004211042349>
- Motala, S., Abo-Al-Ez, K. M., & Adonis, M. (2023). The subjectification of black engineering educators: A posthumanist cartography. *Qualitative Inquiry*, 29(1), 147-156. <https://doi.org/10.1177/10778004221099570>
- Musungu, K., Motala, S., & Smit, J. (2012). Using multi-criteria evaluation and GIS for flood risk analysis in informal settlements of Cape Town: The case of Graveyard Pond. *South African Journal of Geomatics*, 1(1), 77–91. <https://www.ajol.info/index.php/sajg/article/view/107039>
- Peluso, N.L. (1995). Whose woods are these? Counter-mapping forest territories in Kalimantan, Indonesia. *Antipode*, 27, 383-406. <https://doi.org/10.1111/j.1467-8330.1995.tb00286.x>
- Plumwood, V. (1993). *Feminism and the mastery of nature*. Routledge.
- Rubel, L. H., & Nicol, C. (2020). The power of place: Spatializing critical mathematics education. *Mathematical Thinking and Learning*, 22(3), 173-194. <https://doi.org/10.1080/10986065.2020.1709938>
- Rubel, L., Herbel-Eisenmann, B., Peralta, L. M., Lim, V., Jiang, S., & Kahn, J. (2022). Intersectional feminism to reenvision mathematical literacies & precarity. *Research in Mathematics Education*, 24(2), 224-248. <https://doi.org/10.1080/14794802.2022.2089908>
- Swanson, D., & le Roux, K. (in press). Sharing conceptual gifts by bringing into dialogue sociopolitical mathematics education, decolonial thought, and critical global citizenship education. In A. Chronaki, & A. Yolcu (Eds.), *Mathematics education and the boundaries of global citizenship*. Routledge.
- Wallace, D.A., & Motala, S. (2022, November 7). *Micro-mapping apartheid: Archives, stories and AR*. *The Writing Platform*. <https://thewritingplatform.com/2022/11/micro-mapping-apartheid-archives-stories-and-ar/>
- Wessels, S., Maape, S., Schoville, B. J., & Wilkins, J. (2023). The drone, the snake, and the crystal: Manifesting potency in 3D digital replicas of living heritage and archaeological places. *Archaeologies*, 19(2), 395-429. <https://doi.org/10.1007/s11759-022-09460-3>
- Yasukawa, K., Skovsmose, O., & Ravn, O. (2012). Shaping and being shaped by mathematics: Examining a technology of rationality. In O. Skovsmose, & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 265-283). Sense.
- Zembylas, M., Bozalek, V., & Motala, S. (2021). A pedagogy of hauntology: Decolonising the curriculum with GIS. In V. Bozalek, M. Zembylas, S. Motala, & D. Hölscher (Eds.), *Higher education hauntologies: Living with ghosts for a justice-to-come* (pp. 11-28). Routledge.

CONTRIBUTIONS FOR A SOCIO-ECOLOGICAL PERSPECTIVE WITHIN THE FRAMEWORK OF A CRITICAL STOCHASTIC MODELING SCENARIO

Adriana N. Magallanes¹, Cristina B. Esteley²

¹Universidad Nacional de Río Cuarto, Argentina; ²Universidad Nacional de Córdoba, Argentina

In this paper, we present the results of an action research conducted in collaboration with teachers and students at a secondary school in Córdoba, Argentina. We provide detailed information about the research and some of its findings to contribute to Subject B, Subtheme B1. We selected this topic while considering the inherent complexity of ICMI Study 27. To fulfill the objective of this paper, we highlight contributions from this research that illustrate educational, socio-political, and ecological interventions extending beyond the school's boundaries. We emphasize certain interactions between different spaces, individuals, and bodies of knowledge within a local context. As we progress in our discussion, local interpretations of questions related to subtheme B1 are presented. Finally, we conclude by sharing the voices of students recounting their own experiences.

ASPECTS OF A LOCAL RESEARCH

The previously mentioned research corresponds to the doctoral thesis of the first author (Magallanes, 2021), which was supervised by the second author and conducted at the National University of Córdoba. The thesis emerges with the interest of studying an educational practice envisioned by a group of teachers to overcome critical aspects and perceived senselessness in their own teaching practices framed within local curriculum and institutional policy conditions (Greco et al., 2008; Duschatzky & Corea, 2002). The group that perceived a crisis was made up of the mathematics teacher (later thesis author), the geography teacher and technological education teacher. To overcome this challenge, the group imagines and plans a lesson centered in mathematical modeling (MM) that enables interdisciplinary work, involves all three subject areas, and promotes spaces for empowerment to encourage critical citizenship (Skovmose & Valero, 2012). In the context of the mathematics course, priority will be given to stochastic knowledge. The complete practical planning, which covers the period from 2012 to 2014 and involves one single group of students, the three teachers mentioned before, researchers of the public university and agents of the municipality to which the school belongs, surpassed the expectations envisioned. In her dual role as teacher and researcher, the thesis student follows the complete process of the schoolwork. In an academic field, teachers focalize on promoting critical citizenship and spaces for empowerment. By the end of 2012, students freely select a socio-ecological topic of their interest to study and model, referred to problems of water contamination in the area near the school. In 2013, considering the chosen topic, there were multiple outings beyond the borders of the school; many moments to think and rethink uncertainties on site, in order to finally formulate the problem. This was accompanied by the gathering and analysis of pertinent information and interaction with researchers and/or municipality agents. In 2014, they communicated the results of their work at various events. The teacher-researcher gathers -with field notes, videos, photos and all the information related to the experience. The research is organized considering an action research cycle (Kemmis et al., 2014). While writing the thesis and considering the guiding questions of the group and the novelty of the intervention process, it becomes necessary

to develop theoretical tools to characterize the educational environment and describe the empowerment of the involved parties. (e.g., see Magallanes & Esteley, 2022).

While reflecting on the synthesis of schoolwork and research, we observed that the aforementioned illustrates and informs about the interrelationship or tension between curricular policies and a group of teachers who perceive themselves to be in crisis. This tension appears to be resolved when the group initiates change in their practices. This initiative enables them to transcend limits and push boundaries. New interactions begin, placing at the center, the study of a local problem of socio-ecological nature. The issue, as will be illustrated in more detail shortly, necessitates the use of stochastic knowledge and other disciplines for modelling. Initially, it could be argued that the synergy between the selected MM problem and the collaboration of both human and non-human elements enables the approach and comprehension of the complex implications—educational, managerial, research-related, relational, and emotional—arising from social issues.

The intervention and investigation are generated, understood, and comprehended through interaction with theoretical assumptions, selectin and/or constructing the notions that were necessary. In the next section, theoretical stances and new constructs are summarized.

THEORETICAL DIAGOCALLY CONSTRUCTED TOOLS

To explain the experience, and based on the questions that guide the thesis, two theoretical tools, among others, stand out: Critical Stochastic Modeling Scenario (CSMS) and empowerment spaces. We point out that we considered the educational work of the teacher within a network that involves the classroom, the school, and the society (Esteley, 2014). Regarding CSMS and to ponder the stochastic modeling, we mainly drew on contributions from Campos (2016) and Pfannkuch et al. (2018) and the idea of modeling scenario (Esteley, 2014). To consider the cyclic process of the MM, we took into account contributions of Borromeo (2018). To ponder the critical and social aspect in education we turned to Skovsmose and Valero (2012) and when focusing on the stochastic we retrieved Jacobini and Wodewotzki (2006) and Campos (2016). These contributions allow us to propose the notion of Critical Stochastic Modeling (CSM) and the associated cycle (Magallanes, 2021; Magallanes & Esteley, 2022) as shown in Figure 1 (English translation of the original source).

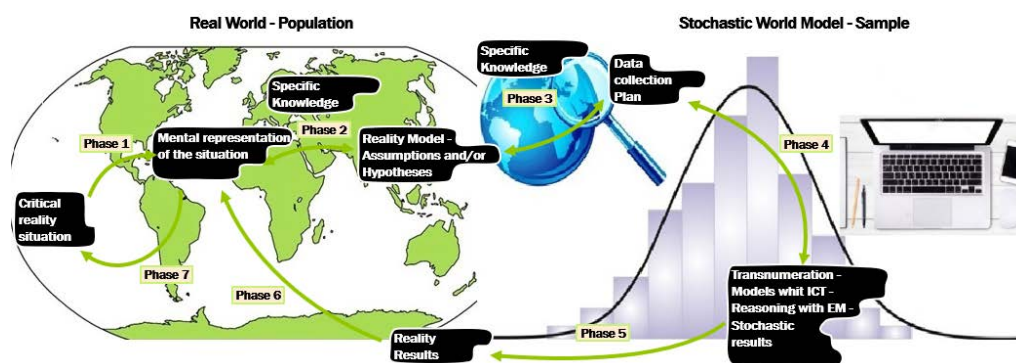


Figure 1: Cycle of Critical Stochastic Modeling (Original source: Magallanes, 2021)

The cycle of CSM interconnects seven phases of work. In Phase 1, a problematic situation is chosen, involving a formulation and mental representation of a problem; variability aspects and randomization inherent to the situation are recognized (Pfannkuch et al., 2018). In Phase 2, by appealing to specific knowledge linked to the situation, it should be described in a simple manner,

selecting relevant aspects to capture the fundament of the situation. In Phase 3, a transition is carried out from reality to stochastic and mathematical knowledge and the need of counting with data to study variability is acknowledged. In this phase, a plan is developed, involving procedures to carry out the research; selection of pertinent variables; definition of possible types of experiments, necessary measurements, and sampling design. In Phase 4 stochastic work is conducted to obtain results, enabling stochastic thinking: transnumeration, consideration of variability, among others. By capturing significant elements of reality, data is systematized, and models of stochastic frameworks are retrieved, to think in a generic manner about what has been done. In Phase 5, the results are interpreted by comparing them with reality, through discussion and argumentation about the interpretations of the contextualized stochastic information. Phase 6 is the moment to validate the model constructed. In Phase 7, a space is enabled for transformation and/or so for socialization. So the students integrate, write and communicate on the achieved and sustained understanding of stochastic knowledge and others.

We acknowledge that the notion of empowerment varies in accordance to the perspective that is adopted and the focus of the project with which one is working (Czuba & Page, 1999; Soler et al., 2014). So, for the school context chosen, we choose to refer the idea of educative space that promotes empowerment. Such a space is characterized as one that fosters interactions that contribute to making visible problems or conflicts, in order to advance in the search of resolutions through revision, analysis and/or contributions of ideas to transform that which is problematic. In this space, teachers as well as students may come to recognize the potentiality of knowledge in order to provide a proposal of transformation generated collectively, considering it to be partial, not definitive and open to criticism (Magallanes, 2021). With contributions from Ernest (2002), empowerment is distributed in three domains: the mathematical-stochastic, the social-political, and the epistemological. The theoretical tools presented, as well as Figure 1, represent the local voices of the researchers that become useful resources to comprehend and recount the experience. They are the voices that emerge in response to a polyphony of voices generated in the classroom. It seems as though, in the complexity surrounding a socio-ecological situation, it is necessary to reformulate its meaning and thus the voices.

THE WORK OF CRITICAL STOCHASTIC MODELING CONTEXTUALIZED IN ITS TERRITORY

The school where the work is carried out is a secondary school attended by approximately a total of 120 students, distributed in 6 courses (20 students per course), comprising the six years of secondary education. During the last five years, the orientation of the studies is focused on tourism. This orientation is selected due to the beauty of the mountains and an abundant river (which we will identify as *Río*) surrounding the town (which we will identify as *La Localidad*) where the school is located. In each course, we can find students that live in areas nearby to the school, while others live in areas much farther, reason for which they live in the school from Monday to Friday. The students' families engage in agricultural, tourism or commercial activities. Some teachers live close to the school while others, like the mathematics teacher, travels from a city that has a national university and is 70 kilometers away. The school is surrounded by mountains and some native trees that still resist the rapid advance of progress that *La Localidad* is going through after the main street of access was paved. The two tributaries of *Río* (tributaries 1 and 2) are situated at the highest parts of the

mountains. *La Localidad* and the surrounding areas do not have sewer systems. Homes have an on-site sanitation system. The municipality, extracts water from *Río*, fills two tanks for purification before it is distributed to homes.

In the school and its surroundings, schoolwork (including teachers, students and other humans and non-human) takes place in stages, which implicates different phases of the CSM cycle. At first, in Phase 1 (June to December 2012), during the second-year course (students 13 to 14 years old), due to lack of a sewer system and potential consequences for the *Río*, students propose a project for 2013 to study the relation between the lack of sewers and the contamination of the water in the river. From this problem (in the sense of Valero & Rayn, 2017), teachers plan an intervention project. In the second stage (March to December 2013), students are now in their third year and presenting conjectures (Phase 2) which leads them to formulate the problem, relating it to the significant development of tourism and the negligence of some businesses that dispose of their waste or other elements into the *Río*. So, teachers and students go on a field trip to find evidence to support their conjectures. Having not found empirical evidence, they propose to consult with researchers who study the connection between the lack of sewer systems and the quality of the *Río's* water. Later on, geologists from a university nearby organize a seminar and contribute specific expertise about contamination from on-site sanitation systems. After which, they organize a field outing in which they taught how to take samples of water. Ultimately, the geologists express that in order to determine if there exists a contamination that affects the river's water due to on-site sanitation system, specific equipment and knowledge beyond the resources available in the school environment would be required. Therefore, (Phase 2) specific knowledge emerges and is incorporated, that lead to modifications in the questions proposed initially in Phase 1, returning in order to create a mental representation of a different situation such as the study of the quality of the water (superficial and drinkable water) in *La Localidad*. The geologists also provide details on how to gather information regarding the quality of the river's water. They indicate that, in order to study the *natural quality* of a river's water, It Is necessary to take sample from an area with minimal human Influence (for example Tributaries 1 and 2, Image 1a) and use them as references for the *natural quality* of that river. They also emphasized the importance of identifying the bacteria *Escherichia Coli* (*E. Coli*) as an indicator of water contamination and point out that, in 2003, the World Health Organization accepted the presence of *E. coli* as the best marker of fecal contamination. The students advance in Phase 2, retrieving specific information offered to them by the geologists (surface water, groundwater, natural background water quality) and deciding on the type of contamination they wish to study (urban and domestic contamination). Also, they request collaboration from the local municipality (by writing a note) and conducting a visit as to collect pertinent information (maps of water distribution, water sampling locations in the river, water treatment system, copies of chemical and bacteriological analyses from the previous months). This information is compared to the results of the chemical and bacteriological analyses conducted by the geologists after their participation in the school. It is noted that the chemical analysis falls within standard range, while the same does not occur with the bacteriological ones. In the latter-mentioned, there is presence of both *E. coli* and fecal and total coliforms, whereas the municipality's data appear within values permitted by provincial organisms. The students advance once again towards Phase 3, in which they define the variability, select relevant variables (presence of *E. Coli*; quantity of fecal coliforms) to investigate the bacteriological quality of the water, advancing in the development of a plan and in data collection.

They define a study area of the water from the *Río of La Localidad* (which includes tributaries 1 and 2) and to study the quality of the drinking water. Based on these settings, they propose methods to select sampling points (Figure 1-a), which they choose randomly and mark those points on the map as is observed in Figure 1-b.



Figure 1-a: Selection of sampling points (Source: Magallanes, 2021)

Figure 1-b: Sampling points in surface water (Source: Magallanes, 2021)

Later on, students and teachers conduct two field trips, one in winter and another in spring, to take samples of river water, its tributaries and the drinking water in homes. In all cases, GPS is used to pinpoint the location of the samples (see photos in Figure 2).



Figure 2: Field trips for water sampling (Source: Magallanes, 2021)

The samples are analyzed in laboratories of a nearby university. These data are entered into InfoStat, by the students, and so initiating an exploratory analysis in Phase 4, which leads to the development of various types of graphs and Phase 5, where they interpret the findings back in a reality setting. So, for example, they construct bar graphs with the averages of coliforms and then express that the quantity of fecal and totals found per 100ml of water in the river is within the limits allowed by the province for surface waters. However, if this result is compared to the coliforms observed in samples 1 and 2 (tributaries 1 and 2) of the natural quality (areas assumed to be without urban contamination), the values are considerably higher than the rest of the collected samples. This is particularly

noteworthy, except at the point located in an intermediate area between the two urban areas, where the river has the opportunity to regain its *natural quality*. They also create histograms with data from the river water and pie charts with data on the presence of *E. coli* in drinking water. They then highlight the presence of *E. coli* in drinking water at a rate of 75% in the first sampling (winter) and 17% in the second sampling (in spring) after measures were taken by the municipality. As part of Phase 6, the students validate their results with the local municipality (see Figure 3).



Figure 3: Municipal authority visit (Source: Magallanes, 2021)

The Secretary of Government (see Figure 3) visits the school to express gratitude for the inquiry conducted by the students. For the results of their work allowed them to identify deficiencies in the private laboratory that conducted water quality inspections and take measures to address this issue. Similarly, he thanked them for the Excel spreadsheet they donated to the Municipality to record information.

Phase 7 takes place the following year (2014) when the students are in their fourth year. The student body participates in the local, regional and provincial Science Fair, in the mathematics category, where they present an exhibit, a presentation, models and a brochure to communicate the entire experience. It should be noted that, at these fairs, students present the idea of implementing a sanitation system in each home by installing bio digesters. These devices enable the treatment of sewage, and once treated, the reclaimed water is used for irrigation. At the local and regional fairs, they received positive evaluations; however, they did not win awards at the provincial fair because, although the project incorporated stochastic elements, the jury considered that it did not fit into the mathematics category. To conclude the project, the students requested the local authorities to create an ordinance for the protection of water in *La Localidad*. This proposed ordinance would require the implementation of a sanitation system similar to the one presented by them at the science fairs, both in homes and in businesses. This student initiative led to the approval of a municipal ordinance (Municipal Ordinance No. 740, 2019). It is noteworthy that within the framework of the CSMS, it was observed that the students experienced moments of empowerment in expression; planning-organization-decision-making; action-inquiry-search; criticism-self-criticism; reflection, and formulation-modification. In the next section, we will present testimonies that reflect their experiences, assessments, and learnings acquired, voices that emerge in the classroom but transcend its boundaries.

CLASSROOM VOICES THAT TRANSCEND BOUNDARIES

As noted at the beginning of this article, in our process of analysis and composition, we had to find manners, words, or ideas to capture the complexity of the experiences. Therefore, we proposed the

notion of Critical Stochastic Modeling Scenario, which evolved through successive phases of characterization, transitioning from a predominantly localized characterization to a more expansive depiction that encompasses the local, while remaining aligned to the underlying motivations. Beyond the classroom and the experience in the Critical Stochastic Modeling Scenario it is possible to identify how a small rural community of adolescents, along with teachers and specialists, develop and sustain a CSM project focused on a socio-ecological situation and how this contributes to the field of mathematics education. This is because, within the scenario, critical stochastic literacy is recognized; understanding it as a process in which students use tables, graphs, stochastic tools, and technology to decipher the reality underlying the numerical information. At the same time, this literacy glimpses a political process that enables students to communicate their experiences, work in an environment of respectful social relationships under a principle of equality, and approach knowledge that promotes a better quality of life for the rural community.

The voices written at the beginning of 2015 by Laura and Beto, participants in the CSMS, exemplify this. Both acknowledge the importance of the project for them and their community, as it allowed them to apply knowledge and achieve more than they imagined. Laura states: “[...] through samples and studies, we discovered that the water in our town was not in good condition. We were able to apply mathematics to a social problem in the community and understand it in a way we hadn’t thought possible. Whereas, Beto recognizes that the project helped him become more aware of environmental protection and points out: “[...] thanks to the help of professionals and the teachers who worked with us on the project, we acquired new knowledge [...]” He further emphasizes: “Together, we were inspired to move forward and learned a great deal of values. We learned to work as a team, and to share ideas, perspectives, among other things.” He also mentions that they learned to discuss to reach conclusions. Like Laura, he also expresses that they achieved more than they thought possible: “At first, we never thought we would go so far, but over time, with much effort, determination, and knowledge, we pushed really hard and achieved tremendous results [...]”

In the voices of Laura and Beto, there is a recognition of mathematics as a political and ecological activity that allowed them to study a social problem. They also express pride because, as students, they were able to identify an issue affecting the community: the poor condition of the town’s water. Additionally, their ecological activity involved conducting water sampling studies, analyzing the problem, and seeking alternatives to solve it and prevent further damages. They also highlight the human activity within a stochastic mathematical framework when they mention being inspired to move forward and learning values, teamwork, and sharing ideas and perspectives. Now, we aim to move beyond the boundaries of school and research work. To this end, we engage in a discussion on whether the proposed CSM cycle can become a flexible tool for some teachers to design, manage, and support scenarios of this nature. Scenarios where critical stochastic literacy intertwines with empowerment, particularly sociopolitical empowerment (Skovsmose, 2011), as students recognize their power to discuss topics of great social and political interest. In this sense, students are considered political actors.

Finally, we would like to highlight that, through what is presented and illustrated in this paper, it is possible to rescue knowledge and experiences that highlight not only cognitive, pedagogical, stochastic and mathematical dimensions of MM, but also socio-ecological and political dimensions (D’Ambrosio, 2009) in synergy with the particular CSM process.

References

- Borromeo, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Cham, Suiza: Springer.
- Campos, C. R. (2016). La educación estadística y la educación crítica. *Segundo Encuentro Colombiano de Educación Estocástica* (2 ECEE). Bogotá, Colombia.
- Czuba, C. E. & Page, N. (1999). Empowerment: What Is It? *Journal of Extension*, 37(5).
- D'Ambrosio, U. (2009). Mathematical modeling: Cognitive, pedagogical, historical and political dimensions. *Journal of mathematical modelling and application*, 1(1), 89-98.
- Duschatzky, S. & Corea, C. (2002). *Chicos en banda: los caminos de la subjetividad en el declive de las instituciones*. Buenos Aires, Argentina: Paidós.
- Ernest, P. (2002). Empowerment in mathematics education. *The Philosophy of Mathematics Education Journal*, 15. <https://www.exeter.ac.uk/research/groups/education/pmej/pome15/empowerment.htm>
- Esteley, C. (2014). *Desarrollo profesional en escenarios de modelización matemática: Voces y Sentidos*. Universidad Nacional de Córdoba. <https://ffyh.unc.edu.ar/publicaciones/tienda/publicaciones-de-investigacion/seicyt-posgrado/tesis/desarrollo-profesional-en-escenarios-de-modelizacion-matematica-voces-y-sentidos/>
- Greco, M. B., Pérez, A. & Toscano, A. (2008). Crisis, sentido y experiencia: conceptos para pensar las prácticas escolares. En Baquero, R., Pérez, A. & Toscano, A. *Construyendo posibilidad. Apropiación y sentido de la experiencia escolar*. Homo Sapiens.
- Jacobini, O. R. & Wodewotzki, M. L. L. (2006). Uma reflexão sobre a modelagem matemática no contexto da Educação Matemática Crítica. *Boletim de Educação Matemática*, 19(25), 1-16.
- Kemmis, S., McTaggart, R., & Nixon, R. (2014). *The Action Research Planner: Doing Critical Participatory Action Research*. Springer.
- Chapman, O. (2003). Facilitating peer interactions in learning mathematics: teachers' practical knowledge. In M. J. Høines & A. B. Fuglestad (Eds.), *Proc. 28th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 191-198). PME.
- Magallanes, A. N. (2021). *Empoderamiento de estudiantes en el contexto de un Escenario de Modelización Estocástico Crítico*. [Unpublished doctoral dissertation]. Universidad Nacional de Córdoba.
- Magallanes, A. & Esteley, C. (2022). Caracterización de un escenario de modelización estocástico crítico y del ciclo de modelización vinculado. In S. A. Peters, L. Zapata-Cardona, F. Bonafini, & A. Fan (Eds.), *Bridging the Gap: Empowering & Educating Today's Learners in Statistics. Proceedings of the 11th International Conference on Teaching Statistics*. Rosario, Argentina. International Association for Statistical Education. 10.52041/iase.icots11.T14F2
- Pfannkuch, M., Ben-Zvi, D. & Budgett, S. (2018). Innovations in statistical recognize to connect data, chance and context. *ZDM, The International Journal on Mathematics Education*, 50(7), 1113-1123.
- Skovsmose, O. & Valero, P. (2012). Rompimiento de la neutralidad política: el compromiso crítico de la educación matemática con la democracia. En Valero, P. & Skovsmose, O. (Eds.). *Educación matemática crítica: Una visión socio-política del aprendizaje y enseñanza de las matemáticas* (pp. 25-61). Uniandes.
- Soler, M., Planas, A., Ciraso-Calí, A. & Ribot-Horas, A. (2014). Empowerment in the community: the design of an open indicators system from participatory evaluation processes. *Pedagogía Social. Revista Interuniversitaria*, 24, 49-77.
- Valero, P. & Ravn, O. (2017). Recontextualizaciones y ensamblajes: ABP y matemáticas universitarias. *Didacticae*, 1, 4-25.

WHO OWNS THE NARRATIVE: REFLECTIONS ON SOCIO- ECOLOGICAL INTEGRATION OF MATHEMATICS EDUCATION FROM THE EGYPTIAN CONTEXT

Mariam Makramalla

University of NewGiza

Mathematics education, as a narrative, unfolds from the depiction of its owner and is replicated from the experience of its user. This paper presents itself as a call for a larger study, which seeks to critically disentangle different narrative threads of the Egyptian teaching and learning context in the interest of understanding the socio-cultural anticipation of a mathematics education experience that would transcend the traditional classroom and engage in complex questions pertaining to the socio-ecological of a learner's stationed environment. The paper mainly focuses on three perspectives. It starts by counter-balancing the mainstream opinion of internationalization by presenting an argument about the relevance and importance of a local narrative. The next part conceptualizes the transdisciplinary integration of the socio-ecological within the context of K-12 mathematics education. Finally, the third part focuses on the case of Egypt, presenting various threads that make up the Egyptian narrative. The paper concludes with a call for intertwining these three perspectives to find impactful ways of integrating the socio-ecological into the Egyptian mathematics classroom.

THE POWER OF A NARRATIVE

An African tribe wisdom states that: “If you want someone’s behaviour to change give them new stories.” [unknown origin]. Literature repeatedly discusses the power and influence of local narratives on the evolution and development of communities (Evans et al., 2013), the way they relate to their own histories and the way they relate to the other, who might have been part in shaping that history (Greene, 2007). In a sense the unfolding of local community narratives affect the socio-ideological relatedness of communities within themselves (Evans et al., 2012) and how they relate to the larger national and global reality in which they are situated (Calvera, 2005). The educational narrative could be claimed to be communicated, transported and orchestrated in a very similar way (Walker, 2012).

Best ways of knowing are defined and dictated across centuries by the party that at this point of time is considered culturally superior (Apple, 2012; Walker, 2012). Learners, who have experienced ways of knowing in a certain way grow to become educators or caregivers that propagate their experienced narrative of best practices to forthcoming generations (Bang et al., 2016). Over time, the quality of ways of knowing becomes tightly entangled to this historically communicated narrative, which in turn is rooted in a culturally superior power game of the past.

As a result, unspoken norms and power structures to narratives are formed that are often deemed as unquestionable and impenetrable (Apple, 2012). With the growing trend of globalization and exposure to “other ways of knowing”, disruptive narratives become more common and often more challenging to deal with. For example, across generations of educators, many educators find themselves in the position of being exposed to different forms of knowledge construction. When confronted with the locally superior narrative, many of them often seek to challenge, question or disrupt the mainstream narrative that over the years has been considered as best practice.

This act of disruption is not uncommon to the field of mathematics education (Nolan, 2020). The literature shows how ways of knowing in mathematics education have often been dictated by the culturally superior (Apple, 2012), local voices have often been intentionally muted (Makramalla & Stylianides, 2021) and disruptions have often been considered unwelcome in the interest of maintaining the societal, political or cultural status quo (Apple, 2012). With this in view, the case of Egypt becomes very interesting to explore. With its entangled history of governance, power and rulership, it becomes interesting to explore how educational narratives have been formed and propagated.

In this work, I want to shed particular light on local perceptions to ways of knowing in mathematics education, particularly in relation to the transdisciplinary connection established between mathematics education and the socio-ecological. In light of the overarching exploration of narrative and power structures, I seek to ask questions that connect the past to the present and challenges the present in view of the future. This work-in-progress study is simply decomposed of three main sections and a call for action. In the next section, I conceptualise the way I frame the transdisciplinary connection between mathematics education and the socio-ecological. In the section that follows, I dwell on histories of narratives and the power play attached to these for the particular context of K-12 education in Egypt. Rooted in this historical understanding of narrative governance and formation, this paper calls for an investigation of perceptions towards disruption in ways of knowing across different teaching and learning contexts. The work-in-progress paper concludes with an open call for an extended Investigation across unspoken culturally varying national narrative boundary lines.

THE IMPACT A TRANSDISCIPLINARY NARRATIVE IN MATHEMATICS EDUCATION

Mathematics education does not exist in isolation. Across contexts, mathematics is perceived differently by educators and is accordingly experienced differently by learners (Bishop, 2013). With the rise of the global crisis that we are facing, it is impossible to view mathematics as a discipline that is isolated from its problem solving based nature; a nature that would seek to capture and actively part-take in such crisis.

Conceptualising transdisciplinarity in mathematics education for the purposes of this study

While the construct of transdisciplinary integration has been differently defined by scholars, in this work I choose to align myself to the working definition that has been coined by the International Society of Transdisciplinary Engineering (ISTE). In their work (originally focused on engineering), scholars have sought to make a distinction between the way engineering is traditionally taught and how it would be taught if coined around a transdisciplinary framework that integrates social and environmental issues into the solving of daily engineering challenges. While it is difficult to claim that the traditional approach to teaching and learning can be unanimously defined across contexts, the comparison between the traditional and the transdisciplinary is still helpful in terms of shedding greater light on the complexity that is enabled in the teaching and learning process through – what the ISTE refers to – as a transdisciplinary integration. The working definition has been designed with an engineering focus In mind. For the purposes of this study, I have Incorporated minimal variations to make it applicable to mathematics education. Table 1, hence shows an iteration to the ISTE’s working definition, in order to incorporate a mathematics education dimension.

Traditional Mathematics Education	Transdisciplinary Mathematics Education
Solves the problem presented as a mathematical problem only	Asks questions of how the problem fits into wider social and environmental systems
Applies only technical criteria and procedural knowledge to identify a fixed numerical or algorithmic solution	Incorporates social and environmental criteria alongside the technical and establishes connections in order to identify a good solution set
Uses mathematical modeling only to make design and other decisions	Incorporates various methodologies and ideologies (including those from social sciences) to present a contextually fit decision rationale
Collaborates only across other mathematical teams to solve problems	Collaborates across the disciplines to solve complex problems and is open to seek critical input from external bodies, if available
Sees only value in numerical and quantitative figures	Seeks to see the quantitative in the context of the wider qualitative setting

Table 1: Traditional vs transdisciplinary mathematics education (adapted from ISTE)

Table 1 presents a variation of the ISTE’s working definition on transdisciplinary integration. It utilizes the problem-solving context as an underlying pedagogical context to better illustrate the potential of an integration that goes beyond the technical disciplines (such as the languages and the sciences) to incorporate larger societal and environmental questions. With this working definition in mind, the next section particularly focuses on the integration of the socio-ecological into the wider discipline of mathematics education.

Transdisciplinary integration of the socio-ecological in mathematics education

The world we live in today is fragile, broken and precarious in nature. With the rise of the internet, social media networks and other technology enabled exposure platforms, the students of this generation can no longer be expected to be taught in a setup that is void of these interactions. Being potentially the most affected generation by the consequences of the current precarity, it becomes crucial to enable students to become informed and equipped problem solvers and change agents. At the heart of much of the aforementioned fragility are socio-ecological questions (Makramalla et al., 2023).

In line with Table 1, a transdisciplinary approach to teaching and learning mathematics would seek to embed mathematical problems within the explicit local manifestation of the socio-ecological as unfolding in each local context. Coles et al. (2024) argue that this integration could take different levels of depth. Through mathematics, educators could enable students to develop a greater sensitivity towards listening to the cues and indicators of potential socio-ecological entanglements. At a deeper level, educators could enable students to utilize the mathematics to attend to socio-ecological

entanglements that they have correctly pinpointed. Finally, socio-ecological entanglements could be lived through, complexified and critically questioned through an active and empowering pedagogy adopted by mathematics educators.

In order for educators to take up this role, they need to firstly relate to the discipline of mathematics as a non-static discipline that seeks to solve real and complex problems (Kohen et al., 2019). Secondly, they need to relate to their own role as mathematics educators as empowering the next generation of problem solvers and critical thinkers. In other words, they need not be intimidated or feel threatened in terms of upholding classroom discipline and superiority of knowledge, when their students raise and engage with complex questions in the classroom (Makramalla, 2022). Finally, educators need to be sufficiently trained to adopt a transdisciplinary approach in their classrooms (Collado-Ruano et al., 2019).

While the transdisciplinary integration of mathematics education with the socio-ecological does seem very timely and essential, yet considering the wider politically formed and historically rooted power schemes, this approach to mathematics education might be considered as unwelcome and potentially threatening to existing educational stakeholders. This will be further elaborated in the next section.

The potential and threat of transdisciplinary integration

The act of effectively solving problems transforms a mathematics student from being in a position of passivity to an active stance of change agency (Makramalla & Stylianides, 2021). This transformation, when practiced responsively by educators has the potential to enable students to become members of society who change the way policies are formed, challenge stereotypical associations that often limit societies' functioning, thereby effectively creating new stories that disrupt the dominant narrative. Educators that empower students in this way need to however be prepared to reconsider the way they perceive their own authority in the classroom (Makramalla, 2022). This is often a challenge to teachers' perceived self-identity and role.

Beyond the perceived identity, role and impact of an educator, an educational system that endorses change agency needs to be prepared to empower autonomous thinkers at each level of the system. This is often challenging as it might imply a potential disruption to the system's stability, discipline, control and functioning (Makramalla & Stylianides, 2021). Local educational systems are often subtly governed at larger by unspoken cross border relationships. These are often rooted in post-colonial histories (as will be more elaborately presented in the next section) and often aim to dictate an imbalanced power play in the interest of the mainstay of the considered culturally superior party (Popkewitz, 2014).

In light of this local and global understanding, it could hence be better understood that the transdisciplinary integration of the socio-ecological in mathematics education, particularly in historically and culturally labelled inferior regions might be intentionally repressed. In the next section, I shed more light on the particular context of K-12 mathematics education in the Egyptian context.

THE NARRATIVE OF THE EGYPTIAN CONTEXT

The educational landscape in Egypt is complex and very diverse. It is out of scope of this paper to present a holistic understanding of the varying forces that affect the unfolding of the narrative. In the

following, I choose to focus on three particular narrative perspectives, namely the historical, the social and the cultural. Through intertwining these, my intention is to problematize a potential disruptive narrative of mathematics education that would allow the transdisciplinary integration of the socio-ecological in mathematics education to unfold from within.

The historically formed narrative

To unpack the complexity of the narrative, it is important to understand the history of modern Egypt. Figure 1 presents a very abstract timeline of the modern Egyptian era, highlighting the main influencing sources of power, which to a great extent have influenced the educational landscape.

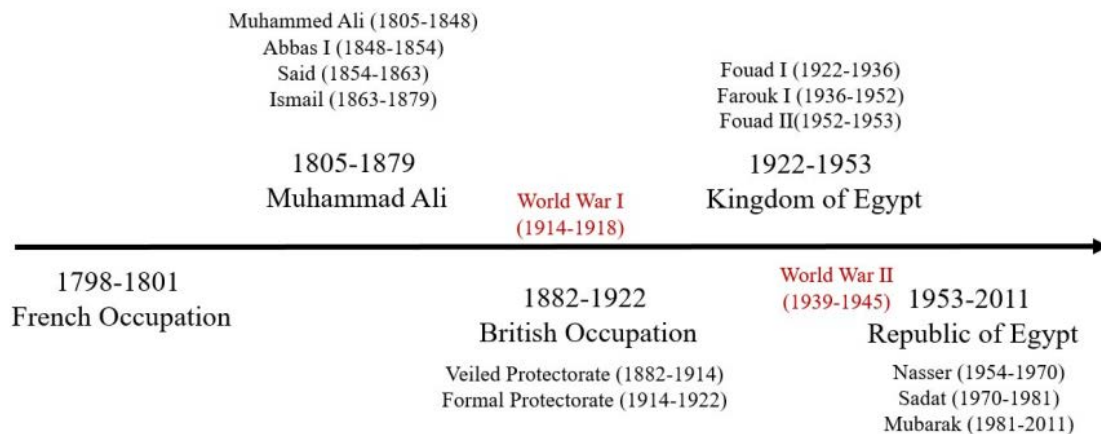


Figure 1: History of modern Egypt at a glance (Makramalla, 2020)

As illustrated in Figure 1, the modern Egypt has experienced varying systems of governance structures including colonial, monarchical, socialist and republican. Each of these governance structures influenced the power dynamic of education and knowledge dissemination in the interest of maintaining their own authority. Resulting from that is a baggage of uneven social distributions (Makramalla, 2020).

The socially populated narrative

A larger study (Makramalla, 2020) has considered in depth the socio-historical mapping and its influence on the unfolding of the educational landscape. The research shows that throughout the monarchical and colonialist rule, the mainstay of an uneven social divide was intentional in the interest of better control and governance. As a result, mathematical education programs that encouraged critical thinking, creativity and problem solving were only available to the elite minority (Makramalla, 2020). The majority had a static, procedural and traditional experience of mathematics. The latter was cascaded through the different generations. With the rise of the socialist movement that started during Nasser’s era, a national system of education that would cater for all alike was called for. Since the traditional mathematics education structure was more socially propagated, it was taken up as the national dogma for teaching and learning mathematics at the time (Makramalla, 2020). The socialist elite – mainstream divide, however still had its traces throughout this time, reflecting itself on the cultural narrative, as will be briefly discussed in the next section.

The culturally populated narrative: Schooling typologies

Resulting from the socially populated cultural divide was a typology of schooling, each following a different dogma of mathematics education that stands to this day. This topology includes (at large) three schooling types, namely the national public schooling type, the national private schooling type and the international schooling type (Biltagy, 2012). It is out of scope of this paper to discuss sub-typologies that also exist to each schooling type (such as experimental schools, STEM schools etc.). The important thing is that across the three typologies varying perceptions of mathematics education exist ranging from a purely traditional view to a connection making view to a problem solving transdisciplinary view (Biltagy, 2012). It is worth mentioning that the amount of student access and exposure across the three typologies is not equal. As a result, transdisciplinary mathematics as conceptualized above, would only be experienced by a small minority of learners. The power dynamics coupled to that reality are subtle, complex and have societal and historical roots that are worth untangling.

The potential of a disruptive narrative

The intention is for the expansion of this work to untangle the history of modern Egypt and to map out the societal power dynamics that resulted from that. In studying the resulting societal power dynamics, it becomes much clearer why a transdisciplinary approach that would engage each learner in the solving of complex socio-ecological problems that pertain to their stationed societal environment would be subtly unwelcome. This paper acts as a proposal to a more extended study that would analyse existing power structures. The proposed study would more deeply analyse the contextually suitable features of a mathematics education experience that targets to disrupt the current societal narrative, which in turn is rooted in a historical post-colonial and post-monarchial context.

CONCLUDING THOUGHTS

This paper presents some highlights of the author's wider in depth investigation of the Egyptian context. It calls for a deep dive into societal structures and cultural affordances that have resulted from a complex historical baggage in order to deeply understand the system structure of the mathematics educational landscape. If mathematics education is to have a disruptive role in transcending the discipline itself and critically engaging learners in socio-ecological questions that pertain to their stationed environment, it is important to unpack the unique local educational identity. It is only through this deep understanding of intertwined power threads that make up the socio-cultural context, that we can design a mathematics education experience that is contextually welcome and globally impactful.

References

- Apple, M. (2012). *Education and power*. Routledge.
- Bang, M., Faber, L., Gurneau, J., Marin, A., & Soto, C. (2016). Community based design research: Learning across generations and strategic transformations of institutional relations towards axiological innovations. *Journal of Mind, Culture and Activity*, 23(1), 28-41.
- Bishop, A. (2013). *Mathematics education and culture*. Springer.
- Biltagy, M. (2012). The optimal schooling level in Egypt. *International Journal of Social Science and Humanity*, 2(2), 186-192.

- Calvera, A. (2005). Local, regional, national, global and feedback: Several issues to be faced with constructing regional narratives. *Journal of Design History*, 18(4), 371-383.
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- Collado-Ruano, J., Mandronero-Morillo, M., & Alvarez-Gonzalez, F. (2019). Training transdisciplinary educators: Intercultural learning and regenerative practices in Ecuador. *Studies in Philosophy and Education*, 38, 177-194.
- Evans, K., Murphy, L., & de Jang, W. (2013). Global versus local narratives of REDD: A case study from Peru's Amazon. *Journal for Environmental Science and Policy*, 35(1), 98-108.
- Greene, J. (2007). Colonial history and national history: Reflections on a continuous problem. *The William and Mary Quarterly Journal*, 64(2), 235-250.
- Kohen, Z., Amram, M., Dagan, M., & Miranda, T. (2022). Self-efficacy and problem solving skills in mathematics: The effect of instruction based dynamic versus static visualization. *Journal of Interactive Learning Environments*, 30(2), 759-778.
- Makramalla, M. (2020). *Contextualising educational reform in Egypt: An embedded case study of mathematics teachers' perceptions about problem solving* [Unpublished doctoral dissertation]. University of Cambridge.
- Makramalla, M. & Stylianides, A. (2021). "I have started this new trend at the end of [students'] notebooks": A case study of a mathematics teacher caught within a reproductive cycle of hierarchical cascading of power. In D. Kollosche (Ed.), *Exploring ways to connect. Proceedings of the Eleventh International Mathematics Education and Society Conference* (Vol. 2, pp. 641-650). MES.
- Makramalla, M. (2022). Student intimidation in the mathematics classroom. In Fernandez, C., Llianers, S., Gutierrez, A., & Planas, N. (Eds.). *Proceedings of the 45th conference of the international group for the psychology of mathematics education* (Vol. 3, pp. 155-162). PME.
- Makramalla, M., le Roux, K., Rubel, L., Chronaki, A. & Guitierrez, R. (2023). Social just us: Towards the socio-ecological in and through mathematics education. In R. Marcone, P. Linardi, R. Milani, J.P.A. de Paulo, A.M. Queiroz, M.T da Salva (Eds.) *Proceedings of the 12th International Conference in Mathematics Education and Society* (Vol 1, pp. 212-217). MES.
- Nolan, K. (2020). Conceptualising a methodology for reframing mathematics/teacher education through a new (disruptive) form of culturally responsive pedagogy. In P. White, R. Tytler, J. Ferguson & J.C. Clark (Eds.). *Methodological approaches to STEM Education Research* (pp. 133-154). Cambridge Scholars Publishing.
- Walker, M. (2012). A capital or capabilities education narrative in a world of staggering inequalities. *International Journal of Educational Development*, 32(3), 384-393.
- Popkewitz, T. (2014). *The reason of schooling: Historicizing curriculum studies, pedagogy and teacher education*. Routledge.

AUTHENTICITY AND MATHEMATICAL CONTENT IN CONTEXTS OF MARGINALIZATION: A PROSPECTIVE TEACHER'S SHIFTS

Sigal-Hava Rotem¹, Alf Coles²

Utrecht University, ²University of Bristol

This paper aims to explore how aspects of authenticity and mathematical content come into play when a prospective teacher designs and teaches a task relating to socio-ecological issues central to her life. The project, from which we report, aims to broaden whose stories are included in mathematics education. We investigate Nadin's participation in a task design-teach-reflect cycle, which includes: 1) taking a photo of an issue from her everyday life and discussing it in class; 2) making a collaborative decision with 3-4 peers on an issue around which they would like to work, designing a mathematical task relating to that issue, and having a peer-assessment of that task; 3) teaching the task they designed to a group of students, and reflecting on their teaching and learning. The data analysis illustrates Nadin's shifts between aspects of authenticity and mathematical content. We show how Nadin, who is a prospective mathematics teacher from a marginalized community in Israel, the Bedouin community, shifts to introducing more authentic issues from her life, e.g., religion and traditions, while also thinking about the mathematical content she wishes to teach her students.

RATIONALE

Recent trends in mathematics education call for teaching mathematics while engaging students in tasks that introduce the use of mathematics in different real-life contexts. Typically, marginalized communities' challenges cannot be found in common textbooks and school materials worldwide (e.g., Gutiérrez, 2013). Mathematics offers a way to understand and act upon socio-ecological challenges (Barwell et al., 2022), by transforming local socio-cultural and environmental issues into mathematical tasks that could be used in mathematics lessons. One repeatedly reported challenge of designing mathematical tasks around socio-ecological issues, in a way that also promotes action, is to negotiate authenticity with meaningful mathematical content (e.g., Marco & Palatnik, 2024). Indeed, some tasks are connected to reality in a way that can be easily ignored, such as “wrapper problems” (Stillman, 1998) (i.e., once the mathematics is unwrapped, the context can be left behind), while others may be genuinely authentic but with oversimplified mathematics so, again, when solving the task, students are not meaningfully engaged with the original context.

This study is part of a larger research project in which we aim to bring authentic real-life socio-ecological issues and mathematical content closer together by asking prospective teachers (PTs) from marginalized communities to design and teach tasks around local issues from their communities' lives. We are interested in “Whose stories/histories are included in mathematics (education), at what scales, and how does that relate to who is considered in the presents or futures of mathematics (education)?” (ICMI Study 27, Discussion Document, B1.1). We explore how PTs might come to view their own local, or global, concerns as relevant to their futures as mathematics teachers. The participants of the extensive research are twenty-three PTs pursuing their B.Ed. in secondary school mathematics at Achva Academic College, a regional college in Israel. The participants are from two communities: the Jewish and Bedouin communities. Many live in the socio-economic periphery and are first-generation in higher education. i.e., the first in their family to earn a post-secondary school

degree. Notably, students from the Bedouin communities (who are Muslim) live in villages in the desert; some are not recognized by the state, i.e., their villages lack basic amenities such as water, permanently built houses, and so forth. Here, we take the case study approach and focus on one PT, Nadin, a PT from the Bedouin community who lives in a Jewish city in the desert (and belongs to a minority community within that city). Nadin is an expressive young woman, both in writing and in class. Additionally, she was motivated to engage in the design-teaching-reflect cycle. Indeed, her expressiveness and engagement in the cycle made her shifts in the cycle most noticeable, which in turn allow us to shed light on shifts that might occur in terms of authenticity and mathematical content when a PT designs-teaches-reflects on tasks centered on socio-ecological issues central to their lives.

We see our work as a starting point allowing us to recognize shifts through this cycle, which then will serve as the basis for building a model to further analyze real-life authentic mathematical tasks, gathered through similar design-teach-reflect cycles, among other marginalized communities.

LITERATURE BACKGROUND

Recently many countries are changing their mathematics curriculum and textbook to include more authentic mathematical tasks. These changes are pushing forward mathematical problem-solving as a way to address real-life problems while also raising awareness toward current burning issues such as the climate crisis and the lack of social justice (Santos-Trigo, 2024). Indeed, mathematics can be seen as a way to (i) “picture” socio-cultural and environmental issues by portraying these issues through modelling, (ii) “constitute” the issues by offering models that affect action in real-time, and (iii) “format” the issues through a model that allows realization and actions regarding these issues, which in turn formats both how we understand these issues and act upon them (Skovsmose, 2021).

Yet, in order for classroom teachers to use authentic real-life problems in their mathematics classroom, they need to see the importance and benefits of using them. It seems that a PT’s preparation program is a good starting point to learn about new reforms in mathematics education. Additionally, and even more importantly, for pushing forward mathematical problem-solving as way to address real-life problems while also raising awareness toward socio-ecological issues, these issues should be central to the PTs’ lives. However, marginalized communities’ issues do not appear in traditional authentic real-life mathematical tasks, as tasks often reflect designers’ ideas. Our work aims to surface the stories and concerns of marginalized actors and support their relevance for the future of the teaching and learning of mathematics.

Teacher as tasks designers

Our literature review indicates that there are not many studies where teacher (in-service and PTs) design tasks concerning socio-ecological issues taken from their lives, and then teach them in their own classrooms. Marco and Palatnik’s (2024) study is one of the few studies addressing teachers’ task design. Broadly, their characterization of teachers’ design can be divided into two types: (1) the task’s mathematical content, such as the mathematical diversity of topics within the different subsections of the task, the level of generalization the task requires, and the level of precision the task requires; (2) task authenticity in terms of its relevancy while also being on a non-artificial topic and which is original in the sense that is not overused in textbooks. Here, we draw on their work by making these characteristics a lens to analyze and explore shifts in Nadin’s designing and teaching a task centered on socio-ecological issue she chose. This designing, teaching and reflecting was done

in a cycle inspired by the work of Clarke and Hollingsworth (2002) who showed the benefits of combining practice and reflection in a cycle as part of teacher professional growth. Hence, here we call this a *designing-teaching-reflecting* cycle, and we explore how aspects of authenticity and mathematical content come to play around socio-ecological issues central to her life. We recognize there are often tensions when teachers aim to bring authentic questions into the mathematics classroom. Consequently, here we ask: *How do aspects of authenticity and mathematical content come into play when prospective teachers design-teach and reflect on a task around socio-ecological issues central to their lives, and what tensions are visible?*

The Bedouin Community In Israel

As Nadin is a part of the Bedouin community in Israel, we provide here some context details. This section is based on Amit and Abu Qouder's paper (2017), which are among the only scholars writing about the Bedouin community in Israel in connection to mathematics education. The Bedouins are part of the Arab population from Mandatory Palestine who remained in Israel after its establishment in 1948. As Muslims, they are considered an ethnic minority in Israel, distinct both geographically and culturally from other Israeli Arabs and Jews. Most Bedouins in Israel reside in the Negev desert. For centuries, they led a traditional lifestyle centered around seasonal migration. However, in recent years, their way of life has notably changed as they transitioned to permanent settlements, influenced by their proximity to other populations and state regulations restricting movement between temporary settlements. This transition is a gradual move towards Israel's modern mainstream, yet the Bedouin community is still quite distinct from the rest of the country. The Bedouin villages in the Negev are from the lowest socio-economic scale in Israel. Further, the percentage of Bedouin holding a post-secondary degree is lower than other communities in Israel. Achva Academic College admits students from the Bedouin community to study toward a Bachelor in Education without considering scores in matriculation exams, depending solely on their college's scores, while providing appropriate support.

METHODS

The study was conducted in the context of a problem-solving year-long course in a B.Ed. program for secondary school mathematics at Achva Academic College. The course included 20 lessons of 1.5 hour each and was taught by the first author. It was the participants' third course on problem solving in the B.Ed. program. The course focused on solving problems that align with the new reform in secondary school mathematics, which assert that an important goal of the program is to "create interest and a sense of relevance of mathematics in them [the students]. The program is based on 'mathematics in everyday life'" (Ministry of Education, 2023). Further, we wanted the tasks the PTs designed to reflect their community's everyday concerns and life. Research shows that the photovoice method (Liebenberg, 2018) allows participants to "provide insight into stories of mathematics that are not typically recognized in classrooms" of marginalized communities (Hunter, 2022). Thus, the photovoice method was the first phase in each of the course' design-teach-reflect cycles (Clarke & Hollingsworth, 2002). The course included two design-teach-reflect cycles (Clarke & Hollingsworth, 2002). Each cycle duration was one semester (10 lessons) and comprised of the following three major phases: 1) taking a photo (photovoice method: Liebenberg, 2018) of an issue from everyday lives and discussing it in class; 2) making a collaborative decision with 3-4 peers on an issue around which they would like to work, designing a mathematical task around that issue, and having a peer-

assessment of that task; 3) teaching the task they designed to a group of students, and writing an individual reflection on their teaching and learning.

The participants of this course are 23 pre-services students (eight students from the Jewish community and 15 from the Bedouin community). Here, we focus on Nadin, a PT from the Bedouin community. We noticed the most significant shifts in her approach to the work, compared to the other PTs.

Data collection and Analysis

In this case study we focus on Nadin’s first design-teach-reflect cycle. We collected all the data regarding every part of the cycle. This data collection yielded the data corpus detailed in Table 1.

Cycle’s phase	Data that was gathered in this phase	Data format
Photovoice: taking a photo of a socio-ecological issue	Photo	Two pictures
	A discussion around the photo in class	Lesson transcripts. An excerpt of four turn-talking between Nadin and the teacher educator.
Choosing an issue and designing a task	The mathematical task	One written task
	Peer-assessment discussion in class	Lesson transcripts. An excerpt of 133 turn-talking between Nadin, the teacher educator and the other participating PTs.
Teaching a lesson around that task and reflecting on their experience	A description of Nadin’s lesson around the task and her written reflection on it	Written report (inspired by Rotem & Ayalon, 2022) including description of the classroom occurrences and analysis in terms of the students’ thinking and teacher’s responses.

Table 1: Data collection according to the course cycle and data format.

As seen in Table 1, the data collection varies in its format. Thus, to allow the data systematic analysis we ordered the data from all phases in their written formats (i.e., the transcripts and the written reflection) and “chunked the protocol” according to idea unit (Bamberger & Schön, 1983). Each idea unit is about authenticity, mathematical content or both. Overall, this process yielded 50 units. Then we coded each idea unit according to its authenticity and mathematical content (Table 2). As authors, we discussed the attribution of these characteristics to the wider set of data and agreed on our analysis of the work of Nadin, described here, in terms of mathematical content and authenticity.

Authenticity	Aspects concerning relevancy, appeal and students’ motivation, while also non-artificial topics that are original in the sense they are not overused in textbooks.
Mathematical content	The mathematical content is defined by diversity, generalization, and precision. <ul style="list-style-type: none"> - Mathematical diversity is defined by the various topics combined in the task. - Generalization is defined by the task request from students to “formulate a general statement or concept that generalizes a relationship between some mathematical entities (e.g., quantities, variables, shapes)” (p. 236). - Precision is defined by how accurate or estimated the answer is required by the task. Do students perform some evaluation in the solution process? Or do they primarily expected to provide a single precise answer?

Table 2: Definitions of authenticity and mathematical content (following Marco & Palatnik, 2022)

RESULTS

Our data analysis process indicates that there are places where Nadin seems to negotiate between authenticity and the mathematical content. We regard these places as shifts. The first shift occurs from the first to the second phase, and the second shift occurs between the second and the third.

Initially, in the photovoice phase, Nadin chose a Tom and Jerry picture she found on-line. After the teacher educator (the first author) urged her to take the photo herself, Nadin changed the photo to two images of real cats. In the class discussion in the course, she explained her photo decision: “Okay, so here you can see that we have two cats, which I noticed that they are both looking for something, so I connected this search and Tom and Jerry’s chase together, why specifically did I want to connect Tom and Jerry? [...] I always have in mind this connection between my everyday life and, with a little laugh, and mathematics. So, I thought it would be a direct and cute connection, to connect cats and Tom and Jerry and math.”

In terms of authenticity, it seems that Nadin thinks that this issue is appealing and motivating for students. She is not addressing whether this is an artificial topic. Nadin thinks that this is something casual in children’s everyday lives, that could be fun for them and to which they can relate. The underneath issue has no mathematical content, but it seems from the excerpt that Nadin keeps in mind that soon she will connect this to secondary school mathematics. The theme of Tom and Jerry is a relatively globalized one and, given our research question, we see no link to Nadin’s local community.

First shift – From the first to the second cycle’s phase

In the second cycle, Nadin presented a task around Ramadan (Figure 1). It includes an introduction section, which gives a background story about Sarah and her family. Similar to Nadin’s students, Sarah is a 7th grade student, and the three tasks are about her life and her family’s during Ramadan.

[...] Sarah’s family gathers every evening for the fast-breaking dinner. Sara [...] has 4 brothers and 5 sisters.

Section A: In the middle of the fasting month, the family decided to order a family-size pizza so that there would be enough for everyone. Therefore, they asked for a 45.72 cm pizza diameter. They want to divide the pizza equally among the family members. Thus, the question arises, what is the amount of pizza that each family member will receive?

Section B: During Ramadan, Sarah wants to calculate her total fasting time. She plans to fast from sunrise to sunset every day. To figure out the total fasting time, she considers the varying length of daylight. • On the first day of Ramadan, the daylight length is 12 hours and 30 mins. • On the second day, there is an increase of 3 mins during the daylight. • On the third day, there is a decrease of 5 mins and on the fourth day, the daylight length increases by 7 mins. How many hours and minutes did Sarah fast in total during the first four days of the fasting month?

Section C: 1. The number of pages of the Koran is 604 pages, Sarah reads 7 pages during the day and 8 pages after breakfast. How many days will it take Sarah to complete the Quran in the month of Ramadan? 2. And if Sarah is unable to complete it during the month of Ramadan, how many pages will Sarah need to read a day as to complete reading the entire Qur’an? 3. After breaking the fast, Sarah eats 3 dates. How many dates does Sarah eat a week? [...] month?

Figure 1: Nadin’s task around Ramadan

In terms of authenticity, Nadin sees the relevancy of Ramadan to her life and to the students’ lives. She explained the decision to design a task around Ramadan both in the peer-assessment lesson and in her report, where she described the lesson she taught. In the peer-assessment lesson Nadin said:

I introduced [in the task] the month of Ramadan, why? Because the month of Ramadan is very soon, and I really want [to have] something realistic from my world, and the students’ world.

Here we see Nadin’s own concern to surface voices and stories, perhaps rarely present, in the mathematics classroom. Given the relative lack of such stories, we view her shift as significant. She reflects on the task’s authenticity in her reflective writing after teaching:

The influence was that we had to design problems that are taken from the students’ reality, related to the curriculum, that is, not to ask them for a formula... In addition, to look for interesting topics related to each student, we therefore came up with the idea of connecting the family, and religious obedience and worship, and food culture and fasting hours in Ramadan.

Here she addresses other aspects of authenticity, besides relevancy. She sees this issue as appealing and motivating for students, while also being on a non- artificial topic and which is original in the sense that is not overused in textbooks.

In terms of the mathematical content, the task has mathematical diversity as it introduces two mathematical topics combined in the task, fractions and the area of a circle while the other sections can be seen as word problems that also deal with conversions. The task does not require any generalization, but rather includes small calculations with great precision.

Second shift – From the second to the third cycle’s phase

The second shift occurred when Nadin used this task in class. As mentioned above, Nadin reported on her experience teaching the task she designed in a structured form (i.e., Rotem & Ayalon, 2022). Nadin started her report with a general description of how she introduced the task – asking students “to go through the problems, and anyone who comes up with an idea, a blessing, habits they do in Ramadan, go to the board and draw what they think.” Figure 2 presents the students’ drawings.

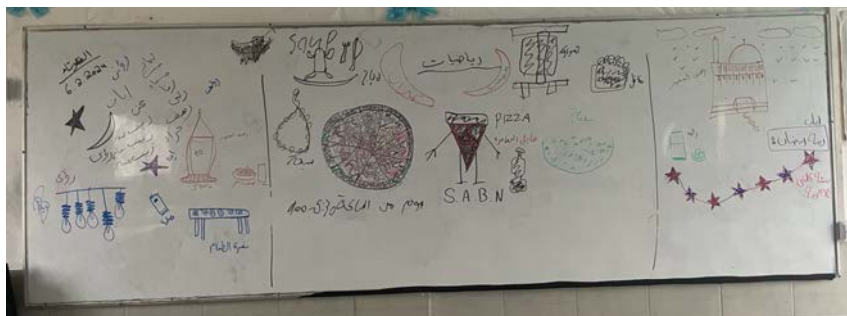


Figure 2: The students’ drawing on the board of Ramadan themes

After presenting the students’ drawing on the board, Nadin described the moment where she noticed students’ statements to further build their mathematical understanding (Table 3).

Speaker	Contributions
Student	Teacher, I calculated the hours Sarah fasted in the first four days of Ramadan and got 48 hours and 146 minutes...
Teacher	... [and] how many minutes are there in an hour?
Student	Has 60 minutes.
Teacher	Great, now in the exercise you got 146 minutes, and you know very well that there are 60 minutes in an hour, what does that imply to you?
Student	Maybe we can divide 146 minutes by 60 minutes? But, wait ... it comes out to 2.43333333. What is an hour anyway??

Table 3: Nadin’s description of the lesson she taught around the task she previously designed

Nadin’s lesson has two parts. The first is about the authenticity, as the students draw Ramadan related aspects on the board. In that part, Nadin tries to make the task relevant, appealing and not artificial to the students. In a way, this section echoes the photovoice phase in the beginning of the cycle. This part has no mathematical content. The second part is about the students’ mathematics where a student had difficult time to convert minutes into hours. Here, precision is required and there is no mathematical diversity. Yet it seems that the student is struggling with the generalization of the conversation, going back and forth between the general way of converting minutes to hours (by dividing by 60 minutes per hour) and the particular number she received. Her struggle comes forward as she says, “what is an hour anyway?”. We view the comment as an example of “evaluation” (Table 2) and the student wanting to make sense of a mathematical answer that is provoking some tension.

Although it seems that the two parts of the lesson are in tension, in Nadin’s experience teaching the task, she appreciates the potential Ramadan has in terms of mathematical content. This comes forward in her reflection after the lesson she taught. Nadin wrote in her written reflection:

The month of Ramadan will be in about two weeks, and that’s all we discuss [...] at home, at school, and also at work. Planning what to do, how to invest in ourselves more this month [...] in the month of Ramadan [...] you can see that there are many numbers, and you can build all kinds of math exercises on them. For example: How many prayers a day? How many will there be during the month? How many total hours do you fast during the month? How many pages are there in the Qur’an? How many pages did you read [...]

Overall, we regard in these two shifts as Nadin’s transitioning from a purely authentic theme she chose, cats, to thinking about how to merge the two aspects together, authenticity and the mathematical content, as a first step in introducing socio-ecological issues in a genuine way for learning in the mathematics classroom. We discuss this transitioning and its meaning next.

DISCUSSION

Because the scale of the socio-ecological issues that Nadin brings is local to her marginalized community, this study’s possible contributions are a starting point towards the potential for promoting social and ecological justice among those communities – particularly among the Bedouin community, whose stories, along with other marginalized communities, receive little attention from the mathematics education research community. Nadin’s initial idea was a context (cats) that was authentic to her life, but perhaps (considering the beginning with Tom and Jerry) not deeply felt, and where there were no obvious links to mathematics. Her move to consider Ramadan provoked the two shifts we document. Firstly, her task design included some mathematical diversity (division of area, calculation of day lengths, etc). Secondly, in her actual teaching, she invited the students to “go to the board and draw what they think” as they worked on the exercises about Ramadan. The invitation was taken up by students (Figure 2) and offers one image of authenticity in a mathematics classroom, in the sense of relevance, appeal and motivation. There remain tensions in terms of aims and the integration of context with mathematics. The students were asked to work on what could be seen as “wrapper tasks” (Stillman, 1998), where the mathematics is unravelled and the context is left behind, and yet there was the invitation to pay attention to what associations arise. The sacred nature of Ramadan is given space through the open invitation to draw on the board. Introducing the sacred into the mathematics classroom invokes a tension of scales, as learners are invited to bring their own personal work into contact with something beyond the individual. We want to argue that the sacred is relevant to the socio-ecological and, in fact, can offer an example of (human/human as well as

human/non-human) relationships that are non-exploitative and supportive of flourishing. In other words, invoking students' sense of the sacred in mathematics seems to us to offer a potential route into mathematics becoming a discipline that does not alienate learners from the living world.

We believe we have evidence of the effectiveness of photovoice, and a design-teach-reflect cycle, in terms of prompting PTs to consider how socio-ecological issues, relevant to historically marginalized communities, might enter their own mathematics teaching. Task design is a complex skill, even with a focus solely on mathematical content. Task design which allows mathematical development and combines authentic problems is even harder. So, while we might question different aspects of what Nadin has done, we find hope in the fact that she has brought an issue she cares about deeply into the classroom and in the fact that her students have responded by offering images from, we assume, deeply important aspects of their lives. We are continuing to explore the potential for using photovoice with PTs, aiming at generalizing our work. Nadin's work serves us as a starting point, offering an image of how the sacred might be surfaced in a mathematics classroom.

References

- Amit, M., & Abu Qouder, F. (2017). Weaving culture and mathematics in the classroom: The case of Bedouin ethnomathematics. In R. Milton, S. Lawrence, M. E. Gavarrete, W. V. Alangu (Eds.), *Ethnomathematics and its diverse approaches for mathematics education*, (pp. 23-50). Springer.
- Bamberger, J., & Schön, D. A. (1983). Learning as reflective conversation with materials: Notes from work in progress. *Art Education*, 36(2), 68-73.
- Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013.
- Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, 18(8), 947-967.
- Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, 44(1), 37-68.
- Hunter, J. (2022). Challenging and disrupting deficit discourses in mathematics education: positioning young diverse learners to document and share their mathematical funds of knowledge. *Research in Mathematics Education*, 24(2), 187-201.
- Liebenberg, L. (2018). Thinking critically about photovoice: Achieving empowerment and social change. *International Journal of Qualitative Methods*, 17(1), 1609406918757631.
- Marco, N., & Palatnik, A. (2024). Teachers pose and design context-based mathematics tasks: what can be learned from product evolution? *Educational Studies in Mathematics*, 115(2), 223-246.
- Ministry of Education (2023). Introduction for the mathematics curriculum for 3 point. Retrieved from [mavo-math.pdf\(education.gov.il\)](https://mavo-math.pdf(education.gov.il)) (in Hebrew).
- Rotem, S. H., & Ayalon, M. (2022). Building a model for characterizing critical events: Noticing classroom situations using multiple dimensions. *The Journal of Mathematical Behavior*, 66, 100947.
- Santos-Trigo, M. (2024). Problem solving in mathematics education: tracing its foundations and current research-practice trends. *ZDM—Mathematics Education*, 1-12.
- Skovsmose, O. (2021). A philosophy of Critical Mathematics Education. *Philosophy of Mathematics Education Journal*, 37, 1–24.
- Stillman, G. (1998). The emperor's new clothes? Teaching and assessment of mathematical applications at the senior level. In P. Galbraith, W. Blum, G. Booker, & D. Huntley (Eds.), *Mathematical modelling: Teaching and assessment in a technology-rich world* (pp. 243–253). Horwood.

EXPLORING CHILDREN'S SPATIAL KNOWLEDGE THROUGH MAP MAKING FOR REPRESENTING THEIR TERRITORY

Armando Solares-Rojas¹, Tatiana Mendoza von der Borch²

¹Mathematics Education Department, Cinvestav, ²Department of Psychology and Education,
Autonomous University of Querétaro

Our research examines how children construct meanings about their territory when faced with disputes and displacement due to socio-ecological devastation. To do this, we took a territorial and spatial approach and asked elementary school students to create maps of their communities' environment. We focused on the work of children from two schools in the Atoyac River basin. Our initial findings indicate that this mapping process helps children grasp their connection to their surroundings and encourages the use of mathematical concepts related to space, significantly when their territory is affected by socio-ecological devastation.

INTRODUCTION AND PROBLEM

The crisis produced by climate change, biodiversity loss, and increasing chemical pollution has produced several effects on human and non-human systems. In the Latin American region, more than 650 million people are facing the pressures of poverty, no health access, industrialization, loss of ecosystems, no water supply, pollution, and corruption, amongst others (Carriquiriborde et al., 2014).

The research we present is situated in the heart of one of Mexico's most polluted regions, the Atoyac River basin, in central Mexico, near Mexico City. The environmental degradation here is alarmingly evident, primarily due to the excessive pollution levels stemming from the discharge of wastewater from the region's factories and urban centers. The toxic substances, including benzene, toluene, and xylenes, dumped into the Atoyac River have led to severe health issues such as teratogenic damage, increased genotoxicity, and oxidative stress in both children and adults (Montero et al., 2020). This socio-ecological crisis has grave implications for the health of the Atoyac River basin communities. Figure 1, a mortality risk map, illustrates the path of the Atoyac River and how it carries the pollution.

Building from the experience of the Atoyac River Memorial Museum school project (Solares-Rojas et al., 2022), this study articulates and gives continuity to the actions of local environmental networks and organizations. We intend to explore the meanings children construct about their territory when it is the object of dispute and dispossession due to environmental devastation by creating maps at school. We seek to detonate and guide discussions about the contributions of these mapping processes to understanding relationships with the territory and, in turn, to the meaningful learning of spatial mathematical notions.

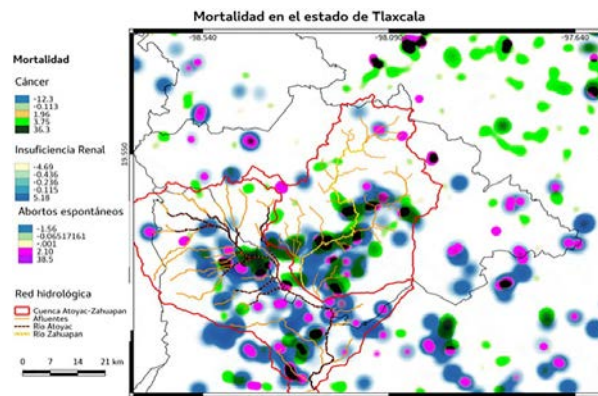


Figure 1. Mortality risk map of the Atoyac basin, Mexico (cancer, renal insufficiency, spontaneous abortion) (Rosado Zaidi, 2018)

The notion of territory allows us, on the one hand, to consider communities' stories, myths, emotions, and, in general, the network of dense relationships between communities and human and non-human entities (e.g., animals, plants, rivers, mountains, spirits); and, on the other hand, to build bridges with the spatial mathematical knowledge required in the construction of representations of socio-ecological spaces (environmental maps).

PAST RESEARCH ON MATHEMATICS EDUCATION AND COMMUNITY MAPPINGS EDUCATIVE PROJECTS FOR DEALING WITH ECOLOGICAL ISSUES

The past two decades have seen the emergence and growth in education scholarship that attends to and conceptualizes the “ecological” and its relation to education and society. Below, we briefly review recent research on mathematics education and community mappings educative projects.

Mathematics education and the socio-ecological: an emerging research area

We adhere to the current efforts that, from very different theoretical perspectives, prompt an active role of mathematics education in attending to the current socio-ecological crises (Coles et al., 2024). Critical mathematics education studies have approached the impact of the social world on the ecological (e.g., Steffensen et al., 2023). Concerning mathematical modeling, some studies promote modeling as a practice for critically understanding the world, focusing on social inequities and exploring human action on ecologies of the planet (e.g., Yanagimoto & Yoshimura, 2013). Perspectives from Ethnomathematics and Indigenous knowledge have also dealt with how diverse socially and culturally situated ways of knowing and doing mathematics are used in social, cultural, political, economic, and environmental practices (e.g., Eglash, 2023; Gutiérrez, 2022; Kulago et al., 2021). These studies offer our project directions on how mathematics can support understanding the issues and taking an active role in the defense of culture and ecology.

Communities' territory knowledge and community mappings

Community-based organizations have implemented various actions to address environmental issues, including community mapping, social cartography, and participatory mapping activities. These activities highlight the connections between communities and their territories, aiming to provide communities with the tools to protect their territories and propose educational programs to preserve and restore people's cultural, historical, social, and environmental connections with their communities' environment (Briseño-Roa, 2023). We benefit from the numerous educational

proposals for participatory mapping developed in America and Latin America for school and non-school spaces, which generally seek to bring to light, recover, and appreciate the knowledge of diverse communities, including children and young people, through social and cultural geography approaches (e.g., Bautista et al., 2021).

CONCEPTUAL FRAMEWORK

To deal with the research problem, in this paper we construct a conceptual framework consisting of two main components: a territory approach and a “spatializing” approach for school mathematical knowledge.

Territory, socio-ecological problems and mapping

We make use of the notion of territory, understood as the web of relationships between people and their environment, which includes other people, animals, hills, water, land, and also the symbolic world, inseparable from the material world (Haesbaert, 2011).

From the “Atoyac River project”, we know that mapping is a point of confluence of curricular knowledge (contents on science and environment, history, geography, and mathematics), scientific knowledge (risk communication maps) and community knowledge (narratives of the places and environments that make up their local cultural history). The notion of territory provides us with an arena to bridge the community territorial relationships and the representations of socio-ecological spaces for giving account of specific local socio-ecological.

Space and territory

We propose to use mapping activities to address socio-ecological problems from a territorialized education point of view. By incorporating educational activities centered on mapping, we aim to utilize school knowledge, such as mathematics and environmental sciences, to study local socio-ecological problems. This approach will integrate people’s connections to the environment, their narratives, and power dynamics (Rubel & Nicol, 2020). To achieve this, we will draw on previous research in mathematics education that explores the potential of linking the creation of maps and environmental drawings with mathematical spatial concepts (Gálvez, 1985).

METHODOLOGICAL ASPECTS

The communities and schools of the study

Our study includes schools from three towns along the Atoyac basin. The first school is in San Rafael Tenanyecac, in the municipality of Nativitas, near the Atoyac industrial zone. The productions of these children come from the writings, audio recordings, and drawings they did for the Atoyac River Memorial Museum school project.

Besides, we went to two primary schools along the basin: one in the Cuatla neighborhood in Teolocholco and another in San Mateo Huexoyucan in the mountains of the Panotla municipality (see Figure 2). We collected data through field notes and photographs of children’s production during three visits (three days in March 2023, and one week in June 2023, and one week in June 2024).

We used ethnographic techniques to observe classroom lessons and talk with teachers about environmental problems and how they address them in their classrooms. We also proposed activities for students to describe their environment and build maps.



Figure 2. San Rafael, Cuatla, and San Mateo, three towns along the Atoyac River in Tlaxcala. Map created by Erik Said Lara Corro

FIRST RESULTS

In a previous paper (Mendoza & Rockwell, 2023), we presented students’ narratives on the rivers within the Atoyac Basin. The study aimed to gain insight into their understanding of their communities’ socio-ecological state. In the current study, we are taking a step forward by developing a territoriality and mathematical spatialization framework to explore students’ knowledge further and to develop (in the future) educational activities that integrate children’s, community’s, and school’s knowledge. For this paper, our analysis centers on the work of students from two schools in San Rafael and San Mateo.

The children of San Rafael Tenanyecac: The Atoyac River between environmental deterioration and the historical memory of the community

The children of San Rafael Tenanyecac describe a river with black or dark blue water, which has turned into a big garbage dump where there are dead animals (see Figure 3), and where “everyone who walks by runs away because of its awful smell.”



Figure 3. Drawing by a third-grade student for the Atoyac River Memorial Museum. Retrieved from: <https://red-comunidadcienciaeducacion.org/index.php/museo/>

Thanks to the stories their parents, aunts and uncles, and grandparents tell them, they know the river has not always been this way. A boy wrote: “The river that was so beautiful was lost.” In their texts and drawings, the children explain that, up until four decades ago, the families cooked meals from the river animals, like the crayfish known as *acociles* that are endemic to Mexico. Additionally, they had domesticated animals that they would take to the river to graze and drink water. The families also ate the native plants, such as the wild black cherries, and their vegetables. The children drew the tools and objects that were once used for planting, including the armadillo shell for gathering seeds. There “even the sun was happy,” wrote a girl.

The children talk of their grandparents’ nostalgia and fondness for the times before the Industries arrived. They express their sadness for not having had experiences like those of their grandparents, how they would have liked to have dipped their feet in the water that, as shown in many images in the churches of Tlaxcala, had once been considered miraculous.

The views of the children of San Mateo Huexoyucan: The Atoyac River as a flow of crystal-clear water and life in nature

The two-teacher primary school of San Mateo, in the municipality of Panotla, is in a very green, hilly region less than 30 kilometers from San Rafael. It has a group of first to third graders and another one of fourth to sixth graders. When asked what it is like to live there, the first thing the older students replied was that, at home, they have bulls, horses, sheep, fighting cocks. Suddenly, a boy shouted: “There’s a river!”

The two groups mentioned the river very quickly. They all shared that they go there often, that they celebrated Father’s Day there, that they get in the water: “It’s really nice. I feel free here.” They pointed to several places to show where it runs by: “There’s a river over here... Another one over there... A waterfall that way... There’s another one, but I don’t know where.” In fact, numerous tributaries of the Atoyac River flow down to San Mateo, so many that their names are not even known.

Researcher: And is the river far from here?

Student: You can get there in five minutes.



Figure 4. Map of the San Mateo Huexoyucan community (the waterfall, the kindergarten and elementary schools, the church and the medical center)

The children gave many directions, making gestures with their arms and explaining how to get there from the school: “...then you go down... Cars can go up to this point, and then you have to walk, or

you slide down...you turn at don Antonio's. There, you'll see a dog tied up, and that's where my house is, and from there you keep going..."

When asked to draw a place they like to go, almost all of them chose the river (for instance, see Figure 4). They put rocks in it and trees that provide ample shade. "You forgot the sun!" one girl told another. Suddenly, one of the younger girls changed her mind and decided to draw the beach; she had never been there but said she would really like to go. She drew sea shells, sharks, turtles, and palm trees. The children spoke a lot about the animals in San Mateo. They knew the names of many birds and, with the tone and expressions of someone who knows it is a privilege, shared that sometimes they come across eagles. In the group of younger students, they all cut in to share about some kind of encounter with snakes, highlighting different details from those encounters. One girl stressed that she was the one who saw the snake before her aunt, who was with her, and that they decided to take off running because if they threw a rock at it, it would get mad and chase them. At recess, several children went to play in the black cherry tree that was planted next to the school's courtyard: a tree that has been part of the region's flora since before the Spanish conquest.

DISCUSSION AND FINAL REMARKS

The study we are presenting is ongoing. The first results show us the mathematical knowledge and practices that children put into play when constructing maps of their environment. Among them, we find *spatial notions* (Gálvez, 1985) that children use to locate places based on the route to get from one place to another. For example, they locate places by describing the route from the waterfall to the primary school, following the course of the river waters, "You should go down from the waterfall, and first you'll find the kindergarten...". They also locate places based on positional relationships; for example, to say that "the kindergarten is between the primary school and the river."

Another mathematical notion is that of *closeness*, not metricized yet. For example, in San Rafael, proximity to the river becomes problematic; third-grade children point out: "Everyone runs when they pass (near the river) because of its stinky smell." Moreover, a girl asks one of the researchers from the Atoyac River project, "Could it be that my cousin has leukemia because he lives so close to the river?" In the second-grade group, a girl comments to the teacher and her classmates, "Did you know that there are other continents where there are rivers that even have little fish?" In this case, the vast distance illustrates the magnitude of the environmental problem: a distance as great as that between continents is necessary for a river to have fish.

We often find children's considerations of size when deciding what places to include in the map. Children often ask, "Will it fit on the map?". We observe a potential dialogical relationship between the territory and a specific mathematical concept, *scale*, which emerges qualitatively before being quantified with a numerical value. For instance, some children choose to include the Malintzi volcano on their map, even though it is more than a hundred kilometers away, even though the "canonical mathematical scale" would not allow it to "fit" on their map.

Behind these considerations lies a characteristic of socio-ecological problems: these problems disrupt the social fabric of everyday relationships between people and their immediate environment and are also intertwined with regional, national, and global dynamics. A didactical opportunity to support significant meaning for the mathematical ratio between sizes emerges in these relationships between local and global scales.

Unlike some positions that conceptualize “space” without the presence of people, their stories, or their relationships, in this study, children construct maps to speak, represent, and think collectively about their territory. We focus on how children construct maps to express and understand their experiences with others, such as people, animals, hills, and rivers. We follow Rubel and Nicol’s (2020) proposal to use mathematics to explore spatial and social relations shaped by the place where people live and the people who create that place.

Based on these initial and exploratory results, we plan to continue our study by designing educational activities that, on the one hand, allow students to learn and consolidate mathematical knowledge (like spatial location, proximity, and scale) through the construction and interpretation of maps to represent their territory; and, on the other hand, activities that involve the exchange of maps between children from different communities in the Atoyac, taking into account the differences, similarities, and risks of the socio-ecological crisis that affects them. We seek to provide tools to empower them to take an active position in protecting and recovering their environment.

Acknowledgements

We acknowledge Conahcyt-Mexico for Tatiana Mendoza’s postdoctoral stay grant (CVU167541). And to the Engineering and Physical Sciences Research Council–United Kingdom Research and Innovation (Grant Ref: EP/T003545/1) for the founding of the “Atoyac River project” through the Community, Science and Education Network.

References

- Barwell, R. (2013). The mathematical formatting of climate change: Critical Mathematics Education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16
<https://doi.org/10.1080/14794802.2012.756633>
- Bautista, A., Pedraza-Jiménez, Y., & Díaz-Marquez, F. (2021). Reconocimiento de los saberes campesinos a través del mapeo comunitario [Recognition of peasant knowledge through community mapping]. *Cuadernos Geográficos*, 60(2), 297-313. DOI: <http://dx.doi.org/10.30827/cuadgeo.v60i2.9647>
- Briseño-Roa, J. (2023). Indigenous land-based education in Mexico: practices to perpetuate communal knowledge of territory. *AlterNative: An International Journal of Indigenous Peoples*, 19(3), 593-602.
<https://doi.org/10.1177/11771801231188181>
- Carriquiriborde, P., Mirabella, P., Waichman, A., Solomon, K., Van den Brink, P., & Maund, S. (2014). Aquatic risk assessment of pesticides in Latin America. *Integrated Environmental Assessment and Management*, 10(4), 539–542. <https://doi.org/10.1002/ieam.1561>
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- Eglash, R. (2023). Ethno-biomathematics: A decolonial approach to mathematics at the intersection of human and nonhuman design. In M. C. Borba & D. C. Orey (Eds.), *Ubiratan D’Ambrosio and mathematics education: Trajectory, legacy and future* (pp. 289-303). Springer.
- Gálvez, G. (1985). *El aprendizaje de la orientación en el espacio urbano. Una proposición para la enseñanza de la geometría en la escuela primaria* [Spatial orientation learning in urban space. A proposal for teaching geometry in primary school] [Unpublished Doctoral dissertation]. CINVESTAV.
- Gutierrez, R. (2022). A spiritual turn: Towards desire-based research and Indigenous futurity in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 379-88.
<https://doi.org/10.5951/jresmetheduc-2022-0005>
- Haesbaert, R. (2011). El mito de la desterritorialización [The myth of deterritorialization]. Siglo XXI.

- Kulago, H. A., Wapeemukwa, W., Guernsey, P. J., & Black, M. (2021). Land, water, mathematics, and relationships: What does creating decolonizing and Indigenous curricula ask of us? *Educational Studies*, 57(3), 345-363. <https://doi.org/10.1080/00131946.2021.1892690>
- Mendoza von der Borch, T., & Rockwell, E. (2023). Ríos de memoria, pérdida y vida. Relatos de niñas y niños sobre la cuenca del Atoyac [Rivers of memory, loss and life. Stories from children about the Atoyac basin]. *Educa ren al diversidad*, 9, 13-24. ISBN: 978-987-813-662-2
- Montero, R. D., López, R., Méndez, A., Galicia, I., García, G., Serrano, L., Beltrán, R., Rosado, S., Albores, A., Oropeza, L., Hernández, L., Mercado, F., Alvarado, E., Herrera, S., & Arellano, O. (2020). Increased micronucleus frequencies in reticulocytes of children exposed to industrial pollution: Oxidative stress and the OGG1 S326C polymorphism. *Mutation Research*, 853, 503170. <https://doi.org/10.1016/j.mrgentox.2020.503170>
- Rubel, L. H., & Nicol, C. (2020): The power of place: spatializing critical mathematics education, *Mathematical Thinking and Learning*, 1-20. DOI:10.1080/10986065.2020.1709938
- Rosado Zaidi, S. J. (2018). *La transformación del hábitat humano en el modo de producción capitalista: una aproximación teórico metodológica de la fractura metabólica entre el sujeto y la naturaleza* [Human habitat transformation in the capitalist mode of production: A theoretical and methodological approach to the metabolic fracture between subject and nature] [Unpublished bachelor thesis]. National Autonomous University of Mexico.
- Skovsmose, O. (2023). *Critical Mathematics Education*. Springer International Publishing.
- Solares-Rojas, A., Arellano-Aguilar, O. García González, M. M., López-Vargas, M. R., Coles, A. & Méndez Serrano, A. (2022). Mathematics education and social-environmental crises: an interdisciplinary proposal for didactic innovation with rural communities in Mexico. *Research in Mathematics Education*, 24(2), 202-223. <https://doi.org/10.1080/14794802.2022.2062781>
- Steffensen, L., Herheim, R., & Rangnes, T. E. (2023). Wicked problems and critical judgment inschool mathematics. In N.S. Kennedy & E. Marsal (Eds.), *Dialogical inquiry in mathematics teaching and learning: A philosophical approach* (pp. 79-102). Lit Verlag.
- Yanagimoto, A., & Yoshimura, N. (2013). Mathematical modelling of a real-world problem: The decreasing number of bluefin tuna. In G.A. Stillman, G. Kaiser, W. Blum, & J.P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 229-239). Springer.

NETWORK OF COLLABORATIONS AND SOCIAL PRACTICES TO ENGAGE STUDENTS IN SOCIO-ECOLOGICAL ISSUES OF PEATLANDS

Lisa Steffensen

Western Norway University of Applied Sciences

Here, I focus on identifying qualities in a network of collaboration and social practices aiming to contribute to awareness about socio-ecological issues in peatlands. Theoretical perspectives involving networks of mathematics education practice and communities of practices are used to discuss the qualities. The context of this study is an interdisciplinary project about peatlands in lower-secondary schools, where four teachers and researchers design a teaching activity about peatlands' socio-ecological issues in students' municipalities. I discuss three identified qualities. The first is the joint enterprise, where collaborators have a shared understanding of their aims, and everyone considers it a learning situation. The second is mutual engagement. The various collaborators showed genuine enthusiasm throughout the teaching and learning. The third relates to a shared repertoire, where the collaborators developed and jointly pursued common resources and where the diversity of expertise contributed to relevant artifacts. A network of collaboration can contribute to students' engagement in socio-ecological issues; however, the qualities are essential to consider.

INTRODUCTION

Peatlands play an essential role in socio-ecological issues; they store vast amounts of carbon, provide a unique habitat for (specialized) plants and wildlife, and their capacity to hold water reduces the risk of flooding. However, peatlands have been degraded throughout history for reasons such as peat extraction for fuel or soil, drainage for agriculture, and infrastructure and housing. Mathematics can play an essential role in raising awareness of peatland's qualities, for instance, by providing estimates of peatland's carbon storage potential. Such estimation is not straightforward and requires knowledge about peatlands typically found in other subject areas, and most mathematics teachers probably would not know how to do this. However, suppose teachers engage students in estimating peatlands' carbon storage potential. In that case, this should not be done isolated without considering the full scale of socio-ecological issues involved in the degradation of peatlands. Historically, the economic benefits of using areas with peatlands for infrastructure or housing have often trumped preservation concerns. Therefore, students should also reflect on the complexity of the socio-ecological issues of peatlands by being encouraged to weigh various concerns and to reflect upon potential actions. By using the concept of *socio-ecological*, I emphasize the relationship between social and ecological systems, their interdependencies, and the essential role humans play in these issues. In line with researchers such as Coles et al. (2024), Amico et al. (2023), and Coles (2022), I understand the first part of the concept, "socio" as including politically (often tensed) issues. Sustainability issues can involve injustice towards groups of marginalized people or towards nature itself; thus, awareness and action are required. For instance, Coles (2022) suggests that socio-ecological can be linked to political activism in education. A first step towards any political activism in school is to bring awareness of the socio-ecological issue in play, combined with engaging students in the full complexity of the issue.

In the Norwegian core curriculum, sustainable development is one of three interdisciplinary topics (MER, 2017) and is defined as protecting life on earth and providing for the needs of people without

destroying the possibilities for future generations to fulfill their needs. It is based on social, economic, and environmental conditions and where students should engage in societal challenges demanding engagement from individuals and local communities and “understand basic dilemmas” in society (p. 16). The management of peatland in students’ municipalities can constitute multiple dilemmas. For instance, should the municipality allow peatland degradation to accommodate the building of houses and cottages, bringing much-needed finances into a strained economy and providing public services to older people or young people in schools? In the mathematics curriculum, there is no explicit mention of sustainability (MER, 2019). However, it is highlighted that students should explore and analyze real datasets from nature and society, formulate arguments, and “participate in public debate” (p. 4). Therefore, to meet the demands from the core curriculum’s sustainability aims and the mathematics curricula’s aims, there are arguments for including socio-ecological issues in school mathematics. The degradation of peatlands could be one way for students to use real sustainability datasets and where they practice to formulate mathematics-based argumentation in debates.

In this study, I take the starting point in a teaching activity aiming to engage students in the socio-ecological issues of peatlands. The activity was designed and carried out by a collaboration of four teachers: one from lower secondary school (mathematics and natural science) and three from higher education (two mathematics teacher educators and one from environmental sciences). Also, collaborators, such as experts and school leaders, came into play before, during, and after the activity. In this article, I *identify qualities in a network of collaboration and social practices that aim to contribute to awareness about socio-ecological issues in peatlands.*

THEORY

Learning can be considered a process of social participation in *communities of practice*. This notion is described by Wenger (1998) as a theory of learning where engagement in social practices is fundamental. It is often characterized by professional teaching communities in collaboration with researchers. He highlighted three interrelated dimensions when describing communities of practice. The first dimension concerns *mutual engagement*. Participants are not part of a community of practice just because they are structured into a group or have personal relationships with some people. Instead, it requires true interaction and engagement from participants. Participants should strive towards the inclusion of all. Mutual engagement does not mean homogeneity; rather, participants could reflect diversity, for instance, in expertise. Also, mutual engagement creates relationships among participants. The second dimension involves a *joint enterprise*. It includes collective processes of negotiations; the processes belong to the participants, and the goals are created amongst participants. The third dimension is a *shared repertoire*. Members’ joint pursuit contributes to creating resources for negotiating and developing tools and artifacts.

Valero (2010) highlighted that “our understanding of mathematics education as a field of research practices needed to be enlarged” (p. LXVII) and questioned which practices have the focus of research. She introduced the notion of the *network of mathematics education practices* as “a complex, multilayered space of social practice where the meanings of the teaching and learning of mathematics are constituted (p. LIV). This network can involve a series of social practices, where different people in different sites negotiate the meaning of teaching and learning mathematics. Social practices can be found inside and outside the classroom where students and teachers interact with mathematics: for instance, textbook practices, teacher education practices, families’ expectations of school

mathematics, youth culture practices, communities' educational needs, labor-marked practices, requirements for qualifications, public views and discourses of mathematics, and international comparison of achievement (see Figure 1). Valero also described how research on mathematics education has evolved, taking the starting point from the didactic triad. It has developed from viewing research on mathematics education where mathematics (M) is the center of the field of practice and research towards focusing on the didactic triad of mathematics (M), teaching (T) and learning (L), towards taking into account the classroom context, both inside and outside the classroom.

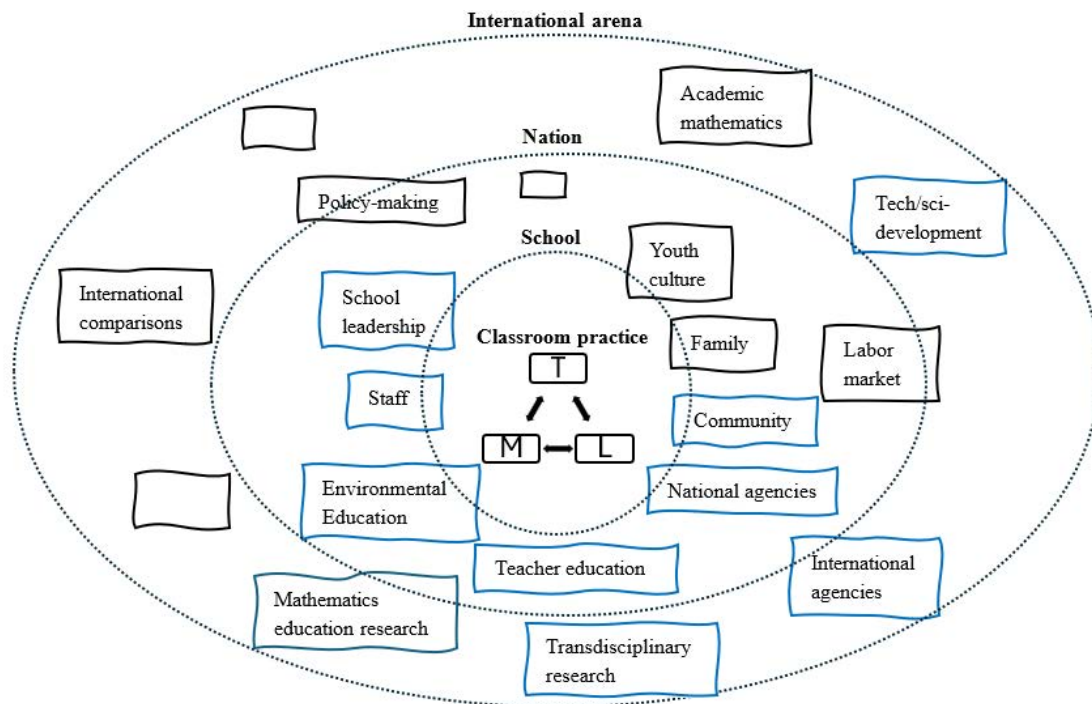


Figure 3: Network of mathematics education practice (Valero, 2010). Blue boxes illustrated how this study interacts at various levels, inside and outside the classroom

The didactic triad, seen in a context, illustrates that factors outside the classroom also influence the teaching and learning of mathematics and challenges the idea that classrooms are a privileged place for teaching and learning. In line with the ideas presented in the network of mathematics education practice, I consider the scales of mathematics education to interact within and across various sites (e.g., the classroom and experiences from the peatlands; the school, and life outside schools). It can operate at multiple scales simultaneously, such as if students' awareness and engagement in local peatlands contribute to a broader national or global engagement or if participants in the network connect to people or sites they would not typically come across in the traditional physical classroom. Valero highlighted that research through a network of mathematics education demands a collective effort from interdisciplinary collaboration with partners from various research fields and expertise.

THE CONTEXT OF THIS STUDY

This study is part of a larger research, Rethinking sustainable land use of PEATlands (Repeat, 2024). The larger project aims to collect data on peatland and restoration with peatland management. In this interdisciplinary study (mathematics and natural science), we focus on how teachers and students can engage in the socio-ecological issues of peatland. To do this, we formed a community of practices (Wenger, 1998): Two mathematics teacher educators (I am one of these), a teacher from

environmental sciences, and a lower secondary school teacher (mathematics and natural science). Some of us have collaborated for more than ten years on various projects and interact regularly, while others are new to each other. Initially, it was not our intention to research our collaborative network. However, during and after completing the teaching activities, we had joint reflections about the collaborative aspects of this study, and this paper discusses some of our reflections.

Combined, we developed a teaching plan, taking courses over five weeks in two classes with students aged 14-15. It consisted of several parts, such as teacher-led classroom activities about peatlands and modeling, an excursion to a local peatland where students measured peat depth and identified plant species typical for this ecosystem, hands-on activities in the laboratory, group work on modeling tasks, the inclusion of experts, students' video-blogs, and students' poster-presentation. In the activities, we expected students to estimate their municipality peatlands' potential carbon storage and consider the attached uncertainties; engage in its associated controversies and provide recommendations to local authorities on the management of their community's peatlands; and form their opinions and make well-grounded decisions as responsible citizens. We used theoretical perspectives from critical mathematics education (Skovsmose, 2023) and mathematical modeling (Geiger et al., 2022; Maass et al., 2023) when designing the task (Steffensen & Auestad, forthcoming). We expected that peatland management might be unfamiliar to students, and to make it mathematically and scientifically accessible to students' investigations, we developed worksheets with information about biodiversity and mathematical formulas. We used methods described by Rydgren (2022) to estimate peatlands' carbon storage potential.

In addition to the four collaborators already mentioned, other collaborators were involved. For instance, the school leader approved the project and facilitated plenary discussions among all school teachers in that particular school, colleagues of all four teachers, and a second mathematics teacher were present in the classroom assisting the school teacher with students. The landowner contributed by locating the particular peat and allowing us to use the area. Also, we contacted a local representative from the organization Naturvernforbundet (Friends of the Earth Norway). This organization aims to protect nature so that human activity does not exceed the planet's tolerance limits (Naturvernforbundet, 2024). We wanted the representative to focus on local knowledge about the peatlands of the municipality, showing some examples of local peatlands and some of the pressure areas and where various interests were at play (e.g., building of infrastructure on areas of peatland).

In the following, I use Wenger's (1998) three dimensions, mutual engagement, joint enterprise, and shared repertoire, to discuss identified qualities in the network of collaboration and social practices in this study, inspired by Valero's (2010) descriptions of network.

QUALITIES OF NETWORKS OF COLLABORATIONS AND SOCIAL PRACTICES

The first quality relates to what Wenger (1998) calls *joint enterprise*. The collaborators all shared a common understanding of goals: Learning about the socio-ecological issues of peatlands, including students and our own learning. One example of students' learning involves how the representative from Naturvernforbundet contributed to increased awareness of the uncertainty in numbers that students retrieved from websites. During the representative's PowerPoint presentation, he highlighted that their municipality has a lot of peatlands and that the name originated from peatlands (Fenring, which refers to a ring of fens). In Norway, the percentage of peatland is about 9%, while a worldwide

estimate of about 3% (Rydgren, 2022). Students asked how much, but he did not know the numbers. However, when he later observed the students during their group work, he noticed that they used a percentage of peatlands varying from 1.7% to 3%. They obtained the percentages from recommended databases and websites from the Norwegian Institute of Bioeconomy Research (NIBIO) (2024) and Statistics Norway (2024). Both these percentages are relatively low compared to national numbers, and the representative was slightly surprised by this relatively low percentage. Following his classroom visit, he contacted NIBIO in several e-mails for clarification. For instance, he asked why the numbers differed, what the various categories included, and whether it was a minimum area size for a peatland to be registered. NIBIO responded that 1.7% referred to peatlands without trees, 3.1% referred to peatlands with trees, and peatlands with an area of less than 0.2 hectares were typically not registered. The representative from Naturvernforbundet forwarded the e-mail correspondence between him and NIBIO to me, and the school teachers and I decided to keep the school students oriented about his findings. As a result of these inquiries, the students became aware of how categorizing various types of peatlands (with and without trees) and that not all peatlands were measured impacted the percentages and the uncertainty in the numbers from the websites. This awareness was observed during students' group discussions and on their posters. Some students pointed to the uncertainty by writing in their poster: "1.7% of Askøy is peatlands. (It is not exactly 1.7% peatland on Askøy, but this is what has been calculated except for some peatlands)".

Another example involves how students' work potentially influenced the practices of Naturvernforbundet and how this could influence political decisions in their local community. When students estimated the carbon storage potential in peatlands in their municipality, they started by estimating one particular peat using the formula: $Carbon\ storage\ potential = Volume \cdot Volumetric\ weight \cdot Carbon\ Concentration$. To find the volume, they first identified the area using online map tools. To find the depth, they did hands-on measuring, using a specific tool (a measuring stick consisting of several units, each one meter long) to get to the bottom of the peat. They measured depths in a coordinate system of 5 meters. Afterward, they found an average depth and multiplied this by the area. The students estimated the volumetric weight and the carbon concentration from some given numbers (depending on the type of peat), calling for judgment about the condition of the peat (Steffensen & Auestad, forthcoming). The representative noticed how students had estimated; he saw their calculations and pictures from the fieldwork activity, where they used the measuring sticks and the measuring band to draw coordinates in the peat. He became curious and asked the students, the teacher, and the researchers how this measurement occurred.

Following the representative classroom visit, he asked me whether it was possible to borrow the measuring sticks because he wanted to do a similar estimation. He intended to estimate the potential carbon storage of two peatlands in the municipality and where the authorities planned to build infrastructure, which meant degrading the peatland. During e-mail correspondences and conversations, I volunteered to participate in the Naturvernforbundet measuring. I asked whether it would be okay to include students if they wanted to join, to which he responded positively. I was unsure whether the students would take this invite; however, some students expressed interest in participating (at the time of the writing, the measurement from Naturvernforbundet had not yet taken place). He later wrote: "I'll give you a word when we have a day. It is your merit that we now have got carried away". I do not consider this a personal attribution; rather, it was the students' work inside and outside the classroom initiated through the collaboration of networks and social practices.

The second quality relates to what Wenger (1998) described as *a mutual engagement*. In this network of collaborators, we experienced an omnipresent enthusiasm for learning and genuine engagement among the other collaborators. For instance, initially, the representative from Naturvernforbundet was considered just as a visit to the classroom by an external, local collaborator who talked about local site-specific and socio-ecological issues regarding peatlands. However, throughout this project, he showed a genuine interest in students' work and their findings, and on his own initiative, he engaged in several follow-up interactions. He was critical of the peatland numbers and made inquiries into what these numbers represented from other collaborators (NIBIO) in society. As a result, he increased students' awareness of what the peatland numbers represented. Also, he became inspired by students' practices and expressed interest in learning so he later could use similar methods to influence the politicians in their municipality. Similarly, students expressed interest in participating in voluntary work that involved taking action to preserve peatlands. Thus, a quality from this network of collaborators was a genuine engagement in the degradation of peatlands and acknowledging that learning is not just a one-way street but that one also can learn from the network of collaboration. Our mutual engagement did not mean we were homogeneous. Instead, our collaborative network reflected diversity, and we considered this a strength, matching the interdisciplinary nature of the topic. However, having collaborators with various competencies does not automatically generate quality. Acknowledging each other's assets and playing on this can contribute to the learning and teaching of mathematics or interdisciplinary competencies. For instance, the two mathematics teacher educators know mathematical modeling and how complex real-world problems can be used to engage students in socio-ecological issues, while the school teacher knows the students, their foreground and background, appropriate level of tasks, and curricula. We used our combined knowledge to make the activity suitable for the students. In our social practices, we valued the combined competencies each of us held from various subject- and site-specificities. We consider these an essential quality that can facilitate students' (and ours) learning of interdisciplinary and socio-ecological issues.

Mutual engagement was also identified in how the broader school community interacted with our network of collaborators. We contacted the school leadership in the initial phases for formal approval and received positive responses. Later, the principal organized a workshop for all school teachers. In this, the school teacher and I gave a plenary talk about the project. Afterward, the teachers discussed and suggested similar projects and identified opportunities to engage students in local challenges. One example offered by one group involved how the issue of fish farming could contribute to raising awareness about the pollution of water and wildlife and weighing such concerns against potential local economic growth. By inviting all school teachers to participate in such a workshop, the principal acknowledges the importance of the school teachers' work and brings it to the fore for her colleagues, which could potentially inspire others to do similar projects. The principal became enthusiastic about collaborating with the wider community outside the school and invited me (as a representative of teacher education) to collaborate further. Such supportive behavior from the school leadership (and colleagues) is essential if school teachers should engage students in local and socio-ecological issues and spend time and effort on these often complex challenges. Later, the school teacher presented the project in a larger setting, involving interest organizations and multiple schools and teachers in her municipality. In this way, she contributed to sharing solutions with other communities while not ignoring local specificity.

A third quality identified in this network of collaboration and social practices concerns what Wenger (1998) described as *a shared repertoire*. While preparing for the field activity, we held several meetings where we shared resources (such as PowerPoint presentations with factual information and field trip experiments) and jointly pursued creating common resources. The school teacher and I (the local ones) had several attempts to locate an appropriate site for the fieldwork activity. It should be within walking distance of the school and be qualified as peatlands. The environmental teacher is an ecologist with field experience in peatlands. However, she was geographically located a five-hour drive from us, which meant that during this process, the three of us negotiated online, using online map tools and our own pictures before deciding on a location. Other collaborators also provided valuable insight during this process. For instance, the local landowner advised us about a local peatland. Socio-ecological issues taking a local approach requires local knowledge about the site-specificities, and it is not given that mathematics teachers have such knowledge in advance. Thus, this joint pursuit contributed to a shared repertoire of tools and artifacts. Other examples of such artifacts included using measuring sticks to find the depth of the peat, formulas to estimate the carbon storage potential of the peat, and designing lab experiments (e.g., we decided to do an experiment where students should test how much water moss could hold). We also decided that students should present their findings in video blogs and posters, resulting in student-designed artifacts.

CONCLUDING COMMENTS

Here, I have discussed some qualities identified in a network of collaboration and social practices. I used the work of Valero (2010) to identify how mathematics learning takes place in complex and multilayered spaces in the network of mathematics education practices, within and across site-specificities and various scales of mathematics education. Further, I used the three dimensions described by Wenger (1998) to structure these discussions. The first quality relates to the dimension of joint enterprise and its relevance when engaging the socio-ecological. Our network of social practices and collaborations shared a common understanding where we aimed at learning and teaching about the socio-ecological issues of peatlands. In educational settings, students are typically expected to be receivers of learning (e.g., in the didactic triad). However, networks of collaborations can be considered as social practices where everyone involved is learning through interactions. The examples illustrated that social practices and a network of collaborators can impact students inside the classroom by bringing them awareness of issues relevant to their school work and to issues relevant to local and global challenges. It also demonstrated how students' school work can impact society by bringing awareness about specific approaches that can help organizations in their work.

The second quality relates to the relevance of mutual engagement when engaging in socio-ecological issues. The examples from the network of collaborators demonstrated in several ways how true engagement permeated the learners, from the students who engaged in the learning activities inside and outside the classroom and volunteered to contribute to Naturvernforbundets work in their municipality, the representative follow-ups and interactions on own initiative, and the positive responses from the broader school community and where the principals and other school teachers became engaged to inquire how they could implement socio-ecological issues in their municipality. The diversity among the collaborators regarding competencies and expertise was considered a strength. The third quality relates to a shared repertoire, where the collaborators developed and jointly pursued common resources relevant to the socio-ecological issues of peatlands. Without the diversity

of expertise, the artifacts might have been less relevant for students to connect to the wider society. Collaborating with teachers and students on socio-ecological issues about peatlands in their community was a process of learning for all involved. The network started with four teachers but expanded during planning and teaching to include a wider network. The network of collaboration and social practices worked not only within the didactic triad; it included classroom practices inside and outside the classroom; it involved the school community (leaders, staff); it related to national aspects (e.g., how curricula are formed or national agencies like Naturvernforbundet), and it connected to national and international research on education and sustainability. Although we have briefly observed how the activity contributed to students' awareness and knowledge of socio-ecological issues in peatlands, the next step in our research is to analyze the students' work in more detail.

References

- Amico, A., Andrà, C., Chorney, S., Coles, A., Doria, L., Helliwell, T., Hunter, J., Makramalla, M., Pezzutto, M., Rubel, L. & Solares, A. (2023). How socio-ecological issues are urging changes in curriculum (and beyond). In M. Ayalon, B. Koichu, R. Leikin, L. Rubel and M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol.1, pp. 71–102). PME.
- Coles, A. (2022). A socio-ecological turn in mathematics education: Reflecting on curriculum innovation. *Revista Paradigma*, 43 (Edición temática 1), 207–228.
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- Geiger, V., Galbraith, P., Niss, M., & Delzoppo, C. (2022). Developing a task design and implementation framework for fostering mathematical modelling competencies. *Educational Studies in Mathematics*, 109, 313–336
- MER Ministry of Education and Research. (2017). *Values and principles for primary and secondary education*. Retrieved from <https://tinyurl.com/p6ahpb4m>
- MER Ministry of Education and Research. (2019). *Læreplan I matematikk 1.–10. Trinn [Curriculum for Mathematics year 1–10]*. Retrieved from <https://www.udir.no/lk20/MAT01-05>
- Maass, K., Zehetmeier, S., Weihberger, A., & Flößer, K. (2023). Analysing mathematical modelling tasks in light of citizenship education using the COVID-19 pandemic as a case study. *ZDM – Mathematics Education*, 55(1), 133–145.
- Naturvernforbundet. (2024). *Naturvernforbundet*. <https://naturvernforbundet.no/about-naturvernforbundet/>
- Norwegian Institute of Bioeconomy Research (NIBIO). (2024). <https://www.nibio.no/>
- Repeat. (2024). *Repeat, Rethinking sustainable land use of PEATlands*. <https://www.hvl.no/en/research/project/repeat/>
- Rydgren, K. (2022). Samlet karbonlager i myrene i Viken fylke. *HVL-notat*. <https://bit.ly/48ja4ub>
- Skovsmose, O. (2023). *Critical mathematics education*. Springer.
- Statistics Norway (SSB). (2024). <https://www.ssb.no/en>
- Steffensen, L., & Auestad, I. (forthcoming). *Designing a modelling task about peatlands* ICME15, Sydney.
- Valero, P. (2010). Mathematics education as a network of social practices. In V. Durand-Guerrier, S. Soury-Lavergne, F. Arzarello (Eds), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. LIV–LXXX). CERME
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press

THEME C – RESOURCES OF AND FOR MATHEMATICS EDUCATION

BIO AND MATH IN SOCIAL SCIENCE: USING BIOLOGY AND MATHEMATICAL CONTENT IN COMICS TO INTRODUCE SOCIOSCIENTIFIC ISSUES

Fadhlan Muchlas Abrori, Zsolt Lavicza, Branko Andjic

Johannes Kepler University Linz

Transdisciplinarity in learning content development is currently being widely developed to provide holistic learning. In this study, we apply transdisciplinarity in developing educational comic content for elementary school students, combining biology and mathematics to introduce socioscientific issues. Through the rule-of-five frameworks, we unify biology and mathematics through three types of interconnection approaches: numerical to visual, verbal to visual, and experiential to visual interpretations. In this paper, we provide four examples of comic samples through variations in the interconnection approach to interpretation, such as topics related to tobacco and smoking, environmental damage (mining problems, waste and air pollution), and COVID-19 prevention. The comic design developed shows that transdisciplinarity in the development of learning media is able to continuously provide a more accessible and engaging abstract concept for students.

INTRODUCTION

The context of transdisciplinarity in various scientific fields has been widely introduced to illustrate that disciplines are not isolated but connected to each other (Daneshpour & Kwegyir-Afful, 2022). One of the most famous examples of transdisciplinarity is the concept of STEAM (Science, Technology, Engineering, Arts, and Mathematics), which illustrates that the five disciplines can be related to each other (Perignat & Katz-Buonincontro, 2019). The most common form of transdisciplinarity involves the integration of mathematics and science (especially in biology). A previous study has explored how mathematics can be applied to biology and vice versa, illustrating that the two sciences are closely related and mutually supportive (Cox et al., 2016), providing a strong foundation for the two being connected.

Generally, educational comics only focus on explaining the basic concepts of monodisciplines (e.g., Dewi & Harini, 2021). In our work, we have applied several interdisciplinary and transdisciplinary approaches to biology and mathematics in educational comics, which is part of the design and construction design-based research (DBR) (Abrori et al., 2024). Interdisciplinarity involves aligning relationships between different disciplines into a unified and interconnected framework (Choi & Pak, 2006). Transdisciplinarity, however, goes a step further by not only focusing on integrating scientific disciplines but also incorporating humanities perspectives and moving beyond traditional boundaries (Choi & Pak, 2006). For instance, some comics we developed in our previous study combine biology and mathematics in the context of microbiology and evolution, representing interdisciplinarity (Abrori et al., 2023). In contrast, transdisciplinary examples explore topics like earthquakes and mitigation, approached through both biological and mathematical lenses (Abrori et al., 2024). This paper presents additional examples of how mathematics and biology are integrated to support transdisciplinarity, particularly through the development of socioscientific issues. *Socioscientific issues* here are defined as social issues related to science (Zeidler, 2014).

Why is it important to introduce socioscientific issues through both biology and mathematics? This is a crucial question that underpins our study. Typically, socioscientific issues are presented in classrooms from a purely scientific perspective, particularly focusing on biology (Lenz & Willcox, 2012). However, many aspects of these issues are deeply intertwined with numerical data, which brings mathematics into the picture (Abrori et al., 2024). Elements such as descriptive statistics, regression, correlation, and risk analysis are integral to understanding and analyzing socioscientific issues (Abrori et al., 2024). By integrating these mathematical components, we can enhance the biological perspective and provide a more comprehensive approach to introducing socioscientific issues in educational settings. This integration can enrich students' learning experiences by offering a multi-faceted view of complex social phenomena.

This paper presents some examples of educational comics we have designed and developed with teachers. These educational comics are part of products in the design and construction stage in DBR. We have roles as designers and storytellers. Meanwhile, the teachers involved in this study act as idea conceptualizers, linking the content to the curriculum.

THE RULE-OF-FIVE FRAMEWORK AS A BASIS FOR DEVELOPING BIOLOGY AND MATHEMATICS CONTENT

Before discussing the comics we designed further, it is important to explain how mathematics and biology are developed in our comic content. This development refers to the rule-of-five framework, which emphasizes five types of representation: experiential, verbal, numerical, visual, and symbolic (Diaz Eaton et al., 2019). Diaz Eaton et al. (2019) developed this framework, which aims to unify biology and mathematics in the context of learning.

Experiential refers to direct experience, such as scientific experiments (Diaz Eaton et al., 2019). Verbal refers to hypotheses, predictions, assumptions, and qualitative data (Diaz Eaton et al., 2019). Numerical includes quantitative data obtained from simulations, derivative data, and experimental data in numbers (Diaz Eaton et al., 2019). Visual includes graphs, schemes, and data visualizations. Finally, symbolic refers to formal mathematical constructs (Diaz Eaton et al., 2019). The five types of representation are interconnected.

In designing educational comics, we use several approaches based on the relationship between representations. For example, in some comics that emphasize data, we use numerical → visual to visualize the data. Another example is the experiential → visual approach, which describes the design of an experiment, and the verbal → visual, which describes comics as part of the dissemination. We describe the comic development in detail in the next section.

HOW ARE COMICS DEVELOPED?

The development of comics in this study follows a collaborative process between the authors of this paper, acting as researchers, and teachers as practitioners. The authors are responsible for creating the visuals and crafting the stories for the comics. Meanwhile, the teachers serve as concept developers, offering ideas for socioscientific issues that align with the curriculum (in this study, the Indonesian curriculum). This division of roles is based on our research (Abrori et al., 2024), which found that teachers often struggle with visualizing images and storytelling, making these tasks the authors' responsibility.

The comics are designed in two styles, depending on their use in learning. The first style is comic strips, which feature limited panels and are used during short learning stages, such as introductions. The second style is comic books, which have more panels and are used throughout the broader learning stages.

As mentioned earlier, the content focuses on socioscientific issues, enriched with the integration of transdisciplinary biology and mathematics. Socioscientific issues are closely tied to socio-ecological systems, the relationship between humans in society and ecosystems (Everard, 2020). These issues intersect with social and scientific concerns and align with this concept.

The comic content and style are developed using the rule-of-five framework and adjusted in consultation with teachers, who contribute as idea conceptors.

COMIC DESIGN EXAMPLES

This section presents several comics we have developed, illustrating types of interconnections between representations. First, we employ the numeric \rightarrow visual relationship through graphs for data visualization. Second, we continue with the numeric \rightarrow visual relationship and utilize a concrete-pictorial-abstract approach combined with graphs. Third, we utilize the verbal \rightarrow visual relationship in creating comic-based posters. Fourth, we showcase the experiential \rightarrow visual relationship by providing examples related to experimental stages.

Numerical \rightarrow visual: using graphs for data visualization

We first explore the relationship between numerical and visual representation through data visualization in the form of infographics. Figures 1 and 2 illustrate two types of comics that use this representational relationship approach. Figure 1 illustrates a comic that discusses the phenomenon of tobacco as a trade commodity and its negative impacts on health. We use a pie chart that presents quantitative data related to the impact of tobacco on the social-health aspect as a mathematical representation. Meanwhile, from a biological aspect, this data contains links to a summary of health statistics from various sources (Tobacco Control Support Center, 2020), including the age at which individuals start smoking and non-communicable diseases due to smoking habits. We use two pie charts, a single pie chart, and a comparative pie chart.

Figure 2 illustrates environmental damage due to mining activities. Similar to the first comic, we use a pie chart as part of the mathematical representation. Meanwhile, the biological context is taken from data on damage to primary and non-primary vegetation due to mining. The packaging of the interconnection of mathematics and biology in this comic is to make it easier for students to understand the storyline about environmental damage, which in the context of the main storyline in Figure 3 tells the story of a pair of ants, a story which will later be supported by inserted data related to vegetation damage to illustrate that this incident has an impact in the social and environmental context.

Numerical \rightarrow Visual: Data revisualization through concrete-pictorial-abstract and graphics

Another notable example of utilizing the numeric \rightarrow visual connection in our comic visualization applies to design through a concrete-pictorial-abstract (CPA) approach. Originally introduced by Jerome Bruner, this approach highlights the importance of scaffolding for children by using tangible

props to learn abstract concepts (Bruner & Kenney, 1965). Mathematics, being naturally abstract, often needs contextual instances for sufficient comprehension. In our comic shown in Figure 3, we use the CPA approach integrated with graphical representation. The comic's theme centres on the garbage problem. Rather than conventional graphs, we use visual representations of garbage piles that increase in each comic panel (notably panels 3-5), illustrating the data through CPA.

Similar to the first comic, which utilizes pie charts to present data on socioscientific issues, this comic also aims to describe the yearly accumulation of garbage. However, we modify the graph through CPA by visually representing it as increasing piles of garbage. This innovative approach transforms a bar chart into a more relatable and visually engaging format, making the abstract concept of data accumulation more concrete and understandable for students.

Verbal → Visual: Dissemination of comics-based posters

We use a verbal→visual approach in illustrating a dissemination poster related to preventing COVID-19. In Figure 4, we illustrate measures for COVID-19 prevention, such as keeping physical distance between people. The comic employs mathematical representations to describe the safe distance needed for social distancing, while from a biological viewpoint, it emphasizes the importance of distancing in preventing virus transmission.

This verbal→visual approach alters verbal health literacy messages into engaging visual content for students. By visualizing information, we aim to capture students' interest and help them better understand the recommended physical distance during the COVID-19 pandemic. The comic uses a measuring tape to visually represent the required distance, making the information more accessible and comprehensible.

Experiential → Visual: Experimental activities

The last comic in Figure 5 depicts the phases of a simple experiment designed to illustrate air pollution. This comic outlines the experimental technique, using an experiential-to-visual approach to represent experiential activities visually. The experimental activity constitutes the biological part of the comic. Meanwhile, the fourth panel depicts the mathematical element, where the experiment results are reported. It includes using nominal data to categorize the appearance of bottles and the color of tissues, emphasizing the importance of accurate data representation in scientific reporting.

HOW WILL COMICS BE IMPLEMENTED?

The comics we developed are intended for use in elementary schools and are designed to align with the Indonesian Curriculum. Recent changes in the elementary education curriculum have merged natural and social sciences into a single transdisciplinary subject called *IPAS (Ilmu Pengetahuan Alam dan Sosial)*, or in English terms, the Natural and Social Sciences subject (Kemendikbud, 2023).

As mentioned earlier, we apply a transdisciplinary approach by integrating biology and mathematics (as components of natural sciences) to explore social phenomena (a part of social sciences) in addressing socioscientific issues.

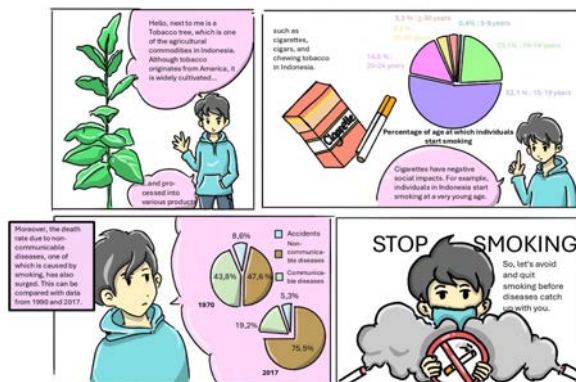


Figure 1. Related comic about the dangers of smoking. A single pie chart (panel 2) depicts the age of individuals starting smoking, and a comparison pie chart (panel 3) depicts the causes of death in 1970 and 2017 (Source: Digital drawing by first author)

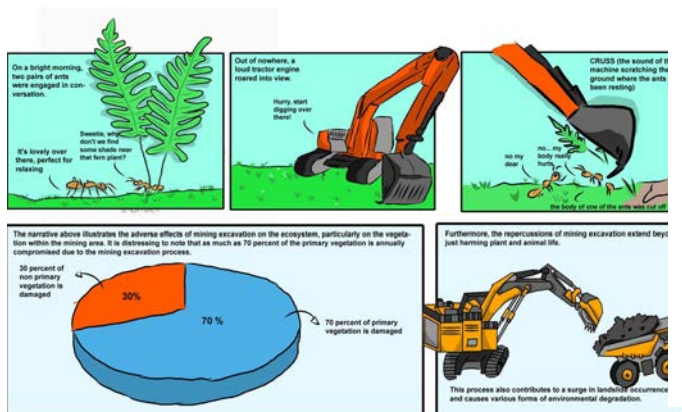


Figure 2. The comic is about environmental damage, the first panel related to the comic characters, a pair of ants. The comic also has a pie chart (panel 4) that illustrates damage to primary and non-primary vegetation (Source: Digital drawing by first author)

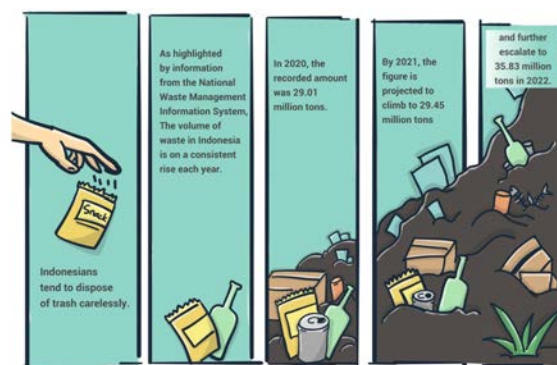


Figure 3. Visualization of the amount of waste per year using the CPA approach is seen in panels 3 to 5 (Source: Digital drawing by first author)

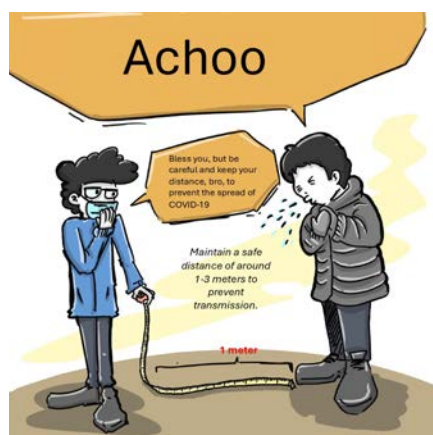


Figure 4. Comic depicting social distancing, the image of a meter measuring tape is a mathematical representation depicting distance (Source: Digital drawing by first author)



Figure 5. Visualization of the experimental stages in comic form (Source: Digital drawing by first author)

As described in the section “How Are Comics Developed?”, comics come in two styles with distinct purposes. Comic strips are typically used for shorter learning segments, such as introductions, while comic books are employed throughout broader learning stages. These comics are not intended to introduce fundamental concepts in mathematics or biology; they aim to enhance students’ understanding of socioscientific reasoning. Socioscientific reasoning refers to students’ ability to address socioscientific issues, specifically how they process information from biology and mathematics when tackling them. For example, in our previous study (Abrori, Lavicza, & Anđić, 2023), we analyzed this aspect; in this context, we are working to develop similar approaches.

DISCUSSION

This section focuses on two primary aspects that are crucial for educational comic design. First, we discuss integrating biology and mathematics in learning content. Second, we explore the potential of this interdisciplinary approach to enrich information related to socioscientific issues and influence students’ understanding of real-world problems.

Content Development through Biology and Mathematics Integration

In the comics we developed, biology and mathematics are used interconnectedly to introduce socioscientific issues as social phenomena. The integration of these disciplines in education is not new; it ranges from complicated applications, such as mathematical modeling of biological processes (López-Ruiz, 2022), to simpler implementations, such as the use of graphs to illustrate plant size (Abrori et al., 2023) or the use of addition and subtraction with examples from living organisms (Lindner, 2020). These examples display that in real-world contexts, scientific disciplines are fundamentally interconnected.

This interdisciplinary integration helps students understand abstract mathematical concepts through a contextual approach or improves quantitative problem-solving in biology using mathematical principles. For example, a study by Cox et al. (2016) revealed positive student perceptions when mathematics was applied to solve intermediate biology problems. That study underlines the benefits of a transdisciplinary approach to education.

In line with the current research, developing comic content that integrates mathematics and biology offers significant potential beyond single-disciplinary comics. Introducing socioscientific issues from a biological and mathematical perspective not only enriches students’ understanding but also provides real-world data related to these issues, thus fostering a deeper and more comprehensive educational experience.

Mathematics and Biology lens in comics: Contextualization of socioscientific issues

The integration of biology and mathematics in introducing socioscientific issues in comics has the potential to enrich students’ understanding of real-world problems that are more contextual. As an illustration in our comic about tobacco and smoking, we not only describe the negative impacts of smoking but also present real data related to the social-health context through numerical to visual representation. Another example in the comic related to the environment and the impact of mining on vegetation is that we also use the same approach, where we present real data on damage to primary

and non-primary vegetation. From this example, it can be seen that the combination of biology and mathematics makes the content more relevant and data-based.

In addition, the verbal-to-visual approach helps in transforming the dissemination approach, which is usually verbal, into a visual form, as seen in the comic related to preventing COVID-19, where the dissemination is changed in such a way in a visual form through the integration of mathematics and biology. The same thing can be seen in the comic related to the air pollution experiment, which adapts experiential to visual, contextualizing the stages of the experiment to make it easier to follow.

Our previous studies highlight the significant role of combining disciplines in comic content to enrich students' information (Abrori et al., 2023). In addition, this interdisciplinary approach also enhances student reasoning, making learning more engaging and efficient (Abrori et al., 2023). In brief, the transdisciplinary comics design can potentially improve learning outcomes holistically and contextually.

CONCLUSION AND FUTURE RECOMMENDATIONS

In conclusion, this paper illustrates the integration of biology and mathematics in comic content to introduce socioscientific issues. This interdisciplinary approach offers a contextual and holistic method to present social-scientific phenomena, enhancing students' understanding and engagement. The comic design is a potential model for incorporating transdisciplinary content into curricula and developing versatile, visual-based learning media.

ACKNOWLEDGMENT

We acknowledge the use of Large Language Models, Chat GPT 3.5, in the final editing of this manuscript for grammar checking, sentence structure refinement, and enhancing academic language.

REFERENCES

- Abrori, F. M., Lavicza, Z., & Anđić, B. (2023). Enhancing socio-scientific reasoning of elementary school students through educational comics: A comprehensive exploration across diverse domain of knowledge. *Education 3-13*, in press, 1–22.
- Abrori, F. M., Lavicza, Z., Anđić, B., Annisa, M., & Qamariyah, N. (2023). Enhancing elementary school student debates on socio-scientific issues through multiple storyline comics: An exploratory study. *2nd International Conference on Education Innovation and Social Science (ICEISS 2023)*, 21–33.
- Abrori, F. M., Lavicza, Z., Tejera, M., & Anđić, B. (2023). Experts' perception on the usage of mathematical representations in biology comics. *International Journal for Technology in Mathematics Education*, 30(3), 171–180.
- Abrori, F. M., Prodromou, T., Alagic, M., Livits, R., Kasti, H., Lavicza, Z., & Anđić, B. (2024). Integrating mathematics and science to explain socioscientific issues in educational comics for elementary school students. *Journal of Graphic Novels and Comics*, 15(4), 508–524.
- Abrori, F. M., Saimon, M., Lavicza, Z., & Anđić, B. (2024). Challenges and opportunities of training teachers to develop comics for teaching socio-scientific issues. *Media Practice and Education*, 25(1), 56–76.
- Bruner, J. S., & Kenney, H. J. (1965). Representation and mathematics learning. *Monographs of the Society for Research in Child Development*, 30(1), 50–59.
- Choi, B. C. K., & Pak, A. W. P. (2006). Multidisciplinarity, interdisciplinarity and transdisciplinarity in health research, services, education and policy: 1. Definitions, objectives, and evidence of effectiveness. *Clinical and Investigative Medicine. Medecine Clinique Et Experimentale*, 29(6), 351–364.

- Cox, C. D., Reynolds, B., Schuchardt, A., & Schunn, C. D. (2016). Using mathematics and engineering to solve problems in secondary level biology. *Journal of STEM Education: Innovations and Research*, 17(1), 22-29.
- Daneshpour, H., & Kwegyir-Afful, E. (2022). Analysing transdisciplinary education: A scoping review. *Science & Education*, 31(4), 1047–1074.
- Dewi, L. K., & Harini, N. W. (2021). Development of digital comics as a media for biology learning of environmental change for senior high school students. *Journal of Biology Education*, 10(3), 296–302.
- Diaz Eaton, C., Highlander, H. C., Dahlquist, K. D., Ledder, G., LaMar, M. D., & Schugart, R. C. (2019). A “rule-of-five” framework for models and modeling to unify mathematicians and biologists and improve student learning. *PRIMUS*, 29(8), 799–829.
- Everard, M. (2020). Managing socio-ecological systems: Who, what and how much? The case of the Banas river, Rajasthan, India. *Current Opinion in Environmental Sustainability*, 44, 16–25.
- Kemendikbud. (2023). *Kurikulum Merdeka: Keleluasaan Pendidik dan Pembelajaran Berkualitas*. <https://kurikulum.kemdikbud.go.id/kurikulum-merdeka/>
- Lenz, L., & Willcox, M. K. (2012). Issue-oriented science: Using socioscientific issues to engage biology students. *The American Biology Teacher*, 74(8), 551–556.
- Lindner, M. A. (2020). Representational and decorative pictures in science and mathematics tests: Do they make a difference? *Learning and Instruction*, 68, 101345.
- López-Ruiz, R. (2022). Mathematical biology: Modeling, analysis, and simulations. *Mathematics*, 10(20), 3892.
- Perignat, E., & Katz-Buonincontro, J. (2019). STEAM in practice and research: An integrative literature review. *Thinking Skills and Creativity*, 31, 31–43.
- Tobacco Control Support Center. (2020). *Atlas Tembakau Indonesia (Indonesia Tobacco Atlas)*. Ikatan Ahli Kesehatan Masyarakat Indonesia.
- Zeidler, D. L. (2014). Socioscientific issues as a curriculum emphasis. Theory, Research, and Practice. In N.G. Lederman & S.K. Abell (Eds.), *Handbook of research on science education*, 2nd Edition (Vol. 2, pp. 97–726). Routledge

VIDEO FESTIVAL, MATHEMATICAL EDUCATION, AND SOCIO- ECOLOGY

Marcelo de Carvalho Borba¹, Fernanda Martins da Silva², Aldo Peres Campos e Lopes³

^{1,2,3}São Paulo State University, ³Federal University of Itajubá

The article discusses the integration of socio-ecological perspectives into mathematical education through the use of digital video production. Using the metaphor of Janus to illustrate the interconnectedness of past and future in socio-ecology, the article emphasizes the holistic understanding of social and ecological systems. The Digital Video Festival and Mathematical Education project in Brazil serves as a case study, showcasing videos produced by students and teachers that address real-world socio-ecological issues, such as consumerism, and water waste. The theoretical framework “humans-with-media” underpins the collective production of knowledge, emphasizing the intershaping relationship of humans and media. By incorporating socio-ecological themes and promoting innovative, student-centered learning, the article highlights the potential of video production to enrich the educational process, foster environmental awareness, and support marginalized voices. The addition of the “Indigenous and Traditional Peoples” category in the Festival underscores the importance of cultural diversity.

INTRODUCTION²⁴

The Roman metaphor of Janus, with its two faces, one facing the past and the other the future, can be applicable to socio-ecology. Just as Janus integrates different perspectives, socio-ecology seeks to holistically understand the interactions between social and ecological systems, both from the past and in terms of their evolution towards the future (Borba & Orey, 2023). This approach balances historical and prospective analysis, promoting an integrated vision aimed at environmental and social sustainability. Thus, the metaphor of Janus provides an effective visual representation of the complexity and interconnection between society and the environment in the field of socio-ecology.

The interrelationship between the social and the ecological highlights the interdependence and mutual influence between subject and object. A relational approach analyzes networks of relations that encompass culture, politics, history, biology, and ecology. In mathematical education, the activity is socially organized and connected to technological innovation, applied to understand issues such as biodiversity and climate change. Mathematics is perceived interconnected with both human activity and technology (Borba, 2021). Therefore, a transformative curriculum in mathematical education should integrate the living world into all educational aspects, promoting healthy relationships and granting a central role to citizens in mathematics and the science of the living world (Boylan, 2023).

Immersive educational experiences with videos (which can be instructional and/or interactive) can be advantageous for students who prefer affective or self-directed learning, as well as providing new ways to understand complex systems (Merkt et al., 2011). According to the authors, videos can enhance learning by allowing greater interaction between students and content and offering flexibility

²⁴ The ideas presented here represent an expansion of the abstract published at the ICMI Symposium on March 20, 2023 (Borba & Silva, 2023), which was presented online.

in the study pace. Merkt et al. (2011) also point out that students in e-learning environments with instructional and interactive videos show better performance and satisfaction, as these videos allow proactive and non-linear access to content.

Engelbrecht et al. (2020) argue that there are concerns about the use of instructional videos in the mathematics classroom, because learning is not always enhanced, as design, context, and viewing patterns are critical factors. For them, watching videos passively does not promote significant engagement with mathematics, as it pushes the content to the student, so it is important to consider students' perceptions of the use of instructional videos, as they desire more voice in how they are taught and the content they learn.

A socio-ecological perspective in mathematical education, which can be related to video production, highlights the importance of considering the perspectives and experiences of marginalized groups, as well as the ecological challenges they face, such as pollution and climate change, and the adaptations they make to address these issues (Coles et al., 2022). The teaching, research, and extension project, Festival of Digital Video and Mathematical Education (<https://www.festivalvideomat.com/>), which is a virtual and in-person project based in Brazil, exemplifies this relationship through videos produced by teachers and students (Domingues & Borba, 2021). These videos address mathematical content while discussing real-world themes, such as the social and environmental impacts of the mining dam collapse in Brumadinho, Brazil, in 2018 (for example, Elementary Education – Sea of Mud: Modeling in Mathematics Education – III *Festival of Digital Video and Mathematical Education*, available at <https://www.youtube.com/watch?v=YpCteGqjxd0&t=5s>, accessed September 12, 2024). Real-world themes presented in the videos illustrate the potential of video production in the mathematics classroom, making it a valuable environment for integrating the socio-ecological perspective into mathematical education.

In the contemporary educational context, integrating innovative pedagogical practices is essential to engage students and promote meaningful learning. One such practice is video production, a teaching methodology that can be used to explore socio-ecological themes in a dynamic and interactive way. This article aims to discuss the potential of socio-ecological discussion through mathematical videos produced by students and teachers for the Digital Video and Mathematical Education Festivals, highlighting how this approach can enrich the educational process and foster environmental and social awareness.

THEORETICAL FRAMEWORKS

Humans-with-media

In this research, we adopt the theoretical construct “humans-with-media” from Borba and Villarreal (2005) and Borba et al. (2023) to underpin the production of mathematical knowledge with technologies. The authors discuss a collective composed of human actors (teachers, students, researchers) and non-human actors (cameras, video production software, for example) that are interconnected and shaping each other reciprocally. This notion of intershaping relationship is understood as the intersection between humans and media. In this context, human thought reorganizes based on the nature of the media used, while the media itself can be reshaped by human actions. This relationship suggests that the minimum unit of analysis is a collective of humans and non-humans.

From this perspective, knowledge is a product of both humans and media, with both having agency in its creation (Borba & Villarreal, 2005). This theoretical framework highlights the co-production of knowledge by media and humans, proposing discussions on how available technology shapes our understanding of what it means to be human (Borba, 2021; Borba & Villarreal, 2005).

Borba and Villarreal (2005) argue that educational practice should consider this reorganization of thought and discuss problem-solving performed by humans-with-media. Research results, such as those by Oechsler and Borba (2020), indicate that different media express different mathematical ideas, and by expanding this notion to multimodality, they transform the possibilities for producing mathematical knowledge, as they allow the use of different modes to produce meaning. Relating this construct to videos, Souto and Borba (2016, p. 56) argue that the proposal is to make explicit that learning is a collective construction that involves “[...] in addition to emotional, social, historical, and cultural dimensions, new ways of thinking, which presuppose the participation of human and non-human actors as protagonists in the process.” This collective knowledge production of humans with media through video production reflects a curriculum in which students engage with teachers about possibilities for discussing mathematics with issues they consider important. An example of this, which will be explored in this article, is the relationship of two videos shared at the Festival of Digital Video and Mathematical Education with the socioecological perspective in mathematical education.

Socio-Ecological

The socio-ecological perspective in mathematical education highlights mathematics beyond numbers, considering the perspectives of marginalized actors and the ecological challenges they face (Coles et al., 2022). The authors reflect on challenges such as climate change, pandemics, poverty, unemployment, loss of biodiversity, migration, and totalitarianism, among others. According to Coles (2021), any social system is composed of natural agents, making it impossible to delineate where a natural and ecological system ends and a social one begins. The social is necessary for the ecological and vice versa. Just as presented in the section on humans-with-media, where we analyze the interconnected system of humans and media, Coles (2021) asserts that the term socio-ecological also reflects that the social and ecological cannot be disconnected.

In mathematical education, possibilities for including a socio-ecological perspective include listening to marginalized actors and the issues they raise; addressing the ecological precariousness of communities and the adaptations made to tackle issues such as pollution or climate change; and questioning the role of mathematics in conceptualizing the ecological, for example (Coles et al., 2022). Specifically in this research, we will seek to recognize socio-ecological aspects in a video from the Festival of Digital Video and Mathematical Education, provoking potential discussions for mathematics teachers to reflect on their classes based on the production of the videos.

Festival of Digital Videos and Mathematics Education

The Festival of Digital Videos and Mathematical Education is an educational, research, and outreach project at São Paulo State University (UNESP), Rio Claro Campus, supported by the Brazilian Society for Mathematical Education (SBEM) and the National Council for Scientific and Technological Development (CNPq). Organized by the Research Group on Information, Other Media, and Mathematical Education (GPIMEM), it aims to foster the collective production and

dissemination of mathematical knowledge through educational videos. Since its inception in 2017, the Festival has engaged teachers, students, and the general community across various educational levels, achieving national reach. By 2023, it featured six categories and shared over 800 videos. An experimental category for the initial years of elementary school was added, with expectations to reach 900 videos in the eighth edition.

The new “Indigenous and Traditional Peoples” category, introduced in the 2023 Festival, celebrates cultural diversity and ancestral mathematical knowledge. It underscores the importance of integrating Indigenous and traditional wisdom into mathematical education, offering an inclusive perspective that enriches learning for all. By encouraging the exploration of mathematical themes through these cultural lenses, the Festival promotes intercultural dialogue and honors the contributions of these peoples to global mathematics. This category broadens educational horizons, strengthens cultural identity, and fosters respect for diversity, illustrating that mathematics is a universal language expressed uniquely in each culture. An example is the winning video where an Indigenous math teacher connects body paintings to traditional Euclidean geometry, linking the drawings to nature and cultural heritage (https://youtu.be/OtHm4r4_KK0?feature=shared, accessed September 12, 2024). While this category is new, with limited videos, it is inherently tied to socio-ecological issues and serves as a platform for Indigenous and traditional peoples, with future studies planned to focus on its development.

Carvalho et al. (2022) argue that Festival videos on socio-political and environmental issues reveal students’ critical perspectives, emphasizing interdisciplinary concepts and student reflection as agents of change. The authors highlight that video production supports interdisciplinary teaching and promotes a positive image of mathematics (Carvalho et al., 2022). The Festival of Digital Video and Mathematical Education fosters an environment for socio-ecological reflections by encouraging video production in mathematics classrooms (Domingues & Borba, 2021).

METHODOLOGY

According to D’Ambrosio (2019), qualitative research focuses on people and their ideas, seeking meanings that might be hidden in narratives and discourses. The author emphasizes that the goal of this approach is to interpret and understand the discourses and data produced. Goldenberg (2004) adds that qualitative research seeks an interpretative understanding based on the context in which individuals experience their lives. In this sense, such research investigates the motivations, beliefs, feelings, and values underlying these experiences. Based on these authors, the research presented in this article is characterized as qualitative, as it aims to explore the potential of socio-ecological discussions in the production of mathematical videos by students and teachers. The investigation was conducted based on videos selected and shared over the seven editions of the Festival of Digital Videos and Mathematical Education. During the selection and analysis process of the videos, the authors’ interpretation was necessary to understand this research object. Two videos were analyzed, as presented in Table 1.

The selection of the videos was initially based on their being from different editions and categories of the Festival of Digital Video and Mathematical Education. After this first selection, we watched the playlists of finalist videos from each edition in the categories of final years of elementary school and high school, which means the chosen videos were produced respectively by teachers and students

from these levels.²⁵ The videos were chosen for addressing topics that allow connections with socio-ecology. Video 1 represents the high school category from the 4th edition; and Video 2 from the final years of elementary school category from the 2nd edition²⁶.

For video data analysis, we used the research model described by Powell et al. (2003). According to them, this model was developed to analyze video data investigating the mathematical thinking of students engaged in ideas, reasoning, and mathematical investigations. They present seven interactive and non-linear phases: 1. Carefully viewing the video data, which involves watching the video multiple times and familiarizing oneself with the content; 2. Describing the video data, which requires writing details of the videos so that an external person can get a clear idea of the video; 3. Identifying critical events, which includes clarifying an important event in the video according to the research objective; 4. Transcribing speech and gestures; 5. Coding based on the research objective, which could include mathematical ideas, explanations, and representations, for example; 6. Constructing the narrative line, to understand the data; and 7. Composing the narrative, relating the produced data to the research objective. Thus, in the next section, the videos were analyzed based on this described model.



Video 1: Consumismo X Consumo consciente (Consumerism vs. Conscious Consumption)	Video 2: O desperdício de água (Water Waste)
	
https://youtu.be/qKoBanYOIg4?feature=shared	https://youtu.be/OhobEmnSiNc?feature=shared

Table 1: Videos selected for data analysis

VIDEO ANALYSIS

Video 1: Consumerism vs. Conscious Consumption

The educational video on consumerism discusses the consequences of consumerism and strategies to become a conscious consumer. The content is divided into two main parts: the definition of consumption and consumerism, and proposals to reduce excessive consumption. Identifying critical events, such as defining the terms and presenting data on consumerism in Brazil, helps establish the foundation for discussion. Proposed solutions, such as practices of reducing, recycling, and reusing, are essential to promote conscious consumption. The content coding includes mathematical ideas, conceptual explanations, social and environmental impacts, and practical strategies. The video's narrative follows a logical progression from the problem to the solution, aligned with the goal of educating about consumerism. For Coles et al. (2022), discussing socio-ecology in mathematical education requires allowing students to share their challenges. In this video, the choice for discussion involved reflecting on practices related to consumerism and the importance of sustainable practices.

²⁵ The playlists are available at GPIMEM's YouTube Channel: <https://www.youtube.com/@gpimem/playlists>, accessed September 12, 2024.

²⁶ Ethics Committee Case Number 5,107,147

In this group, students used digital technologies to explore definitions, statistical data, and practical solutions to promote conscious consumption. This reflects the dynamic interaction between students and digital media to produce mathematical knowledge (Borba & Villarreal, 2005). Thus, the video production not only expresses mathematical ideas but also stimulates dialogue about environmental and social issues associated with unchecked consumerism.

The first video emphasizes the relationship between consumerism and sustainability, using statistical data and practical solutions to promote conscious consumption. From the perspective of humans-with-media, the interaction between students and digital media allows for the co-production of knowledge, where mathematical ideas intersect with social and environmental concerns. The video medium, as a non-human actor, shapes how the information is organized and shared, while the students actively shape the content to reflect critical issues such as recycling and reducing waste. The multimodal nature of the video allows for a dynamic expression of mathematical ideas intertwined with socio-ecological concerns, such as the impact of consumerism on the environment. The socio-ecological perspective is evident as the video connects mathematics to real-world ecological problems, promoting an awareness of sustainable practices that address both the social and environmental challenges the community faces.

Video 2: Water Waste

In the second video analyzed, students initially discuss mapping the school's water sources, identifying three main sources. From this point, they perform practical measurements of water flow using buckets and timers, applying mathematical concepts to construct flow graphs over time. This step is crucial as it allows for a detailed analysis of the collected data, where they observe a decrease in water flow during certain periods. Following the analysis of the school's water consumption, they estimate a daily consumption of 7,500 liters on weekdays, highlighting the need to reduce this value to maintain the school's seal. To achieve this goal, students propose sustainable solutions, such as using electric pumps powered by solar panels and utilizing non-potable water for activities that do not require potable water. They conclude that even using all the water from the spring, the excess will be released into the stormwater system, demonstrating a practical and sustainable approach to the problem. The project concludes with a proposal to present to the school administration, illustrating the effective application of mathematical and scientific concepts in solving real problems and promoting meaningful and sustainable learning.

As highlighted by Coles et al. (2022), bringing socio-ecological aspects into mathematics classes involves questioning the role of this mathematics in conceptualizing the ecological. Thus, through practical actions within the school environment, this modeling activity that uses video production has the opportunity to consider the sustainability of water usage practices, proposing solutions like solar energy use and the reuse of non-potable water resources. The mathematical modeling used by the group was an opportunity to develop critical thinking by connecting social and critical aspects in the application of a real-world problem (Lopes, 2023). Through the video, they verbalize their knowledge and use multimodal resources to share what they have learned and produced (Oechsler & Borba, 2020). This approach demonstrates the interconnections between social and ecological systems, promoting learning that transcends numbers to encompass significant environmental issues.

In this video, the focus is on quantifying water usage within the school and proposing sustainable solutions, such as solar-powered pumps and non-potable water reuse. Here, the humans-with-media concept is reflected through the students' use of digital tools to analyze data and visualize solutions to environmental issues. The video, acting as a medium for multimodal expression, enables students to verbalize their findings and propose solutions that integrate mathematics with sustainability goals. The socio-ecological perspective is clearly demonstrated as the students recognize the interdependence of social practices (water consumption) and ecological impacts (sustainability of water resources). The technological tools not only mold how the students understand and present these issues but are themselves shaped by the students' critical thinking and problem-solving approaches, illustrating the reciprocal relationship between human agency and non-human tools.

FINAL REMARKS

Many of the Festival's videos engage in socio-ecological discussions, showing how students and teachers can transform and generate mathematics through a bottom-up curriculum. These videos, as discussed in this article, offer practical tools for integrating a socio-ecological perspective into mathematics classes. We understand that incorporating longer projects, like producing mathematical videos, is not always easy due to constraints in the school curriculum. Despite the challenges, the nearly 1,000 videos produced across various Festival editions demonstrate that such initiatives are not only feasible but also yield important results. The new "Indigenous and Traditional Peoples" category celebrates cultural diversity and ancestral knowledge, integrating this wisdom into mathematical education to enrich learning and foster intercultural dialogue. This category broadens educational horizons, strengthens cultural identity, and promotes respect for diversity, illustrating that mathematics manifests uniquely in each culture. The Festival plays a significant role in enabling a critical, inclusive, and culturally connected bottom-up curriculum.

By the end of 2024, students and teachers, using various media, have produced and showcased nearly 1,000 videos that express their knowledge and research in a broad sense. These videos represent new collectives of humans-with-media, contributing to the production of knowledge in educational environments. As categories such as Indigenous and traditional peoples are highlighted, the metaphor of Janus materializes in the virtual world: ethnomathematics and native languages can be preserved through such a Festival, aiding Indigenous peoples in maintaining ownership of their territories, cultures, and ethnomathematics (Borba & Orey, 2023). Preserving culture and territory for Indigenous peoples in countries like Brazil can also mean preserving ecological environments while addressing social inequality. Indigenous people who lose their culture and are absorbed into urban areas often only gain access to the poorer segments of Western culture. Ethnomathematics captured by digital videos is the way Janus sees the world. Collectives of humans-with-media are always transforming and can be integrated into socio-ecological concerns.

Acknowledgements

This study has been supported by the following Brazilian research agencies: Coordination for the Improvement of Higher Education Personnel (CAPES) – Finance Code 001, and CAPES-PRINT, and CNPq – Process Number 309992/2020-6.

References

- Borba, M. C. (2021) The future of mathematics education since COVID-19: Humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics*, 108(1-2), 385–400.
- Borba, M. C., & Orey, D. C. (2023). Past and future: Ubi's way of seeing education in the present. In M. C. Borba, & D. C Orey. (Eds.), *Ubiratan D'Ambrosio and mathematics education. Advances in mathematics education*. Springer.
- Borba, M. C., & Silva, F. M. (2023, March). Socio-ecological possibilities in video production by mathematics teacher and students. *Proceedings of the International Commission on Mathematical Instruction (ICMI) Symposium: Mathematics Education and the Socio-Ecological*. https://www.mathunion.org/fileadmin/ICMI/Conferences/Socio%20Ecological/Proceedings-ICMI-symposium-20th-march-2023.pdf
- Borba, M., & Villarreal, M. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, visualization and experimentation*. Springer.
- Boylan, M. (2017). Towards a mathematics education for ecological selves: Pedagogies for relational knowing and being. *Philosophy of Mathematics Education Journal*, 32, 1–23.
- Carvalho, G. S., Costa, R. F., & Borba, M. C. (2022). O Festival de Vídeos Digitais e Educação Matemática: uma “cultura” em movimento. *Anais... Simpósio Internacional de Tecnologias em Educação Matemática-SITEM*, Rio Claro –SP, 06 a 08 de outubro.
- Carvalho, G. S., Gimenez, H., & Borba, M. C. (2022). Ciclo do Conhecimento e a ação das mídias: o que o vídeo “A Geometria das Abelhas” pode expressar. *Anais... XIV Encontro Nacional de Educação Matemática (ENEM)*, online, jul.
- Coles, A., le Roux, K., & Solares-Rojas, A. (2022). Towards a socio-ecological perspective of mathematics education. In C. Fernández, et al. (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol 2., pp. 171-178). Alicante: PME.
- D'Ambrosio, U. (2019). *Prefácio*. In Borba, M.C., Araújo, J.L. (Orgs.), *Pesquisa qualitativa em educação matemática: Coleção tendências em educação matemática* (6^a edição). Autêntica.
- Domingues, N. S., & Borba, M. C. (2021). Digital video festivals and mathematics: Changes in the classroom of the 21st century. *Journal of Educational Research in Mathematics*, 31(3), 257–275.
- Engelbrecht, J., Llinares, S., & Borba, M. C. (2020). Transformation of the mathematics classroom with the internet. *ZDM Mathematics Education*, 52(5), 825–841.
- Goldenberg, M (2004). *A arte de pesquisar: Como fazer pesquisa qualitativa em ciências sociais*. Record.
- Lopes, A. P. C. (2023). Countering the ideology of certainty through reflective knowledge in mathematical modeling. *Bolema: Boletim de Educação Matemática*, 37(77), 958-979.
- Merkt, M., Weigand, S., Heier, A., & Schwan, S. (2011). Learning with videos vs. learning with print: The role of interactive features. *Learning and Instruction*, 21, 687–704.
- Oechsler V., & Borba, M. C. (2020) Mathematical videos, social semiotics and the changing classroom. *ZDM Mathematics Education*, 52(5), 9891001.
- Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior*, 22, 405-435.
- Souto, D. L. P., & Borba, M. D. C. (2016). Humans-with-internet or Internet-with-humans: An exchange of function? *Revista Latinoamericana de Investigación en Matemática Educativa*, 19(2), 217-242.

MATHEMATICAL MODELLING AS A KEY PRACTISE FOR PROMOTING LEARNERS' COMPETENCIES TARGETING THE TRANSFORMATION OF GLOBAL SUSTAINABILITY

Rita Borromeo Ferri

University of Kassel

The transformation to global sustainability with the fulfillment of the 17 Sustainable Development Goals (SDGs) is a worldwide consensus. Sub-goal 4.7, Education for Sustainable Development (ESD), states that schools must also provide education about sustainability. This also includes the ESD competencies: recognize, evaluate and act. Mathematical modelling (MM) is presented here as a possible key practice for the teaching of ESD and promoting its competencies. ESD-modelling tasks were already developed by the author in previous studies, but one of those is illustrated showing its potential. Finally, in a pre-post intervention study with N=105 secondary school learners, the change in sustainability and environmental awareness through ESD-MM tasks is examined. The results are significant. In a case study, one class of the sample, observation and analysis of the learners' solutions made it clear that ESD competencies can be promoted also through ESD-MM tasks. Finally, the results are discussed in the context of mathematics education and the socio-ecological.

INTRODUCTION AND RESEARCH QUESTIONS

Our earth, our life and our coexistence on our planet are characterized by a constant succession of different events at various levels. These events with consequences include, for example, extreme weather events caused by climate change, health crises, hunger, forced migration or growing poverty and inequality (Coles et al., 2024). These topics and more should and must also be addressed in schools and in teacher training, ideally in an integrative and interdisciplinary manner. As set out in detail in the ICMI Study 27 Discussion Document (Coles et al., 2024), the pressing questions for the (new) research area of mathematics education and the socio-ecological can be answered from different perspectives. Many researchers have already dealt extensively with socio-ecological aspects – taking theoretically reflective positions on the one hand and proposing practical approaches with regard to mathematics teacher education and mathematics teaching on the other (e.g., Barwell, 2013; Coles & Helliwell, 2023; Skovsmose, 2023).

This contribution focuses on the guiding principle of sustainable development with the 17 Sustainable Development Goals (United Nations, UN, 2015) and the concept of Education for Sustainable Development (ESD) (ibid.). With the ESD concept as defined by the UN (2015), certain goals are also set regarding to the practical implementation in the classroom in terms of the fulfillment of ESD competencies. It is therefore relatively clear how ESD is characterized (see below), how the criteria for ESD activities are designed and which ESD competencies are to be promoted (UN, 2015). Furthermore, in the research presented here, the resource or practice of mathematical modelling is used to teach ESD in an integrative way and from a mathematics educational perspective (Wiegand & Borromeo Ferri, 2024). In Germany, a nationwide analysis (Holst & Brock, 2020) has shown that many educational approaches to practically relevant ESD learning tasks tend to originate from the subjects of biology, geography, ethics and political science. Due to the educational concepts of environmental biology and global learning, these subjects – in contrast to mathematics – can draw on

a longer tradition of topic-oriented tasks. This makes the promotion of mathematics as a core discipline in the context of ESD and global learning even more important. Specifically mathematical modelling is a key practice for the author. Mathematical modelling is understood as a process of translating real-world problems into the language of mathematics, solving them with the help of mathematics, interpreting and validating the results and thus translating them back into reality (Niss & Blum, 2020). In addition to the inherent interdisciplinarity (Borromeo Ferri & Mousoulides, 2017), modelling activities offer the opportunity to know and recognize mathematics to answer current real-world questions in a fact-based and critically questioning manner (Wiegand & Borromeo Ferri, 2023).

Education for Sustainable Development (UN 2015) is a required cross-cutting task in the context of the transformation to global sustainability with the fulfillment of the 17 Sustainable Development Goals (SDGs). Sub-goal 4.7, Education for Sustainable Development (ESD), states that schools must also provide education *about* sustainability. According to the Ministry of Education in Germany (Standing Conference of Germany, 2015), this includes teaching the SDGs in a concrete manner. This also means out of a school subject, like mathematics, while promoting the core competencies of ESD: *recognizing, evaluating and acting*. Although ESD is of great importance to education policy and knowledge about the SDGs and ESD forms the basis for teaching in schools, ESD is not anchored in (mathematics) teacher training and it is not an explicit and visible component of (mathematics) school curricula worldwide (Dahl, 2019). Within the author's two-year study, 40 pre-service mathematics teachers were trained to teach ESD integratively with modelling. About 700 learners solved ESD modelling tasks. Using a mixed-method approach, various research questions have so far been investigated in sub-studies (e.g. Wiegand & Borromeo Ferri, 2023; Borromeo Ferri & Wiegand, 2023). This paper presents the current results of a sub-study with secondary school learners (N=105), which focused on the following research questions:

1. Can a difference of sustainability and environmental awareness of secondary school learners be determined statistically before and after solving an ESD modelling task? (total sample)
2. Do the learners' solutions of the ESD modelling task, their reflective descriptions and the observation protocol of the researcher reveal the ESD core competencies of recognizing, evaluating and acting? (case study, one class)

The study presented here aims to contribute to providing answers to the question posed in the ICMI Discussion Document within "Theme C. Resources for and of Mathematics Education, Sub-Theme 1": "How are mathematical resources used in understanding, interpreting and responding to the socio-ecological?" (Coles et al., 2024, p. 12).

THEORETICAL BACKGROUND

Guiding principles of sustainable development and education for sustainable development

With the aim of transforming national economies in line with global sustainable development, the 70th UN General Assembly (UN, 2015) adopted the 2030 Agenda with its 17 SDGs. With the guiding principle of sustainable development, the international community is pursuing the goal of reconciling economic progress with social justice worldwide within the Earth's ecological limits. The aim is therefore to achieve a balance between the three dimensions of sustainability – ecological, economic

and social – as well as intra- and intergenerational justice by fulfilling the 17 SDGs, according to the UN (2015). As already mentioned in the introduction, ESD is the result of political will, but it is the responsibility of the education ministries of the countries, educational experts, teacher educators, schools and teachers to structurally integrate ESD into the subjects, including mathematics lessons, and also to develop practical examples (Wiegand & Borromeo Ferri, 2023). The UN (2015) defines ESD as follows: (1) Key topics of sustainable development should be included in teaching and learning (e.g., climate change, poverty reduction), which also incorporate the dimensions of sustainability; (2) ESD calls for participatory teaching and learning methods that motivate and empower learners to reflect on their behavior and take action for (non)sustainable issues; (3) ESD should therefore promote competencies such as critical thinking, imagining future scenarios, and making decisions cooperatively. Moreover, the three core competencies of ESD, recognizing, evaluating and acting, should also be focused on (see also de Haan, 2002).

According to de Haan (2002) and the already existing OECD DeSeCo key competencies (Rychen et al., 2003), ESD-oriented teaching should meet certain ESD criteria in order to be able to convey (non)sustainable thinking and action. Therefore, an ESD topic or activity must be (a) a central, (b) local and/or global topic for sustainable development processes, (c) with long-term significance, (d) a subject over wide claim to knowledge, and (e) potential for action. ESD thus aims to (f) develop visions, and (g) provide creative solutions, while (h) demanding a participatory, inter- and transdisciplinary teaching and school culture. During the ESD-activity, learners must also be encouraged to adopt a “First-person perspective” in order to be able to formulate precisely these visions and options for action (in the long-term). These criteria also formed the basis for the development and implementation of ESD modelling tasks with pre-service mathematics teachers and ultimately in schools (Wiegand & Borromeo Ferri, 2023).

As a researcher I do not see any in-principle problems with the guiding principle of sustainability or the ESD framework. Rather, as always when something is politically driven, it is a general challenge for people working and researching in education, for those of us in mathematics education in particular, to implement it accordingly into school practice, based on empirical evidence.

Mathematical modelling in the context of ESD

With the help of mathematics, problems and phenomena in reality can be better understood and solved. In the process known as mathematical modelling, a real-world problem is simplified and transformed into a mathematical problem, which is then solved and brought back to reality (Niss & Blum, 2020). Wiegand and Borromeo Ferri (2023) have argued in detail why mathematical modelling is suitable for teaching and learning ESD “concretely” from a mathematical or mathematics educational perspective: The focus of an ESD goes beyond isolated subject knowledge, towards thinking in contexts and in alternatives, towards systemic and transformative thinking. In order for this integrated and interdisciplinary learning approach (Hobbs et al., 2017) of ESD to be conveyed, one obvious possibility is that the teaching and learning of ESD takes place through real questions or task formats with consideration of age-appropriate mathematical content. Mathematical modelling represents an excellent practice for promoting sustainability goals and socio-ecological issues from the subject of mathematics and across disciplines (English, 2009). The current empirical research situation in relation to the definition of ESD is scarce in mathematics education. There are still too

few studies on teacher education of ESD through mathematics education. We know little about how and whether learners can grasp the importance of sustainability through mathematical working and thinking. However, good tasks and, specifically in relation to modelling, good modelling problems (Borromeo Ferri, 2018) that meet the criteria of being realistic, open, authentic, problem-based and going through the phases of the modelling process, are central to quality mathematics teaching (Maaß, 2010). In summary, an ESD modelling task is defined, if it combines and features the ESD criteria and the criteria for modelling tasks (Wiegand & Borromeo Ferri, 2023).

METHODOLOGY AND DESIGN OF THE STUDY

The sub-study presented here, consisting of quantitative and qualitative methods, is a part of a larger study by the author on “Learning and Teaching ESD through mathematical modelling” in school and teacher education. A pre-post intervention study was conducted with N=105 secondary school learners aged between 14 and 16 years in a total of five classes. The learners participated voluntarily, and parents consented to learners’ participation in the study so that ethical guidelines were complied with. The design of the study was as follows: Learners were first given a general introductory lesson (45 min) on modelling, sustainability and the SDGs in the first part of the intervention phase. The aim was to bring the learners’ knowledge up to a common level. On the following day, the learners worked in the second part of the intervention phase, on a 90-minute ESD modelling task, with appropriate support from the teachers in line with the ESD criteria. Specifically, the lesson was usually introduced by showing the learners a short documentary of around three to four minutes on the topic. Alternatively, there were always pictures and information texts. This gave the learners an insight into the problems of water supply in many countries, for example. This is an emotional approach, but it ultimately introduced the learners to the topic intellectually. The question of the ESD modelling task was presented and questions were clarified. Then the learners worked in groups of four, results were presented, and finally discussed critically from a modelling and ESD perspective. All teachers were trained by the author to teach ESD and modelling integratively. The learners completed a questionnaire before and after the intervention.

Survey and evaluation methods

The pre- and post-questionnaires were deliberately kept short. In addition to asking for demographic data, the focus was on the scale of subjective assessment of sustainability and environmental awareness (Preisendörfer, 1999) with 12 closed items. The learners had to mark the statements on a 4-point Likert scale from “completely agree” to “strongly disagree.” Example items are: “If we carry on as we are, we are heading for an environmental catastrophe” or “There should be fair trade between the rich countries of the world and the developing countries.” The aim was to find out whether a change in the learners’ attitudes could be statistically recorded after the short intervention. The scales were evaluated using inferential statistics. Cohen’s *d* was calculated using a *t*-test based on the mean values (Creswell, 2008). For the case study and thus the qualitative part of the study, a seventh grade class with 25 learners was selected from the overall sample and the lessons were also observed and recorded by the author. This class has an average performance level in mathematics. The aim was once again to specifically record how mathematical modelling forms the basis for talking about sustainability problems and to what extent ESD skills become apparent in this process. Finally, the

learners’ written solutions were evaluated by the author and a further researcher to determine whether the ESD skills of recognizing, evaluating and acting were present.

ESD modelling task

Table 1 presents one of the ESD modelling tasks that was the key survey instrument in the study. The task was developed according to the ESD and mathematical modelling (ESD-MM) criteria.

ESD-MM-Task:	Explanation
<p>How much water do you need per year for showering and bathing – how sustainable are you really? – The Karl-Heinz Böhm Foundation report states: “An Ethiopian has an average of 25 liters of water per day, in rural areas it is sometimes only 5 to 10 liters...” Not so little at first glance, is it?</p> <p>(The following additional activity was included in the task text for the learners):</p> <p>Scan the QR code and think about your own consumption when you find out about water scarcity in Africa. Write down what makes you think the most! What options for action and visions do you have for yourself and for us as a society to reduce water?</p>  <p>Further comments: The learners had to <i>recognize</i> what the local and global problem of water scarcity means. Through the modelling, the learners have further <i>recognized</i> the importance of the topic, but above all, the solutions have provided them with an <i>evaluation basis</i> for critical reflection. The possible realistic results (e.g. 288ecogni. 22,000 liters for showers per year) must be embedded in the context by the learners! What does this “22,000 liters” mean in comparison (e.g., to water consumption in one’s own country) and is it only a part of their own water consumption per year? On this basis, the learners can then develop <i>options for action</i> and reflect on them themselves, again using the information text.</p> <p>The modelling process and especially the validation of the results are the central evaluation basis for critical reflection. The possible results were discussed in plenary with the teacher, as multiple solutions were also found based on the different assumptions.</p>	<p>The ESD-MM task focuses mostly on SDGs 6 (clean water) and 12 (sustainable consumption). This makes it a <i>central key topic</i> (see def. ESD 1). The topic of water scarcity is central and worthy of discussion, especially as seventh graders can question numerical values. The learners were given the quote of the Karl-Heinz Böhm Foundation in order to think directly about the <i>global</i> problem of water use from their <i>local</i> perspective, as water scarcity has a longer-term significance. In addition to mathematics, the cross-curricular knowledge requirement in the task is also based on dealing with the living conditions of people in Ethiopia (de Haan, 2002, criteria a,b,c,d). The duration of the shower and the associated loss of water often only become apparent through restriction. Solutions of up to 200 liters of water for one shower are thought-provoking. Although the question relates only to showering and bathing, it is an open task that can be posed more broadly at any time. The learners made assumptions and researched the water consumption for showering, even for washing their hands. The mathematical models can be presented as pure calculations or functional relationships, dealing with quantities was required, solutions for days, weeks and years had to be extrapolated if necessary. Overall, the data had to be recorded and then interpreted and validated during modelling. The second part of the task fulfills the two def. points of ESD (2) and (3) as well as the other ESD criteria (c, f, g, h) according to de Haan (2002), as it asks about <i>participation and options for action</i>. All criteria for a modelling task are also met.</p>

Table. 1: ESD-MM Task with explanation

RESULTS OF THE STUDY

Research question 1 is answered first. The reliability of the sustainability and environmental awareness scale has a Cronbach's alpha of .73, which is satisfactory. The t -test with $N=105$ for paired samples with $p < .001$ (p -value 2-sided) results in a Cohen's $d = .204$. According to Cohen (2013), this is a small effect. Nevertheless, the significant increase in sustainability and environmental awareness among the learners in the overall sample is a surprising result after the short intervention.

Results for research question 2, provides a deeper insight into the effect of the intervention beside the statistic result, although only the central findings can be presented here. A prototype example is the citation of Lukas (15 years) on his worksheet before he started working on the ESD-modelling task: "I don't deal with topics like sustainability." After the intervention he wrote during reflection: "I have learned that sustainability is important for the world!". What progress, so one could assume, that the topic of the task has motivated him to think more about sustainability issues. Since the learners generated individual solutions despite the group work, a good analysis was possible. All learners achieved realistic modelling results, partly modified by the validation and discussion in plenum. The ESD competencies recognizing, evaluating, and acting were evident in different ways among the learners. All learners (25/25) showed that they recognized the problem of the question and all (25/25) were ultimately able to evaluate it through their own calculations. Fifteen of the 25 learners succeeded in formulating concrete options for action and visions in writing, based on their own reflections. These included, for example, the consistent use of energy-saving shower heads that use less water, not taking baths, or not showering for 20 minutes anymore. The learners therefore adopted the "First-person perspective" and reflected on their future actions. In particular, the text to be read (see QR code, Table 1) quickly stimulated conversations among the participants. The reason for this was the mathematical modelling problem they had solved by themselves and the resulting realization that they could compare their own solutions with those from reality in other countries. Based on the author's observation protocol, all learners took part in the feedback round and expressed critical thoughts on as well as dismay at the topic of water scarcity in other countries. Learners initially reported being astonished that they would consume such a large amount of water. Some reported that they would use much more water over a normal day. It was also exciting that one learner, who is not normally involved in mathematics lessons, talked about his family's own experiences. He talked about his family in Africa: "They use just one bucket of water for washing with the whole family."

The following is an insight into a learner solution: Lilli (14 years) had researched that six minutes of showering requires around 72 liters of water. Since Lilli, according to her assumptions, showers on average three times a week for about six minutes, she came up with 216 liters per week. She extrapolated this to the year, but increased the duration and number of showers in some summer months, so that she finally got a solution of approximately 14,900 liters per year. She was shocked by this solution and wrote:

I find it incredibly scary and disconcerting how much water we consume! When I first read that people have only 5-25 liters of water available per day, I thought, "that's not so little..." Now I've modelled it myself and I know it better! I also think it's terrible that so many people die (every day) because they don't have (clean) water!

In her first model, Lilli only referred to showering, later she also included washing hands and laundry. Another learner, Tim, wrote: "I have learned that we use far too much water and that we urgently

need to change things so that we don't run out of water. I pay attention and turn the tap off again quickly!" Anna wrote: "We should also eat less meat, it also uses up water."

DISCUSSION

"How are mathematical resources used in understanding, interpreting and responding to the socio-ecological?" is a question from the ICMI Study 27-Discussion Document (Coles et al., 2024, p. 12). Based on my own previous research and the current study, I can offer a possible answer: mathematical modelling in the context of ESD represents a key practice through which learners can critically question sustainability issues based on their own mathematical/realistic results through the modelling process, and thus also acquire ESD competencies. In a sub-study as part of a larger study, a short intervention with ESD-MM tasks increased the learners' awareness of sustainability and environmental issues. Longer interventions with further tasks targeting more SDGs are in the research agenda of the author and in progress.

However, a further point for discussion arises from my examination of the ICMI Study 27-Discussion Document (Coles et al., 2024): the topic of ICMI Study 27 relates to mathematics education and the socio-ecological (MaSoEc). From the perspective of the UN guiding principle of sustainability, the dimension of the economy, which undisputedly has a strong interrelated role with the other two dimensions, is missing in my view. The author has thus come to the necessary realization that a clarification of the terms and aims is important, although the economy is inherently present in the Discussion Document. For instance, can all best-practice examples of climate education per se be classified ESD, even though the described research does not comply with the ESD definition and the ESD criteria and aims of a corresponding activity? Is ESD a part of MaSoEc or vice versa? In my opinion, this needs to be clarified because it is important for research. There is a large overlap between the characterization of MaSoEc to date and ESD, but it is not the same. Consequently, an important aspect to think about is: from an educational policy perspective, the ESD framework is strongly promoted. Educational institutions will try to implement ESD accordingly. How will MaSoEc position itself in the educational context or pursue the goal of practice implementation? Summarizing, I therefore cautiously argue that with the ESD-MM approach I am contributing to answering the above question in the context of mathematics education and the socio-ecological.

References

- Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1-16.
- Borromeo Ferri, R. (2018). *Learning how to teach mathematical modelling in school and teacher education*. Springer International Publishing.
- Borromeo Ferri, R., & Mousoulides, N. (2017). Mathematical modelling as a prototype for interdisciplinary mathematics education? – Theoretical reflections. In *CERME 10, Feb 2017*. Hal-01933490
- Borromeo Ferri, R., & Wiegand, S. (2023). Teaching Education for Sustainable Development – Challenges and successes of pre-service mathematics teachers. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 131-138). PME 46.
- Coles, A., & Helliwell, T. (2023). The role of mathematics teacher educators in preparing teachers of mathematics to respond to global challenges within their classrooms. *London Review of Education*, 21(1). <https://doi.org/10.14324/LRE.21.1.02>

- Cohen, J. (2013). *Statistical power analysis for the behavioral sciences*. Elsevier Science. <https://ebookcentral.proquest.com/lib/kxp/detail.action?docID=188284>
- Creswell, J. W. (2008). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage.
- Dahl, T. (2019). Prepared to teach for sustainable development? Student teachers' beliefs in their ability to teach for sustainable development. *Sustainability*, 11(7), 1993.
- De Haan, G. (2002). The core themes of education for sustainable development. Advance online publication.
- English, L. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM*, 41(1-2), 161-181.
- Hobbs, L., Clark, J. C., & Plant, B. (2017). Successful students – STEM program: Teacher learning through a multifaceted vision for STEM education. In R. Jorgensen & K. Larkin (Eds.), *STEM education in the junior secondary: The state of play* (pp. 133-168). Springer Singapore.
- Holst, J. & Brock, A. (2020). Bildung für nachhaltige Entwicklung (BNE) in der 291ecogn. Strukturelle verankerung in schulgesetzen, lehrplänen und der lehrerbildung. https://www.ewi-psy.fu-berlin.de/einrichtungen/weitere/institutfutur/Projekte/Dateien/2020_BNE_Dokumentenanalyse_Schule.pdf, last proof 20.05. 204
- Coles, A., le Roux, K., Borba, M., Bose, A., Geiger, V., Gutiérrez, R., Makramalla, M., Solares-Rojas, A., Sinclair, N., Valero, P., Vistro-Yu, C., Leung, F.K.S., & Dorier, J-L. (2024). *ICMI Study 27 discussion document. Mathematics education and the socio-ecological*. ICMI Study 27.
- Maaß, K. (2010). Classification scheme for modelling tasks. *Journal Für Mathematik-Didaktik*, 31(2), 285-311.
- Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling. IMPACT: Interweaving mathematics pedagogy and content for teaching*. Routledge.
- Preisendörfer, P. (1999). *Environmental attitudes and environmental behavior in Germany: Empirical findings and analyses based on the population surveys "Environmental awareness in Germany 1991-1998"*. VS Verlag für Sozialwissenschaften GmbH. <https://ebookcentral.proquest.com/lib/kxp/detail.action?docID=6673053>
- Rycken D.S., Salganik L.H., & McLaughlin M.E. (Eds.) (2003). *Contributions to the Second DeSeCo Symposium*. Neuchâtel, Switzerland: Swiss Federal Statistical Office.
- Skovsmose, O. (2023). *Critical mathematics education*. Springer.
- Standing Conference of Germany. (2015). *Framework for Global Development Education: In the context of Education for Sustainable Development* (2nd ed.). Cornelsen. https://www.kmk.org/fileadmin/Dateien/veroeffentlichungen_beschluesse/2015/2015_06_00-Orientierungsrahmen-Globale-Entwicklung.pdf
- United Nations. (2015). *Transforming our world: The 2030 agenda for sustainable Development*. <https://sdgs.un.org/publications/transforming-our-world-2030-agenda-sustainable-development-17981>
- Wiegand, S., & Borromeo Ferri, R. (2023). Promoting pre-service teachers' professionalism in STEAM education and Education for Sustainable Development through mathematical modelling activities. *ZDM*. Advance online publication.
- Wiegand, S., & Borromeo Ferri, R. (2024). Teaching and learning of Education for Sustainable Development through Modelling activities with an integrative teaching approach. In H.-S. Siller, G. Kaiser & V. Geiger (Eds.) *Researching mathematical modelling education in disruptive/challenging times* (ICTMA 20). (pp.665-675). Cham: Springer. https://doi.org/10.1007/978-3-031-53322-8_55

DIGITAL VIDEOS AND EDUCATION FOR SUSTAINABILITY: A POSSIBILITY FOR THE DEVELOPMENT OF SOCIO-ENVIRONMENTAL AWARENESS

Geciara Carvalho¹, Marcelo Borba²

¹Universidade Estadual de Feira de Santana, ²São Paulo State University (UNESP)

This work aims to understand the mathematical discourse of the video “Sea of Mud” as a socio-ecological gesture in mathematics education, effectively integrating education with environmental sustainability. Based on multimodal discourse analysis, grounded in social semiotics and the construct of humans-with-media, we analyzed the narrative in question as a social production. Finally, we consider that the producers-with-media mobilize the digital discourse in an interdisciplinary and technological manner, involving various levels of world knowledge that are recontextualized to foster socio-environmental awareness.

INTRODUCTION²⁷

The phrase by Paulo Freire, “There is a mathematical way of being in the world” (Freire et al., 1997), underscores the importance of qualitative approaches in understanding and addressing social, ecological, health, economic, and political challenges, especially in the context of education for sustainability. Environmental sustainability, the central theme of various global agendas, demands precise tools and robust models to tackle complex crises such as climate change and biodiversity loss. Climate models, based on mathematical equations, allow for the simulation of Earth’s climate behavior and prediction of the impacts of greenhouse gas emissions, while ecological models map interactions between species and their environments, providing crucial insights for conservation strategies.

Data analysis, particularly through big data and advanced analytics, emerges as an indispensable tool in identifying patterns and trends that inform sustainable policies and actions. Sustainability indicators, based on mathematical metrics, are essential for monitoring progress toward goals such as reducing CO₂ emissions and conserving natural resources. Additionally, mathematical optimization techniques applied in natural resource management help minimize waste and maximize economic and ecological benefits, while life cycle analysis assesses the environmental impacts of products and processes throughout their entire life cycle.

In decision making, the use of decision models and risk analysis allows for a careful evaluation of different policy options and actions, considering their costs, benefits, and uncertainties. Environmental economics applies economic models to quantify the costs and benefits of sustainable interventions, thus justifying investments in this area. Finally, education and communication greatly benefit from mathematical tools such as graphs, maps, and simulators, which make complex information more accessible and promote awareness about sustainability issues.

²⁷ This work is based on data from the doctoral thesis titled *Digital Video Festival and Critical Mathematical Education* (Carvalho, 2023).

These perspectives suggest that we increasingly shape a “calculable” world, which can sustain the notion that mathematical truths are universal and immutable, and that mathematics offers privileged access to an objective and unquestionable reality (Borba & Skovsmose, 2005).

However, “Numbers have contexts!” This is what elementary school students asserted when producing a digital discourse, the video “Sea of Mud,” depicting one of the largest environmental disasters in Brazil in 2019, at the Festival of Digital Videos and Mathematics Education (FVDEM). This highlights the relevance of a more critical and contextual view of mathematics, where the social issue at hand is inseparable from an ecological dimension, and the ecological issue is inseparable from a social dimension, recognizing its complexities and cultural influences. This concept has profound implications for mathematics education, especially in students’ learning experiences. Therefore, this article explores how digital videos can be effectively integrated into education for sustainability and aims to understand the mathematical discourse of the video “Sea of Mud” as a socio-ecological gesture in mathematics education.

THEORETICAL FRAMEWORKS

According to Carvalho (2023), the FVDEM²⁸ is a space for the creation of mathematical ideas, configured in a hybrid form as a means by which mathematics can represent and transform contexts. This festival has been gaining prominence in the current context of Brazilian education, with the participation of elementary and secondary school students, mathematics graduates, elementary and higher education teachers, Indigenous peoples, as well as the community representing all Brazilian states and the Federal District. From our perspective, the festival’s videos play an active role in building human consciousness change, both for those who produce them and those who watch them, meaning the videos have the potential to drive social transformation. Therefore, we affirm that videos and humans interact and have the power to transform consciousness. Thus, the FVDEM plays a significant role in fostering socio-ecological awareness by incorporating video production into mathematics classes, encouraging reflections on sustainability (Domingues & Borba, 2021). Consequently, some of these videos bring critical issues in their discourse, addressing climate change, deforestation, water and food insecurity, biodiversity loss, pandemics, poverty, inequality, among other topics of interest to mathematics education, as Borba (2021) pointed out about the future agenda involving digital technologies, critical mathematics education, and the Philosophy of mathematics education.

These videos contain digital discourses, which are computational, multisemiotic, and multimodal in nature, co-produced with digital technologies such as cameras, editing software, computers, etc. This is based on a vision of technologies where humans and non-humans, in a collective process, produce knowledge (Borba & Villareal, 2005; Borba, 2021). Based on the understanding of language as social semiotics (Halliday, 1978), discourses are “socially constructed knowledge of (some aspect of) reality” (Kress & Van Leeuwen, 2001, p. 4). Social semiotics raises the issue of meaning as a social construction. This implies that meaning is affected by the different social contexts in which individuals interact in the world, constructing different discourses in the digital medium. Thus, in the scope of this work, media-producers are discourse producers, whose authorship is a collective action (Carvalho, 2023).

²⁸ <https://www.festivalvideomat.com/>, accessed June 15, 2024.

Therefore, the choices made by media-producers through semiotic resources for composing their discourses—such as words, images, colors, music, gestures, photographs, among others—are articulated in the production of meaning and integrated through sensory modalities (e.g., visual, auditory, tactile, olfactory) in multimodal phenomena like video. Hence, we can say that these resources express and manifest indications of the occurrence of an action-reflection-action process expressed in the reconstitution of concrete reality when humans and media interact collectively in video production for FVDEM, constituting the agency of human and non-human actors in the humans-with-media (H-W-M) collective.

Cunha (2023) and Borba et al. (2023), when investigating the process of agency in humans-with-media collectives, suggested that, in these contexts, agency manifests in a systemic and omnipresent manner. In his research, Cunha (2023) evidenced that students' agentive actions, such as taking measures and committing to solve mathematical problems, involved co-participation with digital media. Based on this finding, the author suggests that "certain agencies may not be intrinsically human, but result from co-participative agencies in a human-with-things collective, thus constituting ubiquitous agency" (Cunha, 2023, p. 86).

This work presents the video "Sea of Mud" as a socio-ecological gesture in mathematics education. According to Coles et al. (2024), we understand its production as an investigative practice that layers knowledge within diverse social activities and power relations. It also functions as a curricular resource, serving as learning material (Oechsler, 2018; Gimenez, 2023). When the video gains visibility through social networks and FVDEM, this aspect further strengthens its role as a socio-ecological gesture in educational spaces, as it engages broader audiences in discussions on socio-environmental themes within mathematics education. "The term 'socio-ecological' points to the totality of relationships involving human, non-human, living, and non-living actors" (Coles et al., 2024, p. 171), not separating the social from the ecological in discourse and listening. The understanding here is of a social and ecological entanglement, in a form of responsive listening expressed by the video as a gesture that can operate across various layers of abstraction or generality (Coles et al., 2024).

METHODOLOGY

The methodology employed is based on the analytical model developed by Carvalho (2023), focusing on the analysis of scene sequences from the video "Sea of Mud." The study examines the multimodal elements present, such as images, graphs, mathematical symbolism, animations, verbal language, and special effects, which give form and expression to the work, considering a situational and sociocultural context. This approach aligns with a critical perspective of mathematics education, encompassing a semiotic landscape in the composition of the video's digital discourse.

The methodological steps align with the social semiotics approach in multimodal research: data collection and recording; visualization and gathering of samples; and data transcription and analysis (Bezemer & Jewitt, 2010). The technique used was based on Taylor's (2013) multimodal transcription, which involves inserting images of static frames and meta-textual descriptions into a table to establish meaning in a multimodal text. This process facilitates the identification of relationships that occur as information is modified in on-screen captions and voice narration, which, in the case of the studied video, is directed in dialogue directly at the audience.

To develop this qualitative study, we segmented the narrative “Sea of Mud” through moments characterized by one or more sequences of scenes, aiming to decompose the video by dividing it into parts while simultaneously understanding it in detail from an “internal view,” from the inside out, to grasp the whole (Carvalho, 2023). This procedure is based on multimodal discourse analysis—systemic-functional (O’Halloran, 2004) for moving images, and Freire’s culture circle (2022) as a means of action-reflection-action on reality.

VIDEO ANALYSIS

The background of the narrative “Sea of Mud” is the tragedy caused by the collapse of Dam 1 (B1), which occurred on January 25, 2019, at the Córrego do Feijão Mine, controlled by the mining company Vale, in the municipality of Brumadinho, Minas Gerais, Brazil. In this tragedy, 270 people lost their lives in the mud, and the survivors face irreparable losses resulting from the environmental collapse. The video was produced with narrators who are sixth-grade students from a public school in Belo Horizonte, Minas Gerais, who experienced and narrated the stages of a mathematical modeling project carried out in the classroom. In this process, the students demonstrated connections with the viewers, participants of the FVDEM, and social networks, discussing their perceptions, emotions, understandings, and positions in responsive action through the video.

The analysis done here has a broad pedagogical effect, aiming to go beyond the literal reading of the expressed content to uncover what may be implicit in the text, extrapolating the mathematical discourse by viewing the narrative discourse as a whole. In analyzing its discourse, we identified five key moments throughout the video, which are:

MOMENT I: Collective Responsibility for Reality (00’– 20’)

MOMENT II: Statement of the Situational Context (21’– 56’)

MOMENT III: Simulation of Reality Coding / Learning Based on Reality Involving Prior Knowledge and the Search for New Knowledge (57’– 3’17’)

MOMENT IV: Mathematics in Action – Numbers in Context (3’18” – 4’42’)

MOMENT V: Intellectual and Social Responsibility in Promoting Social Justice (4’42” – 5’50”).

Moment I is marked by a sequence of scenes that emphasize *collective responsibility for reality*, as expressed in the students’ discourse. They state:

It is 5,330,000 cubic meters. It’s 57 hectares, right? These are not just numbers without context. Do you know how much this is worth? It cost many lives! A lot of destruction! And many stories... It was a tragedy! We are from Minas Gerais; we will not remain silent!

And they position themselves critically by problematizing reality, as depicted in Figure 1. The students see themselves as part of a historical context of dam ruptures in the state of Minas Gerais, denouncing that it was not an accident, but a tragedy exacerbated by the state’s inertia in the face of similar previous events, thus critiquing the “disaster capitalism,” the destruction of lives, and the environment. Here, the social and ecological dimensions are not separated.



Figure 4 – Problematization of reality. Source: Carvalho (2023)

At this point, based on the observation and identification of a socio-ecological problem, the *situational context* is articulated through short videos inserted as parts of the edited video, depicting real scenes from the moment the dam rupture occurred. Let us refer to Figure 2, which clarifies Moment II, through the investigation: “How much waste from the rupture of the Córrego do Feijão dam in the city of Brumadinho needs to be removed to start an environmental recovery project?” At that time, the combination of image, oral, and written language highlights human responsibility in environmental degradation. The media’s multimodal nature (image and sound) acts as a powerful co-producer of discourse, mobilizing viewers and provoking emotional and cognitive responses that can lead to collective action, as noted by Carvalho (2023). The agency of the students-with-media collective is evident in shaping reality and investigating the modeling project, as discussed by Cunha (2023) and Borba et al. (2023).

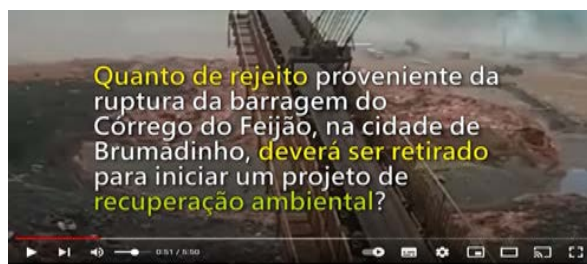


Figure 5 – Thematic coding. Source: Carvalho (2023)

Moment III (see Figure 3), on the other hand, consists of scenes that associate the concept with the image, completing the construction of meanings. In this sequence, we observe a transdisciplinary approach *grounded in reality, involving prior knowledge and the development of new knowledge* through the multiplicity of channels that integrate theory and practice: satellite visualization, a model of the mud containing the elevation contours, and a topological map of the affected region. The image of the area affected by the sea of mud, accessed via satellite, real-time photos and videos, combined with the construction of models and topographic maps, and integrated with narration in the video, creates an understanding of the magnitude of the environmental problem. This representation engages viewers not just passively, but actively in constructing collective knowledge about the crisis. This knowledge, which encompasses both mathematical and ethical dimensions, is crucial for raising awareness (Carvalho, 2023). The emotional impact of viewing a concrete problem prompts viewers to reflect on their own responsibility and engage in actions to mitigate the issue. This can lead to the formation of action networks, where individuals unite around common goals, such as advocating for stricter environmental policies.

This allows for the understanding of the socio-environmental situation in a collective construction and the development of concepts from multiple perspectives, transcending disciplinary barriers. This

process transforms a set of information, discourses, and facts, including those from the internet and newspapers, into recontextualized knowledge, resulting from awareness arising from the agency of human-with-media (Borba et al., 2023).



Figure 6 – Multiplicity of channels. Source: Carvalho (2023)

Moment IV is marked by *mathematics in action – numbers in context*, aiming to estimate the calculation of the mud, resulting from a simulation of the approximate integration calculation of an irregular area, making it possible to determine the amount of mud that should be extracted from the affected area, as detailed in Figure 4. This thus displays a process of experimentation with technologies marked by questioning and acknowledgment of the uncertainty of mathematical precision in calculating irregular areas.

...Sabemos calcular a área de quadrados, como calcular essa área esquisita?

Cada grupo escolheu a área de um quadrado para preencher a área da cota estudada.

Quando não era mais possível, utilizaram a estratégia de usar a metade do quadrado anterior para preencher os espaços não contemplados ainda no preenchimento em curso.

Novamente, repetia o processo com áreas ¼ de área da área anterior até se obter o preenchimento de toda área e assim, sucessivamente.

UMA NOÇÃO DE INTEGRALIDADE da área irregular!

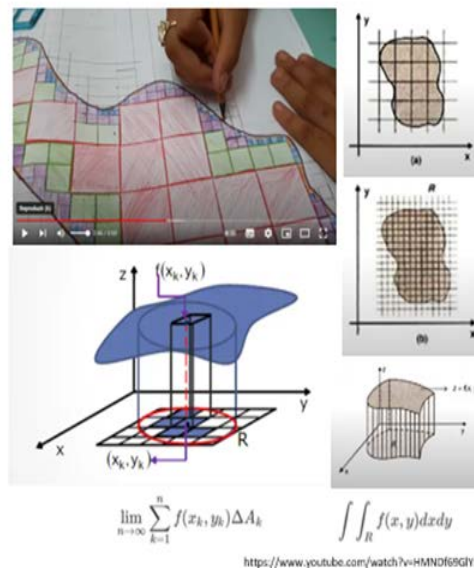


Figure 7 – Mathematical investigation. Source: Carvalho (2023)

Finally, Moment V expresses *intellectual and social responsibility in promoting social justice*, as the storyline reveals the students’ interest in helping the residents of Brumadinho, calling for actions from the authorities for environmental recovery. In this context, the students mobilize to simulate ways to address complex everyday demands and the full exercise of citizenship in their digital discourse, explicitly articulating a political stance (see Figure 5):

With the data from our work, we hope to help the residents of Brumadinho. But how? By demanding from the authorities strategies for environmental recovery. To achieve environmental recovery, we need to remove the mud from there! For example, placing it in a waste pile... Will the authorities do that?



Figure 8 – Political stance. Source: Carvalho (2023)

Thus, the narrative critically reconstructs the media discourse, highlighting those multimodal resources combined in the video media, as a mirror of reality connected to the real world. This mirror of reality helps us understand and produce mathematical knowledge and socio-environmental awareness among students. Therefore, the digital discourse, the “Sea of Mud” video, produced through the interaction of students-teacher-with-media, recreates reality and fosters socio-ecological debate in both concrete and virtual situations simultaneously through the semiotic power of the video. This entire approach transcends the boundaries of the classroom, the school, and the FVDEM, promoting a dynamic interaction between theory and practice. The specific connection between the elements analyzed in the video and the collective action of humans-with-media demonstrates how this combination can generate knowledge and mobilize society to address socio-ecological challenges. The collective action resulting from this mobilization of consciousness through the digital mathematics video as a gesture can have significant socio-ecological impacts. For example, awareness and public pressure can lead to the implementation of measures for river restoration or to prevent the recurrence of practices that caused pollution or environmental destruction. Thus, students-with-media not only inform but also transform reality by enabling the organization of collective efforts in favor of environmental sustainability.

FINAL CONSIDERATIONS

Why is the analysis developed above important for socio-ecological issues? This article suggested that a collective effort between students, the teacher, and media facilitated the creation of a digital discourse, a mathematics video, that takes a side on the environmental sustainability debate. Digital technologies were used in a critical, meaningful, reflective, and ethical manner in school practices to communicate a mathematical modeling project. The video was not a neutral way of storytelling, on the contrary it has agency (Borba et al., 2023). The video’s discourse showcased the totality of relationships involving human and non-human actors, fostering the development of political and socio-environmental awareness. The theoretical construct collectives of humans-with-media, stresses the agency of humans and non-humans. In this paper, we emphasize the power of video, produced by humans, that in six minutes can discuss high level mathematics and socio-ecological issues. This allowed us to understand the “Sea of Mud” video as a socio-ecological gesture of mathematics education, produced by sixth graders and a teacher. The collective of human-with-media defended their ideas and viewpoints, making common decisions that respect and promote human rights and

socio-environmental awareness, with an ethical stance towards caring for others and nature as if it were directed to oneself.

References

- Borba, M. C. (2021). The future of mathematics education since COVID-19: Humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics*, 108(1-2), 385–400.
- Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, experimentation and visualization*. Springer.
- Borba, M. C., & Skovsmose, O. (2004). A ideologia da certeza em educação matemática. In O. Skovsmose (Ed.), *Educação matemática crítica: A questão da democracia* (2nd ed., pp. 127-148). Papirus.
- Borba, M. C., Souto, D. L. P., Cunha, J. F. T., & Domingues, N. S. (2023). Humans-with-media: Twenty-five years of a theoretical construct in mathematics education. In B. Pepin, G. Gueudet, & J. Choppin (Eds.), *Handbook of digital resources in mathematics education* (pp. 1-23). Springer.
- Carvalho, G. S. (2023). *Festival de vídeos digitais e educação matemática crítica*. [Tese de Doutorado]. Universidade Estadual Paulista. <https://hdl.handle.net/11449/250946>
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116(1), 165–183.
- Cunha, J. F. (2023). *Licenciatura híbrida em matemática: Quais são os papéis dos vídeos digitais* (Doctoral dissertation, Tese de doutorado, (Doutorado em Educação em Ciências e Matemática)-Rede Amazônica de Educação em Ciências e Matemática, Cuiabá).
- Domingues, N. S., & Borba, M. C. (2021). Digital video festivals and mathematics: Changes in the classroom of the 21st century. *Journal of Educational Research in Mathematics*, 31(3), 257–275.
- Freire, P. (2020). *Pedagogia do oprimido* (74 ed.). Rio de Janeiro: Editora Paz e terra.
- Freire, P., D'Ambrosio, U., & Mendonça, M. D. C. (1997). A conversation with Paulo Freire. *For the Learning of Mathematics*, 17(3), 7–10.
- Gimenez, H. (2023). *Vídeos digitais e Educação Matemática: Uma possibilidade de pesquisa educacional baseada em arte* [Doctoral dissertation, Universidade Estadual Paulista]. Universidade Estadual Paulista Repository. <http://hdl.handle.net/11449/242350>
- GPIMEM. (2020, December 16). *Ensino Fundamental – Mar de Lama: Modelagem na Educação Matemática – III FVDEM (Legends in English)* [Video]. YouTube. <https://www.youtube.com/watch?v=YpCteGqjxd0&t=5s>
- Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Edward Arnold.
- Kress, G., & Van Leeuwen, T. (2001). *Multimodal discourse: The modes and media of contemporary communication*. Arnold.
- O'Halloran, K. L. (2004). Visual semiosis in film. In K. L. O'Halloran (Ed.), *Multimodal discourse analysis: Systemic functional perspectives* (pp. 109-130). Continuum.
- Oechsler, V. (2018). *Comunicação multimodal: Produção de vídeos em aulas de Matemática* [Doctoral dissertation, Universidade Estadual Paulista]. Universidade Estadual Paulista Repository. <http://hdl.handle.net/11449/154093>
- Taylor, C. J. (2013). Multimodality and audiovisual translation. In Y. Gambier & L. Van Doorslaer (Eds.), *Handbook of translation studies* (Vol. 4, pp. 98-104). John Benjamins Publishing Company.

PROBLEM-BASED LEARNING AND MATHEMATICS IN HIGHER EDUCATION TO RAISE AWARENESS ABOUT ENVIRONMENTAL DISASTERS

Denner Dias Barros¹, Johnny Nazareth dos Santos²

¹University of São Paulo (USP), ²Federal University of Rio de Janeiro

Over time, society suffered several changes, like the ways of living and dressing, the way of thinking about social relationships, among others. Recently, climate change has generated a lot of reflection in different segments of society due to the extreme events that have occurred around the world. Cities are being destroyed, people are dying and animal and plant species are being extinguished by the neoliberal advance. We believe that people's awareness of socio-environmental issues occurs in different environments and that one of them is the educational space. In that regard, this article will present an activity developed with students from various postgraduate courses at the University of São Paulo in a discipline entitled Pedagogical Preparation that aims to prepare them for teaching in higher education. The activity was performed using the active methodology Problem-Based Learning, which opposes the traditional teaching model and allows learning to occur in an innovative, active way and based on the resolution of a social problem.

CONCERNS AND AWARENESS

Extreme weather events have hit several continents and raised alarm bells among a large part of the world population. Researchers from all over the world are concerned about the signs that nature has given, and the slowness on the part of society in providing effective responses in the proportion necessary to combat the advancement of environmental destruction and the catastrophic effects on people, especially those who belong to the less favored classes. It is undeniable that all citizens must make their contribution to the current climate situation and rethink attitudes that do not contribute positively to the environment.

Thinking about our daily relationships with other human beings and animal and plant species and seeking to change them (in negative cases) or expand them (in positive cases) from a perspective that guarantees the possibility of living with dignity is a process (pedagogical and political) fundamental and which characterizes this perspective of education (Reigota, 2016, p. 13).

According to Krenak (2020), the era characterized as the Anthropocene represents this distance that we human beings maintained from the environment, degrading it as if the effects of these actions would not fall on ourselves. And, therefore, we must re-establish our relationships with nature by understanding that we are beings belonging to/integrated with it.

When people sometimes talk to me about imagining another possible world, it is in the sense of reordering relationships and spaces, of new understandings about how we can relate to what is admitted to be nature, as if we were not nature (Krenak, 2020, p. 67).

The author also emphasizes that the propagated skeptical future does not represent the only possibility for this humanity, and he argues that we have the opportunity to tell other stories, as, in this way, it will be possible to postpone the end of the world (Krenak, 2020).

Due to the recent tragic consequences and the pessimistic projections for the coming decades, environmental issues are present in countless social discussions and guide several international agreements both in the private market and between countries that maintain commercial relations. In this sense, Chomsky and Pollin (2020) defend the Green New Deal, that is, a new agreement to be signed between people and the environment through economic decisions. This is a package of measures proposed by North American parliamentarians and which aims, as a priority, to initiate an energy transition process and reduce the emission of polluting gases onto the planet.

It is highlighted by Chomsky and Pollin (2020) that such economic measures can generate discomfort for business people, as it will require a loss of profits and a decrease in revenue in companies. However, they highlight that joint action between public and private entities will be necessary to continue seeking a reversal of the environmental problems we currently face. In this context, Marques Filho (2018) points out that the ideals of capitalism as a socio-economic system have directed us towards environmental collapse and that a possibility to re-establish new relationships would be in a mixed way, in which the population and the state are in equal positions to take decisions that affect everyone's lives.

Therefore, it is important that, through people's engagement, there is a collective social movement in search of reversing or mitigating the extreme climate effects that have been plaguing the entire world. This change in posture is not random or involuntary, it is necessary for each individual to become aware of the situation in which they are inserted. To transform the oppressive reality it is necessary to change people. Social change depends on everyone, each individual must assume their position as the subject of history and act (Freire, 2016). It is important to be fully convinced that

[...] the social structure is the work of men and that, if so, its transformation will also be the work of men. This means that their fundamental task is to be subjects and not objects of transformation. [...] Therefore, the social worker cannot be a neutral man in the face of the world, a neutral man in the face of dehumanization or humanization, in the face of the permanence of what no longer represents human paths or the change of these paths. The social worker, as a man, has to make his choice. Either adhere to the change that occurs towards the true humanization of man, of his being more, or be in favor of permanence (Freire, 2023, p. 63).

From this perspective, Freire (2023) leads us to reflect that professionals, regardless of their area of activity, must commit to society. The author highlights that it is not about assuming a I stance, on the contrary, to make the commitment, the individual must have the ability to act and reflect in search of a more humanizing society and seek to achieve social transformation. In this sense, it is not understandable to think about the professional performance of an individual who has not previously established a commitment as an individual inserted in the world, that is, a person capable of critically understanding reality and aiming to change it.

Therefore, we advocate that this awareness process be initiated in the training of teachers, at all levels of education, who will contribute to the preparation of other professionals who will work in the world. Every educational practice must be thought of and executed in this direction.

ANALYZING AN EXPERIENCE IN HIGHER EDUCATION

Given the context presented, it is essential that actions are taken to transform reality. Next, we will share an experience in an activity, designed and developed by the first author of this article, with students of a Pedagogical Preparation discipline offered to Master's and Doctoral students at the

University of São Paulo (USP) in the areas of Mathematics, Statistics and Sciences of Computing, in the first semester of 2024.

The University of São Paulo Instituted the offering of a subject entitled Pedagogical Preparation which is offered to students of postgraduate courses on all campuses seeking to prepare future higher education teachers for teaching, bringing themes, for example: didactics, active methodologies, digital technologies for teaching and inclusion in higher education.

One of the active methodologies presented was Problem-Based Learning (PBL), which represents a counterpoint to traditional teaching. Dias (2012) highlights that universities need to adapt to the needs and realities of today's society, therefore they need to reinvent themselves to fulfil their role of collaborating with the construction of a more just and democratic society. PBL, in addition to presenting an alternative way of conducting classes, also allows for a new meaning in the recognition of content itself, which is often restricted within disciplines, without allowing its contextualization and its interdisciplinary character to be explored.

The use of PBL itself arose from the demands of higher education in the search for educational innovations that began in the 1970s and gained space in universities around the world. Working with PBL involves specific practice in the classroom.

In PBL, students work in small groups and learn what they need to know in order to solve a problem. The teacher acts as a facilitator to guide student learning [...] the students are presented with a problem scenario. This fact-identification step helps students represent the problem. As students understand the problem better, they generate hypotheses about possible solutions (Hmelo-Silver, 2004, p.236).

The PBL structure was presented to the 50 students in the Pedagogical Preparation discipline who were able to see, through a practical example, the work carried out by Souza-Carneiro (2021) who developed activities with PBL in two higher education institutions: one in a group of studies for first-year Civil Engineering students and at another university for Differential and Integral Calculus II students in the second year of the Ecology course. The problem that motivated the discussions was the same in both classes, and is based on an environmental disaster that occurred in Brazil in 2015.

In Souza-Carneiro's thesis (2015), the itinerary of activities is presented based on the situation of a fire that occurred in the city of Santos, on the coast of the state of São Paulo in Brazil. The disaster began with an explosion in a fuel tank at a company that stores chemicals, petrochemicals, biofuels and vegetable oil. The fire took 192 hours to completely extinguish and caused extensive damage to the environment and the lives of people around the site. Many families who depended on fishing as a means of subsistence were no longer able to carry out their activities due to the pollution generated in the water that was used to cool the tanks. The discussion provoked by Souza-Carneiro's (2015) work is how to think about fair compensation considering all the damage caused by such a disaster.

After presenting this experience to Pedagogical Preparation students, the first author/researcher inspired them with the recent disaster that also happened in Brazil in the state of Rio Grande do Sul. On April 27, 2024, heavy rain hit the Region of Valleys in the extreme south of the country. The rains lasted for more than ten days and caused the waters of several rivers to overflow and the water to invade several municipalities, levelling cities and destroying lives. 471 cities were hit, expelling more than 600,000 people from their homes and taking more than 170 lives. After putting on the agenda this issue that was plaguing the country at that time, the teacher asked the students what our

responsibility as educators is in contrast to the disasters we faced. The students seemed surprised by the question. The provocation continued with the proposal that students, in groups, following the precepts of PBL, idealize an activity that could be applied to undergraduate students and that was related to their teaching areas for the prevention of disasters such as fire and flood previously described, in addition to actions that could be taken during moments of crisis and/or after tragedies.

The students organised themselves into groups of three to five students to prepare proposals that were later presented to the class and discussed with the teacher. Below we will present three examples of proposals prepared by the groups. The images in Figures 1 and 2 are part of the work of the students, who authorized their use in this paper.

Group 1 decided to investigate the collapse of a dam called 14 de Julho, which worsened the situation in many cities during the Rio Grande do Sul disaster. The large volume of water caused the structure to collapse and the people living in risk areas had to leave their houses immediately. The question proposed by the group is about the implementation of a monitoring system at the maximum level of dams to monitor and predict risks. In Figure 1, we can see the location of the dam that was broken and a photograph of the location.



Figure 1: Dam 14 de Julho.

The group's proposed activity envisages the implementation of a contactless liquid level sensing system, equipment that already exists and is intended for use in industrial wastewater, pumping stations, and reservoirs, among others. The activity was designed to be applied to a Computer Engineering course. Students should start from a certain measurement of the water level in meters, provided by the average readings from the dam sensors obtained every hour and would have a maximum level of overflow (M), the maximum capacity and the percentage of approach to the maximum level (A) which indicates the level of the dam where warning of the level increase speed is necessary and which represents a risk factor for the dam. From there, students should use a programming language of their choice to write code that continuously receives the water flow measured at the last minute and the precipitation forecast. Furthermore, implement the logic that determines the entry of M and A information and that identifies the need to issue an alert when necessary.

Group 1 highlighted that the activity will aim to mobilize knowledge of logic and programming to think about preventing an environmental problem, with emphasis on reflections on the importance of technological solutions in facing real challenges through innovative strategies.

The work carried out by Group 2 was concerned with the food supply chain, thinking about agriculture in Rio Grande do Sul. The intense rains also affected local agricultural production and industry. After the disaster, cities' food supply logistics were severely disrupted, causing problems in several cities. Submerged bridges and roads made it difficult for products and resources to be transported, in addition to the destruction of many crops and stored goods. The group's proposed activity involves the distribution of milk in the region that has large national producing industries. Starting from a real scenario and assuming the need to transport milk in the region, the group created a graph with the representation In an ordered pair $G = (V, E)$, where V is the set of vertices and E is the set of edges with an imagined milk distribution chain, considering that the sections in yellow were damaged by rain. The situation is represented in Figure 2.

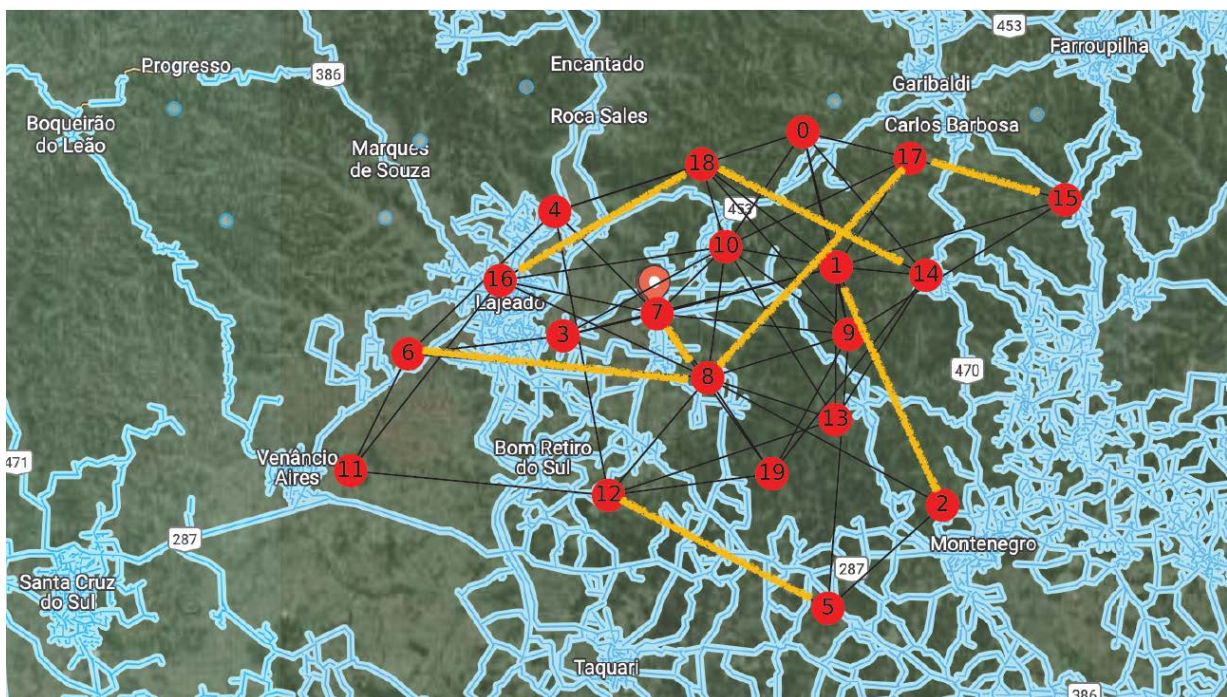


Figure 2: Graph G with the milk distribution network and highlighting the affected routes.

Assuming that the milk industry is at vertex 7 and that there are two trucks to simultaneously collect the milk produced at the other vertices and that there is no maximum limit of milk that they can carry, the proposal is for students to write an algorithm that indicates the shortest path paths must take to traverse all vertices using graph theory.

Group 3's proposal, in turn, was to propose the investigation of a mathematical model to predict soil fertility after floods. The proposal is that students use as parameters the initial quantity of residues in the soil (qi), the final quantity of residues in the soil allowed for it to be suitable for cultivation (qf), the soil's capacity to degrade residues – rate of degradation I , and the time required for the soil to reach the permitted amount of waste (t).

A possible mathematical model that would result from what was proposed by group 3 is given by the function $f(t)$, where:

$$f(t + 1) = (1 - c)(f(t))$$

$$f(0) = qi$$

The function in question describes the reduction of waste over time due to the soil's degradation capacity and to find out the time y necessary for it to reach the allowed value qf , we have to solve:

$$qf = f(y)$$

Using the recursive equation, iterations can be made until the value that satisfies expectations is found and, thus, the soil can be used with the guarantee that it will not cause harm to those who consume its products.

The three proposals prepared by the Pedagogical Preparation students demonstrate the mobilization of knowledge from their areas to solve problems involving prevention, emergency action and resolution of situations following an environmental disaster. Furthermore, they show care in raising students' awareness of the problem and their commitment to collaborating with their resolution beyond just identification.

SOME CONSIDERATIONS

The environmental crisis needs to be faced. To achieve this, it is important that everyone assumes their role in confronting and seeking awareness for transformation. The present work sought to reflect on a proposal for an activity developed within the scope of teacher training for higher education, where postgraduate students in the exact sciences developed proposals for activities inspired by prevention, action during a crisis and subsequent solutions for floods that took place in the state of Rio Grande do Sul, Brazil in 2024. Such activities were designed considering the ideals of Problem-Based Learning and showed the potential that different areas have to contribute and assume their social role in the face of the dilemmas we face in the search for a society responsible for the environment and critical in relation to the actions necessary to fight against the end of the world.

PBL is a teaching methodology that makes students the protagonists, which makes it possible for them to commit to creating activities that would involve classes with more active students who could engage in dialogue and present new ideas, as opposed to traditional teaching in which only teachers talk and students listen and obey. The activities developed in this research by the groups were

designed for students to analyze situations, mobilize different knowledge and apply them in a contextualized way.

Having more creative, critical citizens and professionals who know how to work in groups is one of the goals of working with PBL in universities. We believe that people with this profile can contribute to the transformation of the world that we seek so that we can overcome crises and enable the construction of a new context from a social and environmental point of view for future generations.

References

- Chomsky, N., & Pollin, R. (2020). *Crise climática e o Green New Deal global: A economia política para salvar o planeta*. Roça Nova.
- Dias, M. A. R. (2012). Inovações na educação superior: tendências mundiais. 2012. In P. Speller, F. Robl, & S. M. Meneghel (Eds.), *Desafios e perspectivas da educação superior brasileira para a próxima década*. UNESCO, CNE, MEC.
- Freire, P. (2016). *Conscientização*. Tradução de Tiago José Risi Leme. Cortez.
- Freire, P. (2023). *Educação e mudança*. Tradução de Lilian Lopes Martin. Paz e Terra.
- Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn? *Educational Psychology Review*, 16(3), 235-266.
- Krenak, A. (2020). *Ideias para adiar o fim do mundo*. Companhia das Letras.
- Marques Filho, L. C. (2018). *Capitalismo e colapso ambiental*. Editora da Unicamp.
- Reigota, M. (2016). *O que é educação ambiental*. Brasiliense.
- Souza-Carneiro, D. V. (2021). *A matemática em ação no ensino superior: possibilidades por meio do problem-based learning*. PHD dissertation presented in the Postgraduate program in Mathematics Education at São Paulo State University.

CRITICAL MATHEMATICAL THINKING: A FRAMEWORK AND ILLUSTRATIVE EXAMPLES

Vince Geiger¹, Thorsten Scheiner¹, Kim Beswick², Jill Fielding³, Merrilyn Goos⁴, Gabriele Kaiser⁵,
Katherine Fernandez¹, Sabahat Arwan¹

¹Australian Catholic University, ²University of New South Wales, ³University of New England,
⁴University of the Sunshine Coast, ⁵University of Hamburg

In this paper we explore the importance of critical mathematical thinking (CMT) in mathematics education and present a framework for teachers' support of CMT development. The CMT framework consists of five key dimensions: mathematical capability, critical capability, evaluating, reasoning, and disposition. Together, these dimensions address the knowledge, capabilities, and dispositions necessary for students to engage critically and ethically with real-world challenges. We also describe the CMT Noticing Instrument (CMTNI), designed to foster instructional capability by capturing and reflecting lack awareness of the capabilities associated with teacher noticing of CMT. Excerpts from teachers' responses to the CMTNI are discussed against the dimensions of the CMT framework. This paper contributes to the reimagination of mathematics education in the context of an increasingly complex world, highlighting the CMT's importance and pointing to directions for development.

INTRODUCTION

The development of criticality in mathematical thinking has emerged as a crucial goal in mathematics education in response to pressing social and ecological challenges (Geiger, 2024; Gutstein, 2006; Skovsmose, 2023). Critical mathematical thinking (CMT), a term coined by Gutstein (1997), goes beyond mere mastery of mathematical concepts and procedures as it encompasses the capacity to critically analyze and evaluate mathematical claims, consider the ethical implications of decisions, and use mathematical reasoning to envision and work towards more just and sustainable futures (Geiger & Schmid, 2024; Frankenstein, 1983). CMT recognizes that mathematics is not a value-free discipline, but is deeply embedded in social, political, and ecological contexts (Valero, 2004). How mathematical knowledge is constructed and applied can have profound consequences for individuals, communities, and the planet (Gutiérrez, 2013). Thus, developing students' CMT involves fostering a critical awareness of the values underlying mathematical practices, in addition to promoting mathematical capability—empowering students to use mathematics within social, political, and ecological milieux (Skovsmose, 2012)—fostering critical, responsible citizenship (Geiger et al., 2023).

A key dimension of CMT is the ability to critically evaluate mathematical claims and arguments, especially those presented in public discourse and decision-making contexts. In an era of 'fake news', misinformation, and competing narratives from experts and non-experts alike, students need to be equipped with the capabilities to assess the validity of mathematical evidence, question the assumptions and biases that may underlie claims, and consider alternative interpretations and perspectives (Gal & Geiger, 2022). This requires both mathematical capability and a critical disposition that questions taken-for-granted assumptions and seeks multiple points of view.

Another important aspect of CMT is the capacity to use mathematical reasoning to analyze complex real-world problems and work toward more just and sustainable futures. This often involves the use of mathematical models and simulations to explore the potential consequences of different courses of action (Jablonka, 2003). It also requires a willingness to engage with the ethical dimensions of mathematical decision-making, to consider differential impacts on different stakeholders, and to prioritize the needs of those who are marginalized and vulnerable (Skovsmose & Greer, 2012).

Developing students' CMT therefore requires a reimagining of the goals and methods of mathematics education. Rather than focusing only on the acquisition of discipline knowledge and technical procedures, mathematics education must cultivate a critical and ethical orientation (Valero, 2004). This will necessarily involve embedding the exploration of real-world challenges into teaching/learning—providing opportunities for fostering classroom cultures that value multiple perspectives and encourage respectful dissent (Gutstein, 2006). Yet, we argue, there is limited research into how to identify and foster the teaching capabilities necessary to promote students' CMT. The purpose of this paper is to describe a framework for conceptualizing CMT, introduce the CMT Noticing Instrument, Instrument (CMTNI), designed to capture teacher awareness of the capabilities needed to foster CMT, and to present insights from the implementation of this instrument.

CMT FRAMEWORK

As part of a significant nationally funded project in Australia (Geiger and colleagues, 2022-2025), we have developed the CMT framework in seeking to define and investigate both cognitive and affective aspects of thinking that students must acquire to use mathematics when addressing complex real-world situations which have potential consequences for equity, inclusiveness and social justice—particularly in the context of the socio-ecological. Our CMT framework is based on five key dimensions: mathematical capability, critical capability, evaluating, reasoning, and disposition. Taken together, these dimensions provide an account of the knowledge, capabilities, and dispositions required to engage critically and ethically with problems related to problematic or challenging real-world contexts. Thus, we see the CMT framework as a step towards the recognition of socio-ecological perspectives as central to the role of a mathematics education.

The first dimension of the CMT framework, mathematical capability, refers to the discipline knowledge and capabilities required for mathematical thinking and problem-solving. This includes understanding mathematical concepts and principles, identifying patterns and relationships, and manipulating mathematical symbols and expressions. While these capabilities are essential for engaging with mathematical ideas, they are not sufficient on their own and must be integrated with the other framework dimensions to enable students to use mathematics in critical and ethical ways.

Critical capability, the second dimension, involves examining ideas, drawing conclusions, and considering the consequences of decisions from societal/political/ecological/economic perspectives. In particular, this dimension is concerned with understanding the differential impacts of mathematics-based decisions on different communities and ecosystems. This requires that students learn to question assumptions, analyze arguments, and consider multiple perspectives.

Dimension 3, evaluating, is concerned with assessing claims and arguments based on the strength of evidence (e.g., quality of data, reliability of sources). In a world where mathematical evidence is often used to justify policy decisions, restrict or enhance personal and community freedoms, and shape

public opinion, it is essential that students be equipped with the capabilities to critically evaluate the validity and trustworthiness of claims in the media, by authorities, and from industry. This requires an understanding of the biases inherent in different methods of data collection and analysis.

The fourth dimension, reasoning, includes logical thinking, inferring, proposing and checking hypotheses, and generating evidence-based arguments. This dimension emphasizes the importance of using mathematical reasoning to analyze complex problems and to develop solutions that are grounded in empirical evidence. By engaging in reasoning processes, students learn to construct logical arguments and draw valid conclusions from data.

The final dimension, disposition, includes a positive attitude toward, and belief in, the relevance and importance of mathematics for addressing real-world problems, a willingness to take intellectual risks and be persistent, and a commitment to using mathematics in ethical and responsible ways. This dimension is key for the ongoing engagement of students with socio-ecological issues, which are complex and take time and patience to resolve.

The CMT framework has been designed as a guide for teachers when engaging students with problematic real-world situations that require both the application of mathematics and consideration of associated consequences—social, political, environmental, and economic.

CMT NOTICING INSTRUMENT

Teacher noticing is a construct that has gained significant attention in recent years as a key aspect of effective teaching practice. It is a multi-dimensional activity that encompasses various perceptual and cognitive processes (König et al., 2022). While the core components of teacher noticing have typically included *attending* to particular classroom events and making sense of such events (Sherin et al., 2011), more recent conceptualizations have expanded to include *responding* to classroom events (Jacobs et al., 2010) or decision making, either by anticipating a response to students' activities or proposing alternative instructional moves (Kaiser et al., 2015). The processes—*attending*, *interpreting*, and *responding*—are considered to be intricately intertwined and form the basis of the noticing part of this study (Scheiner, 2016).

Despite the growing recognition of the importance of teacher noticing, there are only a few standardized instruments designed to capture this capability (Weyers et al., 2023). Moreover, to our knowledge, no instrument exists that specifically focuses on teacher noticing in the context of CMT. The instrument has been designed to capture teachers' abilities to attend to, interpret, and respond to classroom events that demonstrate students' engagement with CMT.

The development of the CMT Noticing Instrument (CMTNI) was based on a classroom implementation of the Flood Buyback Program task, a real-world problem designed to engage students in CMT by challenging them to apply their mathematical knowledge and skills to a socially relevant context. The task was based on the consequences of severe flooding in Brisbane, Australia, which affected thousands of homes, many built on flood plains. In response, the government offered a 'buy-back' scheme, providing homeowners with the market value of their houses to facilitate relocation. The task was presented through two contrasting cases: a young mother willing to accept the buy-back offer to move after recent flood damage, and a 90-year-old man, having experienced many floods, who sees no point in relocating. These perspectives allowed students to explore the practical implications and social consequences of the scheme while applying mathematical thinking.

The task, presented to a Year 9 mathematics class at a school in Brisbane, was implemented over two 80-minute lessons, allowing students to work on the task for a total of 160 minutes. During these lessons, the interactions and discussions of two groups of students were videotaped. The lessons were semi-scripted, with the teacher being provided with broad, non-prescriptive guidelines for the conduct of the lesson—seeking to ensure genuine classroom dynamics. The video data collected during the implementation of the Flood Buyback Program task served as the foundation for developing the CMTNI. Through analysis of the recorded interactions, we identified key incidents that exemplified students' engagement with CMT and used these to create authentic vignettes for the instrument.

The CMTNI consists of six vignettes, each including a video clip (2-3 minutes) showing teacher-student and/or student-student interactions. The vignettes also provide contextual information to help teachers situate the events within the broader classroom setting. To simulate real classroom observation, the video clips can be viewed only once before teachers are asked to respond to a series of questions related to noticing in the context of CMT. The Instrument was implemented in 2023 during an online symposium that involved a diverse group of educators, including teachers, curriculum developers, and curriculum leaders from different Australian states. The symposium provided a platform for participants to work through the task and engage with the CMTNI, in addition to presentations on CMT and teacher noticing. A total of thirty-six teachers completed the Instrument.

In the following section, we present selected excerpts from two teachers' responses, to the CMTNI to exemplify their observations about the activity in the vignettes against the CMT dimensions. We have limited the discussion to two teachers' responses because of space limitations.

SELECTED EXCERPTS FROM TEACHERS' RESPONSES TO THE CMTNI

As an example of an attending question, teachers were asked—In what ways did students use the CMT dimensions in identifying factors such as flooding, cost, crime, and education? (Question 5.1)

Teacher A and Teacher B, responded as follows:

Reasoning—inferring what factors would be a priority. (Teacher A)

They used critical capability to identify and analyse factors relevant to the decision of where to move. The evaluating and dispositions dimensions were also at play. (Teacher B)

Teacher A's response focuses solely on the reasoning dimension of the CMT framework, noting that students were inferring and prioritizing relevant factors to make informed decisions. This response, however, does not provide evidence of the other CMT dimensions. In contrast, Teacher B's response highlights three dimensions of the CMT framework—critical capability, evaluating, and disposition. Students used critical capability to identify and analyze factors relevant to the decision. The evaluating dimension was evident as students assessed the relevance and reliability of information, while the disposition dimension was reflected in their engagement with the problem and consideration of multiple factors that might impact on decision-making.

Although both responses provide insight into students' thinking, Teacher B's response offers a more comprehensive view of how multiple CMT dimensions were utilized in thinking about the problem.

Another question was designed to elicit responses about *interpreting*—*How does the teacher's approach to addressing the problem differ, in terms of promoting the dimension(s) of CMT, when interacting with group 1 compared to group 2?* (Question 2.5)

In response, Teacher A and Teacher B, provided the following:

The teacher explicitly asked Group 1 to think about the mathematics they were using to complete the task —mathematical capability. The teacher was more encouraging of critical capability in Group 2 by asking students to consider the social implications of the problem. (Teacher A)

This is an approach that is more in line with encouraging deeper exploration with a greater emphasis on CMT. (Teacher B)

In Teacher A's response, there is evidence of the teacher in the vignette promoting two CMT dimensions — mathematical capability and critical capability. When interacting with Group 1, the teacher asked students to think about the mathematics they were using to complete the task — the mathematical concepts and problem-solving strategies that align with the mathematical capability dimension of the CMT framework. By encouraging students to reflect on their mathematical thinking, the teacher is promoting the development of foundational knowledge and skills required for effective mathematical reasoning. In contrast, when interacting with group 2, the teacher placed a greater emphasis on critical capability. The teacher asked students to consider the social implications of the problem, which encourages them to examine the issue from a socio-ecological perspective. This approach promotes the development of critical thinking skills, as students are prompted to analyze the broader context and potential consequences of their decisions.

Teacher A's response indicates they had seen the teacher promoting different CMT dimensions when interacting with each group, although why they did so is unclear. Teacher B's response, however, is brief and does not provide specific examples of how the teacher's approach differed between group 1 and group 2. At the same time, Teacher B does mention that the approach is more in line with encouraging deeper exploration with a greater emphasis on CMT.

In terms of *responding*, teachers were asked to respond to the following question — *Suppose you are teaching this lesson, how would you promote critical mathematical thinking in relation to the task (e.g., expanding on what the teacher is already doing or taking a different approach)?* (Question 1.5)

Teachers A and B, responded as follows:

Ask students to justify their reasons, e.g. when students suggested moving to a less expensive area ask students to explain why they think this is a viable option and prompt students to think about other factors (e.g. what makes it cheaper—house sizes, access to parks/facilities—critical capability). Ask students to share their sources of evidence...and whether students believe these are credible sources. (Teacher A)

I think a different approach would encourage thinking more critically. Using more open questions to help consider different points of view. The teacher is helping explore one channel more deeply at the expense of balancing the approach with alternate approaches. There is less development of critical capability but is still opportunity for the development of reasoning and encouragement of positive dispositions as to the value of mathematics in solving the problem... (Teacher B)

In Teacher A's response, there is evidence of two dimensions of CMT—critical capability and evaluating. The critical capability dimension is the most prominent in the teacher's response. They suggest asking students to justify their reasons for proposing moving to a less expensive area. By prompting students to consider other factors that might contribute to an area being cheaper, the teacher is encouraging thinking related to a broader social/economic perspective—the potential consequences of their proposed solution, a key aspect of critical capability. The evaluating dimension is also evident in the teacher's response when they suggest asking students to share their sources of evidence, such as websites, and to consider whether these sources are credible. This encourages

students to assess the quality of the data they are using to inform their decision-making process, which is an essential component of the evaluating dimension.

Teacher B's response provides evidence of several dimensions of CMT, with a particular emphasis on critical capability and reasoning. They indicate that a different approach would encourage deeper critical thinking (critical capability dimension). By using more open questions to help consider different points of view, they appear to wish to encourage broader student perspectives on the issue and encourage them to examine alternative approaches. This promotes the development of critical capability by fostering an awareness of the limitations associated with a focus on a single perspective and the importance of considering multiple viewpoints when addressing complex issues. Teacher B also acknowledges that the approach used in the vignette, which focuses on exploring one "channel" more deeply at the expense of balancing it with alternates, may limit the development of critical capability. At the same time, they recognize that there is still an opportunity for the development of reasoning, another dimension of CMT, through logical thinking, inference, and the generation of evidence-based arguments. Finally, Teacher B notes possibilities for the development of positive dispositions towards the use of mathematics in solving the problem. This aligns with the disposition dimension of CMT, which emphasizes the importance of fostering positive attitudes and beliefs about the relevance and importance of mathematics in addressing real-world issues.

CONCLUSION

The CMT framework presented in this paper offers a multidimensional approach to fostering students' critical engagement with mathematical ideas and their applications to real-world problems in ways that promote students' attention to issues related to equity, inclusion and social justice. By integrating the dimensions of mathematical capability, critical capability, evaluating, reasoning, and disposition, the CMT framework provides a guide for teachers attempting to develop students' capacity to use mathematics as a tool for analyzing complex social/political/ecological/economic issues and positioning mathematics as a means to achieve a more just and sustainable future.

The development and Initial Implementation of the CMTNI represents a significant step forward in operationalizing the CMT framework by supporting teachers in developing their capacity to attend to, interpret, and respond to students' CMT. Teachers' responses to the instrument provide insight into how the different dimensions of CMT manifest in authentic classroom settings and how teachers might adapt their instructional approaches in ways that promote CMT.

One key insight that emerges from the teachers' responses to items in the CMTNI is the importance of explicitly *attending* to and promoting multiple dimensions of CMT in the classroom. While some teachers focused primarily on one or two dimensions, such as mathematical capability or reasoning, others demonstrated a broader understanding of inter-relatedness of the dimensions and how they interact with each other. For example, Teacher B's response to the *attending* question highlighted how students' critical capability, evaluating skills, and dispositions were all at play as they identified and analyzed factors relevant to the decision-making task.

Another insight from the teachers' responses to the CMTNI is the need for flexibility and adaptability in promoting CMT in the classroom. As evident in Teacher A's response to the *interpreting* question, effective teachers adjust their instructional approaches to emphasize different dimensions of CMT depending on the needs and strengths of individual students or groups. By asking one group to reflect

on their mathematical thinking and another group to consider the social implications of the problem, the teacher demonstrated a nuanced understanding of how to tailor support for students' CMT.

The teachers' responses also underscore the Importance of fostering positive dispositions towards mathematics and its role in addressing real-world problems. As Teacher B noted in their answer to the *responding* question, even when focusing primarily on one perspective, such as the financial aspects of the problem, there are still opportunities to encourage students to see the value and relevance of mathematics in solving complex issues. Cultivating these positive dispositions is essential for motivating students to engage with mathematical ideas and to remain persistent.

While the initial implementation of the CMTNI provides valuable insight into teachers' views on how to promote CMT, it also highlights the need for further research on professional learning that supports teachers in promoting CMT. Our research will now be extended to longitudinal case studies of 10 teachers in five schools who will engage with CMT focused professional learning through four cycles of classroom observation and collaborative reflection on observed practices over a 12-month period. A key question through this longitudinal approach will be how teachers' noticing capabilities and instructional practices evolve over time as they engage with the CMT framework and receive feedback and support from both researchers and their peers in the project. Another area for future investigation is the ways in which students' CMT develops in response to different instructional approaches and how this development can be assessed and supported over time. These and other questions will provide direction for our project going forward (Geiger and colleagues, 2022-2025).

In terms of teacher professional learning, we see the CMT framework and the CMTNI as potentially powerful means of engaging with teachers and facilitating reflective practice. By analyzing classroom recordings and discussing their noticing, teachers can deepen their understanding of the different dimensions of CMT and explore strategies for its promotion in their own classrooms. Professional learning programs that incorporate these aspects of the research, and provide ongoing support for teachers as they implement new practices, could have a significant impact on students' CMT and their ability to use mathematics to address the pressing current and future challenges faced by societies and humanity at large.

Ultimately, the development of critical mathematical thinking is essential for preparing students to become active and engaged citizens in an increasingly complex and interconnected world. By providing a framework and implementation guidelines for promoting CMT in the classroom, we aim to contribute to ongoing efforts to reimagine mathematics education as a catalyst for positive change in the context of the socio-ecological perspective. As we continue to grapple with the urgent challenges of our time, from climate change to social inequality, the cultivation of critical mathematical thinking must be a central priority for educators, researchers, and policymakers alike.

Acknowledgements

This paper is an outcome of the project *Enabling students' critical mathematical thinking* (DP220101015; Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., & Goos, M.) funded by the Australian Research Council (2022-2025).

References

Frankenstein, M. (1983). Critical mathematics education: An application of Paulo Freire's epistemology. *Journal of Education*, 165(4), 315–339. <https://www.jstor.org/stable/42772808>

- Gal, I., & Geiger, V. (2022). Welcome to the era of vague news: A study of the demands of statistical and mathematical products in the COVID-19 pandemic media. *Educational Studies in Mathematics*, 111(1), 5–28. <https://doi.org/10.1007/s10649-022-10151-7>
- Geiger, V. (2024). The need for critical approaches to mathematical modelling in the time of disruptive events. In H-S. Siller, G. Kaiser, & V. Geiger (Eds), *Researching mathematical modelling education in disruptive/challenging times* (pp. 15-34). Springer. https://doi.org/10.1007/978-3-031-53322-8_2
- Geiger, V., Gal, I., & Graven, M. (2023). The connections between mathematics education and citizenship education. *ZDM Mathematics Education*, 55, 923–940. <https://doi.org/10.1007/s11858-023-01521-3>
- Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., Goos, M., & Fernandez, K. (2023). Investigating critical mathematical thinking when applying mathematics to real-world problems. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education* (pp. 1177–1184). Alfréd Rényi Institute of Mathematics and ERME.
- Geiger, V., & Schmid, M. (2024). A critical turn in numeracy education and practice. *Frontiers in Education*, 9, 1363566. <https://doi.org/10.3389/educ.2024.1363566>
- Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education*, 44(1), 37–68. <https://doi.org/10.5951/jresmetheduc.44.1.0037>
- Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Routledge. <https://doi.org/10.4324/9780203112946>
- Gutstein, E., Lipman, P., Hernandez, P., & de los Reyes, R. (1997). Culturally relevant mathematics teaching in a Mexican American context. *Journal of Research in Mathematics Education*, 28, 709–737. <https://doi.org/10.2307/749639>
- Jablonka, E. (2003). Mathematical literacy. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 75–102). Springer. https://doi.org/10.1007/978-94-010-0273-8_4
- Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children’s mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202. <https://doi.org/10.5951/jresmetheduc.41.2.0169>
- König, J., Santagata, R., Scheiner, T., Adleff, A.-K., Yang, X., & Kaiser, G. (2022). Teacher noticing: A systematic literature review on conceptualizations, research designs, and findings on learning to notice. *Educational Research Review*, 36, 100453. <https://doi.org/10.1016/j.edurev.2022.100453>
- Scheiner, T. (2016). Teacher noticing: Enlightening or blinding? *ZDM Mathematics Education*, 48(1), 227–238. <https://doi.org/10.1007/s11858-016-0771-2>
- Skovsmose, O. (2023). *Critical mathematics education*. Springer. <https://doi.org/10.1007/978-3-031-26242-5>
- Skovsmose, O. (2012). *An invitation to critical mathematics education*. Sense. <https://doi.org/10.1007/978-94-6091-442-3>
- Skovsmose, O., & Greer, B. (Eds.). (2012). *Opening the cage: Critique and politics of mathematics education*. Sense. <https://doi.org/10.1007/978-94-6091-808-7>
- Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 5–23). Springer. https://doi.org/10.1007/1-4020-7914-1_2
- Weyers, J., König, J., Santagata, R., Scheiner, T., & Kaiser, G. (2023). Measuring teacher noticing: A scoping review of standardized instruments. *Teaching and Teacher Education*, 122, 103970. <https://doi.org/10.1016/j.tate.2022.103970>

EXPLORING CRITICAL MATHEMATICS EDUCATION IN TEACHING STATISTICS IN THE PHILIPPINES

Eric Loyd Hilario, Ma. Nympha Joaquin

University of the Philippines Diliman

In the Philippines, statistics are often manipulated by those in power to mislead the public and obscure realities faced by ordinary Filipinos. To counter this, teachers should reorient the learning of statistics as a tool to critique societal issues. This paper employs Critical Mathematics Education as a teaching approach, incorporating elements of crisis, critique, and democratization, to use statistics to address social issues in the country. Post-lesson interviews revealed that while most students supported this approach, some preferred focusing solely on mathematical concepts. This highlights both an appreciation for the role of mathematics in societal critique and the challenge of moving beyond the traditional notion that mathematics is only about solving problems and computations. The findings suggest that Critical Mathematics Education bridges the gap between objective data and subjective experiences preparing students to become critical citizens in the future.

INTRODUCTION

According to the Philippine Statistics Authority (2024), poverty incidence decreased significantly in 11 out of 18 regions in the Philippines in 2023 compared to 2021. This is promising news, suggesting that more Filipinos are experiencing a better quality of life. However, a closer examination of the statistics reveals some concerning details. The government agency set the poverty threshold at only 13,873 pesos per month. Hence, it can be said that these government statistics do not reflect the reality on the ground. IBON Foundation (2023) claims that in May 2023, a family of five in the National Capital Region (NCR) needed 25,239 pesos per month to sustain a decent living. This disparity highlights how statistics can be manipulated to mislead the public and obscure the challenges faced by ordinary people.

Given this, it is crucial for educators to teach students to critically engage with data—how to read, organize, and analyze it properly to become informed citizens. This can be achieved by integrating discussions of social issues into the teaching of statistics, exposing students to the application of mathematics in addressing societal concerns. Critical mathematics education emerges as a pedagogical approach that facilitates the incorporation of discussions surrounding social issues within the mathematics classroom. It is rooted in critical theory and pedagogy, underscoring the necessity for students to nurture critical citizenship (Avci, 2020). This entails the ability to critically reflect on personal lives and engage in efforts to effect transformative change in society, with a commitment to social justice (Steffensen, 2017). This paper will discuss the implementation of Critical Mathematics Education in teaching statistics, emphasizing students' critique of policies implemented in the country and other social issues Filipinos face.

DEVELOPING CRITICAL MATHEMATICS LESSONS

According to Powell and Brantlinger (2008), Critical Mathematics Education may involve integrating explicit political messages into teaching mathematics and addressing social justice issues. The lesson plans incorporate crisis, critique, and democratization as key features.

Crisis

Ernest (2016) contends that Critical Mathematics Education should involve a sense of crisis, wherein problems necessitate action that determines whether conditions will improve or worsen. This can be achieved by discussing socio-political issues in the classroom, enabling students to understand how these issues impact their lives and society. By doing so, students gain insights into how institutional structures control societal systems and recognize that mathematics is not a neutral subject, as it may serve the interests of certain groups (Chartres, 2008). A critical mathematics teacher should create relevant learning experiences that connect students' lived experiences to mathematics. This approach helps students understand their role in society as individuals directly affected by sociopolitical issues, which in turn may guide them in deciding whether to contribute to maintaining or changing the current system.

Critique

According to Ernest (2016), Critical Mathematics Education must employ critique, which involves analyzing the strengths and weaknesses of a subject and utilizing mathematics as a tool to understand issues of social justice. To achieve this, students should be given opportunities to pose questions, investigate, seek diverse explanation, and question the inclusion or exclusion of certain aspects. Additionally, students must learn to scrutinize the basis of presented data, underlying premises, data gathering methods, and how the use of mathematics can shape perspectives and decisions (Steffensen, 2021). By doing so, students will be able to view societal problems not just as issues they must accept and live with, but as constructs they can critique and challenge. Critique may be done through answering reflective questions. During reflection, students learn to acknowledge their own biases, which can lead to differing views among their peers when interpreting social issues due to their diverse backgrounds. After recognizing their biases, students face the challenge of deciding whether to prioritize their own interests or act for the greater good of more people. A critical mathematics educator must aim to teach students to make decisions that benefit the majority.

Democratization

Another essential element is democratization which emphasizes shifting power dynamics in the classroom. Educators should foster discussions and encourage diverse viewpoints among students (Ernest, 2002), moving away from the traditional role of being the sole authority on knowledge. To achieve this, teachers must establish a classroom community that incorporates group activities (Breunig, 2009). The classroom should be a democratic space that fosters a safe and comfortable environment for students to engage in dialogue and process their thoughts. This approach allows students' differing opinions on certain issues to be debated. By providing an environment where students can explore various perspectives, they may arrive at decisions that are in the best interests of society and supported by the majority. To ensure democratization, a critical mathematics teacher must be mindful of their own biases. It is essential to ensure that these biases do not hinder students from finding the best solutions when addressing social issues. Furthermore, these biases should be used constructively to guide students toward decisions that promote the common good rather than serve the interests of those who exploit and oppress others.

IMPLEMENTATION OF CRITICAL MATHEMATICS LESSONS

A Grade 10 class with 30 students at a laboratory school in Metro Manila, Philippines, was selected to participate in the study. The students came from diverse backgrounds and were heterogeneously grouped, which allowed for a variety of perspectives on issues that could aid in decision-making. The students were randomly assigned to six groups with five members each before the start of the implementation, and these groups remained consistent throughout the study. This approach aimed to help students build rapport with one another and develop empathy by listening to their peers' experiences and understanding how they address social issues.

Before each lesson, students were asked to study the mathematics concept in advance and read an article about a specific social issue. This approach enabled students to reflect on the social issue to be discussed and prepare to share their opinions. It also provided an opportunity for them to examine any biases they might have related to the topic. During class, the focus was on understanding the social issue by applying the mathematical concept. Students were first invited to share their thoughts on the social issue, followed by a teacher-led review of the mathematical concept. Listening to students' initial opinions about social issues helps demonstrate how their views might change when presented with data and engage in discussion with peers.

Afterward, students participated in a group activity where they answered guide questions, using the mathematical concept as a tool to deepen their understanding of the social issue. It is crucial for students to learn how to present, organize, and analyze data to form an educated opinion on the matter. For the summary of the discussion, the class revisited the mathematical concept and shared insights about the social issues. This structure allowed students to share their informed opinions from small group discussions with the whole class, broadening their perspectives by listening to other groups. This process could lead to constructive arguments that help identify the best ways to address the social issue.

In the following subsections, four social issues and students' perceptions of them will be discussed.

Teenage Pregnancy

In 2022, a local government unit in the Philippines issued an ordinance mandating comprehensive sexuality education at all school levels, prompted by the rate of teenage pregnancies in the area. Given the country's predominantly Christian background, this ordinance likely elicited mixed reactions. Students, acting as members of the local government, were asked to agree or disagree with the ordinance. To aid their decision-making, they were provided with data on the ages of pregnant females over three years, as shown in Figure 1.

From the data, all groups observed that teenage years did not account for the highest number of pregnancies. Nonetheless, they supported the ordinance, arguing that even a single case of teenage pregnancy must be addressed. Notably, concerns related to religious beliefs were not raised, likely due to the school's non-sectarian nature, allowing students to focus on the data. Two groups suggested making contraceptives accessible to prevent teenage pregnancies—a proposal that, while unopposed in the classroom, might be controversial elsewhere due to religious beliefs and the students being minors.

Mother's Age	Number of Birth		
	YR 2020	YR 2019	YR 2018
Total	47,715	56,037	54,803
0 - 9	-	-	-
10 - 19	3,680	4,839	5,151
20 - 29	24,619	29,011	28,685
30 - 39	17,457	19,891	18,839
40 - 49	1,934	2,270	2,081
50 - 59	3	8	11
60 - 69	-	-	-
70 - 79	-	-	-
80 - 89	-	-	-
90 - 99	-	-	-
Not Stated	22	18	36

Source: Philippine Statistics Authority, Civil Registration Service, Vital Statistics Division

Figure 1: Registered live births by age of mother in the local government unit per year [Data sourced from Philippine Statistics Authority (2021, 2022)]

This activity illustrates that students did not base their opinions solely on the data. Students also considered their biases, viewing comprehensive sexuality education as important despite declining numbers. This underscores that both statistics and personal beliefs play roles in decision-making. However, it is important to weigh which should take precedence. This demonstrates the value of integrating social realities into teaching statistics, as responses might differ if the context were reduced to a mere word problem without real-world relevance.

Press Freedom

In 2023, the Philippine President promised to protect the rights of media practitioners. However, many groups argue that this statement was merely for show, as violations of press freedom, including recorded killings of media practitioners, continue to be widespread. To assess whether the promise was fulfilled, students were asked to use measures of position to compute the country’s percentile rank and construct a boxplot to visualize the Philippines’ standing in the 2023 World Press Freedom Index, compiled by Reporters Without Borders (n.d.).

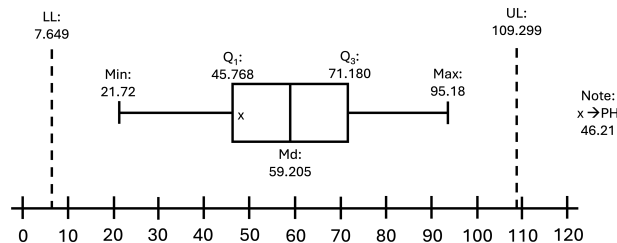


Figure 2: Boxplot of World Press Freedom Index 2023 [Data sourced from Reporters Without Borders (n.d.)]

After completing the statistical tasks, four groups concluded that the President had not upheld his promise, noting that the Philippines ranked below the median compared to other nations. This exercise also reminded students of recent incidents, such as the killing of a broadcaster during a live broadcast and ongoing media censorship, making them realize the issue’s proximity to their own experiences and concerns about freedom of expression. Conversely, the remaining two groups argued that the President had upheld his promise by closely examining his exact words, which mentioned his goal of improving the country’s ranking. This argument was supported by the fact that the Philippines improved its ranking to 138 in 2023 from 147 the previous year. This closer examination of the President’s statement led these students to conclude that, while he may have technically kept his

promise, the intent behind it might not have been genuinely aimed at improving the conditions of press freedom in the country.

This case illustrates that relying solely on numbers without additional context can be dangerous and misleading. Some groups believed the President fulfilled his promise based on the improved ranking, while others, drawing on personal experiences and moral judgment, concluded that any form of violence against media practitioners remains unacceptable. This highlights the role of personal experience and morality in interpreting statistics. This case demonstrates the importance of Critical Mathematics Education in statistics. It shows that numbers alone are insufficient; they must be supported by context such as facts and evidence to provide a stronger foundation for sound and valid conclusions.

Teachers’ Salaries in Public and Private Schools

The Philippine Vice President, who was also the Secretary of Education at the time, claimed that there is a significant disparity between the salaries of public and private school teachers. The starting salary for public school teachers is around 27,000 pesos, while salaries for private school teachers range from 6,500 to 33,000 pesos. This disparity has often been cited by the government to deny salary increases for public school teachers, arguing that private schools cannot match the salaries offered in public schools. To verify the existence of this disparity, the students were asked to use measures of variability. All groups concluded that there is indeed a disparity, noting the number of standard deviations between the starting salary of public school teachers and the average starting salary of private school teachers. After establishing the disparity, students were asked to provide suggestions on how to address it.

6,500.00	12,000.00	15,000.00	16,000.00	18,500.00	24,000.00
8,000.00	12,500.00	15,000.00	17,000.00	20,000.00	25,000.00
10,000.00	13,000.00	15,000.00	17,500.00	20,000.00	26,500.00
10,000.00	15,000.00	16,000.00	18,000.00	20,000.00	27,000.00
11,500.00	15,000.00	16,000.00	18,500.00	22,000.00	30,000.00

Figure 3: List of starting monthly salaries of 29 private schools and 1 public school (Hilario, 2023)

It was evident that the groups had differing opinions on the matter. Half of the groups agreed on the need to standardize private school teachers’ salaries, bringing them in line with public school teachers’ salaries. One group even suggested that the salaries must be a livable wage. Conversely, the other half supported the variation in salaries, arguing that they are based on the tuition fees paid by students. In this example, it is noteworthy that statistics were not used to definitively resolve the issue but rather to initiate a discussion about it. It is also notable that students’ biases were evident in their insights, as they felt they might be the aggrieved party if tuition fees needed to be increased to raise private school teachers’ salaries. This indicates that decision-making was influenced not only by statistics but also by personal interests.

While the first two cases highlighted the importance of students’ personal experiences and judgment, this case illustrates the limitations of such an approach, as personal biases can affect decision-making even when provided with statistical data. Thus, Critical Mathematics Education addresses this issue by combining objective data with the subjective opinions of students, allowing for a more balanced and appropriate resolution to the problem.

Employees’ Salaries in a Water District

Students were given data on the monthly salaries of employees in a water district and asked to comment on the fairness of the salary distribution. This distribution reflects a common pattern in many organizations, where managerial positions receive significantly higher salaries than those performing manual labor. The students used measures of central tendency and skewness to evaluate whether the salary distribution was fair.



Figure 4: Frequency histogram of the salaries of the water district employees [Data sourced from Cabuyao Water District, (2020)].

Even before performing calculations, five out of the six groups noticed outliers in the distribution, observing that managerial positions had significantly higher salaries than other employees. After computing, all groups found that the mean salary was higher than the median, due to these outliers. They interpreted this as meaning that more than half of the employees were paid below the “average” salary. They also noted that the distribution was positively skewed, indicating that more employees received lower compensation compared to others.

All groups agreed that there was a significant pay gap between the highest and lowest paid employees, but their opinions differed on the fairness of this situation. One group argued that variations in compensation between different job titles were natural due to differences in roles, responsibilities, and tenure. This perspective raises concerns, as it shows the limitations of using statistics to address social issues when people are led to believe that certain systems are fixed and cannot be changed. In contrast, the rest of the groups argued that the pay gaps were too large, noting that the highest-paid employee earned almost ten times more than the lowest-paid employee. One group also observed disparities in pay among employees with the same job titles. Another group contended that more than half of the employees were not receiving livable wages. When asked how to address the salary distribution, all groups suggested reducing the disparity and basing compensation more closely on the work performed. Half of the groups proposed lowering the salaries of those in managerial positions and redistributing the savings to other employees. One group emphasized the need to increase compensation for most employees due to rising living costs caused by inflation.

The students' suggestions for addressing salary gaps in the water district are promising, reflecting broader concerns shared by other institutions. It is commendable that they used statistics to expose unfair salary distributions and considered the experiences of Filipinos, such as the rising cost of living, in their decision-making process to determine what is in the best interest of the majority.

However, this case also highlights the limitations of the discussion, as the students had a limited understanding of the issue itself. In the first three cases, students relied heavily on their personal

experiences and core beliefs when analyzing the statistics and social issues. In this case, however, they struggled to reach a sound conclusion, as they lacked knowledge about the differences in salaries and responsibilities between managers and rank-and-file employees. This demonstrates that when teaching statistics through Critical Mathematics Education, the choice of social issue is important, as students may better understand topics that are closer to their own experiences.

DISCUSSION AND CONCLUSION

After the nine lessons, 11 students agreed to be interviewed to assess their perceptions of integrating social issues in the Philippine context into learning statistics. Nine students supported the integration, arguing that discussing social issues made them more engaged in class discussions, as they could see how mathematical concepts applied to real life. This feedback highlights that students learned about the role of mathematics in critiquing and shaping society, countering the notion that mathematics is only for those pursuing careers in the sciences and engineering. However, two students commented that learning about social issues in mathematics class confused them, as they were accustomed to focusing solely on mathematical concepts. This feedback shows the challenge for critical mathematics educators to shift away from the traditional view that mathematics is solely about solving problems and computations. It also demonstrates that students have seemingly become accustomed to the idea that mathematics is just about learning numbers without fully understanding its context. Thus, it should be emphasized to students that mathematics is all around them and must be used to connect their experiences when analyzing the current system.

While all students recommended continuing the incorporation of social issues into mathematics lessons, three suggested that this integration should start at earlier grade levels and be introduced gradually. This recommendation aligns with the previous feedback, as it would be more effective if students developed critical thinking skills from a young age, preparing them to be critical citizens in adulthood. Additionally, this implies the need for changes not just in the curriculum but in the entire education system. Based on the interviews, it became evident that although students were engaged in discussing social issues, they were also concerned about comprehending the mathematical concepts, especially with upcoming exams in mind. It is disheartening that, in the current education system, priority is given more to achieving good grades than to applying concepts to real-life experiences. Critical Mathematics Education also proposes a solution to the grade-conscious mindset of students, as it may make understanding mathematical concepts easier when applied to relevant real-world issues instead of being confined to abstract scenarios.

Given that changing the entire education system may still be out of reach, it appears that critical mathematics educators must work within the current system until a time when broader changes are possible. This study explored the possibility of incorporating Critical Mathematics Education into consecutive lessons within the standard curriculum. This approach involves “creative subversion,” as suggested by Ambrose (2013), where teachers find ways for their students to engage in creative exploration in mathematics— in this case, by integrating social issues into the classroom while adhering to the current curriculum. Aside from creative subversion, openness to different views and opinions must be present in a critical mathematics classroom. However, this openness must have boundaries, as a critical mathematics teacher must guide students toward a bias in favor of the best interests of the majority, particularly those who are oppressed and exploited.

Most importantly, this study has strongly demonstrated that Critical Mathematics Education is essential, especially when teaching statistics, as it bridges the gap between objective numbers and the subjective experiences of students. It shows how both are equally important in interpreting statistics and solving real-world problems. The numbers provide the “what” of the problems, but the context of the social issues provides the “why” it is important to solve these problems, which should be the primary purpose of mathematics education.

Acknowledgments

The researchers extend their deepest gratitude and appreciation to the Philippines’ Department of Science and Technology – Science Education Institute for funding this research study.

References

- Ambrose, D. (2013). Dogmatic neoliberal ideology suppressing talent development in mathematics: Implication for teacher education. In L. J. Jacobsen, J. Mistele, & B. Sriraman (Eds.), *Mathematics teacher education in the public interest: Equity and social justice* (pp. 83–97). Information Age Publishing, Inc.
- Avci, B. (2020). Research methodology in Critical Mathematics Education. *International Journal of Research & Method in Education*, 44(2), 135-150.
- Breunig, M. (2009). Teaching for and about critical pedagogy in the post-secondary classroom. *Studies in Social Justice*, 3(2), 247-262.
- Cabuyao Water District. (2020). *Regular 322 recognize position for CY 2020*. <https://cabuyaowaterdistrict.gov.ph/wp-content/uploads/2021/02/Annex-C-List-of-Plantilla-of-Personnel-for-FY-2020-compressed.pdf>
- Chartres, M. (2008). Are my students engaged with critical mathematics education? In J. F. Matos, P. Valero, & K. Yasukawa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 186-196). Publishing Department of the University of Algarve.
- Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal*, 15, 1–16.
- Ernest, P. (2016). The scope and limits of critical mathematics education. In P. Ernest, B. Sriraman, & N. Ernest (Eds.), *Critical Mathematics Education: Theory, praxis, and reality* (pp. 99-126). Information Age Publishing.
- Hilario, E. L. P. (2023). Unpublished raw data on starting salary of teachers in the Philippines.
- IBON Foundation. (2023, June 21). *NCR family living wage as of May 2023*. <https://www.ibon.org/ncr-flw-ao-2305/>
- Philippine Statistics Authority. (2021, December 10). *Birth statistics of Quezon City: Year of 2019*. <http://rssoncr.psa.gov.ph/sites/default/files/Birth%20Statistics%20of%20Quezon%20City%2C%202019.pdf>
- Philippine Statistics Authority. (2022, March 15). *Birth statistics of Quezon City: Year of 2020*. http://rssoncr.psa.gov.ph/sites/default/files/Birth%20Statistics%20in%20Quezon%20City%2C%202020_0.pdf
- Philippine Statistics Authority. (2024, August 15). *11 out of 18 regions recorded significant decreases in poverty incidence in 2023* [Press release]. <https://www.psa.gov.ph/statistics/poverty>
- Powell, A. B., & Brantlinger, A. (2008). A pluralistic view of critical mathematics. In J. F. Matos, P. Valero, & K. Yasukawa (Eds.), *Proceedings of the Fifth International Mathematics Education and Society Conference* (pp. 186-196). Publishing Department of the University of Algarve.
- Reporters Without Borders. (n.d.). *List of Scores 2023*. <https://rsf.org/en/index?year=2023>
- Steffensen, L. (2017). Critical mathematics education and post-normal science: A literature overview. *Philosophy of Mathematics Education Journal*, 32.
- Steffensen, L. (2021). *Critical mathematics education and climate change: A teaching and research partnership in lower-secondary school* [Doctoral dissertation, Western Norway University of Applied Sciences]. HVL Open.

“GREENWASHING”, DISINFORMATION, OR CLIMATE PROTECTION? HYBRID TASK DESIGN WITH A FOCUS ON THE COMPETENCE OF DATA-BASED ARGUMENTATION

Sebastian Kuntze, Jens Krummenauer

Ludwigsburg University of Education, Germany

Disinformation being one of the key challenges for present and future democratic societies in their response to, e.g., threats related to climate change, mathematics education should foreground competence development goals for strengthening learners with this respect: We argue that in particular data-based argumentation can help learners to deal with fact-denying claims and other phenomena aiming at citizens' disinformation. The development of the competence of data-based argumentation can be supported from the primary school level on in the mathematics classroom, as found in prior empirical research. However, the development of focused learning material is needed, in particular for the mathematics classroom in secondary schools. Under the perspective of hybrid task design, which combines mathematics-related with subject-overarching learning goals, there is a need to explore corresponding task design possibilities: we discuss the learning potentials of tasks in which data-based argumentation and skills in dealing with disinformation is the key to understanding climate change and evaluating climate protection measures.

INTRODUCTION

Democratic societies are faced with the need to respond to a number of threats, including the need of introducing effective climate protection measures. However, disinformation of citizens can play a very detrimental role for the process of finding such responses, as democratic discourse and decision-making may be influenced by fact-denying or manipulative claims, for example in the form of “greenwashing” driven by short term economic interests. Consequently, for enabling citizens' problem-oriented participation in decision-making processes, individual competences related to dealing with information and disinformation are key. From the perspective of mathematics education, the competence of *data-based argumentation* can be seen as a key competence in this regard, as discourses such as those mentioned above are (or should be) based on statistical information. The mathematics classroom can contribute to the development of this competence: If learners learn to probe claims against available data, to reject them if the data does not support the claims, and to produce argumentations based on data from early school years on, this can be expected to restrict the impact of attempts of disinformation.

However, besides competence in data-based argumentation, subject-overarching content knowledge, for example climate-related or sustainability-related, is needed together with skills in dealing with disinformation with the help of such knowledge. At first sight, such learning goals could be seen as being in concurrence with mathematics education goals, for instance as far as learning and instruction time is concerned. There is hence a need to reconcile such subject-overarching with mathematics-related learning goals, which raises questions regarding the design of appropriate learning tasks.

Responding to this research and development need, this paper uses the concept of so-called *hybrid tasks* to combine a focus on the support of learners' data-based argumentation with a focus on learning

goals related to dealing with disinformation in the area of climate change and sustainability. We discuss corresponding tasks and analyse their learning potential with respect of both goal domains.

In the following, we will introduce to the theoretical background of this paper (1), deduce the research and development aim (2), provide insight into task design and sample tasks (3), and analyse and discuss the associated learning potentials in a concluding section (4).

THEORETICAL BACKGROUND

Mathematics is a discipline which is marked by its specific argumentation culture (e.g., Heintz, 2000), also beyond formal proof. Dealing with data and statistics has become an important content domain in the mathematics classroom. Being able to generate arguments on the base of statistical data is considered as an important element of, for example, statistical literacy (Wallman, 1993; Watson & Callingham, 2003), and it is also highly relevant for responsibly participating in social discourse. However, the focus on data-based argumentation, its structure, and ways of fostering this competence is rather recent (Krummenauer & Kuntze, 2018, 2021, accepted; Krummenauer et al., 2022). A *data-based argument* connects an interpretation of the data or a claim in a consistent way with data, so that the interpretation or claim is supported (Krummenauer & Kuntze, 2018, 2021). Theory elements related to scientific reasoning (Kuhn, 1988; Kuhn et al., 1989; Koerber et al., 2005; Zimmerman, 2007; Sodian et al., 1991; Bullock & Ziegler, 1994) can be used to describe data-based argumentation: the coordination of *hypotheses* and *evidence* in scientific reasoning corresponds to the coordination of *interpretations of data* and *data* in data-based argumentation: In case the evidence contradicts the hypothesis, the hypothesis has to be rejected, just as claims or interpretations of data which are not supported by the data. Research into scientific reasoning has pointed to pre-adolescent restrictions in scientific reasoning (e.g. Tschirigi, 1980; Dunbar & Klahr, 2012; for example, learners only tend to seek conforming evidence for hypotheses or modify evidence so as to fit to the hypothesis) – evidence of such restrictions can also be found in examples of unsuccessful data-based argumentation (Krummenauer & Kuntze, 2018). Fortunately, data-based argumentation can be supported and fostered, as empirical findings from an intervention study have shown (Krummenauer & Kuntze, accepted): The intervention was centred in scientific reasoning strategies within data-based argumentation, and it produced an increase in data-based argumentation which was significantly higher than the non-significant developments in a control group and in a further intervention group with a data reading focus. The results call for an increased emphasis on data-based argumentation in the school classroom, and they further suggest that competence in data-based argumentation can be beneficial to learners in order to prepare them to deal reflectively with disinformation, as seeking for potentially conflicting evidence in data and rejecting unsubstantiated claims is a core of data-based argumentation.

However, dealing with disinformation attempts has various sides and extends beyond data-based argumentation competence. Non-mathematical topic-related knowledge can be an important base which can help to judge claims. Moreover, higher order thinking strategies such as identifying potential intentions or economic interests behind disinformation elements, or actively considering socio-economic systems in interaction with disinformation elements are connected with further goals of the classroom, in a school subject-overarching way (c.f. e.g., Coles, 2023; Ödmo et al., 2023). In the case of dealing with disinformation attempts through greenwashing, for instance, sustainability-related and climate protection-related knowledge can be expected to be helpful and to complement

data-based argumentation competence. In a school subject-overarching way, developing such climate protection-related and sustainability-related knowledge is hence an additional goal for the classroom (Coles, 2023), for – among other benefits – being able to successfully deal with disinformation attempts, for example related to greenwashing. For this paper, this raises the question whether the complementary goals can be combined in learning opportunities in the mathematics classroom.

A possible answer to this question is the design of so-called *hybrid tasks* (Kuntze, Aizikovitsh-Udi, & Clarke, 2017) as a way to combine mathematics-related with subject-overarching learning goals. Hybrid tasks aim to promote higher order thinking and subject-overarching knowledge together with mathematical content-related learning, through the same task (cf. Kuntze et al., 2013). In conclusion, the combination of the needs highlighted above lead to research and development aims with a focus on data-based argumentation in the subject-overarching topic of greenwashing and climate protection.

RESEARCH AND DEVELOPMENT AIMS

Hybrid task development in the field of greenwashing, disinformation, and climate protection which is related to data-based argumentation is scarce, so that corresponding task-based learning material needs to be developed for the mathematics classroom. Consequently, this paper focuses on the following research and development question:

How can tasks be designed which combine mathematics-related learning goals regarding data-based argumentation with subject-overarching learning goals related to dealing with disinformation in the area of climate protection and sustainability?

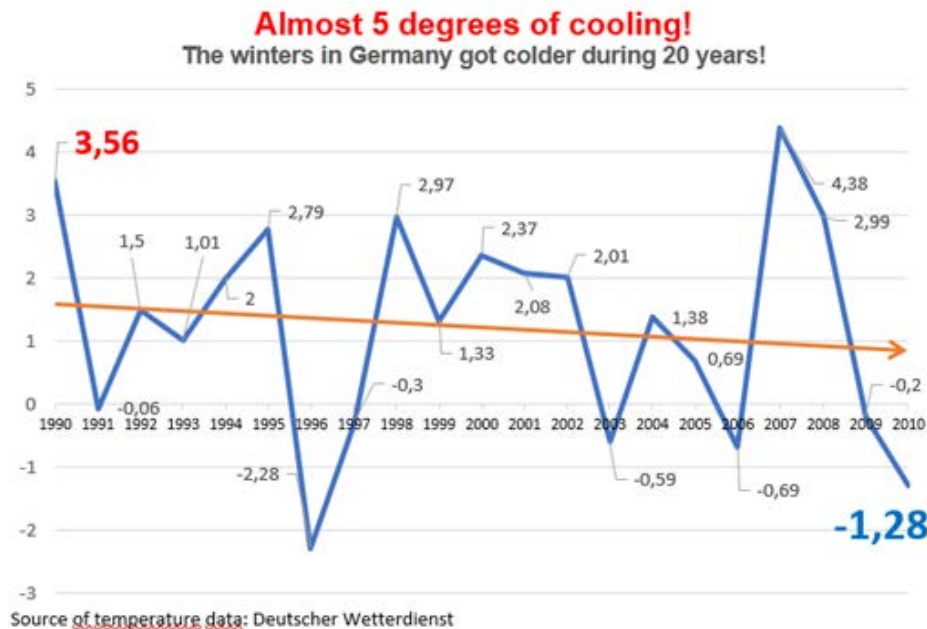
There is a need to explore and to describe the learning opportunities related to such tasks, also for facilitating further task design and supporting corresponding goals in the mathematics classroom.

HYBRID TASK DESIGN AND SAMPLE TASKS

Disinformation related to climate protection can undermine the need felt for protecting the climate, for example by questioning climate change. In such cases, interpretations of data may be used to question scientific findings on which there is a wide-spread consensus in science. Figure 1 shows a task (cf. Kruppenauer et al., 2022) on a claim which suggests that temperatures are rather getting lower than higher in German winters. Like this, global heating is being questioned, even in the form of a data-based argument. From the point of view of learners' dealing with disinformation, learners should be expected to have heard of evidence of global warming, in the media and/or in school, for instance. First of all in class, the task can be used to encourage self-reflection on the effect that the statements in the task may produce on the learners and on others: persons who are confronted with the statements might feel unsure about their prior climate-related knowledge, or they might even be open to the statements in the task, especially in cases where they have a tendency to seek for claims or data which could support practices which are rather not in line with climate protection. Dealing with disinformation often means having to deal with perceived contradictions and the need to weigh conflicting information. In this case, the restricted data base (e.g., selectively chosen years, restriction to German winters, statistical variation in data) opens up ways to question the given statements and to deal with the potential disinformation attempt.

Data-based argumentation can be helpful in this context. On a first level, the statistical variation in the given data could be used to produce counter-arguments, such as the comparison of, for example,

the years 1991 and 2007 or 1996 and 2006, which would, under the same kind of interpretation procedure, yield a rise in average temperatures. On a second level, further data could be considered and examined. This analysis could lead to the inclusion also of recent years' average temperatures, temperature extremes could be considered beyond averages, and a more complete data picture could be the base for a set of data-based arguments, which would allow rejection of the biased interpretations of data given in the task.



Climate change is everywhere in the media, but you have to look at it with more precision. If you take the average winter temperatures from 1990 and 2010, you get a drop in temperature of almost 5 degrees! How can you speak of global heating against this background?

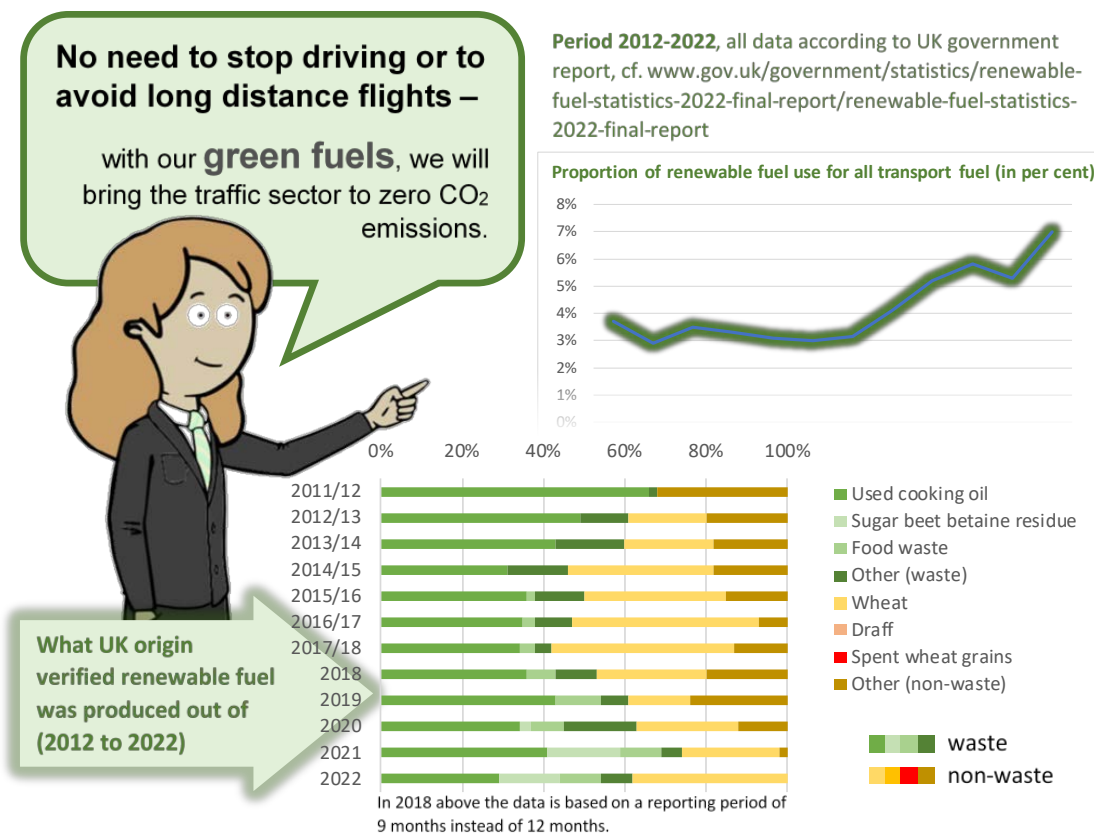
Evaluate this claim – and don't forget to justify your answer.

Figure 1: Task on dealing with a potential climate-related disinformation attempt (Krummenauer et al., 2022, p. 223; translated)

Greenwashing can be a further form of disinformation related to climate protection. In the case of greenwashing, practices which are non-optimal for climate protection are presented as effective measures for climate protection, often with the intention of influencing citizens' practices according to short term economic interests. Such forms of disinformation can contribute to climate-hostile decisions as a consequence of disinformation, so that the design of hybrid tasks for strengthening learners is of high importance. Figure 2 presents such a task for the context of "renewable" fuels. The data used in the task is inspired by data from a UK government report (see weblink in Fig. 2). The claim in the speech bubble does not stem from this report and it may be seen as an exaggeration in the context of greenwashing – however observable statements in the media, in political discussions, or in conversations can have a similar dimension, beyond considering the statement against the

background of purposeful didactic simplification. From the perspective of dealing with disinformation, the task contains the challenge that the given statements might take up and connect with the addressees' hope of receiving good news related to easy and effective climate protection measures. Learners might accept the statement as reliable information about effective climate protection in the traffic sector and – as a consequence – stay with non-climate friendly practices which are associated with climate protection through the given statements. Successfully dealing with misleading information requires taking a distance from the given statements and using climate-related context knowledge, also in this example.

From the perspective of data-based argumentation, the task in Figure 2 offers two levels of appropriate answers, in a somewhat analogous way as in the previous task: The given data can be



Petrol goes green*

* Our company is committed to zero CO₂ emissions from 2050 on

Evaluate the claims given in this (fictitious) advert – and don't forget to justify your answer.

Figure 2: A (fictitious) climate-related disinformation attempt as a learning opportunity in a corresponding hybrid task

used by the learners to produce an argument for a contradiction between the statements and the data: The relative stagnation of waste components out of which regenerative fuels are produced suggests that it might not be possible to replace fossil fuels in the traffic sector without considerable CO₂ impact within a reasonable time scale, so that indeed, any flights and unnecessary car use should be avoided. Moreover, the use of additional data can bring more possibilities of data-based argumentation. Among these could be data-based arguments related to the worldwide availability of

used cooking oil or waste resources, which may only cover a small part of the worldwide fuel need in the upcoming decade and beyond, if the practices, systems, and growth rates in the traffic sector remain unchanged (as implicitly suggested in the task’s statement “no need to stop driving or to avoid flights”). So additional data can be used to challenge the statements given in the task based on corresponding arguments. Also, more specific data, for instance related to the ecological and high CO₂ footprint of palm oil production which may be used for producing “renewable fuels”, connected with media reports on doubtful re-declaration of palm oil products into “used cooking oil” by “certification” procedures (cf. e.g., www.tagesschau.de/328/ecognize/328328_ed/ndr/biodiesel-palmoel-china-deutschland-100.html), can be used by learners if they are given the time to find out and work on such data and to deepen their argumentation.

The task in Figure 2 consequently rather focuses on disinformation in the form of greenwashing. At the same time, data-based argumentation and dealing with disinformation is focused on in this topic context as outlined above.

Summing up, the learning potential of the hybrid tasks in Figures 1 and 2 can be described both (1) on the level of dealing with disinformation related to climate protection together with the learning of subject-overarching knowledge connected to climate change and climate protection, and (2) on the level of data-based argumentation, with possibilities of argumentation directly on the basis of the given data and argumentation based on further data the learners could find out about, for example through an online search. Both on the level of dealing with disinformation and of data-based argumentation, the tasks aim at encouraging the learners to take a distance from the given statements and to critically evaluate them (cf. Krummenauer & Kuntze, 2021), leading to argumentation activities.

DISCUSSION AND CONCLUSIONS

First of all, we would like to recall the limitations of this study. The presented tasks and their analysis do not imply that learners’ competences in dealing with disinformation or in data-based argumentation will necessarily grow – the enactment and framing of corresponding learning opportunities on the one hand, and the learners’ dispositions (e.g., their prior knowledge) on the other hand, have to be expected to play key roles. Empirical studies in this domain are needed in order to gain reliable insight into whether the predicted learning potential both in dealing with disinformation related to climate change and climate protection and in data-based argumentation can be recognized in different learner target groups. Corresponding follow-up questions on possible transfer effects to dealing with disinformation in more generality are of high interest, given today’s challenges (Coles, 2023; Ödmo et al., 2023) for (mathematics) education.

However, the research and development aims of this paper could be reached. Earlier research into data-based argumentation (Krummenauer & Kuntze, in press) shows that data-based argumentation can be fostered through learning opportunities which focus on scientific reasoning strategies (Krummenauer & Kuntze, 2018; Kuntze et al., 2013) in the context of data-based argumentation, and that learners’ potential difficulties can successfully be dealt with (Krummenauer & Kuntze, 2018; Krummenauer et al., 2022). Such strategies can be recognized, challenged, and further developed with tasks like the ones presented above, as learners are encouraged to take a distance from the given statements, to evaluate them against given and/or non-given data, to reject them with arguments based

on data, and to deal with statistical variation in the data when generating arguments (Krummenauer & Kuntze, 2021). Learners can use the data given in the tasks to produce counter-arguments which question the given statements. Moreover, learners can use additional data for further argumentation in order to completely reject the given claims, which corresponds to a learning activity for secondary school, in particular.

The simultaneous focus on dealing with disinformation in the contexts of climate change and greenwashing brings an additional learning potential and can support the learners' experience of relevance, by at the same time profiting from the didactic potentials of hybrid tasks. The tasks design suggested in this paper further unfolds the potentials of hybrid task design on the level of mathematics education theory development (Kuntze et al., 2017).

The recent and increasingly urgent challenges for mathematics education related to environment protection and climate change (e.g., Coles, 2023; Ödmo et al., 2023) are connected to needs to prepare present and future generations of learners for these challenges and for productive social practices of democratic participation as responsible citizens. The hybrid tasks presented in this paper showcase in particular how scientific reasoning (e.g., Kuhn, 1988; Koerber et al., 2005; Zimmerman, 2007; Sodian et al., 1991; Bullock & Ziegler, 1994) can be a core to describe – and to foster – key competences learners should build up in the mathematics classroom (and possibly beyond). Moreover, scientific reasoning can be helpful to conceive of goal descriptions and their analysis in the context of these challenges. As argued in the case of dealing with disinformation and data-based argumentation here, distinguishing claims and interpretations from data and further evidence, as well as evaluating the first against the background of the second, can serve as a key to learners' preparation for the future also beyond mathematics education.

Acknowledgements

Some of the graphical elements in the Figures stem from the project coReflect@maths (www.coreflect.eu), which was co-funded by the European Union in the framework of Erasmus+ (2019-1-DE01-KA203-004947). The European Commission's support for the production of this publication does not constitute an endorsement of the contents, which reflect the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

References

- Bullock, M. & Ziegler, A. (1994). Scientific thinking. In F. Weinert & W. Schneider (Eds.), *The Munich Longitudinal Study on the Genesis of Ind. Competencies*. MPI.
- Coles, A. (2023). Teaching in the new climatic regime: Steps to a socio-ecology of mathematics education. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17–33). PME.
- Dunbar, K. N., & Klahr, D. (2012). Scientific thinking and reasoning. In K. J. Holyoak & R. G. Morrison (Hg.), *The Oxford handbook of thinking and reasoning* (pp. 701–718). Oxford University Press.
- Heintz, B. (2000). *Die innenwelt der 329ecognize329. Zur kultur und praxis einer beweisenden disziplin*. [*The inside world of mathematics. On the culture and practice of a proving science*]. Springer.
- Klahr, D. & Dunbar, K. (1989). Developmental differences in scientific discovery processes. In Klahr, D. & Kotovsky, K. (Eds.), *Complex Information Processing* (pp. 109–143). Erlbaum.

- Koerber, S., Sodian, B., Thoermer, C., & Nett, U. (2005). Scientific reasoning in young children: Preschoolers' ability to evaluate covariation evidence. *Swiss Journal of Psychology*, 64(3), 141–152.
- Krummenauer, J. & Kuntze, S. (2018). Primary student's data-based argumentation – An empirical reanalysis. 2018. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 251–258). PME.
- Krummenauer, J., & Kuntze, S. (2021). The role of critical thinking in data-based argumentation – Empirical findings from studies with primary students. In B. Maj-Tatsis & K. Tatsis (Eds.), *Critical thinking in mathematics: Perspectives and challenges* (pp. 91–100). University of Rzeszów. <http://cme.ur.edu.pl/wp-content/uploads/2022/01/CME'21%20critical%20thinking.pdf>
- Krummenauer, J. & Kuntze, S. (in press). Datenbasiertes argumentieren in der grundschule fördern – Eine interventionsstudie. [*Fostering data-based argumentation in primary school. An intervention study*]. *Journal für Mathematikdidaktik*.
- Krummenauer, J., Gutensohn, F., Aichele, J., Emhart, M. & Kuntze, S. (2022). Argumentation based on statistical data at the very beginning of primary school – Evidence from two empirical studies. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 75–82). PME.
- Krummenauer, J., Stark, L., Schwaderer, F., & Kuntze, S. (2022). Vom faktencheck zum datenbasierten argumentieren. [From fact check to data-based argumentation] *MNU-Journal*, 75(3), 219–224.
- Kuhn, D. (1989). Children and adults as intuitive scientists. *Psychological Review*, 96, 674–689.
- Kuhn, D., Amsel, E. & O'Loughlin, M. (1988). The development of scientific thinking skills. Academic Press.
- Kuntze, S., Aizikovitsh-Udi, E., & Clarke, D. (2013). Strategies for evaluating claims – an aspect that links critical thinking and statistical thinking. In Lindmeier, A. & Heinze, A. (Eds.), *Proceedings of the 37th Conf. of the IGPME*, (Vol. 5, p. 235). PME.
- Kuntze, S., Aizikovitsh-Udi, E., & Clarke, D. (2017). Hybrid task design: Connecting learning opportunities related to critical thinking and statistical thinking. *ZDM Mathematics Education*, 49, 923–935.
- Ödmo, M., Björklund Boistrup, L., & Chronaki, A. (2023). A teacher education statistics course encounters climate change and critical mathematics education: Thinking about controversies. In R. Marcone, P. Linardi, R. Milani, J.P. A. de Paulo; A. Moura Queiroz, & M. T. da Silva (Eds.), *Proceedings of the 12th International Conference of Mathematics Education and Society* (sixth sketch, pp. 1307–1321). MES.
- Sodian, B., Zatchik, D., & Carey, S. (1991). Young children's differentiation of hypothetical beliefs from evidence. *Child Development*, 6, 753–766.
- Tschirigi, J. (1980). Sensible reasoning: A hypothesis about hypotheses. *Child Development*, 51, 1-10.
- Wallman, K. K. (1993). Enhancing statistical literacy: Enriching our society. *Journal of the American Statistical Association*, 88, 1-8.
- Watson, J., & Callingham, R. (2003). Statistical literacy: A complex hierarchical construct. *Statistics Education Research Journal*, 2(2), 3–46.
- Zimmerman, C. (2007). The development of scientific thinking skills in elementary and middle school. *Developmental Review*, 27(2), 172–223.

DO THEMES OF ENVIRONMENTAL COLLAPSE GUARANTEE A SOCIO-ECOLOGICAL DISCUSSION?

Aldo Lopes¹, Marcelo Borba²

¹Federal University of Itajubá, São Paulo State University, ²São Paulo State University

There is an urgent need to address climate change causing extreme weather events. The growing ecological crisis demands attention not only in environmental policies but also in mathematical education. This field needs to devote more attention to environmental and ecological issues, rethinking not only the content but also the epistemologies, ontologies, and pedagogies of the discipline. Mathematical modeling can be a methodological alternative with possibilities for debates on socio-ecological issues. Empirical research with future environmental engineers investigated how they critically engage in modeling practices related to socio-ecological issues. The study suggests that an approach combining mathematical modeling with social and human sciences can be fruitful for discussing the current challenges of the ecological crisis, although not all aspects of reflective knowledge may be manifested.

INTRODUCTION

“The planet is sending an SOS” (New York Times Opinion, December 2021).

“Climate change is the single biggest health threat facing humanity” (World Health Organization, November 2023).

“As the planet warms, the expectation is that we will see more and more intense, more frequent, more severe rainfall events, leading also to more severe flooding” (Stefan Uhlenbrook, Director of hydrology, water and cryosphere at the World Meteorological Organization, July 2023).

In addition, the post mentioned in the former Twitter (now called X) of The New York Times includes a link (<https://nyti.ms/3dITMkx>) to a set of images and short videos. This visual material demonstrates how climate change is already altering daily life in 193 countries. We agree with The New York Times that the world does not need to continue burning, considering that we inhabit a world marked by rising temperatures and increasingly severe weather conditions (New York Times Opinion, 2021). Additionally, heavy rains in April and May 2024 caused severe flooding, leaving thousands homeless in Porto Alegre, the capital of the state of Rio Grande do Sul in Brazil (see, for example, <https://www.bbc.com/news/articles/c2899kqv7zeo> – accessed May 1, 2024). In view of these increasing climate changes, reflections, debates, and actions are needed towards a socio-ecologically sustainable world.

The global ecological crisis relates to mathematics, given its role in describing, predicting, and communicating environmental issues (Barwell et al., 2022). Moreover, mathematics is fundamental to the functioning of technological consumer capitalism, which contributes to environmental degradation.

Despite the scale of the crisis and the role of mathematics within it, environmental, ecological, and sustainability issues have received little attention in the field of mathematical education. The current state of the planet, including within the context of mathematics and mathematical education practices, demands an urgent reassessment of priorities in mathematical education. This includes reconsidering

the meaning of environmental issues for mathematical education and whether this area can contribute to meaningful action in response (Coles et al., 2024). However, there is a body of work suggesting that mathematical education needs to be completely rethought in response to the growing crisis, not only in terms of ‘content’ but also based on new epistemologies, ontologies, ethical frameworks, and pedagogies (Boylan, 2017; Barwell et al., 2022).

Coles et al. (2024) highlight the term socio-ecological to emphasize the interconnectedness between social and ecological issues, arguing that these issues cannot be treated separately because every social issue has an ecological aspect and vice versa. They define socio-ecological as the relationships between humans, non-humans, living, and non-living entities. Based on this definition, we will discuss the significance of socio-ecological issues for mathematical education.

Some recent studies have highlighted that there is still much to be explored about mathematical discourse in classrooms while considering broader ecological issues (Coles et al., 2024; Nicol et al., 2023). For these authors, doing so is a necessary socio-ecological gesture that opens up space for critical discussions on this important theme. The final sentence of Coles et al.’s article (2024) does more than just request more research and dialogue from mathematical education on socioecological issues. It states: “We implore further research and dialogue to conceptualise and exemplify socio-ecological gestures, and how and what mathematics might follow” (Coles et al., 2024, p. 178).

An approach in the classroom towards reflecting on socio-ecological issues, according to Amico et al. (2023), is mathematical modeling. Mathematical models influence the real world, such as when models on the impact of climate change result in commitments to reduce carbon emissions (Barwell et al., 2022; Steffensen, 2021). For Amico et al. (2023), it is necessary to explore how mathematical modeling can be integrated with the qualitative tools of social and human sciences to address current socio-ecological issues.

In order to promote ongoing conversation and contribute to defining and exemplifying socio-ecological gestures (Coles et al., 2024), as well as to explore how mathematics can accompany and develop from them, we conducted empirical research with future engineers. It was analyzed how future environmental engineers critically engage in modeling practices when the topic involves socio-ecological issues. The research question that guided the study was: to what extent can mathematical modeling of socio-ecological issues lead to reflective knowing?

THEORETICAL ASPECTS

Socio-ecology in mathematical education

Mathematical activity, including the production of mathematical meaning, is socially organized and linked to technological innovation (Borba, 2021). Mathematics concerning biodiversity or climate change is part of humans’ understanding of these issues and the actions they take in response. Furthermore, technological innovation influences human systems of economic organization (Barwell et al., 2022). Mathematics can be understood as a complex system that intertwines and is interdependent with humans and technology (Borba, 2021). This approach aligns with a complex understanding of the living world. Mathematics cannot provide definitive descriptions or solve all problems related to the living world. Thus, uncertainty and relationality are key concepts to be considered (Barwell et al., 2022). With this perspective, school mathematics can adopt a transformative curriculum, where the living world is integrated into all aspects of educational

processes. Mathematical learning aligns with healthy relationships within the living world, pointing towards a more central role for citizens in mathematics and the science of the living world. In pursuit of social justice, Boylan (2017) suggests that education should “seek to change both students and society” (p. 370).

Mathematical education related to the living world is globally necessary, but its forms and local meanings vary. It is essential that the ecological responsibility approach of mathematical education expands beyond its current foundations to consider diverse origins and locales (Barwell et al., 2022). Ecosystemic crises are framed not as information to be acquired but as situations to be explored and understood, focusing on active student participation. Mathematical education should go beyond skills and knowledge to prepare students to be engaged citizens capable of dealing with uncertainty, values-based decision-making, and democratic debate. Hauge and Barwell (2022) propose three principles for this approach: exploring situations of risk and uncertainty; exploring scientific/mathematical concepts and social perspectives; and learning through dialogue. Uncertainty and risk are challenging aspects of the ecosystemic crises we face and are often incorporated into political discourses. These elements open up space for the implantation of values, highlighting the need to include a broader and more participatory approach in mathematics education.

In this paragraph, we summarize how Barwell et al. (2022) examine the relationships between mathematics, mathematical education, and the living world from a dialogical perspective. Traditional approaches tend to be utilitarian and exploitative, but the dialogical view seeks to reveal normally invisible aspects. By adopting a dialogical ethics, we propose that human relationships with the ecosystem should be mutually shaping, where both humans and the ecosystem influence each other, rather than a relationship of domination. The relationship between humanity and the ecosystem is dialogical and interdependent, with human action limited by this relationship. Dialogical ethics suggests that mathematical education should engage students in processes of reflection, critique, and debate on ecosystemic and social issues. Mathematics is not a neutral tool but part of a dialogue with the living world. Mathematical education has an ethical responsibility to the living world and can contribute to building reparative relationships with it. It should help us listen to the living world and respond to it consciously. Mathematical educators should consider pedagogical and curricular choices that support a dialogical orientation to the living world and promote human capacity to engage with other beings as citizens. A dialogical perspective suggests considering mathematics as part of human relations within the ecosystem. The response to ecological crises must be intertwined with socio-political and social justice issues. This approach emphasizes an ethical responsibility to the rest of the ecosystem and the disproportionate effects of crises on the poorest and most marginalized.

The aspirations of critical mathematical modeling

Critical Mathematics Education (CME) addresses how mathematics and mathematical education contribute to shaping the world, knowledge, and subjectivities, with the aim of aspiring towards a more democratic and socially just world. It also concerns itself with exploring the potential of mathematics to understand and address environmental issues, especially climate change, thereby promoting environmental sustainability (Borba, 2021; Steffensen, 2021). In recent years, CME has grown by incorporating ecological issues, recognizing the impact of the social world, technology, and mathematics on environmental issues. Skovsmose (2023) emphasizes the need to consider “environmental justice” alongside “social justice” as central concerns of CME. To promote social

justice objectives through the critical agency of those involved, mathematical modeling can be used (Jung & Wickstrom, 2023).

Sociocritical modeling, an emancipatory approach to mathematical modeling (Blum, 2015), has a close relationship with CME. It focuses on students' ability to be critical modelers and recognize their power, rather than just developing mathematical skills (Lopes, 2023). Although still underdeveloped, this approach is growing, allowing students to critically analyze problems around them, such as the unequal social impact and the effects of human action on the planet's ecologies (Coles et al., 2024). Examples include modeling issues such as carbon footprint, global warming, waste management, dams, and fish stocks.

The sociocritical perspective of mathematical modeling does not have stages or cycles. In view of this, to conduct modeling from this perspective, one can adopt three stages/phases (Lopes, 2023). The process begins with intuitive understanding of the phenomenon and identification of relevant variables. Next, the equation is developed and a mathematical model with representative graphs is constructed. Finally, solutions are interpreted in relation to the concrete problem, enabling meaningful critical discussions.

Reflective knowledge (or reflexive knowledge) can be understood as the capacity to comprehend and analyze problems and circumstances, reflecting on their impacts on social-, environmental-, and knowledge-related issues in mathematics, technology, and ecology (Lopes, 2023). To understand the non-neutrality of mathematics, it is crucial to develop a reflective and dialogical knowledge, considering mathematical knowledge as a human production, subject to contingencies and particular values (Skovsmose, 2023). Emancipation occurs when mathematics allows the development of reflective knowledge, creating space for students to critically connect their knowledge to reality. This can overcome the ideology of certainty. The ideology of certainty is based on three premises: mathematics is neutral, objective, and ensures certainty due to its rational nature. The consequences of this ideology are negative as it can lead to decisions falsely perceived as important, necessary, neutral, and objective.

The use of reflective knowledge in the production of a mathematical model involves reclaiming the modeled phenomenon, addressing problems and uncertainties when translating real situations into mathematical models and vice versa. This includes recognizing how mathematics influences (or shapes) society and how models can be affected by specific values and interests. Mathematical modeling should go beyond teaching content, providing opportunities for students to move away from the ideology of certainty and the "exercise paradigm", which involves routine exercises disconnected from reality.

Lopes (2023) highlights steps to analyze reflective knowledge in the context of mathematics, which can be synthesized into three aspects: reflection on the use of applied mathematics, on the validation and interpretation of the studied phenomenon, and finally, on the social, socio-environmental, and socio-ecological impacts of the approach used.

METHODOLOGY

The study reported here was conducted at a Brazilian federal university in the state of Minas Gerais in the first semester of 2023. The participants in this study were engineering students, all of whom were taking Differential Equations courses. In an initial conversation with the students, they were

encouraged to choose topics for the models to be produced. The encouragement was for these themes to be related to everyday, social, or environmental issues. As a result, out of the 15 groups formed, seven chose themes associated with environmental issues (the themes related to the breach of a mining waste dam, the impact of cattle and rabbit population growth, disease spread, urban traffic flow, and cultivation and growth of certain crops). The students chose classmates to form groups of four to six members. We will present the results of one of the groups, which chose the following theme: “Dam breaches and their social influence.” All six members of this group were studying Environmental Engineering. We emphasize that the names used here have been changed. In addition to the reports from each group and the presentations, other data production instruments were used. After the completion of the modeling, each group was interviewed. The interview was recorded and subsequently transcribed. The interview centered on a video showing the breach of the ore tailings dam in the city of Brumadinho, Minas Gerais, Brazil (<https://www.youtube.com/watch?v=RZLD69dD4Sg>, accessed May 1, 2024) – a tailings dam is a structure designed to store waste materials from the mining process, including rock particles, chemicals, and water, to prevent environmental contamination. This tragedy occurred in a city in the same state as the students, and some of them know engineers who worked at the site but were not affected by the incident. The collapse of the ore tailings dam in Brumadinho, Minas Gerais, operated by Vale S.A., happened on January 25, 2019, resulting in 270 deaths and causing significant environmental damage (see, for example, <https://www.bbc.com/news/business-55924743>, accessed May 1, 2024). The dam, constructed using the upstream raising method, released millions of cubic meters of toxic mud that buried administrative areas and nearby communities, contaminating the Paraopeba River. The disaster had profound humanitarian, environmental, economic, and social impacts, leading to a review of dam safety regulations and highlighting the need for safer mining practices to prevent future disasters. After the video screening for students, questions were asked to ascertain reflective knowledge in the face of a socio-ecological crisis situation. Some of the questions were: Do you think mathematics can influence the breach of a dam? If there are influences, what are the social implications and related issues? Justify your answer. What are the possible social issues related to dams? How is mathematics involved in this? Could there be any misuse of mathematics in this issue of dams? Justify.

For data analysis, we used the notions of qualitative case study, content analysis, and triangulation for fidelity (Mayring, 2022).

RESULTS

In the first phase of modeling, the groups chose their topics and familiarized themselves with the content. The group that chose the theme of dams justified their choice based on the background of each member’s course, Environmental Engineering. They added to the report:

Modeling plays a crucial role in understanding and predicting complex events, such as dam breaches. (...) The Brumadinho disaster had significant repercussions for the economy of the region and the country as a whole.

With this, the group made it clear their intention to study the (economic) effects of the Brumadinho dam breach. The group developed the following equation through a conversation with an artificial intelligence (AI) – Engelbrecht and Borba (2024) have recently discussed the potential for integrating

AI into mathematical education – which was used solely for this part of the modeling process. Following this initial discussion, the group formulated the model:

$$\frac{dI}{dt} = C(PA(t) - PE(t))$$

This equation relates the rate of change of the economic impact (dI/dt) over time with the difference between the affected population (PA) and economic losses (PE). The proportionality constant C determines the sensitivity of the economic impact to the number of affected people and economic losses. Using data from IBGE (Brazilian Institute of Geography and Statistics), the group determined the constant C and found a relationship between $PA(t)$ and $PE(t)$ that depended only on time t . Then, the group solved the differential equation and sketched the graph of the solution. Afterwards, the group concluded:

It was observed that the number of affected individuals obtained (27,014) closely approximated the reported number of people assisted by the Public Defender’s Office of Minas Gerais, which was 24,393 from 2019 to 2023.

In the second stage, the students created an equation and solved it using differential equations techniques. They also produced related graphs, evidencing mathematical understanding. Furthermore, we noticed that the group’s earlier comment showed reflection on the data obtained in connection with reality, indicating that the students in this group identified themselves as competent mathematical thinkers.

Finally, in the last modeling stage, the group engaged in a discussion of the results obtained. The group concluded:

Initial losses are typically more severe, causing significant damage to local businesses, infrastructure, and employment. However, as authorities and the community come together to address the crisis and rebuild the city, it is possible to observe a gradual improvement in the economic situation.

In this comment, we perceive the group’s commitment to expanding the interpretation of the model, exploring beyond the achieved results. The group added:

It is important to highlight that rebuilding a city after a disaster is a complex and time-consuming process, requiring joint efforts from various sectors of society. Additionally, the effectiveness of the measures adopted, adequate financial support, and the local economy’s adaptability are key factors that can influence the speed and extent of economic recovery.

Thus, through these comments, we observe that the group engaged in discussing a topic with potential for social transformation. References to “joint efforts” and “effectiveness of measures adopted,” among other aspects, reveal a willingness to understand and intervene critically in the world, given the financial consequences of an environmental issue.

Similarly, the interview evidenced aspects of CME. Note the following excerpt from the interview:

Interviewer: On the issue of dams, how do you think mathematics is involved with social issues?

Charles: The local economy will be negatively impacted, as we saw with the dam breach in Brumadinho. Therefore, considering that the economy is closely related to society, I believe this clearly harms the local community. Additionally, there are deaths caused by the breach. Overall, that’s what I think.

The group recognized the economic implications resulting from the breach of a dam. Additionally, in another part of the interview, the group discussed other factors that influence this context, reflecting a critical understanding of the socio-ecological situation:

- Interviewer: How can mathematics be involved in cases of dam breaches, such as what happened in Brumadinho?
- Charles: In our slide, we showed a modeling approach that can predict possible scenarios. For example, it identifies the locations that will be affected and estimates the number of losses.
- Anne: Usually, dam breaches occur due to excess waste. This is calculated based on the size of the dam and is also regulated.

We observed that the development of the model allowed for the manifestation of two of the three aspects of reflective knowledge. The students used mathematics and interpreted the results obtained. However, they did not establish social connections with these results. They could have, for example, highlighted aspects of social justice and the unequal impact of the dam collapse on different social classes. Additionally, the group focused on the economic problems resulting from the environmental disaster, paying less attention to the ensuing socio-ecological issues.

Thus, it can be inferred that integrating socio-ecological themes into mathematical education does not always result in a critical stance toward mathematics and the ideology of certainty. It is crucial to challenge the supposed neutrality of mathematics, analyze its social impacts, promote social justice, encourage students' critical reflection, and develop socio-environmental awareness (Borba, 2021; Skovsmose, 2023). This involves connecting mathematical concepts with social realities, encouraging critical reflection on the use of mathematics, and fostering students' active participation in interdisciplinary discussions (Lopes, 2024). Without this approach, mathematics risks being seen as neutral, missing the opportunity to confront and address social and environmental inequalities.

FINAL REMARKS

Although a mathematical modeling activity may not fully develop reflective knowledge and a critical stance, this pedagogical practice remains relevant for discussing topics of interest, such as those related to socio-ecology. Mathematical modeling is a methodological possibility for future environmental engineers, enabling them to address socio-ecological issues. They use models to simulate reality, anticipate problems, and formulate mitigation and adaptation strategies, considering environmental and economic factors. This practice not only assesses the impact of decisions on ecosystems but also proposes sustainable solutions. Furthermore, it can be argued that critical reflection on socio-ecological issues that favors conscious and responsible action towards the planet's future is a responsibility that should be considered by all academics, although the intensity and focus of this responsibility may vary according to different fields of study and individual interests. Mathematical education can contribute to this process by promoting a critical and engaged mathematical understanding.

References

- Amico, A., Andra, C., Chorney, S., Coles, A., Doria, L., Helliwell, T., Hunter, J., Makramalla, M., Pezzutto, M., Rubel, L., Solares, A. How socio-ecological issues are urging changes in curriculum (and beyond). In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proceedings of the 46th Conference of the*

- International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 71–102). PME. Retrieved from <https://www.igpme.org/publications/current-proceedings/>
- Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013. <https://doi.org/10.1016/j.jmathb.2022.101013>
- Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 73–96). Springer. ICME
- Borba, M. C. (2021) The future of mathematics education since COVID-19: Humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics* 108(1-2), 385–400. <https://doi.org/10.1007/s10649-021-10043-2>
- Boylan, M. (2017). Towards a mathematics education for ecological selves: Pedagogies for relational knowing and being. *Philosophy of Mathematics Education Journal*, 32, 1–23.
- Engelbrecht, J., & Borba, M. C. (2024) Recent developments in using digital technology in mathematics education. *ZDM Mathematics Education*, 56(2), 281–292. <https://doi.org/10.1007/s11858-023-01530-2>
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116(2), 165–83. <https://doi.org/10.1007/s10649-024-10318-4>
- Hauge, K. H., & Barwell, R. (2022). Education for post-normal times. In R. Herheim, T. C. Werler, & K. H. Hauge (Eds), *Lived democracy in education. Young citizens' democratic lives in kindergarten, school, and higher education* (pp. 65–76). Routledge.
- Jung, H., & Wickstrom, M. H. (2023). Teachers creating mathematical models to fairly distribute school funding. *The Journal of Mathematical Behavior*, 70, 101041. <https://doi.org/10.1016/j.jmathb.2023.101041>
- Lopes, A. P. C. (2023). Critical consciousness in engineering education: Going beyond critical thinking in mathematical modeling. *European Journal of Engineering Education*, 48(6), 991–1014. <https://doi.org/10.1080/03043797.2023.2203082>
- Lopes, A. P. C. (2024, accepted for publication). Mathematical modelling: A methodology favourable to the manifestation of critical mathematical consciousness. *International Journal of Mathematical Education in Science and Technology*.
- Mayring, P. (2022). *Qualitative content analysis: A step-by-step guide*. Sage Publications.
- New York Times Opinion. [@nytopinion]. (2021, December 13). *We live in a world of warmer, more violent weather. Stronger storms, longer droughts, heavier floods, larger fires, notes the* [X post]. X. <https://twitter.com/nytopinion/status/1470423834465607685> (Accessed May 01, 2024)
- Nicol, C., Thom, J.S., Doolittle, E., Glanfield, F. & Ghostkeeper, E. (2023). Mathematics education for STEM as place. *ZDM Mathematics Education*, 55(7), 1231–1242. <https://doi.org/10.1007/s11858-023-01498-z>
- Skovsmose, O. (2023). *Critical mathematics education*. Springer Cham.
- Steffensen, L. (2021). *Critical mathematics education and climate change: A teaching and research partnership in lower-secondary school*. 197p. Thesis (Doctorate in Philosophy) – Western Norway University of Applied Sciences, Bergen.
- World Health Organization (2023, November 2). *Climate change and noncommunicable diseases: Connections*. Retrieved from: <https://www.who.int/news/item/02-11-2023-climate-change-and-noncommunicable-diseases-connections> (Accessed May 01, 2024)
- World Meteorological Organization. (2023, July 18). *Extreme heat, rainfall highlight need for more climate action*. Retrieved from: <https://wmo.int/media/news/extreme-heat-rainfall-highlight-need-more-climate-action-0> (Accessed May 01, 2024)

CITIZENSHIP DEVELOPMENT THROUGH VALUE-BASED CONSENSUS BUILDING IN MATHEMATICAL SCIENCES EDUCATION

Mitsuru Matsushima¹, Keiichi Nishimura², Rintaro Ueda³

¹ Kagawa University, ² Tokyo Gakugei University, ³ Tokyo Metropolitan Oizumi-Sakura High School

The purpose of this paper is to clarify concepts in the field of mathematical sciences education (MSE), identify its content, methods, and techniques, develop teaching materials, and provide examples. The methodology for this study employed a design-based approach involving lesson study. Research methods included literature review and task design. A high school example outlines how students used novel mathematical-scientific methods to build a consensus and solve an authentic problem of replacing items in a vending machine. We argue that the experience of consensus building in MSE leads to the development of citizenship, and that this citizenship develops the foundation for responding to socio-ecological problems.

BACKGROUND

Research focusing on the social aspects of mathematics includes a series of studies in Critical Mathematics Education as well as democracy and citizenship studies in mathematics education (e.g., Skovsmose, 1998, 2023; Ernest, 2002; Geiger et.al., 2023a; Maass et al., 2019). The connection between citizenship development and mathematics can be found in research on STEM education. The role of mathematics in STEM education has been pointed out in three interdisciplinary approaches: twenty-first century skills, mathematical modelling, and education for responsible citizenship (Maass et al., 2019). The objectives of citizenship education are manifold, but their goal is to “develop the knowledge, skills and capacities to be active and informed citizens, capable of participating in their own communities, the nation, and the wider world” (Tudball & Henderson, 2014, p. 10). In recent years, discussions have also begun on whether this competency for citizenship should be viewed as an individual right or competency, or whether it should be viewed as a competency that starts from the individual and includes interactions with others and the local community (Doki, 2022). There are similarities with the co-agency argument advocated by the OECD (2019). In today’s complex and diverse society, which is faced with problems such as climate change and war, in order to solve these problems, it is important that students have the opportunity to interact with people who hold diverse values and develop the competency to build a consensus based primarily on mathematical knowledge and skills. In order to bring mathematics education closer to a socio-ecological approach, we believe it is important to develop students’ competency to build a consensus and to give them experience in implementing the results of that consensus in society. Therefore, we conducted a series of studies (funded by JSPS Grants-in-Aid for Scientific Research) over a period of 12 years with the aim of constructing a learning area called ‘Mathematical Sciences’, which aims not only to think critically about the options and decisions presented by others, but also to develop the competency and attitude to participate in social decision-making through the use of mathematics and to implement the results of those decisions in the real world (Nishimura & Matsushima, 2024; Ueda et al., 2024). In this paper, we present some of the results.

PURPOSE AND METHOD

The purpose of this paper is to clarify the philosophy of the learning area MSE, to identify the targeted content, methods, and techniques, to develop teaching materials, and to introduce a practical example. This study employed a design-based approach, which included a literature review and task design, with lesson study. Specifically, a hypothetical framework is constructed through theoretical considerations and is repeatedly revised and improved through practical considerations. This paper presents the framework as a result of such revisions and improvements and a case of a lesson based on it, and derives issues in this area of research.

CONCEPTUAL FRAMEWORK

Mathematical sciences

To address the issue of a future society in which individuals live better in collaboration with others in a democratic society with the goal of a sustainable planet, it is essential to link the two types of science, *science for science*, which aims to verify *factual propositions* (expressed as ‘...is...’), and *science for society*, which targets *value propositions* (expressed as ‘...wish...’, ‘...better than...’, ‘...ought...’, ‘...desire...’, etc.) and incorporates purpose and value from the start (Science Council of Japan, 2005). In many countries, aspects of applied mathematics such as statistics and mathematical modelling are emphasised as part of the school mathematics curriculum. This is also the case in Japan; however, although textbooks and classes provide realistic problem situations, teaching and learning tend to focus on understanding mathematical concepts and learning how to apply them to problems for which the answer is already prepared, rather than on *science for society* (Yamaguchi et al., 2020). We felt that it was necessary to create a new learning area MSE, as a *science for society*. This area includes both mathematics and statistics/applied mathematics with a focus on knowledge, skills, and student values that are not often addressed in traditional mathematics education to promote decision making (Yamaguchi et al., 2020).

Mathematical scientific decision-making processes

Considering the results of decision-making research in cognitive psychology (e.g. Kobashi, 1988), mathematical scientific decision-making processes were defined as follows:

Mathematically, scientifically formulate real-world problems that require decision-making, perform mathematical processing, follow the process of obtaining mathematical results, and create multiple options, and then build a consensus and make some kind of decision, while clarifying the basis for these options. (Yamaguchi et al., 2020, p.107)

This process is similar to mathematical modelling; however, in mathematical modelling, the focus is on building a mathematical model that better describes the real world. In contrast, the focus of this process is on decision-making based on students’ values. In other words, the process from formulation to mathematical scientific results is iterative, examining criteria, assumptions, risks, etc., because the consideration of a wide range of options leads to high-quality decisions. This process is also characterised by the sharing or creation of new value propositions by clarifying the values behind the options through dialogue in the communities to which they belong. Figure 1 shows the process in which mathematical modelling is carried out based on the students’ values, and the solution creates a new value proposition.

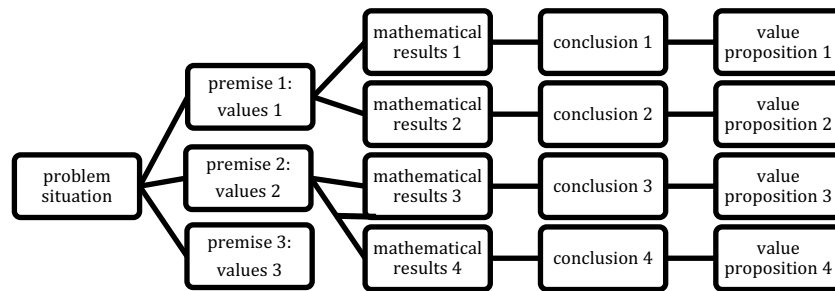


Figure 1: Value proposition generation process (Shimada & Matsushima, 2018, p. 30)

BASIC FRAMEWORK FOR MSE AND BASIC STRUCTURE OF LESSONS

The basic framework for MSE

The basic framework for MSE outlines the main features and guidelines of the new learning area from four perspectives: objectives, methods, content, and assessment (Yamaguchi et al., 2020). This framework was developed through repeated theoretical and practical considerations.

Table 1: The basic framework for MSE (Yamaguchi et al., 2020, p. 108)

A. Objectives	
(A1)	Developing mathematical scientific decision-making competences A11. Process skills A12. Realising mathematical scientific-social value
(A2)	Creating a value proposition
(A3)	Acquisition of knowledge, skills, ways of thinking, etc. related to cutting-edge sciences
(A4)	Arousing interest in contemporary issues
B. Methods	
(B1)	Principles for creating lessons based on the framework for lesson design
(B2)	Principles of basic lesson process
(B3)	Principles of mathematical scientific options
(B4)	Principles of social interaction
C. Contents	
(C1)	Authentic issues related to real life and society
(C2)	Open-ended problems based on diverse values
(C3)	Useful methods for mathematical scientific decision making
D. Assessment	
(D1)	Assessment of mathematical scientific decision-making competences based on the framework for lesson design
(D2)	Assessment of value propositions
(D3)	Assessment of knowledge, skills and ideas related to cutting-edge sciences
(D4)	Assessment of interest in contemporary issues

Process skills and realising mathematical scientific-social value

The development of mathematical scientific decision-making competences (A1) refers to the development of a set of competences that comprise mathematical scientific decision-making competences and is a fundamental objective of MSE. There are two broad categories: process skills (A11), and realising mathematical scientific-social value (A12). In identifying the sub-competences of the process skills that are important in MSE, the framework of the National Curriculum (NC) in England (QCA, 2007) and the Standards (NS) of the NCTM (2000) were taken into account, where ‘process’ is explicitly identified as a dual focus of the curriculum, together with ‘content’. The following six process skills (P0–P5) were used in this study to capture the process skills used in decision-making: (P0) Finding problems: Skills in exploring the real world and finding problems; (P1) Formulating: Skill in translating real-world problems into mathematical scientific problems; (P2) Representing: Skill in expressing decision-making processes, methods, and outcomes using mathematical scientific means of expression; (P3) Reasoning and analysing: Skill in reasoning and analysing the structure of problems based on mathematical scientific procedures and ideas; (P4) Interpreting and validating: Skill in interpreting decision-making processes, methods, and outcomes, and assessing their validity against original real-world problems; and (P5) Communicating: Skill in communicating decision-making processes, methods, and results. These six process skills focus on a set of competences required in the decision-making process.

On the other hand, realising mathematical scientific-social value (A12) is the competence that underpins and significantly influences the demonstration of process skills. This is because the different values of the parties involved in the problem situation are assigned different degrees, which in turn influences the quality of the formulation, the choice of the mathematical sciences to be used, and, ultimately, the decision making (Nishimura et al., 2011). The addition of ‘mathematical scientific’ implies that the goodness and merit of a particular mathematical scientific idea are highlighted by comparing different social values.

Geiger et al. (2023b) conducted a study of Critical Mathematical Thinking (CMT), which aims to empower individuals to engage critically with real-world problems and mathematical applications. CMT also requires the capacity to reflect on the consequences of proposed solutions (e.g., social and ethical) for individuals and society as a whole. The CMT framework comprises five dimensions: Mathematical capability; Critical capability; Evaluating; Reasoning; and Disposition. That study and ours share a similar vision, but our study differs in that it sheds light on the decision-making process based on students’ values and on the development of lessons based on the framework for lesson design and the basic learning process described below.

Framework for lesson design and basic lesson process

Students’ different values about a particular problem situation play the role of premises, and value propositions are formed based on mathematical scientific outcomes. In addition, the value propositions of the whole class community are constructed and shared through dialogue among students. Although the MSE lesson is designed based on the framework for lesson design, due to page limitations, we will not discuss it in this paper and only describe the basic lesson process. When designing our lessons, we use the basic lesson process as a guide. This lesson process is not linear but

progresses back and forth between the various stages, as necessary. It does not necessarily follow the whole process in a single credit hour (45 or 50 minutes) of teaching but can be part of it.

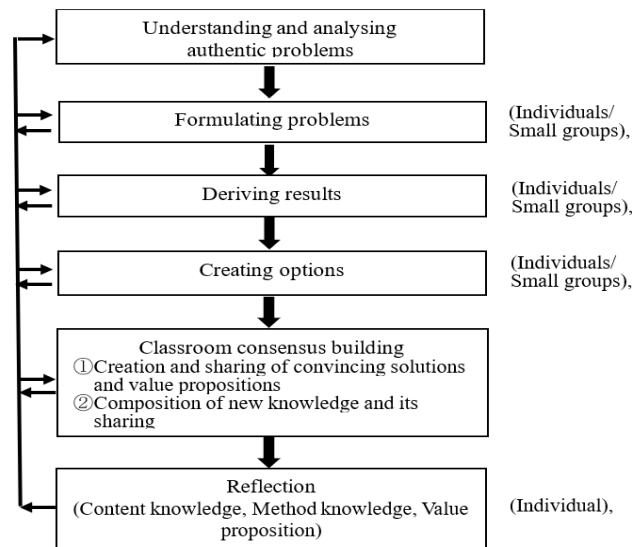


Figure 2: The basic lesson process (Yamaguchi et al., 2020, p.114)

LESSON: WHAT IS THE SELECTION OF THE VENDING MACHINE?

In this section, we introduce a practice using AHP (Analytic Hierarchy Process), a mathematical scientific method that is not often used in mathematics education, and demonstrate the reality of reaching a consensus among students. AHP is a mathematical scientific method for creating scales. There are various activities for creating such scales from elementary school to high school (Yamaguchi et al., 2020), but in this paper, we introduce it as an activity using weights in high school. This practice was conducted for ten hours with third graders in November at a public part-time high school in Tokyo (Ueda et al., 2022).

Authentic problem

This high school has drinking water vending machines on the 1st, 3rd, 5th, and 7th floors. The lineup of products in these vending machines is decided by the contract manufacturer’s staff. However, there are complaints from students, such as “They don’t have what I want to drink,” and “There are types I can’t drink.” Now, it has become possible to make requests for one hot drink except coffee in the vending machine on the 5th floor. How can students create a request that satisfies the students and teachers who use the machine?

Lesson objective

The main goal of these lessons is for students to clarify their own values through dialogue and to build a consensus on the product lineup for the vending machine. In the process, the students also aim to learn mathematical scientific methods such as AHP. AHP is a method used to evaluate multiple options. As shown in Figure 3, AHP makes decisions based on a hierarchical diagram consisting of three elements: objective; evaluation criteria; and options. First, multiple evaluation criteria that encompass multiple options are set. Next, evaluation indicators for each evaluation criterion are created and weights are set between the evaluation criteria. Finally, the options are evaluated based

on the value obtained by multiplying the indicator of each evaluation criterion by the weight of that criterion. AHP calculates relative importance through pairwise comparison. Specifically, when evaluating drinks, evaluation criteria such as ‘heat retention,’ ‘healthiness,’ and ‘satisfaction’ were created and the evaluation was performed.

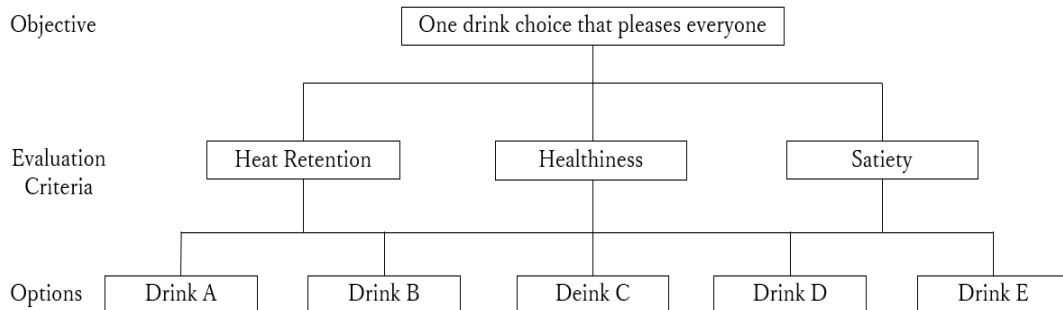


Figure 3: Example of a hierarchy diagram aimed at choosing drinks that will please everyone

Overview of actual classes

In the first and second lessons, the aim was to have students become aware of and think about the various values that customers have when purchasing a drink, such as sweetness, refreshingness, compatibility with food, and a lid that keeps the drink warm. During the third and fourth lessons, the aim of the class was to create a paired comparison perspective as a method of determining the importance of evaluation criteria. Through whole class discussion, a decision-making matrix was created as a method of determining the importance of evaluation items. In the fifth and sixth sessions, the aim was to have students notice the diversity of evaluation criteria and critically examine AHP through an activity in which they examined sugar-free drinks using AHP. AHP was presented as an improved version of the decision-making matrix created in the previous session, and the two methods were compared. The aim of the seventh and eighth lessons was to use AHP to suggest drink requests based on the group’s values. Each group started to set a perspective, which was difficult. Some students suggested that they could just choose subjectively without using AHP, while others said that it was important to use mathematics to make suggestions. They then reached a consensus to use AHP to make a decision from the 12 drinks that the students subjectively selected. In the ninth and tenth lessons, the aim was for each group to compare and consider the options they had created, build a consensus, and reflect on the characteristics of the methods they had used. They had to cut three drinks from the 12 types of drinks. When the teacher asked the students how they would build a consensus, they shared the idea of gradually eliminating the options that each group shared in their choices that they did not want to sell. The students built a consensus across all groups that one drink should be eliminated. They then debated the remaining two. A common view was that drinks with a similar taste should be rated lower. Therefore, rather than whether drinks had a lid or not, the students built a consensus on eliminating drinks that tasted similar to those previously chosen.

DISCUSSION

Creation of knowledge related to mathematical sciences and critical reflection

In this practice, we focused on AHP as the content and methods of mathematical sciences. The instructor presented AHP as a method that refined the decision matrix, and the students critically

reflected on it. Through the activity of comparing the decision matrix and AHP, the students realized that there was arbitrariness in the conversion of quantitative data in the former, and evaluated the good points of the pairwise comparison perspective in AHP as being quantitative and persuasive. It can be interpreted that the authentic social practice of being responsible for explaining the reasons for product selection to the vending machine company helped the students understand the characteristics of AHP.

Creating value propositions oriented towards social practice

This practice is strongly oriented towards social practice. Specifically, students submitted suggestions for items in the school's vending machines, and the company accepted the suggestions and actually changed the display of items. This is reflected in the way the students chose to explain their consensus-building process using mathematics rather than subjectivity when considering how to show it to other students. If we aim for social practice, the extent to which the value proposition and the process of mathematical modelling should be shared by the whole community will be an important perspective in the future. Sharing value propositions allows students to understand that their community is made up of diverse values, and the experience of implementing unique solutions based on these diverse values in society helps develop students' citizenship.

Difficulty in building a consensus

As we have seen, this practice was strongly oriented toward social practice, so the students' values were easily revealed. On the other hand, this led to difficulties in building a consensus. For example, in terms of the volume of the drink, there were conflicting values, such as large volume drinks are good and there are drinks that are satisfying even if they are small volume. In the end, the students focused on values common to each group and decided not to choose drinks with similar tastes twice, but the students expressed the opinion that they could not build a consensus. Students learned AHP in this practice as a method for creating options, which does not deal with methods for building a consensus on one option from the multiple options created. In cases where a social practice such as this one needs a single solution, it will be necessary to deal not only with models for creating options, but also with methods for building a consensus from the options created. It is difficult for students to build a consensus on authentic issues, but our future challenge will be to design learning lessons that allow students to learn more multifaceted methods of building a consensus and gain experience in this area.

Acknowledgement

This study was supported by JSPS KAKENHI (grant numbers 20H01739, 22K13708, and 24H02431).

References

- Doki, K. (2022). *A proposal for a framework for citizenship education in home economics : A case study of dietary education in high school*. Doctoral dissertation. Hokkaido University. <http://hdl.handle.net/2115/87733> (In Japanese)
- Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal*, 15(1), 1–16.

- Geiger, V., Gal, I., & Graven, M. (2023a). The connections between citizenship education and mathematics education. *ZDM*, 55(5), 923–940.
- Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., Goos, M., & Fernandez, K. (2023b). Investigating critical mathematical thinking when applying mathematics to real-world problems. *The 13th Congress of the European Society for Research in Mathematics Education*, Hungary.
- Kobashi, Y. (1988). *Supporting Decisions*. Cognitive Science Selection 18, University of Tokyo Press. (In Japanese)
- Maass, K., Doorman, M., Jonker, V., & Wijers, M. (2019). Promoting active citizenship in mathematics teaching. *ZDM*, 51(6), 991–1003.
- NCTM. (2000). *Principles and Standards for School Mathematics*. NCTM.
- Nishimura, K., Yamaguchi, T., Shimizu, H., & Honda, C. (2011). A study on teaching material and assessment for developing mathematical process skills: An analysis of Bowland Maths in England. *Journal of Japan Society of Mathematical Education*, 93(5), 2–12. (In Japanese)
- Nishimura, K. & Matsushima, M. (2024). Mathematical sciences education to develop citizens capable of participating in democratic activities. *The 15th International Congress on Mathematical Education*.
- Organisation for Economic Cooperation and Development (OECD). (2019). *OECD Future of Education and Skills 2030: OECD Learning Compass 2030 A Series of Concept Notes*. Retrieved from https://www.oecd.org/content/dam/oecd/en/about/projects/edu/education-2040/1-1-learning-compass/OECD_Learning_Compass_2030_Concept_Note_Series.pdf
- QCA. (2007). *Mathematics: Programme of Study for Key Stage 3 and Attainment Targets*.
- Science Council of Japan. (2005). *A New Way of Science: In Search of True Science for Society*. <https://www.scj.go.jp/ja/info/kohyo/pdf/kohyo-18-t995-60-2.pdf> (In Japanese)
- Shimada, I., & Matsushima, M. (2018). Creation of mathematical-scientific education as a new academic field based on the perspective of design science in school mathematics: Focusing on values, value propositions and dialogue. *Japan Society of Mathematical Education Spring Research Conference Proceedings*, 6, 27–34. (In Japanese)
- Skovsmose, O. (1998). Linking mathematics education and democracy: Citizenship, mathematical archaeology, mathemacy and deliberative interaction. *ZDM*, 30(6), 195–203.
- Skovsmose, O. (2023). *Critical Mathematics Education*. Springer.
- Tudball, L., & Henderson, D. (2014). Contested notions of civics and citizenship education as national education in the Australian curriculum. *Curriculum and Teaching*, 29(2), 5.24.
- Ueda, R., Shimizu, H., & Seino, T. (2022). A case study of “Social Practice Process” aimed at learning area “Mathematical Sciences” in upper secondary school: Through the unit “What is the selection of vending machines?” *Proceedings of the 46th Annual Conference, Japan Society for Science Education*, 99–102. (In Japanese)
- Ueda, R., Matsushima, M., & Nishimura, K. (2024). Task design of mathematical sciences education in Japanese upper secondary school. *The 15th International Congress on Mathematical Education*.
- Yamaguchi, T., Nishimura, K., Shimada, I., Matsushima, M., & Matsuzaki, A. (2020). Development of mathematical sciences education in school education: A foundational framework for curriculum design in mathematical sciences education. *Journal of Science Education in Japan*, 44(2), 104–122. (In Japanese)

WATER RESOURCES AND WATER MANAGEMENT: A SOCIAL-POLITICAL-CULTURAL CHALLENGE WHICH MATHEMATICAL MODELLING APPROACH HAVE POTENTIAL TO ADDRESS

Abolfazl Rafiepour

Shahid Bahonar University of Kerman, Nord University

Management of water resources is one of the central issues in societies around the world and UNESCO mentioned it as one of the criteria for sustainable development. This issue will be more important in some countries in the world. In Iran, which is located in west Asia (Middle East), there is a shortage of water at least in some parts of country. This paper presents a modelling task designed based on conceptual framework to evoke students' awareness about the importance and crucial rule of water management in sustainable development of the country. In the conceptual framework, the idea of ethno-mathematics was used for increasing meaningfulness and authenticity of the modelling task. Furthermore, the interpretation phase of the modelling cycle is integrated with Critical Mathematics Education to address a social-political-cultural challenge.

INTRODUCTION

Ultimately, only life educates, and the deeper that life, the real world, burrows into the school, the more dynamic and the more robust will be the educational process. That the school has been locked away and walled in as if by a tall fence from life itself has been its greatest failing. Education is just as meaningless outside the real world as is a fire without oxygen, or as is breathing in a vacuum (Vygotsky, 1997).

In the twenty-first century, climate changes have been happening in many parts of the world (Lowe, Lynch & Lowe 2014). Climate change is a long-term change in the average weather patterns because of human activities, particularly the use of fossil fuel for industrial activities. This climate change increases heat-trapping greenhouse gas levels in the Earth's atmosphere and raises the Earth's average surface temperature. All this has caused extreme weather such as hurricanes, heatwaves, and floods around the world.

Climate changes can cause many problems including water shortage crises in many countries, and the management of water consumption is an essential issue for them. For example, in Iran, which is located in west Asia (Middle East) and has a population of around 85 million, there is a crisis of a water shortage (Madani Larijani, 2005). In this way, the improvement of water management has been announced as a focus through the UNESCO 2030 document (United Nations, 2015). Modifying consumption patterns is one of the water management strategies which appear in literature. Therefore, the modifying of water consumption patterns is an important issue in the real world, and that how to do this using mathematical knowledge is very important. The United Nations (2018) notes that scarcity affects more than 40 percent of the global population and this is projected to rise. The 208 Report defines six target goals for sustainable development of water resources with some of them mostly related to water consumption and water management (see 6.1 and 6.4, Figure 1).

- 6.1 By 2030, achieve universal and equitable access to safe and affordable drinking water for all.
- 6.2 By 2030, achieve access to adequate and equitable sanitation and hygiene for all and end open defecation, paying special attention to the needs of women and girls and those in vulnerable situations
- 6.3 By 2030, improve water quality by reducing pollution, eliminating dumping and minimizing release of hazardous chemicals and materials, halving the proportion of untreated wastewater and substantially increasing recycling and safe reuse globally
- 6.4 By 2030, substantially increase water-use efficiency across all sectors and ensure sustainable withdrawals and supply of freshwater to address water scarcity and substantially reduce the number of people suffering from water scarcity
- 6.5 By 2030, implement integrated water resources management at all levels, including through transboundary cooperation as appropriate
- 6.6 By 2030, protect and restore water-related ecosystems, including mountains, forests, wetlands, rivers, aquifers and lakes
 - 6.A by 2030, expand international cooperation and capacity-building support to developing countries in water- and sanitation-related activities and programmes, including water harvesting, desalination, water efficiency, wastewater treatment, recycling and reuse technologies
 - 6.B Support and strengthen the participation of local communities in improving water and sanitation management.

Figure 1. Target goals for water consumption and management (United Nations, 2018)

This paper focuses on Theme C (Resources of and for mathematics education) and particular Sub-Theme C1 (What and how resources are/may be used in relation to socio-ecological concerns) of the ICMI Study 27. In the ICMI Study 27 Discussion Document introduces socio-ecological concerns and their inter-relatedness with mathematics education (Coles et al., 2024). In the current study, we focused on a social, political and cultural issue of the sustainable development of water resources and its relation with mathematics education through ethno-mathematics experiences. This purpose involved designing a modelling activity as a means of knowledge creation within a learning community.

The main research question in this current study is how we can address social-economic and political issues (in this case water resources and water management) in the mathematics education domain. In this regard, the main claim is that designing suitable meaningful and authentic mathematical modeling activities could address social-economic and political issues in the mathematics education domain. In particular, the interpretation phase of the mathematical modelling cycle could present opportunity for connecting social-economic and political aspects of the task with mathematical ideas, and Ethno-mathematics has potential to guarantee the authenticity of a task when designing modelling tasks. In the rest of paper, in the theoretical framework section, the mathematical modelling approach and ethno-mathematics are discussed and connected to Critical Mathematics Education through a

conceptual frame, then a concrete and practical example is presented for using at mathematics classroom.

THEORETICAL FRAMEWORK

We use the modelling cycle as the main tool for designing activities which address social-political-cultural phenomenon. At the interpretation step, in particular, there is very good potential to discuss social-political-cultural aspects of each phenomenon. In this paper when we use *mathematical modelling cycle*, we mean a round-trip process that connects the real world and the mathematical world (Steelman, 2010). There have been several interpretations of mathematical modelling cycles. The four steps modelling cycle (see figure 2) is one of the well-known (Verschaffel, 2002; Kaiser & Schwartz, 2006, Rafiepour, Stacey, & Gooya, 2012) ones in which the modeling process starts with a problem from the real world and then it is formulated and becomes a mathematical problem; then the mathematical problem is solved by mathematical problem solving techniques and the mathematical answer is interpreted in the real world to measure its compatibility with the real world. Finally, the answer is checked in relation to the real situation to control sense making of the final answer.

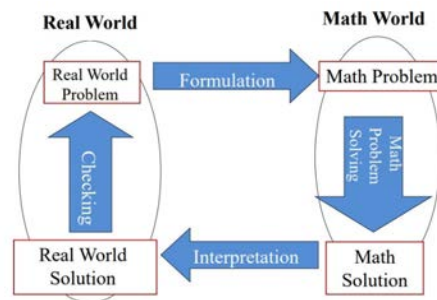


Figure 2. Four steps modelling cycle

As an example of a modelling task, you can consider travelling from one city to another one for the purpose of participating in a workshop when there are different possible ways for travelling including using personal car, train and flight. If you were asked to find the best way in terms of time consumption, or from an economical point of view or with regard to environmental concerns and air pollution, what would be your choice? You have to choose one of these ways for your situation, so this problem starts from the real world, and you have to make it a mathematical problem based on the given information through process of formulation (see Figure 1). Then, in the mathematical world, you have to use your mathematical knowledge and mathematical problem-solving techniques. Then, after finding the answer in the mathematical world, you have to do the interpretation and checking phase of the modelling cycle (see Figure 2).

As mentioned before, the interpretation step of the mathematical modelling cycle (see Figure 2) has good potential for addressing social-political-cultural aspects of each phenomenon. As an example, consider an apple classification modelling task (see Figure 3) which relates to students' everyday life who live in society based on agriculture and horticulture. Teachers can bring different type of apples (size, shape and weight and color) into the mathematics classroom and ask students to help apple farmers to classify these apples for different markets. In this activity, students engage in different interpretations (e.g., what is the best apple based on local people) based on their social cultural context to solve mathematics modelling problem.

You are apple farmers and must classify the apples in your basket for several buyers:

- high quality gift basket distributors (the “best” apples);
- supermarkets (“apples which can be sold”);
- school districts (“smaller apples for lunch”);
- and the least attractive apples which can be used for making purees and juices.

Each group of students has tools for measuring, weighing...and a blank piece of paper for writing the results. You must classify your apples for the different buyers and then explain how you have decided to classify the apples and why. To do this, each group must prepare a presentation for the other apple farmers in order to help them understand how to classify their apples for potential buyers. In the end, we will bring together all your explanations.

Figure 3. Apple classification modelling task (Alsina and Salgado, 2022)

In response to the main research question at this paper, we introduce the mathematical modelling approach for addressing social-economic and political issues in the mathematics education domain. For meaningfulness and authenticity of the modeling task, it would be useful to start from ethno-mathematics. The concept of ethno-mathematics, which was introduced by D’Ambrosio in 1978, is related to Critical Mathematics Education (Tutak, et.al., 2011). D’Ambrosio (1985) criticized Eurocentric models for academic mathematics and promoted de-centering mathematics through recognizing contributions of different cultures in shaping mathematics as a discipline. In mathematics education, ethno-mathematics is sometimes considered as the study of the relationship between mathematics and culture (Bishop, 1988). Ethno-mathematics may also be defined as “the mathematics which is practiced among cultural groups” e.g., by carpet weavers in Iran (Rafiepour & Moradalizadeh, 2022).

There are examples of ethno-mathematics in the context of Iran (e.g., The Persian Qanat which is UNESCO cultural heritage), which show that past developments were compatible with our environment and did not cause climate changes. A Ganat is a system for transporting water from a water well to the surface, through an underground channel; the system originated approximately 3,000 years ago in Iran (Remini, et. Al., 2014). These opportunities can be used to design appropriate modelling activities which have meaning and authenticity for students.

Figure 4, presents a conceptual model for the integration of Critical Mathematics Education with ethno-mathematics and the modelling approach. We use this conceptual model for designing a meaningful and authentic modelling activity. In Critical Mathematics Education there is special attention to the social, political, cultural and economic aspects which can be understood and interpret through mathematical discourses. Skovsmose (1994) believed that in Critical Mathematics Education we can analyze the role that mathematics plays in producing and maintaining potentially oppressive social, political, cultural or economic structures and propose some actions which promote more equitable social, political or economic reform.

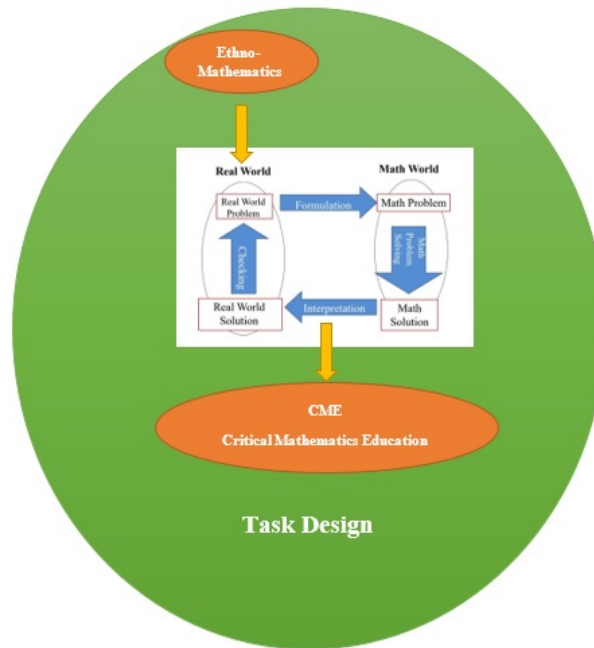


Figure 4. Conceptual model for integration CME with Ethno-mathematics and Modelling

DESIGN A MODELLING ACTIVITY UPON CONCEPTUAL MODEL

As mentioned in the introduction, Iran is a country in west Asia located in the Middle East that has a water crisis and water management would be an important issue for the country. In this situation, several mega projects have been started to solve this water shortage in big cities near deserts. For example, the project of desalination of sea water and transfer water from the Persian Gulf to the central plateau of Iran started some years ago and recently the first phase of this project was completed. Figure 5 shows a modelling task designed to address social-political and cultural issues related to using water obtained through desalination of sea water for agriculture.

According to one of the officials of the country, it costs between 16 and 17 thousand Tomans (Iranian currency) to produce each cubic meter of water through seawater desalination. In this way, the cost of each liter of water will be around 16 to 17 Tomans. In addition, the cost of transfer should also be considered. An environmental expert in Iran has said that the cost of produce and transfer each cubic meter of water obtained through desalination of sea water is about 3 USD.

- Now the question that arises is, is it worth farming with this water? Indeed, Is the cost of producing each liter of water enough to be used for agriculture?

For example, one of the popular export products in agriculture section is sweat watermelon. Because it is so delicious, and it has many enthusiastic customers in neighboring countries. For producing one kilogram of watermelon in Iran, 286 liters of water are used.

- How much should watermelon be sold to justify the consumption of this amount of water? Another popular export product in Iran agriculture section, is pistachio, especially in Kerman province (in southeast of Iran). Math teachers can provide different information about age of

pistachio tree, different season and different type of tree, then ask students to estimate the total water required for the pistachio tree throughout the year.

- Based on this information and price of pistachio at international market, students can discuss about worth of farming pistachio in this situation? Indeed, students should work on this question that is the production and export of pistachios economically justified with considering the cost of water?

In the past, aqueduct (Qanat) water was used to irrigate pistachio trees, and for many years there was no problem with underground water sources. It was an example of sustainable development.

- Research how to use aqueduct (Qanat) water to irrigate pistachio trees and export this product?

Figure 5. Modelling task: using water obtained through desalination of sea water for agriculture

The modelling task related to using water obtained through desalination of sea water for agriculture was designed by writer of this paper based on news released about a mega project which transfers water from the sea to the mainland and current practice of agriculture, especially the case of pistachio trees in Kerman province (a province in south/east of Iran). Iran has a long traditional history of using underground water through Qanats, and sustainable usage of water resources for many years, but recently, with an increasing population, water maltreatment was become one of the main tensions in contemporary Iran. In developing Qanat, there are many mathematical ideas, with some of them discovered and published by Shahraki and Fadaee (2023). So, the mathematical modelling task which is shown In Figure 5 is based on theoretical frame shown in Figure 4. Indeed, this mathematical modelling task could be connected to ethno-mathematics (past experiences of Iranian people) on the one hand, and social, political and cultural issues (contemporary challenges of sustainable water management) on the other hand.

The modelling task In Figure 5 could be used in the ordinary class for secondary students. It also would be useful for implementation in pre-service and in-service teacher education. Iranian teachers of mathematics have experience of solving mathematical modelling problems and tasks through some workshops (which are organized at some national level conferences like IMEC²⁹) and in-service teacher training programs. For example, during the Covid-19 pandemic, Isfahan Mathematics House was the host of a national level in-service teacher education program, and instructors plus participating teachers joined this in-service program through the web from different places of Iran. Iranian students also have some experience with working on modelling problems. Writers of this paper have had several workshops for students in different provinces. In some cities, there are mathematics houses which organize some matches based on the real-world tasks, so some of students have experience of working on modelling tasks through this type of after school program. Furthermore, in the mathematics section of the Iranian national curriculum, it is emphasized that students must have experience of working with real world problems.

The modelling task in Figure 5 was implemented in a class of pre-service mathematics teachers and almost all the prospective teachers found this type of task interesting and worthy for classroom discussion. Because this type of task mathematics could be integrated with social, cultural and

²⁹ Iranian Conference on Mathematics Education (IMEC)

political issues in natural way, which make mathematics meaningful in the context of real world. In some cases, there were some conflicts of interest. For example, one of the teachers who participated in the pre-service class was one of the big owners of a pistachio farm in Kerman province, so that teacher does believe we could, and we should use water obtained through desalination of sea water for developing agriculture. Nevertheless, we lost our export advantages in the world. Now, there is a good market for Kerman pistachio in many countries around the world. One of the important challenges of implementing the modelling task in Figure 5 was that some teachers don't believe in such tasks as mathematical ideas. They believe that such modelling activities don't have relation with mathematical ideas and must be discussed in social science classes instead of mathematics classes.

FINAL REMARK

Climate change and global warming warnings arise from experts and the public around the world. Especially with regard to water resources and water management, several recommendations arise. For example, UNESCO mentioned water management as one of the goals of sustainable development. In this paper we present one mathematical modelling activity designed to address the phenomenon of shortages of water in Iran, encouraging student discussion about this issue to raise public awareness of the inseparability of human and ecological concerns.

There are several examples of ethno-mathematics in Iran (Shahraki & Fadaee, 2023) that show that the past developments were compatible with our environment and did not cause climate changes. These opportunities can be used to design appropriate modelling activities which are meaningful and authentic for students and engage them in real world problems. In this situation, students try to use mathematics to argue about different aspects of the problem. These types of modelling activities could play a major role in raising public awareness about a country's water and environmental problems. This paper, in line with Barwell (2013), Coles et al., (2013), and Skovsmose (2023), shows that Critical Mathematics Education has a crucial role to demonstrate the potential of mathematics to discuss the contemporary challenging situation of the world. Furthermore, in the ethno-mathematics section of the current study, we use the ways of knowing and doing mathematics that were used in social, cultural, political, economic and environmental practices of ancient people in Iran (in line with Rosa et al., 2012) which support sustainable management of water resources.

References

- Alsina, A., & Salgado, M. (2022). Understanding Early Mathematical Modelling: First Steps in the Process of Translation Between Real-world Contexts and Mathematics. *International Journal of Science and Mathematics Education*, 20, 1719–1742.
- Barwell, R. (2013). The mathematical formatting of climate change: Critical Mathematics Education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16.
- Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics*, 19, 179–191.
- Coles, A., Barwell, R., Cotton, T., Winter, J., & Brown, L. (Eds.). (2013). *Teaching secondary mathematics as if the planet matters*. Routledge/Taylor & Francis Group.
- Coles, A., le Roux, K., Borba, M., Bose, A., Geiger, V., Gutiérrez, R., Makramalla, M., Solares-Rojas, A., Sinclair, N., Valero, P., Vistro-Yu, C., Leung, F.K.S., & Dorier, J-L. (2024). *ICMI Study 27 discussion document. Mathematics education and the socio-ecological*. ICMI Study 27.

- D'Ambrosio, U. (1985). Ethno-mathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, 5, 44–8.
- Kaiser, G., & Schwarz, B. (2006). Mathematical modelling as bridge between school and university. *Zentralblatt für Didaktik der Mathematik*, 36(2), 196–208.
- Lowe, B., Lynch, D., & Lowe, J. (2014). The role and application of social marketing in managing water consumption: A case study. *International Journal of Nonprofit and Voluntary Sector Marketing*, 19, 14–26.
- Kaveh Madani Larijani, K. (2005). Iran's water crisis: Inducers, challenges and counter-measures, *ERSA Conference Papers*, ERSA05 p. 563, European Regional Science Association.
- Rafiepour, A., Stacey, K., & Gooya, Z. (2012). Investigating grade nine textbook problems for characteristics related to mathematical literacy. *Mathematics Education Research Journal*, 24(4), 403–421.
- Rafiepour, A., & Moradalizadeh, A. (2022). Mathematics of carpet: Opportunity for designing mathematics activities. *Journal on Mathematics Education*, 13(3), 383–392.
- Remini, B., Kechard, R., & Achour, B. (2014). The collecting of groundwater by the Qanats: A millennium technique decaying, *Larhyss Journal*, 20, 259–277.
- Rosa, M., D'Ambrosio, U., Orey, D. C., Shirley, L., Alangu, W. V., Palhares, P., & Gavarrete, M. E. (2016). Current and future perspectives of ethnomathematics as a program. Springer Nature.
- Shahraki, A., & Fadaee, M. (2023). An inquiry in ethno-mathematics: Extracting traditional mathematics computations used by Qanat Practitioners. *Journal of Curriculum Studies (J. C. S), (Persian language research journal)*, 18(70), 157–176.
- Skovsmose, O. (1994). Towards a critical mathematics education. *Educational Studies in Mathematics*, 27(1), 35–57.
- Skovsmose, O. (2023). *Critical mathematics education*. Springer International Publishing.
- Tutak, F. A., Bondy, E., Adams, T.L (2011). Critical pedagogy for critical mathematics education. *International Journal of Mathematical Education in Science and Technology*, 42(1), 65–74.
- United Nations (2015). *Transforming our world: The 2030 Agenda for Sustainable Development*. <https://sustainabledevelopment.un.org/post2015/transformingourworld>
- United Nations (2018). *Sustainable Development Goal 6 Synthesis Report on Water and Sanitation*. https://www.unwater.org/sites/default/files/app/uploads/2018/12/SDG6_SynthesisReport2018_WaterandSanitation_04122018.pdf
- Verschaffel, L. (2002). Taking the modelling perspective seriously at the elementary school level: Promises and pitfalls. In A. Cockburn, & E. Nardi (Eds.), *Proceedings of 26th Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 64–80). University of East Anglia.
- Vygotsky, L. (1997). *Educational psychology*. CRC Press LLC.

RESOURCES FOR INTEGRATING MATHEMATICS AND SUSTAINABILITY CHALLENGES: A PERSPECTIVE OF THE GREEN MATHEMATICS FRAMEWORK

Soma Salim^{1,2}, Katie Makar², Jana Višňovská²

¹Bosowa University, ²The University of Queensland

Indonesia's new curriculum requires teachers to design interdisciplinary projects on sustainability. The projects can be an excellent opportunity for students to use mathematics to investigate a sustainability challenge. In this exploratory case study, we document how one high school teacher in Indonesia designed and implemented a sustainability project to raise students' awareness and action. Data from classroom observations, teacher interviews, a student group discussion, a student survey, and related documents were analyzed using the characteristics of the green mathematics framework: authenticity, complexity, and projection (Salim, 2023). The findings identify three resources that served to support the integration of mathematics and a sustainability challenge: observations through which students came to view a local environmental problem as being authentic; dialogic discussions through which the teacher helped students unpack the complexity of the problem; and activity of using the collected data to project future situations, through which students became aware of – and surprised by – the future implications of their current lifestyle, and motivated to take action.

INTRODUCTION

The United Nations Educational, Scientific, and Cultural Organization (UNESCO) promotes a roadmap of Education for Sustainable Development (ESD) for 2030, which aims “to raise [students'] knowledge, awareness and action” to achieve the 17 Sustainable Development Goals (UNESCO, 2020, p. 17). According to the roadmap, raising knowledge refers to how students understand sustainability challenges and their complexity, and explore sustainable solutions to address the problems. Raising awareness is about how students are concerned and care for other people and the planet. Raising action refers to how students engage in practical actions as change agents to foster sustainable transformation within personal, societal, and political contexts. To achieve these aims, UNESCO (2020) proposes five priority actions: (1) advancing educational policy, (2) transforming learning environments, (3) enhancing educators' capacities, (4) empowering and mobilizing youth to take community action, and (5) accelerating local-based actions. The roadmap highlights the need for transformative education but lacks resources on how teachers can integrate sustainability into subject areas. While the roadmap uses data and statistics to show the urgency of the priority actions, it does not mention the role of mathematics in sustainability education to achieve the goals.

Regarding educational policy, Indonesia's new curriculum, *Kurikulum Merdeka*, includes sustainability as one of seven themes for co-curricular projects (Anggraena et al., 2022). The curriculum requires teachers to integrate sustainability into classroom activities. Research has shown how teachers in Indonesia implemented co-curricular projects about sustainability (e.g., Anggraena et al., 2022). With respect to mathematics, we know that mathematics can be used as a tool in analyzing and interpreting real-world problems (Schmidt et al., 2022) and can play an essential role in describing, predicting, and communicating complex sustainability challenges like climate change (Barwell, 2013). However, more insight is needed about how the implementations of sustainability

projects involved and could involve mathematics as a tool in raising students' knowledge, awareness, and action. It is to this end that we explore resources (practices) used by one high school teacher in Indonesia, Kartini (all names used are pseudonyms), when she included mathematics in the design and implementation of a co-curricular project on sustainability while aiming to raise her Year 10 students' awareness and action. The data from this exploratory case study were analyzed using the *Green Mathematics Framework* described in the next section.

THE GREEN MATHEMATICS FRAMEWORK

Salim (2023) developed a framework for designing green mathematics tasks, defined as a task that integrates mathematics and sustainability, by investigating word problems related to sustainability and adapting the Theory of Situation Awareness (Endsley, 1995). He argued that a green mathematics task should have these three interdependent elements: sustainability challenges, mathematical ideas, and problem solving (Figure 1). Missing one of these elements may make it difficult to understand the problem and to raise students' awareness and action.

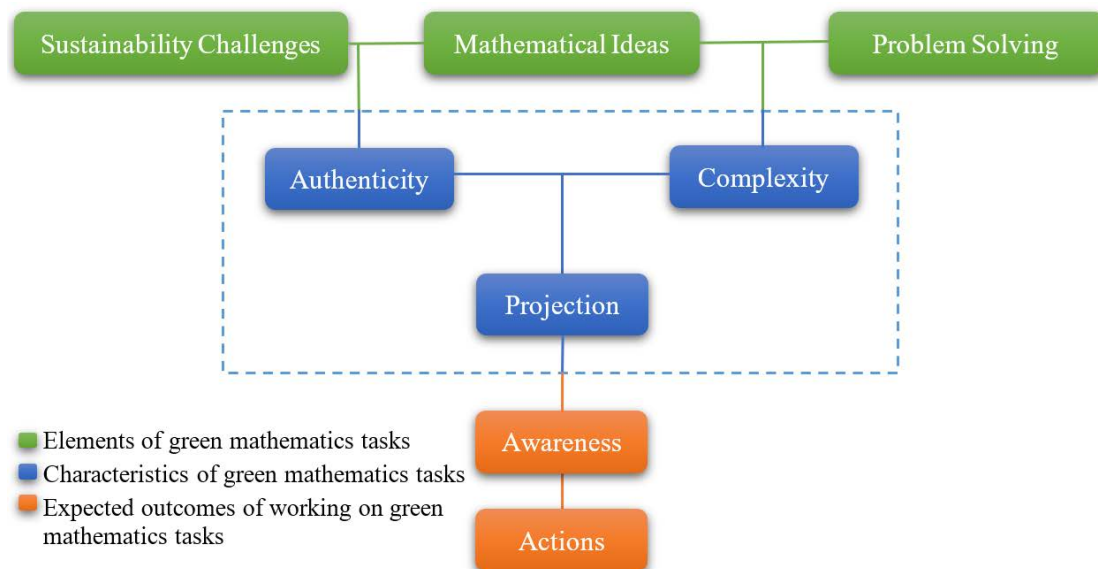


Figure 1: The *Green Mathematics Framework* adapted from Salim (2023)

Some mathematics tasks include a (sustainability) context and (mathematical) problem solving, but students are often enculturated to ignore the context and focus on the intended mathematical procedures (Lubienski, 2000). Palm (2006) proposed that word problems need to be authentic so that students consider the context when making decisions as to how the mathematical procedures are selected, used, and interpreted. Sustainability challenges as a context are – by default – complex. Mathematics can be used to describe the complexity and to envision future situations, such as by making projections from existing data (Barwell, 2013). Considering these arguments and the Theory of Situational Awareness (Endsley, 1995), the *Green Mathematics Framework* was composed to include *authenticity*, *complexity*, and *projection* (Salim, 2023). These characteristics aim to raise students' awareness – or “knowledge and awareness” as defined by UNESCO’s ESD roadmap – of sustainability challenges when these are attended to in the classroom.

Authenticity in the framework refers to the need to use real sustainability situations that have occurred, are currently happening, or are likely to eventuate. Authenticity of the problem allows for

experiencing mathematics in the classroom as being useful in real-life situations. Based on the Theory of Situation Awareness, a person needs to perceive information related to a (sustainability) situation as authentic as possible to become aware of the problem (Endsley, 1995). Palm (2006) proposed eight authenticity aspects in a mathematics task: the event, question, purpose, data/information, presentation, solution strategies, circumstances, and solution requirements. While addressing these aspects is an important start, whether students perceive the task as an authentic, real situation or as a contrived problem ultimately only gets established in the classroom.

Complexity in the framework refers to the extent to which the questions asked are not straightforward (non-routine) for students to solve, including mathematical ideas used in the problem-solving process. A complex task requires students to explore various strategies to approach or solve the problem and may, at times, require developing new strategies. The complexity requirement allows students to investigate the real-life system within a sustainability challenge that would otherwise not be included among suitable mathematical tasks. According to the Theory of Situation Awareness, understanding the complexity of a (sustainability) situation can make a person aware of the problem (Endsley, 1995).

Projection refers to students needing to predict future situations and, in doing so, envision the severity of a sustainability challenge based on their understanding of a current situation (Endsley, 1995). When making future projections, students are required to reason and make sense of patterns (Lim et al., 2010). In addition, the projection can inform a person's decision-making by exploring how possible actions may play out over time and whether they are suitable to address the problem.

METHODOLOGY

This research is an exploratory case study that aims to investigate how a high school teacher in Indonesia designed and implemented an interdisciplinary project, including mathematics, to raise students' awareness of and actions to address sustainability challenges. This paper explores whether and how the characteristics of the *Green Mathematics Framework* (*authenticity, complexity, and projection*) played a role in the project design and implementation.

The school, participants, and the sustainability project

The high school is a co-educational, Islamic boarding school located in a remote village in Indonesia (35 km from the nearest city) surrounded by green mountains and agricultural land. It accommodates approximately 200 students from Year 10 to 12 (15-18 years old), with more than 50 teachers and staff. The school started implementing *Kurikulum Merdeka* in 2022, so the teachers must design and implement co-curricular projects themed on sustainability. A group of teachers designed the sustainability project, but only Kartini (the representative of the teachers who was also the chair) participated in this study. Kartini is a biology teacher with four years of teaching experience. She graduated from a biology education program at an Indonesian public university eight years ago.

Kartini designed the sustainability project for Year 10 students. Twenty-seven students (15-16 years old) participated in this study. They came from different provinces of Indonesia. Kartini chose a local sustainability challenge that she was concerned about as the topic of the project: *the school's waste management*. She wanted the students to investigate how their lifestyle contributed to the problem and what changes could help address the problem. The project was implemented in five classroom sessions (3-4 hours each). First, Kartini *introduced* the project aims and engaged students in discussing waste management around the school. In the *contextualization* session, students worked

in groups to collect and analyze data on different types of waste (e.g., single-use plastic bottles, food waste, and plastic snack wraps). The students then presented their findings to other students in a *mini-exhibition*. After the mini-exhibition, students reflected on their findings and discussed possible solutions to the waste problem. The next session was *action*, where the students enacted aspects of some of the proposed solutions. Finally, the students *reflected* on the whole project implementation.

Figure 2 shows a set of questions that the students were asked to work on. These questions were given to them after Kartini introduced the project's aims and the sustainability challenge that they would investigate. Questions 1 and 2 required students to attend to and gather information about school waste management. Questions 3 and 4 required students to collect and analyze data related to the waste production they identified from the previous questions. Questions 4 and 5 required students to envision future situations (making projections) based on the data that they had collected.

Save The Earth Project	
Bahasa Indonesia	English
1. Menurut Anda, sampah/limbah apa yang diproduksi di lingkungan sekitar Anda?	1. In your opinion, what kind of waste is produced around your (school) environment?
2. Bagaimana pengolahan sampah/limbah yang diproduksi di lingkungan sekitar Anda?	2. How is the waste managed around your (school) environment?
3. Berapa banyak produksi limbah yang dihasilkan tiap hari? (Dapat diperoleh melalui wawancara atau survey ke warga sekolah)	3. How much waste is produced every day? (You can find the data by interviewing or surveying the school community)
4. Buatlah prediksi berapa jumlah sampah yang dihasilkan dalam jangka 5 tahun dan 10 tahun ke depan!	4. Estimate how much waste will be produced in the next 5 and 10 years.
5. Buatlah prediksi apa yang terjadi jika permasalahan tersebut tidak dapat diselesaikan dengan baik!	5. Predict what will happen if the problem cannot be solved properly.

Figure 2: Five questions created by Kartini as part of the project design

Data collection and analysis

The data of this case study were collected through five methods: video-recorded classroom observations; four audio-recorded, semi-structured interviews with Kartini; a focus group discussion with the students; a short, anonymous survey for the students after the whole project implementation; and collecting the copies of related documents (e.g., students' work and the project's plan). Due to the nature of the sustainability project, in which students spent most of their time outside the classrooms and interacted with non-participants, only two sessions were video recorded (the introduction and reflection after the mini-exhibition). One teacher interview was conducted before the project implementation, and three interviews were conducted afterward.

The data were analyzed by Identifying *critical events*, defined as episodes in which mathematics was involved during the project implementation (Powell, Francisco, & Maher, 2003). The transcriptions of these episodes were coded based on *authenticity*, *complexity*, and *projection* as defined in the *Green Mathematics Framework*. For *authenticity*, we analyzed whether and how students came to perceive the waste problem as being authentic. For *complexity*, we analyzed how Kartini assisted the students in understanding the complexity of the problem. For *projection*, we analyzed how making projections using the data that students had collected supported them in becoming aware of the problem and caring for the environment. In addition, we documented actions proposed by students in response to the sustainability challenge they investigated.

FINDINGS

We show that student activity of *observation* was essential for the students to come to perceive the waste problem as being authentic, that *dialogic discussions* in the classroom supported students' understanding of the complexity of the problem, and that *projection* played a key role in envisioning the future situations, in other words, what may happen if different responses to the problem were to be adopted. In combination, these classroom activities became resources for Kartini in integrating mathematics into the sustainability project, which encouraged students in their selection of specific actions that should be taken.

Observation

Before revealing the five questions in Figure 2 to students, Kartini showed them a photograph of the local landfill (Figure 3 illustrates this but uses a similar, generated image to protect privacy). She brought students to observe a local aspect of the waste problem situation: the current, observable quantity of locally produced waste. She aimed to show the urgency of addressing the problem.



Figure 3: An illustration generated by Artificial Intelligence prompted in the Canva platform

- Kartini: So, this is a condition very close to where we live (Kartini shows an image of waste heap in a landfill while asking the students). Do you know what is in the picture?
- Students: A landfill in (a suburb's name).
- Kartini: If we look at the image, the height of the landfill seems like twice the height of this truck (pointing at the truck in the front).
- Ahmed: Four [times].
- Kartini: How many [times]? (Kartini looks surprised as if asking for explanation from Ahmed)
- Ahmed: Four. We can measure it using that [truck] Miss (pointing at another truck behind the truck pointed by Kartini).
- Ali: Using the truck behind.
- Kartini: All right, all right, some say two [times], some say up to four [times], but the point is [that] waste has become a very important issue for us to find the solution. (Introduction Observation, 09:29)

Kartini and Ahmad estimated the landfill's relative height using mathematical ideas (i.e., perspective and scale). Interestingly, they had different estimates because they used different trucks to measure the landfill's height. Despite the differences, by observing the actual situation in the local area, the students had the opportunity to perceive the realness of the waste problem near their school.

Dialogic discussions

Kartini often engaged her students in dialogic discussions to unpack a specific aspect of the complexity of the waste problem. The following excerpt shows how Kartini and the students discussed why plastic waste has become a never-ending problem. In similar discussions, she unpacked issues including waste burning, food waste, or re-use of materials (emphasis shown in italics).

- Kartini: Why do you think the plastic waste problem still exists and has never been solved properly to date? (Ali raises hand) Ali, please.
- Ali: Because there are no places for recycling. Second, because restricting the selling of plastic products can affect plastic companies. Then, [it] will affect the economy of the country.
- Kartini: All right, any other opinions? (Looking around), Nadya perhaps? Why is that?
- Nadya: Because the condition is complex. Factories keep producing plastics.
- Kartini: Why are plastic products still produced? What is the law of economics?
- Ali: Because [it's] simpler and more beneficial.
- Kartini: Other than that, from the economic perspective, the higher the demand, the higher...?
- Fatimah: *The higher the demand, the higher the production.*
- Kartini: All right, what I mean is why is the plastic still produced? Because the demand is still high Where do the demands come from?
- Students: Humans.
- Kartini: Are *we* included?
- Students: Yes. (Introduction Observation, 17:50)

This discussion happened before the students were working on the five questions. The teacher wanted students to notice why removing plastic as a source of waste was complex, including the fact that students themselves were implicated in plastic production.

The dialogic discussion subsequently shaped students' data collection and their interpretation of results. When the students collected data about the school waste production in response to Question 3, they now knew to interview students as producers of plastic waste. For example, Nadya's group interviewed five students to represent each class and calculated the average plastic use per student.

When collecting the data, we finally know that the students at the school produce, um, since I am in the plastic bottle group, so I know that, on average, a student uses one to two bottles per day. (Nadya, Focus Group Discussion, 09:47)

By using statistical processes, the students estimated the total plastic waste production in the school. The dialogic discussion may have helped them to interpret the data (e.g., the data shows how much the students' demand for plastic products contributed to the local landfill).

Projection

In response to Question 4, the students estimated how much waste the school would have produced by a specific date in the future if there were no changes to the consumption. Projecting future waste production may have contributed to students' awareness of the local sustainability challenge. Figure 4 shows some of the students' responses to survey question 4: *After finishing the project, how do you feel about the sustainability issue (waste) at the school?*

<i>Bahasa Indonesia (Original)</i>	
<p>Saya jadi menyadari bahwa sampah merupakan permasalahan yang serius yang jika tidak ditanggapi maka akan menimbulkan masalah yang besar</p>	<p>menurut saya masalah sampah ternyata membawa dampak yang sangat buruk. Jadi kita harus menjaga lingkungan dengan menggunakan mengurangi penggunaan plastik 😊</p>
<i>English</i>	
<p>I became aware that waste is a serious problem that if not addressed, it will lead to a big problem.</p>	<p>In my opinion, the waste problem turns out to have a very bad impact. So, we must protect the environment by reducing the use of plastic.</p>

Figure 4: A couple of students' responses from the short, anonymous survey

Most students noted in the survey that after working on the project, they became aware of the urgency of the waste problem and the need to take action to address the sustainability challenge. For example, the students urged the school to provide water dispensers around their classrooms to reduce the use of single-use plastic bottles. Kartini also noticed that “the students have been cautious about sorting out their plastic waste ..., [and] provided bins for plastic waste” (Kartini, Interview 2, 03:57). The design and implementation of Kartini’s sustainability project had raised students’ awareness and actions and is thus a useful case to study from the perspective of the *Green Mathematics Framework*.

DISCUSSION AND CONCLUSION

The findings identify three resources that supported the integration of mathematics and sustainability challenges: *observation*, *dialogic discussion*, and *projection*. The analysis showed that *observing* the waste problem was essential for students to perceive the sustainability challenge as authentic. When observing, students used mathematics to understand the challenge. The estimation may help students to infer the magnitude of events from pictures of their surroundings and to understand the urgency of the problem. Since the local sustainability challenge was directly related to the students’ lives, they could develop the purpose of working on and addressing the problem (Ainley et al., 2006).

In addition, having *dialogic discussions* regarding the complexity of the problem and its relation to their lives may have helped students to collect and interpret data. They used mathematical ideas (i.e., statistical processes) to estimate the amounts of waste produced in the school. However, without prior discussions about the connection between the supply and demand of plastic products, the waste data may have been less meaningful. Skovmose (2023) also emphasizes the importance of dialogue in critical mathematics education to build connections (students-sustainability challenges).

Making *projections* of waste accumulation into the future may have further strengthened students’ awareness and actions. The sustainability project (questions 4 and 5) required students to predict the school waste production based on the data that they had collected. It appears that exploring different time scales of the problem acted as a resource for integrating mathematics and sustainability. This is how making projections acted as a tool in affecting students’ decision-making about actions they needed to take to prevent unwanted situations from occurring (cf. Endsley, 1995). The students’ projections of the school waste production in the next five and ten years informed their awareness and motivated students to seek sustainable solutions.

To conclude, drawing on the *Green Mathematics Framework*, in this paper, we proposed three resources for teaching mathematics and sustainability: *observation*, *dialogic discussions*, and *projection*. The students had the purpose of observing the phenomena related to the sustainability challenge because it was relevant to them. As a result, they could perceive the problem as authentic. Dialogic discussions were pivotal for students to make sense of the complexity of the sustainability challenge, leading to more meaningful data collection and interpretation. Making projections of future situations may have raised students' awareness and actions to address the challenge.

Acknowledgments

We would like to thank Kartini, her students, and the school principal for making the study possible. Ethics approval 2022/HE002345 was granted by The University of Queensland, and participants gave informed consent.

References

- Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, 32(1), 23-38.
- Anggraena, Y., Felicia, N., Eprijum, D., Pratiwi, I., Utama, B., Alhapip, L., & Wideaswati, D. (2022). *Kajian akademik kurikulum untuk pemulihan pembelajaran*. Pusat Kurikulum dan Pembelajaran, Badan Standar, Kurikulum, dan Asesmen Pendidikan, Kementerian Pendidikan, Kebudayaan, Riset, dan Teknologi, Republik Indonesia.
- Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1-16.
- Endsley, M. R. (1995). Toward a theory of situation awareness in dynamic systems. *Human Factors*, 37(1), 32-64.
- Lim, K. H., Buendía, G., Kim, O.-K., Cordero, F., & Kasmer, L. (2010). The role of prediction in the teaching and learning of mathematics. *International Journal of Mathematical Education in Science and Technology*, 41(5), 595-608.
- Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education*, 31(4), 454-482.
- Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics*, 26(1), 42-47.
- Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, 22(4), 405-435.
- Salim, S. (2023, July 2-6). *A Framework for Designing Green Mathematics Tasks* [Paper presentation]. The 45th annual conference of MERGA, Newcastle. <https://files.eric.ed.gov/fulltext/ED631512.pdf>
- Schmidt, W. H., Houang, R. T., Sullivan, W. F., & Cogan, L. S. (2022). *When practice meets policy in mathematics education: A 19 country/jurisdiction case study*. OECD Publishing. <https://doi.org/10.1787/07d0eb7d-en>
- Skovsmose, O. (2023). *Critical mathematics education*. Springer Nature.
- UNESCO. (2020). *Education for sustainable development: A roadmap*. UNESCO Paris, France. <https://doi.org/10.54675/YFRE1448>

IMPLEMENTATION OF MEDIA AND DATA LITERACY IN STUDENT TASKS – AN ANALYSIS OF DIFFERENT TASKS

Hans-Stefan Siller¹, Nina Unshelm¹, Vincent Geiger², Sarah Digan²

¹Julius-Maximilians-Universität Würzburg, ²Australian Catholic University

The changing needs and increasing challenges of today's societies require new capabilities and an adaptation of capabilities taught in schools. In addressing this issue, we provide an overview of selected capabilities needed to meet the demands of today's challenges. Thereby we outline the essential capabilities needed to evaluate media claims and in using large data sets, for example regarding socio-ecological issues. This is followed by an analysis of textbook tasks, originating from German textbooks for year 9 and 10 of Bavarian Gymnasium, regarding their potential for promoting these capabilities as well as their focus topics, authenticity, and openness. The analysis shows that the tasks include the required capabilities in part but reveal the need for modification to develop the identified capabilities. We then analyze a task from the project "Strengthening Teachers' Instructional Capabilities with Big Data" to reflect on what a task that meets the identified criteria could accommodate. This task thus represents a starting point for the extended requirements and the discussion about how to implement a socio-ecological perspective in mathematics classrooms.

INTRODUCTION

To prepare the future generation for today's global challenges and to enable them to participate in the civic dialogue, an adaptation of the content and capabilities taught is necessary. In response, different approaches of new and necessary content and capabilities, like the use of media and data as mentioned in the 21st Century Skills or Key Competences for Lifelong Learning (Ananiadou & Claro, 2009; European Commission, 2019), have been developed. Furthermore, an increasing implementation of socio-ecological topics in education has gained in importance (e.g., Amico et al., 2023; Coles, 2022).

The increased accessibility to information, i.e., of digitalization, has many benefits, but also risks. In particular, mis- and disinformation are assuming increasing prominence and are influencing public opinion (Lewandowsky et al., 2017). Recent studies have identified different public issues regarding this, for example, an increasing avoidance, by young people, of reading about current affairs because of the challenges associated with understanding the content (Newman et al., 2022). Further, the volume and accessibility of data is increasing, resulting in the need for capabilities that empower individuals to analyze and interpret data (Bureau of Education & Ng, 2019). Such capabilities are often referred to under the heading of data literacy, which Schüller et al. (2019) have identified as key to informed citizenship in the 21st Century. These capabilities include the collection, organization, evaluation, and application of data.

We focus on capabilities needed to link the *evaluation of media claims* (especially regarding sustainability and thus socio-ecological issues), which use mathematical arguments and data to support the claims, through attention to data literacy, in this case, the *analysis of large data sets*. We will address this focus by first identifying the necessary capabilities for current and future civic life regarding these aspects. We then ascertain if these critical capabilities are addressed in current mathematics classrooms or didactic research projects by analyzing a sample of tasks drawn from

German textbooks and a project. Further it will be analyzed to what extent the different tasks meet the requirements for implementing a socio-ecological perspective in mathematics education. Our analysis will be guided by the following research questions: To what extent do tasks presented in mathematics classrooms address the capabilities required in today's society? What might tasks that address capabilities required in today's society look like?

THE IMPORTANCE OF EVALUATIVE AND CRITICAL CAPABILITIES

To evaluate media claims and deal with large data sets, different capabilities are necessary for today's and future civil society. These can be summarized under the terms media literacy (UNESCO, 2015), information literacy (Battelle for Kids, 2019), and data literacy (European Commission, 2019).

The capacity to (critically) evaluate media claims (European Commission, 2019) includes the ability to: find and analyze information; understand and use evidence; and understand ways of communicating information (UNESCO, 2015). Critically evaluating media claims also requires the capacity to access, organize, understand and interpret information (Ananiadou & Claro, 2009; Battelle for Kids, 2019). Further, confidence with the analysis, interpretation and evaluation of large data sets (UNESCO, 2015; European Commission, 2019) is required. In addition to cross-disciplinary capacities needed to evaluate claims in the media and to use large data sets, specific (mathematical) capabilities and knowledge are necessary. This takes into account the broad spectrum of mathematical information that can be found in the media to support claims, arguments and decisions by authorities (Gal & Geiger, 2022). The evaluation of such claims requires critical thinking and problem solving capabilities (Ananiadou & Claro, 2009; Battelle for Kids, 2019) and must include judgements about the quality of evidence, the validity of arguments and the consequences of decisions (Geiger et al., 2023). There is now a growing body of research devoted to the role mathematics education can play in addressing today's socio-ecological challenges (Siller et al., 2024). Geiger et al. (2023), for example, have developed a preliminary framework, they describe as Critical Mathematical Thinking (CMT), that emphasizes not just the cognitive demands of applying mathematics to real world situations, but also the social, environmental, and economic consequences of decisions based on mathematical solutions. The inseparable entanglement of social, political and ecological perspectives is also focused on in socio-ecological approaches (Amico et al., 2023; Coles et al., 2024).

IMPLEMENTATION OF CAPABILITIES IN THE CLASSROOM

Approach

One way of promoting capabilities in an educational context is using tasks. For this reason, we analyzed tasks in relation to criteria developed to serve as the basis for the assessment of the potential of tasks to promote the critical capabilities needed to evaluate media claims and use large data sets, as well as to address the socio-ecological perspective. We analyzed two contrasting types of tasks: current and previous German textbook for Gymnasium tasks (Gymnasium is approximately equivalent to grammar school or high school in Germany; age of students is approximately 10 to 18 years), and a task (see Figure 2) developed in an Australian-German project entitled "Strengthening Teachers' Instructional Capabilities with Big Data". We have chosen the tasks as the former represent typical tasks students are exposed to in the mathematics classroom, while the latter is an example of a task designed for the specific purpose of the project – the development of approaches that combine sustainability issues with mathematics and large data. Both types of tasks are for students in Years 9

and 10. Tasks from textbooks were selected so that they aligned with at least one of the themes explored in the project – such as: use of media articles, use of data, or addressing the sustainability topics CO₂ emissions or poverty.

Table 1: Summary of analysis criteria

	Analysis criterion	Examples
promoted skills	evaluation of media claims	find, identify, and analyze information; understand, organize, interpret, and evaluate media claims
	use of (large) data sets	analyze data; evaluate quality and sources of data
content of tasks	sustainability topic	CO ₂ emissions; poverty
	authenticity	real data and media claims
	openness	given problem and solution tendency; given information

The analysis of the tasks was based on criteria (see Table 1) synthesized from relevant research literature (e.g., Geiger et al., 2023; UNESCO, 2015). The first criterion relates to the evaluation of media claims in general. Examples of the capabilities covered by this criterion include the ability to find, identify and analyze information (UNESCO, 2015) and to understand, organize, interpret and evaluate information and media claims (Ananiadou & Claro, 2009; Battelle for Kids, 2019; Geiger et al., 2023). The second criterion is the use of (large) data sets which we see as key to evaluating claims in the media. This criterion includes the analysis of data (UNESCO, 2015) and the evaluation of quality and sources of data (European Commission, 2019; Geiger et al., 2023). Large data sets are focused on because of the increasing volume and accessibility of data sets in repositories. We included three further criteria related to the nature of tasks: topic, authenticity, and openness. The first criterion identifies whether the topic of a task is related to sustainability. This criterion is based on the need for capabilities to understand issues about sustainability (European Commission, 2019; UNESCO, 2015). As sustainability topics require a combined consideration of social, ecological and political perspectives, tasks that deal with sustainability, have the potential to address the socio-ecological perspective (Amico et al., 2023). The second and third criteria are criteria for the classification of modelling tasks (e.g., Siller & Greefrath, 2020). The authenticity criterion focuses on whether the task uses real data and media claims (Vos, 2011). The openness criterion focuses on the extent to which a problem provides the student with the need to research additional information and to form their own decisions based on the generated evidence, as well as whether the tasks are open regarding the solution. These two criteria are important to clarify the relevance of tasks and the capabilities they promote in relation to current and everyday challenges that students directly encounter. Furthermore, both criteria are decisive for the extent to which authentic socio-ecological issues are addressed by tasks. Overall, the criteria relating to the content of tasks are especially relevant for analyzing the relationship of tasks to socio-ecological contexts.

Analysis of Textbook Tasks

Only a limited number (17) of textbook tasks could be identified that matched the specifications

described earlier. These tasks were analyzed in relation to the criteria outlined in Table 1. The results of this analysis and representative examples are presented below. Textbook tasks were first analyzed as to whether and to what extent they address the capabilities of identifying, analyzing and evaluating media claims. We found that the identified tasks only partially address the criterion. There are textbook tasks, for example, that require students to explain and evaluate the information presented in tasks through media items (see Fig. 1(a)). But we saw in some cases, media items are used only as a motivation to deal with a context or as a basis to create a model, however, without requiring students to evaluate the information (Brendel & Eisentraut, 2021, p. 90). There are tasks that require students to form judgements about manipulative media representations and ask students to evaluate and accept or refute claims. However, such requests do not require student to use data to provide an evidence base of decision making (see Fig. 1(b)). The textbook tasks require capabilities needed to work with (large) data sets: analysis of (mostly graphically represented) data and evaluation of the match between given data and models (see Fig. 1(c)); and the creation of suitable models based on data as well as derivation of claims from these models (Biburger et al., 2022, p. 27). But there are generally limitations on the size of data sets (see Fig. 1(c)). Further, data sets tend to be restricted to relevant information – there are only a few in which students are required to make judgements about the quality of the data, or search for their own data (see Fig. 1(d)). Textbook tasks cover a wide range of sustainability-related topics, for example, CO₂ emissions, population growth, pandemics, and biodiversity. However, in most tasks there is a lack of reflection on results in relation to the social, ecological or political perspectives. Furthermore, these three perspectives are only considered individually and in isolation from each other (see Fig. 1(c)), if at all. These tasks therefore only address the socio-ecological issues to a limited extent. In the case of authenticity, we looked for tasks that include original media items and/or real-world data sets. The textbook tasks rarely include sources for the given media items and data sets (see Fig. 1(a)). This tends to indicate that they are fictitious. Further, there are only a few tasks in which students are asked to search for real-world data (see Fig. 1(d)). In summary, there is little evidence that the authenticity of tasks is seen as important. Finally textbook tasks are closed regarding the solution because the tone of questions belies a view that media claims and representations are usually false (see Fig. 1 (b)). Regarding the second aspect of openness, we found that most of the required information is provided in tasks (see Fig. 1(a)), with only a few requiring students to search for data (see Fig. 1 (d)).

In summary, textbook tasks already provide some starting points for promoting the capabilities needed to evaluate media statements and deal with large data sets. However, across the identified textbook tasks there are limitations: only small data sets are used; the media items and data sets lack authenticity; and the tasks tend to be closed tasks. Further, media items are sometimes included as a form of motivation – adopting the guise of making connections to real-world problems, without requiring the critical evaluation of the presented information. It is particularly noticeable that textbook tasks often include either “realistic” media articles or data sets – we found that a combination of these elements is rare. Although mathematics lessons have an obligation to contribute to socio-ecological education, we do not see the identified tasks as sufficient to support the development of the evaluative capabilities required for critical and responsible citizenship. In particular, the required interlinking of social, ecological and political topics, as well as the reflection of findings and the application of reflected knowledge (Coles, 2022) are often not included. We believe that there is still room for improvement. For this reason, we have developed novel tasks in the project “Strengthening Teachers’

Instructional Capabilities with Big Data” with the aim of promoting the capabilities needed in today’s society. We now analyze a task drawn from a current research project to reflect on what a task that meets the identified criteria could accommodate.

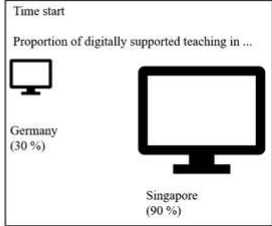
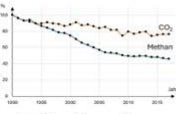
<p>(a) Use the information provided by the public health department to assess which of the newspaper's claims are true. Check your results using a math software.</p> <p><i>Salmonella disaster in egtown: THOUSANDS OF RESIDENTS INFECTED</i> The health department says: "Milk ice cream was infected with salmonella. The number of salmonella in the ice cream grows with $f(t) = 10 \cdot 7^t$ (t in hours)." This means that the salmonella count doubled after 20 minutes. It increases tenfold every hour. After 7 hours, there are more than 10 million salmonella.</p> <p style="text-align: right;">(Biburger et al., 2022, p.37)</p>	<p>(b) An international study found that only around 30% of school lessons in Germany were digitally supported in 2019, compared to 90% in Singapore. An electronics store used this result for an advertising campaign.</p> <p>a) Demonstrate mathematically that the presentation is manipulative. b) Research other ways in which information can be presented manipulatively using diagrams.</p> <div style="text-align: right;">  <p style="text-align: center;">(Brendel & Eisentraut, 2021, p.137)</p> </div>
<p>(c) The diagram shows the emission of carbon dioxide (CO₂) and methane in Germany.</p> <p>a) Explain how CO₂ emissions have developed since 1990.</p> <p>The development of CO₂ emissions can be approximated by the function with $f(x) = 0,02x^2 - 1,5x + 100$ (x: number of years since 1990, $f(x)$ in %).</p> <p>b) Draw the graph G_f of the function f (in a DGS). Describe which values of the graph are not well represented by the modeling. c) Calculate the year in which CO₂ emissions would have reached their lowest point according to the modeling. d) Decide and justify whether a linear function would be more suitable for modeling the situation. e) Research the German government's climate targets for 2030, 2040 and 2050. Explain which properties a quadratic function g must have in order to be able to model the achievement of the targets. (Brendel & Eisentraut, 2021, p.92)</p> <div style="text-align: right;">  <p style="text-align: center;">(Brendel & Eisentraut, 2022, p.36)</p> </div>	<p>(d) At the 1997 World Climate Conference in Kyoto, the industrialized nations committed themselves to reducing greenhouse gas emissions by 5.2 % by 2012 compared to 1990 levels.</p> <p>a) Investigate whether this target can be achieved with an annual percentage reduction of 0.25 %. b) Inform yourself about the development of greenhouse gas emissions and assess whether a model of exponential growth makes sense in this context.</p> <p style="text-align: right;">(Brendel & Eisentraut, 2022, p.36)</p>

Figure 1: Translated examples of textbook tasks

Analysis of a Project Task

The project task is based on two media articles and four data sets on the topic of global CO₂ emissions. Shortened, translated (from English into German) versions of two media items are provided, with the publication sources and origin of the data bases as well as diagrams removed. The items include contradictory statements on the volume of CO₂ emissions. The (numerical) data sets included in the task are taken from reliable sources and are available for public use without restriction. The task is divided into three parts (see Figure 2). First, students are asked to identify different claims in the articles. They are then required to evaluate the claims in the items using at least two of the data sets provided. Students are finally required to explain and justify their approach to evaluate claims and their selection of data sets. The three-part structure of the task addresses the required capabilities regarding the two skill-oriented criteria: The project task requires the use of data sets with large volumes of numerical data, to evaluate the claims in the media items. Students need to make decisions in the selection of appropriate data points or data sets from the four provided. This means that the students must evaluate the relevance of each data set in relation to the task objective – the evaluation of claims. In addition, the design of the task addresses the different requirements regarding the content-oriented criteria. First, the task addresses the sustainability-related topic of global CO₂ emissions. Second, the task meets the criterion of authenticity, because it is based on original media articles and data sets. Finally, the task is open because it is left to students to select data sets that are the most appropriate and no bias is introduced in relation to expected outcome.

In summary the task focuses on the evaluation of media claims about issues related to sustainability through the selection and use of large data sets. Authenticity is ensured using original material. The task satisfies the criterion of openness because students are required to make selections from the

available information and bias in the questions is carefully managed. In total a combined consideration of original media claims and large data sets is given. This is an improvement to the textbook tasks, as during our everyday life we are confronted with the combination of media claims and data, and thus capabilities for a combined use are needed. Further, the use of original media articles, as well as the openness of the task, enhances the relatability of the problem for students and provides opportunity for them to generate solutions — a key mechanism for building student agency in the space of addressing socio-ecological issues.

Reports about climate change and sustainability are in the news nearly every day as headlines, interviews or items in the media. These reports can present different information or conclusions about the same phenomena or event. This means it is important to evaluate what is reported to determine its validity.

This task is based on two different media reports that focus on an issue related to climate change and sustainability, CO₂ emissions. The first article is “*Global CO₂ emissions rose less than initially feared in 2022 as clean energy growth offset much of the impact of greater coal and oil use*” and the second is “*Global CO₂ emissions have been flat for a decade, new data reveals*”. You should notice that these reports discuss different CO₂ emission levels. The overall purpose of this task is to check on the validity of these statements.

Four different data sets related to CO₂ emissions are available via the QR-code below. These data sets are all from reliable sources. Please examine each of the data sets and then respond to the following:

1. Write down the different claims related to CO₂ emissions that are presented in the reports.
2. Determine if the statements in the reports are valid. Justify your response by making reference to at least two of the data sets.
3. Discuss whether your conclusions are dependent on the data sets you selected. If there are differences – please explain how these were resolved. Also explain your selection of data sets.

Figure 2: Project Task

DISCUSSION

The analysis of the textbook tasks has demonstrated that they provide limited support for the development of the evaluative capabilities needed to make judgements about the validity of claims (e.g., regarding sustainability issues) in the media using large data sets. We note, in particular, that textbook tasks tend to make use of items which lack authenticity and closed situations with only one outcome likely. Both are problematic, as the reference to the students’ everyday lives and the challenges they face is partially lost and therefore there is no relatedness as a necessity for dealing with socio-political issues and developing agency. Further, few tasks included combinations of data and media items as well as representations, for example, graphics. These restrictions are problematic as mathematical/statistical information is included in media items (Gal & Geiger, 2022). We further note restrictions on the size of data sets, which stands in contrast to the availability of “big” data from credible, readily accessible, repositories and the resulting needed capabilities (Bureau of Education & Ng, 2019). Overall, these limitations are problematic, as a combined application of the use of large data sets for the evaluation of original media articles is important to prepare students on the existing challenges and thus to address challenges, mentioned in recent studies (Newman et al., 2022). Such limitations restrict the opportunities of students to develop the capabilities needed to be informed and, consequently, to exercise critical and responsible citizenship in their future lives. As a result, such tasks provide limited opportunity to engage students with the capabilities needed to address the current requirements and socio-ecological challenges. Our task, by design, meets such demands and thus represents a promising approach to meet the socio-ecological demands of mathematics teaching.

It became evident that different criteria can be used to clarify the extent to which tasks that address socio-ecological issues differ from “classic” (e.g., social) tasks. The integration of social, political and ecological considerations within socio-ecological tasks (Amico et al., 2023; Coles et al., 2024) facilitates the realization of more authentic tasks, as the three perspectives are no longer separated.

Moreover, the tasks become more open-ended, as they address complex topics with diverse influencing factors and potential solutions.

However, it must be clearly stated that this entails various challenges that makes it difficult to implement corresponding tasks in lessons and textbooks. An issue that became apparent during our analysis of tasks is that the entanglement of the socio-ecological escalates the complexity of task design (e.g., through authenticity or openness) if compared to generating learning activities that focus on only social or ecological concerns. Further the type of task we see as addressing socio-ecological issues often require interdisciplinary capabilities – capacities that are not yet promoted in schools in a common place fashion. Further, in addition to the time restrictions that exist in schools, there are often also restrictions in terms of equipment and accessibility (e.g., to data). Specifically, regarding the use of real data, it is problematic that data sets change and are updated, which makes implementation of data literacy in textbooks more difficult. This challenge points to the need to adopt approaches to learning in schools which are more problem based rather than simply discipline focused. This approach has consequences for both the restructuring of curriculum in schools and approaches to teaching and learning.

The above discussion points to the challenges associated with the design of tasks for engaging with socio-ecological issues. Furthermore, textbooks play a key role due to their ubiquitous use in education. Therefore, they must be seen as a key resource for engaging with socio-ecological issues. Overall, the need for further research that considers the complexity of the requirements while maintaining the accessibility of the tasks for students and focuses on possibilities for implementation in textbooks is apparent. One challenge is the nature of the tasks included in textbooks, which are intended to meet a wide range of needs – from conceptual development to fluency with mathematical techniques to connection with authentic real-world problems. Based on the analysis presented here, we see our task as a starting point for the development of socio-ecological tasks in textbooks. However, it must be critically emphasized that the project task addresses a global problem and is distanced from the local context of the German students by media articles from English-speaking countries. As a result, one of the requirements of the socio-ecological perspectives – the use of local contexts (Coles et al., 2024) – is not considered. Further limitations are the restricted sample of textbooks for Years 9 and 10 in Bavarian Gymnasium and the analysis of only one project task.

CONCLUSION

Developing societal challenges and requirements lead to the need to evolve the capabilities needed for informed, critical, and responsible citizenship. These capabilities should be considered a human right and be promoted through schooling. This contribution focuses on an analysis of textbook and project tasks in terms of promoting these new capabilities needed in today's society and the socio-ecological perspective. We see the project task as illustrative of the type of activity students should be engaging with to promote key data-driven evaluative capabilities. To make the design of such tasks accessible to teachers, guidelines need to be developed that support this activity. We see the criteria used in this paper to evaluate tasks as a starting point to answer the second research question.

References

- Amico, A., Andra, C., Chorney, S., Coles, A., Doria, L., Helliwell, T., Hunter, J., Makramalla, M., Pezzutto, M., Rubel, L., & Solares, A. (2023). How socio-ecological issues are urging changes in curriculum (and beyond). In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings*

- of the 46th Conference of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 71–101). PME
- Ananiadou, K., & Claro, M. (2009). 21st century skills and competences for new millennium learners in OECD Countries. *OECD Education Working Papers, 41*. <https://doi.org/10.1787/218525261154>
- Battelle for Kids. (2019). *Framework for 21st century learning definitions*.
- Biburger, J., Frantzke, A., Herbst, M., Kestler, C., Linde, K., Seibold, C., Sy, B., Thiessen, T., & Weber, M. (2022). *Lambacher Schweizer 10: Mathematik für Gymnasien Bayern*. Ernst Klett Verlag GmbH.
- Brendel, A., & Eisentraut, F. (2021). *mathe.delta 9: Mathematik für das Gymnasium Bayern*. C.C. Buchner Verlag.
- Brendel, A., & Eisentraut, F. (2022). *mathe.delta 10: Mathematik für das Gymnasium Bayern*. C.C. Buchner Verlag.
- Bureau of Education, & Ng, S. B. (2019). *Exploring STEM competences for the 21st century*. UNESCO International Bureau of Education. <https://unesdoc.unesco.org/ark:/48223/pf0000368485>
- Coles, A. (2022). A socio-ecological turn in mathematics education: Reflecting on curriculum innovation. *Paradigma, 43*(1), 207–228. <https://doi.org/10.37618/PARADIGMA.1011-2251.2022.p207-228.id1168>
- Coles, A., Solares-Rojas, A., & Le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics, 116*(2), 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- European Commission. (2019). *Key competences for lifelong learning*. Publications Office. <https://data.europa.eu/doi/10.2766/569540>
- Gal, I., & Geiger, V. (2022). Welcome to the era of vague news: A study of the demands of statistical and mathematical products in the COVID-19 pandemic media. *Educational Studies in Mathematics, 111*(1), 5–28. <https://doi.org/10.1007/s10649-022-10151-7>
- Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., Goos, M., & Fernandez, K. (2023). Investigating critical mathematical thinking when applying mathematics to real-world problems. In *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*. Alfréd Rényi Institute of Mathematics; ERME. <https://hal.science/hal-04415712>
- Lewandowsky, S., Ecker, U. K. H., & Cook, J. (2017). Beyond misinformation: Understanding and coping with the “post-truth” era. *Journal of Applied Research in Memory and Cognition, 6*(4), 353–369. <https://doi.org/10.1016/j.jarmac.2017.07.008>
- Newman, N., Fletcher, R., Robertson, C. T., Eddy, K., & Nielsen, R. K. (2022). *Digital News Report 2022*. <https://reutersinstitute.politics.ox.ac.uk/digital-news-report/2022>
- Schüller, K., Busch, P., & Hindinger, C. (2019). *Future Skills: Ein Framework für Data Literacy*. <https://doi.org/10.5281/ZENODO.3349864>
- Siller, H.-S., & Greefrath, G. (2020). Modelling tasks in central examinations based on the example of Austria. In G. A. Stillman, G. Kaiser, & C. E. Lampen (Eds.), *International perspectives on the teaching and learning of mathematical modelling. Mathematical modelling education and sense-making* (pp. 383–392). Springer. https://doi.org/10.1007/978-3-030-37673-4_33
- Siller, H.-S., Günster, S., & Geiger, V. (2024). Mathematics as a central focus in STEM: Theoretical insights and practical insights from a special study program within pre-service (prospective) teacher education. In Y. Li, Z. Zheng, & N. Song (Eds.), *Disciplinary and interdisciplinary education in STEM: Changes and innovations*. Springer.
- UNESCO. (2015). *Global citizenship education: Topics and learning objectives*. <https://doi.org/10.54675/DRHC3544>
- Vos, P. (2011). What is ‘authentic’ in the teaching and learning of mathematical modelling? *Trends in teaching and learning of mathematical modelling, 1*, 713–722. https://doi.org/10.1007/978-94-007-0910-2_68

DIALOGUES, CONFLICTS, AND ENVIRONMENTAL JUSTICE

Ole Skovsmose¹, Ana Carolina Faustino²

¹Aalborg University, Universidade Estadual Paulista, ²Universidade Federal de Mato Grosso do Sul

In this research proposal we discuss the potential of dialogue across distances for addressing environmental justice in the mathematics classroom. We will elaborate on the dialogic theory of learning mathematics. However, we will consider dialogues taking place across huge distances, and not just in particular classroom settings. We will develop a project for students, around 14-15 years old, concerning questions related to environmental problems. The essential feature of the project is that it will simultaneously take place in three different classroom settings in Brazil: one located in an Indigenous neighbourhood, one located in a favela, and one located in an affluent neighbourhood. The project will include close dialogic interactions between the three settings. In the analyses we will pay particular attention to the importance of developing dialogue across huge distances, whether these distances are physical, cultural, ecological, or socio-economic. We find such dialogues to be essential for constructing conceptions of environmental justice.

INTRODUCTION

On the 25th of January 2019, the city Brumandinho in Minas Gerais, Brazil, was destroyed in a mudslide. At least 270 people were killed. This tragedy also killed non-humans; it contaminated the Paraopeba River and destroyed the ecosystem. The mudslide was caused by the collapse of a dam that was holding a huge lake of mud in place. Doing open mining presupposes that much earth becomes removed, and the removed earth turns into mud to be kept behind huge dams until it, once in the future, might turn into solid ground.

If a dam breaks down, the mudslide will take a predictable route. No wealthy neighbourhoods are to be found along this route. Brumandinho was a poor city, protected only by a cheap dam construction. More secure dams are expensive to construct, and the mining company, Vale, had not found this necessary in the case of Brumandinho.

Approximately 12 million m³ of mud was released during the dam collapse, reaching the Paraopeba River and destroying “133.27 hectares of the native Atlantic Forest (Mata Atlântica) and 70.65 hectares of permanent protection areas” (see Polignano & Lemos, 2020, p. 37, our translation). According to Lacaz, Porto, and Pinheiro (2017), tragedies like this are the result of projects by mega-mining companies, and are not isolated occurrences. In 2015, the Fundão dam, located in the city of Mariana in the state of Minas Gerais, also collapsed. This dam released 40 million m³ of mud reaching the Doce River. “The tragedy affected the Indigenous communities, the Krenak in Minas Gerais and the Tupiniquim and Guarani in the state of Espírito Santos” (see Lacaz, Porto, & Pinheiro, 2017, p. 2, our translation). According to Krenak (2020, p. 45), “the river that has nurtured and nourished our life along a six-hundred-kilometre stretch of the Doce River, from Minas Gerais to Espírito Santo, finds itself today sunk under toxic mud from a burst tailings dam that orphaned our tribes and plunged the river into a coma.”

On the 15th of February 2022, the city of Petrópolis in the state of Rio de Janeiro suffered a mudslide due to intense rainfall. At least 231 people were killed. Previously in Petrópolis, poor neighbourhoods

were located close to the beach, from which it was easy to go fishing. This area was, however, attractive for the tourist industry. Hotels and summerhouses were built on the land, and the local population had to move up into the hills where the land was cheapest. It was also the most risky area, as mudslides posed a constant threat.

By referring to an education for social and environmental justice, we do not mean an education which tries to inform students about what to consider just and what to consider unjust. Instead, we have in mind an education where students get opportunities to articulate their conceptions of what to call just or unjust in dialogue with others, including some that might hold quite different views. Such articulation we see as an important step in formulating and developing a conception of social and environmental justice.

One could assume that while social justice first of all concerns relationships between human beings, environmental justice concerns relationships between human beings and nature. However, as was revealed in the most dramatic way in Brumandinho and Petropolis, there is no sharp distinction to be found between social and environmental justice. In our study we want to explore the close connection between these two forms of justice.

The Indigenous author Ailton Krenak (2020) emphasises that Planet Earth can no longer support the desire for consumption that has destroyed rivers, cut down forests, and displaced animal populations. The author highlights the need to reflect on the logic of merchandise. He calls on us to problematise the way we relate to the planet.

CONCEPTUAL FRAMING

Environmental justice is a constructed and contested concept, and for its further development it is important to engage students in dialogues about equity in access to natural resources, about differences in environmental life conditions, as well as about life conditions for future generations.

A dialogic theory of learning mathematics was presented in *Critical Mathematics Education* (Skovsmose, 2023). We will draw on this theory when we address the formulation and further developments of environmental justice. The dialogic theory of learning mathematics is characterised by six learning interacts: *getting in contact*, *exploring*, *positioning*, *foregrounding*, *externalising*, and *doubting*.

Getting in contact is important for establishing cooperative processes, and establishing dialogic relationships presupposes that one is concerned about the views of the other. Getting in contact refers to emotional elements of such concerns.

Exploring is a learning interact that includes activities such as *presenting*, *listening*, *advocating*, and *questioning*. *Presenting* one's views and insight to others is an integral component of the whole process of learning. *Listening* is an important learning interact, as students must listen to each other and to the teacher, and the teacher must listen to the students. *Advocating* refers to any processes of arguing for a position, and mathematical proof is an example of advocating. *Questioning* can often make up a part of a teacher's control and evaluation of the students, but most often this is not genuine questioning but rather a control procedure. By using questioning as part of exploration, we have in mind a concern for acquiring new insight and for driving an inquiry process forward. Questioning can enable the teacher to understand the paths and strategies used by students in the investigation process. It can also enable the students to understand the paths proposed by the teacher.

In dialogues, different kinds of knowledge and insights are positioned next to each other, and *positioning* is an important learning interact. The traditional mathematics classroom is not an inviting space for positioning; rather, it is regulated as a narrow one-way street leading towards pre-defined mathematical ideas.

It has often been emphasised that it is important that students internalise what they have learnt. Acquired new knowledge should become integrated in order to add to funds of already acquired knowledge. It is, however, equally important that knowledge becomes externalised, which refers to the process of students presenting their acquired knowledge to others. Consequently, we see *externalising* as an important learning interact. Externalising refers to activities that bring the acquired knowledge beyond the horizon of the individual.

As a learning interact, *foregrounding* may refer to students' recognition of their own prospects in life, maybe in terms of new interests, new possibilities for further education, or future job opportunities. Foregrounding might draw students' attention to possibilities of which they were previously unaware. It might also make students recognise obstructions for reaching out towards new possibilities. Foregrounding is not an isolated individual act, but takes place in dialogue with others: students, friends, families, teachers. Foregrounding can concern a whole group of students. It might concern the future living conditions for today's generation of children. It might concern how the unequal distribution of natural resources becomes prolonged into the future. Foregrounding can enable students to build collective future horizons, which are not based on the homogenisation of ideas, but on unity in diversity, aiming for the common good of all who inhabit the planet, both human and non-human.

A dialogic theory of learning mathematics opposes any form of absolutism which tries to place mathematics on an imposing pedestal, out of reach of any form of doubt. We see mathematics as being constructed in a zone of contradictions and conflicts, and consequently we recognise *doubting* as being an important learning interact. Doubt is not to be contrasted with knowledge. Any construction of knowledge is also a construction of doubt. Knowledge and doubt are features of the same learning process (Skovsmose, 2023).

Renuka Vithal (2003) searched for a pedagogy of conflict and dialogue for mathematics education. She was preoccupied by the situation in post-Apartheid South Africa. Here, conflicts were all with dominant phenomena, also in the educational context. Simultaneously, dialogues were necessary. The dialectic relationship between conflict and dialogue is also a permanent condition when we are dealing with environmental issues. Such issues concern the unequal distribution of risks, as illustrated in the most dramatic way by those who get exposed to mudslides; they also concern unequal access to natural resources.

Ole Skovsmose, Amanda Moura, and Manuella Carrijo (2003) introduced the notion of inclusive citizenship, and for developing such a citizenship they highlighted the importance of establishing dialogue amongst differences. These differences might be natural, economic, political, ethnic, or cultural. We find that the conception of inclusive citizenship is closely related to the formation of conceptions of environmental justice, and we are directly inspired by the idea of engaging students in dialogues amongst differences. Dialogues with respect to learning are normally considered features of classroom communication. However, in the present study we are going to enlarge the notion of

dialogue. We are going to consider not only dialogues within classrooms, but also dialogues taking place amongst classrooms located in quite different socio-ecological contexts.

DATA PRODUCTION

The study will be based on a shared project taking place in three different classrooms in Brazil with 14–15-year-old students. One of the classrooms will be located in an Indigenous setting, one in a favela, and one in a wealthy neighbourhood. The content of the project work will be an urgent environmental issue. The importance of environmental issues in critical pedagogies has also been emphasized by Freire (2005) when highlighting:

[...] how urgent it is that we fight for more fundamental ethical principles, such as respect for the life of human beings, the life of other animals, of birds, and for the life of rivers and forests. I do not believe in loving among women and men, among human beings, if we do not become capable of loving the world. Ecology has gained tremendous importance at the end of this century. It must be present in any educational practice of a radical, critical, and liberating nature. (p. 25)

So, the urgent environmental issue of the project could well concern the use, waste, and pollution of water. It could address questions like: Who has access to clean water? Who experiences problems with polluted water? How? How does water get polluted? By whom? However, the specific formulation of the project will be the result of a dialogue between the involved students, teachers, and researchers.

The first classroom will be located in an Indigenous community in the State of Mato Grosso. This part of the project work will be organised by Adailton Alves da Silva, who is an associate professor in the Department of Mathematics at the University of the State of Mato Grosso. Da Silva gave a plenary lecture entitled “Ethnopedagogies: Socio-educational knowledge without looking for the right form!” (da Silva, 2023) at the Mathematics Education and Society Conference (MES 12) that took place in São Paulo in 2023.

The second classroom will be students from a poorly resourced school located in a vulnerable neighbourhood, lacking access to basic sanitary means – specifically, in a public school in the State of Pará. According to data from Instituto Trata Brasil, approximately 91% of the population of Pará State does not have access to sewage collection. This part of the project work will be recognized by Ana Carolina Faustino.

The third classroom will be students from a school located in a wealthy neighbourhood in the State of São Paulo. Here the student will have access to all the means and public resources that make life comfortable: sanitation systems, electricity, educational systems, health care systems, etc. This part of the project work will be organised by Denner Dias Barros. For many years Barros has been associated to the research group *Épura* from the State University of São Paulo, at Rio Claro, directed by Miriam Godoy Penteadó and Ole Skovsmose. The participants in the *Épura* Group are Master’s students, PhD students, and young researchers dedicated to inclusive mathematics education and critical mathematics education.

In the data production we will carefully register what is taking place in the individual classrooms. In particular, we will see how dialogic processes evolve in the different classroom environments. The data will be recorded in audio, video, and through materials created by the students during the investigation.

We will pay particular attention to what is taking place when all three classrooms are interacting. At this stage of data production, interactions will be recorded on Google Meet with the authorisation of all participants. The recording of the meeting will be analysed, and the audio will be transcribed.

An important initial feature of data production is to define the shared project. The overall aim is to address an environmental issue that is experienced as urgent. It is important that the content of the project is decided upon by the students involved.

However, if necessary, we will discuss with students some projects that have already been developed. This dialogue can inspire and support them in identifying their own environmental issue to investigate. We have many resources for dealing with projects concerning water. Faustino (2018) has directed a project taking place among 9–11-year-old students in a poorly resourced school. The project concerned the use of water in everyday practices, including estimations of the quantities that it might be possible to save. Denival Biotto (2008), also associated to the *Épura* Group, has engaged 15–16-year-old students in estimating the amount of water that gets lost in the water supply system in a city in the interior of the São Paulo State. Based on available information from the city council, the students estimated that only 50% of the water that is supplied to the water system in fact reaches the consumer and is paid for. Debora Souza-Carneiro (2021), also associated to the *Épura* Group, has organised project work dealing with water pollution for mathematics students in the initial year of their university studies. So, we have many resources to draw on for developing projects concerning the use, waste, and pollution of water.

ANTICIPATED CONCLUSIONS

At the present moment we cannot outline the content of our analysis nor of our conclusions, but we can point out nine particular issues that we will pay particular attention to, and which will come to frame our conclusions. Furthermore, we will summarise some more general observations to be included in our conclusions.

First, we will consider the dialogic theory of learning mathematics and see how it needs to be further elaborated in order to deal with dialogues amongst differences, and we will consider in what way dialogue is essential for constructing conceptions of environmental justice. Getting in contact is important for establishing cooperative learning processes, and we will study what getting in contact might mean when three different classrooms are involved, and what difficulties might be experienced. In a similar way, we will carefully study the other learning interacts that make up the dialogic theory of learning mathematics.

Second, we will carefully contemplate the mathematical topics that are addressed in the project. We are fully aware of the fact that when a project in mathematics addresses real-life problems, then mathematical content-matter issues frequently get downgraded, if not simply disregarded. We will pay attention to what sort of mathematics is actually used in the project, as well as the possible mathematical topics and techniques that could have been applied, but were ignored. In our conclusions we will try to point out educational possibilities by not only considering what did take place, but also what could come to take place.

Third, a particular challenge is caused by the physical distance between the three classrooms, and we will address the potentials as well as the limitations associated with internet interaction. For instance, “exploring” as a leaning interact includes the activities of presenting, listening, advocating, and

questioning, but how do such learning interacts come to operate in an internet interaction? Presenting comes to reach beyond the walls of the classroom. Listening, advocating, and questioning might also take new forms due to internet communication.

Fourth, the learning interact of positioning will come to assume a wider scope by reaching across cultural, environmental, geographic, and economic differences. This brings stronger contrasts to positioning. In our conclusions we will consider how such contrasts might bring a new dynamic to the dialogic processes, but also what marked conflicts might emerge.

Fifth, we will consider the impact of the learning interact of externalising when it comes to reach beyond the horizon of the individual as well as beyond the particular classroom. Such externalising will come to stretch deep into completely different contexts, meaning that conflicting views might get exposed. Such externalisations, however, we also see as crucial for the further articulation of environmental justice.

Sixth, we will consider how the learning interact of foregrounding might gain new dimensions. Foregrounding could refer to the students' recognition of their own prospects in life, but it could also concern the possibilities of a whole group of students. Foregrounding might concern the future living conditions of an Indigenous groups of students, of a group of students living in a favela, or of a group of well-off students. It might concern the shared future of different groups of students living in Brazil. It might concern our shared future on the planet. In our conclusions, we will be attentive to how a foregrounding stretching across differences might bring about new force to the articulation of environmental justice.

Seventh, in our investigations and in our conclusions we will be aware of the fact that different political positions can emerge in all socio-economic contexts. Leftist opinions might be formulated by students from affluent environments, while conservative positions also find ardent support in poor neighbourhoods.

Eighth, we will be aware of the tensions that might appear between dialogues and conflicts. The further articulation of conceptions of environmental justice will take place in such tensions. We will analyse the possible emergence of dialogic as well as of non-dialogic acts, as investigated by Faustino and Skovsmose (2020). In our conclusions, we will consider carefully what a dialectic between dialogue and conflict might mean for the construction of conceptions of environmental justice.

Ninth, in our conclusions we will point out potentials for future project work that engages students in dialogues amongst differences, with the aim of formulating views on environmental justice. We will highlight the importance of such work for early childhood education, for the early years of elementary school, for secondary school, and for initial and continuing teacher training. We will highlight the importance of establishing learning interacts amongst radically different socio-ecological contexts. We will also point out possibilities for establishing dialogues amongst students located in different countries, bringing still new contours of diversity to the dialogic processes.

In the most dramatic way, the events of Brumandinho and Petropolis showed that environmental issues are existential. Environmental conflicts may concern contradicting discursive practices. They might be rooted in different networks of power, which could be of economic, industrial, political, or military nature. Cases of environmental injustice can concern social exclusion, allocations of tremendous risks to certain groups of people, economic exploitation, and explicit physical violence.

Environmental injustice might destroy the living conditions for some, while ensuring extra profit for others. Environmental injustice might concern life and death.

When addressing environmental injustice, dialogues and conflicts will operate in dialectic complexities. Shared conceptions of environmental justice cannot be expected to be reached through simple conversational compromising. We just have to remember Krenak's (2020) observation that the further destruction of the planet is propelled forward by the logic of merchandise. This is a powerful logic that cannot be overcome by casual conciliation. The economic and industrial interests associated with mining easily come to outweigh environmental preoccupations.

Environmental conflicts might be so deep that dialogue seems impossible. Still, dialogue is necessary. This is the tremendous challenge that a dialogic theory of learning mathematics must face.

Acknowledgements

Some of the formulations we use for describing the dialogic theory of learning mathematics are taken directly from Skovsmose (2023, 2024). We thank Rosalyn Sword for proof reading of the manuscript.

References

- Biotto, D. (2008). *O desenvolvimento da matemática no trabalho com projetos*. Master's thesis. Rio Claro: Universidade Estadual Paulista (Unesp).
- Faustino, A. C. (2018). "Como você chegou a esse resultado?": o processo de dialogar nas aulas de matemática dos anos iniciais do Ensino Fundamental. Doctoral dissertation. Rio Claro: Universidade Estadual Paulista (Unesp).
- Faustino, A. C. & Skovsmose, O. (2020). Dialogic and non-dialogic acts in learning mathematics. *For the Learning of Mathematics*, 40(1), 9-14.
- Freire, P. (2005). *Pedagogy of indignation*. Paradigm Publishers.
- Polignano, M. V., & Lemos, R. S. (2020). Rompimento da barragem da Vale em Brumadinho: impactos socioambientais na Bacia do Rio Paraopeba. *Ciência e Cultura* [online], 72(2), 37-43.
- Krenak, A. (2020). *Ideias to postpone the end of the world*. Trans. A. Doyle. House of Anansi Press.
- Koles, A. (2022). Mudanza socioecológica en educación matemática: Reflexiones acerca de la innovación curricular. *Paradigma*, XLIII(1), 207-228.
- Lacaz, F. A. de C, Porto, M. F. de S, & Pinheiro, T. M. M. (2017). Tragédias brasileiras contemporâneas: O caso do rompimento da barragem de rejeitos de Fundão/Samarco. *Rev Brasileira de Saúde Ocupacional*, 42, e9.
- Silva, A. A. (2023). Ethnopedagogies: Socio-educational knowledge without looking for the right form! In R. Marcone, P. Linardi, R. Milani, J. P. A. de Paulo, A. Q. Moura, & M. T. da Silva (Eds.), *Proceedings of the 12th International Conference of Mathematics Education and Society* (pp. 78-96). MES.
- Skovsmose, O. (2023). *Critical mathematics education*. Springer.
- Skovsmose, O. (2024). *Critical philosophy of mathematics*. Springer.
- Skovsmose, O., Moura, A., & Carrijo, M. (2003). Inclusive citizenship through mathematics education. *ZDM – Mathematics Education*, 55, 941-951.
- Souza-Carneiro, D. V. (2021). *A matemática em ação no ensino superior: Possibilidades por meio do problem-based learning*. Doctoral dissertation. Rio Claro: Universidade Estadual Paulista (Unesp).
- Vithal, R. (2003). *In search of a pedagogy of conflict and dialogue for mathematics education*. Kluwer.

RECONTEXTUALIZING ENGAGEMENT WITH DATA VISUALIZATIONS IN THE CONTEXT OF THE SOCIO-ECOLOGICAL

Eva Thanheiser¹, Laurie Rubel², Beth Herbel-Eisenmann³, Courtney Koestler⁴

¹Portland State University, ²University of Haifa, ³Michigan State University, ⁴Ohio University

We explore how data visualizations (DVs) are representational tools that can be used to narrate certain aspects of a context but silence others. Using multiple DVs allows for contrasting the DVs and likely provides opportunities for critical reading and exploring the entanglement of political and ecological dimensions of an issue. We start by integrating two sets of practices, Notice, Wonder, Feel, Act and Reimagine (NW-FAR) (Kahn, et al., 2022) and critical graph reading, Framing, Formatting, Narrating (FFN) (Rubel, et al., 2021) towards taking a socio-political stance and supporting students to read data visualizations and develop critical data literacy. We look forward to conversations about extending this combined set of practices further toward the socio-ecological.

BACKGROUND

There is increased recognition that preparing people to use mathematics requires going beyond “everyday” contexts to understand complex social, ecological, and political issues (SEPIs), as these are intertwined (e.g., Coles et al., 2022). From such an approach, mathematics in general and data visualizations (DVs), in particular, become tools for understanding, analyzing, critiquing, communicating about and changing the world (Frankenstein, 1990). The focus of the mathematics classroom expands to include using mathematics to learn about socio-ecological issues. Other examples of work in mathematics education on SEPI contexts include understanding systemic racism (Thanheiser, 2023), understanding the world’s diversity (Thanheiser & Koestler, 2021), and exploring state-run lottery industries (Rubel, et al., 2016). In this paper we illustrate an example of reading data visualizations, using a theme of migration as related to climate change.

Communication about current challenges and crises is often accompanied with DVs, and thus DVs play an important role in society. We focus on expanding traditional mathematics lessons by engaging with data, learning to read and create data visualizations, as well as with how data visualizations can support people to communicate with/about the socio-ecological (tell stories and listen to stories). We center how data visualizations are social texts that tell stories from specific points of view, each offering one of multiple perspectives on any given situation (Rubel et al., 2021) which allow for supporting or refuting claims. Building on two previous sets of practices related to data and data visualizations (DVs), we demonstrate an integrated set of practices to support students’ learning to critically read DVs. We attempt to move and open for discussion how this combined set of practices might support students’ engagement with topics captured by DVs in ways that account for not only their socio-political but also their socio-ecological dimensions.

DATA VISUALIZATIONS AND CRITICAL DATA LITERACY

An increasingly vocal recommendation is for a greater focus on mathematical literacy, to better support peoples’ access to DVs, which have become common forms of communication and have shown themselves to be essential in crisis, for personal and collective decision-making. School

mathematics curricula (in the United States) have included data analysis and the interpretation or production of data visualizations (DVs) since the 1990's, usually focusing on scatter plots, bar graphs, and pie charts. Here, DVs indicate a broader set of representations than those conventional graphs. We have argued elsewhere that DVs could serve as important tools in supporting students to use mathematics to understand SEPIs, such as the COVID-19 pandemic, in more explicitly visceral and emotionally engaging ways than conventional bar graphs (Lim et al, 2023). As social texts, DVs narrate particular stories from specific points of view (D'Ignazio & Klein, 2020). Yet their reliance on mathematics in essential ways then endows DVs (and by implication, any narratives they produce) with a guise of objectivity and with authority. Accordingly, *critical* data literacies (Weiland, 2017), that include troubling, and interrogating data and data visualizations are necessary.

Multiple data visualizations about the same topic can provide complementary, or at times divergent, narratives. We propose that sharing multiple DVs allows for contrasting the DVs and likely provides opportunities for critical reading of how they each amplify certain aspects of the socio-ecological context but silence others. For example, Lim and colleagues (2023) provide two examples of DVs about corporate profit during the COVID-19 pandemic. They show how a more conventional bar graph “is ripe for conventional statistical analyses such as finding various measures of central tendency” (p. 115), whereas Mona Chalabi's (2020) unconventional DV, using the same dataset, draws “attention to the inequality, social hierarchy, and inhumanity connected with excess profits” (p. 114) through its inclusion of human bodies and identities.

Next, we bring together ideas from two sets of practices that can support students to read data visualizations. We argue that this combined set of practices and frames can develop critical data literacy and socio-political perspectives, but needs further elaboration to push toward the socio-ecological. We illustrate the use of the combined ideas and open for discussion how to continue to develop a socio-ecological perspective through examples using the context of migration.

COMBINING TWO WAYS OF READING DATA VISUALIZATIONS: NW-FAR AND FFN

Here we bring together two sets of practices – Notice, Wonder, Feel, Act and Reimagine (NW-FAR) (Kahn, et al., 2022) and critical graph reading (Rubel, et al., 2021) – that support critical reading of DVs in different ways. We describe each briefly and propose an integrated set of practices, along with a rationale for their integration. The NW-FAR practices are an extension of the “Notice and Wonder” routine introduced by Burton (1984), developed to emphasize attention to social justice in design work around DVs. NW-FAR provides a collection of possible starting points, rather than a routine that has to be done in a particular sequence. The starting points include: *Notice*: interpreting explicit contexts and identifying hidden assumptions behind data and their representations; *Wonder*: raising questions about data, their representations, and the contexts in which they are situated; *Feel*: engaging with a learning activity or data representation on a physical, emotional, relational, and affective level; *Act*: challenging unequal power structures and inequalities through and within data practices; *Reimagine*: reworking the tools of data science, the structures of the data context, the uses of data, what counts as data, who counts as a data scientist, and who has sovereignty over data (Kahn, et al., 2022, p. 82). We highlight that the focus of activity using NW-FAR is on the *reader* (student) of the DV, for example, by encouraging connections between the student's experiences, questions, and feelings.

The critical graph reading framework presented by Rubel et al. (2021) suggests a second set of practices with which to read DV's. As social texts that are created to accomplish a particular version of the story represented, DVs are not static representations of truth. Critical graph reading requires attention to the authoring process and identification of a graph's: mathematical *formatting*: what is quantified and how, “[casting] a guise of objectivity to what are partial interpretations, [lending] an implicit but authoritative voice, and [enabling] an ease of communication” (Rubel et al., 2021, p. 253); mathematical *framing*: what variables are put in relationship and how, influencing what is made significant and shaping the potential responses to the issue represented in the DV; and *narrating*: what story is being told (Rubel, et al., 2021). Mathematical formatting and framing define and structure the DV and create certain pathways for narration. In this sense, the DV's reader considers: how the DV's narrative gives insight to the problem or context, how the communicated narrative might reinforce or challenge existing power relations, and who benefits from this way of narrating. A graph reader's reflection on a DV's mathematical formatting, framing and narrating focuses on the *author* of the DV, their intended message, positionality, and evidence.

A PROPOSAL AND RATIONALE FOR A COMBINED SET OF PRACTICES

We propose guiding reflection on a DV with the general notice and wonder prompts could lead students to identify aspects of a DV, but perhaps without critically engaging with the *role of mathematics* in the DV. Thus, we suggest that students be asked to consider the formatting, framing, and narrating in both their noticing and their wondering. For example, by asking questions about formatting such as *What is measured and counted? What data proxies are used? How are measurements defined and how are these data generated? Who gathered these data? When and where were these data collected?* (Rubel, et al., 2021, p. 264), students' attention can be drawn to aspects of mathematizing a situation or context. Such questions can support students to reformat the situation. Similarly, students can be engaged in wondering about formatting questions such as *What other data could be collected? What data are missing? What obstacles might there be to gathering these data?* (Rubel, et al., 2021, p. 264). These critical reflections about formatting, framing and narrating direct the attention to the DV and implicit and explicit decisions of the creator of the DV. As such, it can support learning to “read the world” (Freire, 2020) with data and DVs.

To engage with “writing the world,” (Freire, 2020) however, we think it would be important to come back to NW-FAR and in particular, FAR, to connect back to the student. We note that critical graph reading briefly mentions feel and act in re-narrating, but we think these deserve further attention. Feeling is central to resisting the patriarchal view on reason as masculine and emotion as feminine, disrupting the binary logic between reason and emotion, and challenging the notion of universal objectivity of engaging with data. Reflecting on how one feels, too, could provide important context for the development of empathy, something researchers have cited as imperative to equity-related work (e.g., Warren, 2014). Acting supports the proposal to “change the game” through social action (see Gutiérrez, 2009) toward changing systemic oppression. We note that both sets of practices identify complementary ideas related to “re-imagining.” While critical graph reading focuses on the processes of reformatting, reframing, and renarrating, reimagining in NW-FAR embraces, more broadly, new and different ways of doing data practices that are grounded in possibility, resistance, and freedom dreaming. Drawing on the work of Warren (2021), we describe resistance as requiring “critical reflection on how schooling...data and their contexts, necessitate opposition to unjust

authority figures and systems” and dreaming as providing “a means for reimagining the capabilities of Black (and other marginalized) individuals to ‘produce novel creations that respond to or solve a problem of interest to the maker: designs guided primarily by the desire of the producer versus that of the colonizers’ (Warren, 2021, p. 35)” (Kahn et al., 2022, p. 84).

We propose this integration of these two sets of practices as a powerful way to engage students with DVs and support their learning to *critically* read, reflect, and act on the world and relationships to other humans and nonhumans. We integrate a focus on the creator of the data visualization who formats, frames, and narrates with a focus on the student who applies NW-FAR and within NW reformats, reframes, and renarrates the story. We note, however, that centering the reader and creator of the DVs can promote a critical reading of the DV but also centers humans, rather than centering socio-ecological entanglements. Alongside this point, we also agree with scholars who point out that methods of inquiry that “black-box or flatten the consequentiality of identity, culture, race, gender, epistemology, and lands and waters risk overlooking the way educational processes and structures can shape relationality and produce, disrupt and transform persistent inequities (Artiles, 2017) and underexamine Western ontological and epistemological assumptions” (Vossoughi, et al., 2023, p. 226). Thus, we see a socio-political perspective as still underpinning a socio-ecological perspective.

We demonstrate this integration using two paired examples of DVs that narrate stories about migration. Layering multiple DVs about the same topic highlights key differences in mathematical formatting and framing and how these differences, in turn, shape different and alternative narratives.

EXAMPLE: MIGRATION

In 2024, we continue to see large migrations (human and animal), for a range of reasons, yet education rarely focuses on migration in relation to its social and ecological impacts (Pinson & Arnot, 2020). The global population of forcibly displaced people remains at a record high (Edwards, 2016), exacerbated by climate change and conflict. In addition, an “increasing number of species – including non-migratory ones – are leaving their habitats for cooler, more agreeable environments” (Geldard, 2023). While a human-centered focus privileges migration patterns and statistics about people, human and more-than-human migrations, as well as their impacts on the environment, are entangled.

To illustrate our application of the integrated set of practices we present a set of two representations offered by the BBC in a series examining human migration (BBC, 2016) and two representations focusing on fish. For the latter, we provide one by the European Environment Agency (2017) examining Recordings of two tropical fish 1963-1996 and one image (AI generated by the authors of this paper) representing dead fish related to an Associate Press (2024) report on A Climate-Related Mass Die-Off Leaves Over 100 Tons of Dead Fish Collecting at a Greek Port. We selected these representations because of the ways they highlight different aspects of migration. In Table 1 we integrate Formatting, Framing & Narrating (FFN) into Notice and Wonder followed by Feel, Act, and Reimagine. We show how this integrated set of practices produces different narratives. We explore human migration in detail in the table and thereafter discuss the fish. In Figure 1 we share the two sets of data visualizations and in Table 1 we share some of our own engagement with the DVs.

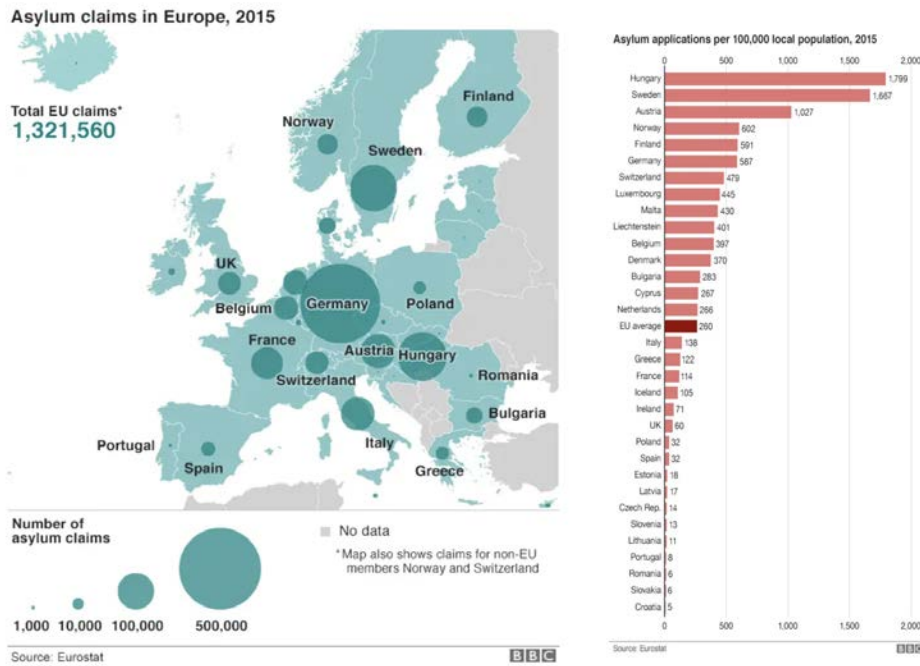


Figure 1: Two DVs on migration from BBC News at [bbc.co.uk/news](https://www.bbc.com/news/world-europe-34131911) specifically at <https://www.bbc.com/news/world-europe-34131911>

Table 1: NW-FAR with Format, Frame and Narrate applied to DVs in Figure 2

Some of the authors' application of the combined set of practices to the DVs in Figure 2		
NW-FFN	Asylum Claims (map)	Asylum Application (bar graph)
Notice: Formatting, Framing & Narrating	<p><i>Formatting:</i> Total asylum claims by country. (At a fixed point in time)</p> <p><i>Framing:</i> Map shown with national borders indicated, only some countries are labeled, and map is centered on Germany. Dots sized proportionally to show distribution.</p> <p><i>Narrating:</i> Many countries received asylum claims, with Germany, Hungary, and Sweden, receiving the most.</p>	<p><i>Formatting:</i> Asylum applications are normalized by the local population. (At a fixed point in time)</p> <p><i>Framing:</i> Vertical bar graph that emphasizes rank ordering of countries.</p> <p><i>Narrating:</i> Hungary and Sweden receive most applications relative to their population sizes. Many countries receive very few applications relative to their population sizes.</p>
Wonder: Formatting, Framing, & Narrating	<p><i>Formatting:</i> How might this mathematical formatting (of absolute numbers of claims) hide other salient variables, such as the different population sizes of these countries? Why are we counting people? What are the unintended consequences of making people into numbers?</p> <p><i>Narrating:</i> Why do people seek asylum? Who tend to be asylum seekers based on</p>	<p><i>Formatting:</i> How might this mathematical formatting, of normalizing by population, hide other salient variables, like absolute counts, land area, or amount of available resources?</p> <p><i>Framing:</i> Why is Germany so big on the map but low on the bar graph?</p> <p><i>Narrating:</i> How do these numbers compare to the US, where I currently live? How many of the asylum</p>

	age, gender, religion, etc.? Why are many going to Germany and few to Portugal?	applications actually get approved, as compared to those who apply?
FAR	Asylum Claims (map)	Asylum Application (bar graph)
Feel	Sad that so many people are being forced out of their countries. Worried about the safety and wellness of so many refugees.	Sad that Hungary closed the border to migrants.
Act	Support refugees (e.g., housing, healthcare, financial support, ...). Counter xenophobia.	Support refugees (e.g., house them, support financially, teach language, ...). Counter xenophobia.
Reimagine	Reimagine places without borders (i.e., a more decolonizing stance about land and ownership).	Reimagine a more open Europe.

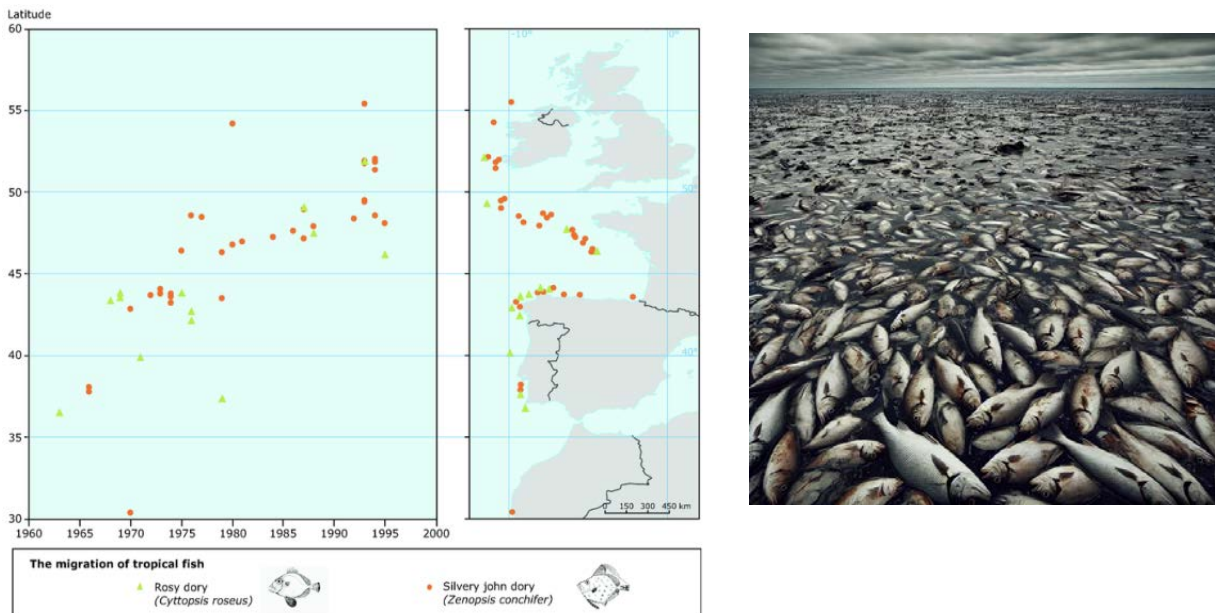


Figure 2: Two DVs regarding fish related to climate change.

The two representations regarding fish (see Figure 2) focus on different story lines about what happens due to climate change. The first representation focuses on fish migrating north to cooler waters. With regard to noticing we can examine: *Formatting*: Recordings of fish by year and latitude, (time is a variable that is included), *Framing*: Data points for recordings of fish connect geographical location and latitude, and *Narrating*: Both fish types are migrating north. With regard to wondering we can examine *Formatting*: How might this mathematical formatting (connecting latitude to geographical location across two visualizations) hide other salient variables such as the size of the fish population? With respect to *Feel*: Sad that fish need to migrate to survive, worried about what the other implications of this migration are. Are there new predators? Are the migrating fish predators for other species? *Act*: Raise awareness of the change of living spaces for all species. *Reimagine*: Reimagine a world where we collectively work to counter climate change. The second representation (an image is a basic data visualization) focusses on a different impact of climate change on fish, namely the fact that they cannot survive the rising temperatures. This affects the fish but also the

ecosystem relying on the fish as well as the human relying on food and tourism in the area. The second representation illustrates the result of global warming on one group of fish at a particular time.

RENARRATING BASED ON APPLYING THE COMBINED SET OF PRACTICES

Engaging with multiple DVs allows a fuller understanding of the stories within and across contexts. It allows us to examine relations among humans, as well between humans and more-than-humans. In this case, within each context, two data visualizations jointly tell a more detailed story than each of the DVs on its own.

In the human migration context, the map communicates absolute numbers with geographic relativity. Yet the map obscures other salient variables, such as how each of the shown countries has a different population size. The bar graph, on the other hand, communicates order comparisons, but obscures geographic context and spatial relationships. The two DVs produce different narratives, the first focuses on the dataset's range across the geography of Europe, and the second normalizes that data using population density. Both DVs depict distributions of aggregate numbers of asylum seekers. Aggregates are useful in how they provide overall trends of a dataset, in this case, the geographic patterns in asylum seekers across Europe. However, what gets lost in these aggregate representations are the individuals and especially, their stories. A third kind of DV includes asylum seekers' stories, of their individual journeys to Europe (e.g., Khalili, 2016). The aggregate displays that communicate the relative sizes of the phenomenon across Europe and the individual display of what it can mean to be an asylum seeker are both important and essentially motivate one another.

With the fish, the two visualizations focus on different aspects; the first connects latitude with geographical location but does not highlight whether the size of the fish population has changed. The story focuses on the move north across time. The second focuses on a whole fish population dying due to climate change. The story focuses on a specific point in time and on the entangled implications of climate change. Together the data visualizations tell a story that is at (at least partially) grounded in the context of climate change and its effects across species. The fish are looking for cooler water to survive and the asylum seekers are fleeing unlivable conditions partially due to climate change.

CONCLUSIONS

In this paper we highlighted the integration of two sets of practices NW-FAR and FFN. Integrating these two sets of practices connects the reader of a DV with the intentions and messages of its author. Reflection about formatting, framing, and narrating engages the DV's reader in making sense of why the DV's author made certain choices and in exploring what other choices could be made. FAR redirects the reflections back to the reader and how the communicated narrative elicits emotions and action. Finally, multiple DVs about a topic invite application of this integrated set of practices. Ultimately, we propose that this integrated set of practices can support students to understand different pieces of a story and thus get a more complete picture of the topic at hand.

References

- Associated Press. (2024, August 29). A Climate-related mass die-off leaves over 100 tons of dead fish collecting at a Greek port. *US News*. <https://www.usnews.com/news/world/articles/2024-08-29/over-100-tons-of-dead-fish-collect-at-greek-port-after-climate-related-mass-die-off>
- Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal for Research in Mathematics Education*, 15(1), 35–49. <https://www.jstor.org/stable/748986>

- Chalabi, M. (2020, December 29). The pandemic has been good to big US retail firms—to their workers, less so. *The Guardian*. <https://www.theguardian.com/news/datablog/2020/dec/29/us-retail-companies-workers-coronavirus-datablog>
- Coles, A., Solares-Rojas, A., & le Roux, K. (2022). Towards a socio-ecological perspective of mathematics education. *Educational Studies in Mathematics*, 116, 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- D’Ignazio, C., & Klein, L. F. (2020). *Data feminism*. The MIT Press,
- Edwards, A. (2016, June 20). Global forced displacement hits record high. UNHCR <https://www.unhcr.org/news/stories/global-forced-displacement-hits-record-high>
- European Environment Agency (2017, May 24). Recordings of two tropical fish 1963-1996. <https://www.eea.europa.eu/data-and-maps/figures/recordings-of-two-tropical-fish-1963-1996>
- Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *The Journal of Negro Education*, 59(3), 336–347. <https://www.jstor.org/stable/2295568>
- Freire, P. (2020). Pedagogy of the oppressed. In J. Beck, C. Jenks, N. Keddie, & M.F.D. Young (Eds.), *Toward a sociology of education* (pp. 374-386). Routledge
- Geldard, R. (2023, October 5). Here’s how extreme weather is affecting animal migration. *World Economic Forum*. <https://www.weforum.org/agenda/2023/10/climate-crisis-impacting-animal-migration/>
- Goldfarb, B. (2017, June 15). Feeling the heat: How fish are migrating from warming waters. *Yale Environment* 360. <https://e360.yale.edu/features/feeling-the-heat-warming-oceans-drive-fish-into-cooler-waters>
- Gutiérrez, R. (2009). Framing equity: Helping students “play the game” and “change the game.” *Teaching for Excellence and Equity in Mathematics*, 1(1).
- Kahn, J. B., Peralta, L. M., Rubel, L. H., Lim, V. Y., Jiang, S., & Herbel-Eisenmann, B. (2022). Notice, wonder, feel, act, and reimagine as a path toward social justice in data science education. *Educational Technology & Society*, 4(25), 80–92. <https://www.jstor.org/stable/48695983>
- Khalili, Bouchra. (2016). *The mapping journey project*. Museum of Modern Art (MOMA). New York.
- Lim, V.Y., Peralta, L. M., & Rubel, L. H., Jiang, S., Kahn, J., Herbel-Eisenmann, B. (2023). Keeping pace with innovations in data visualizations: A commentary for mathematics education in times of crisis, *ZDM-Mathematics Education*, 55, 109–118. <https://doi.org/10.1007/s11858-022-01449-0>
- Pinson, H., & Arnot, M. (2020). Wasteland revisited: Defining an agenda for a sociology of education and migration. *British Journal of Sociology of Education*, 41(6), 830–844. <https://doi.org/10.1080/01425692.2020.1755229>
- Rubel, L., Lim, V., Hall-Wieckert, M., & Sullivan, M. (2016). Teaching mathematics for spatial justice: An investigation of the lottery. *Cognition & Instruction*, 34(1), 1–26. <https://doi.org/10.1080/07370008.2015.1118691>
- Rubel, L.H., Nicol, C., & Chronaki, A. (2021). A critical mathematics perspective on reading data visualizations: Reimagining through reformatting, reframing, and renarrating. *Educational Studies in Mathematics Education*, 108, 249–268. <https://doi.org/10.1007/s10649-021-10087-4>
- Thanheiser, E. (2023). What is the mathematics in mathematics education? *Journal of Mathematical Behavior*, 70, 1-13. <https://doi.org/10.1016/j.jmathb.2023.101033>
- Thanheiser, E. & Koestler, C. (2021). If the world were a village: Learning mathematics while concurrently learning about the world. *Mathematics Teacher Educator*, 9(3), 202–228. <https://doi.org/10.5951/MTE.2020.0021>
- Warren, C. A. (2014). Towards a pedagogy for the application of empathy in culturally diverse classrooms. *The Urban Review*, 46, 395–419. <https://psycnet.apa.org/doi/10.1007/s11256-013-0262-5>
- Warren, C. A. (2021). *About centering possibility in Black education*. Teachers College Press.
- Weiland, T. Problematizing statistical literacy: An intersection of critical and statistical literacies. *Educational Studies in Mathematics*, 96, 33–47. <https://doi.org/10.1007/s10649-017-9764-5>

CREATING A LEARNING EXPERIENCE TOWARDS AGENCY FOR CRITICAL CITIZENSHIP: A SOCIO-CRITICAL MATHEMATICAL MODELLING PERSPECTIVE

Jimbo Juanito B. Villamor^{1,2}, Catherine P. Vistro-Yu¹

¹Ateneo de Manila University, ²Surigao del Norte State University

The study seeks to investigate a mathematical modelling project through a socio-critical lens, as a potential learning resource to develop agency for critical citizenship. An explanatory sequential mixed methods research design was employed to carry out the objectives of the study. Data were collected through survey, document analysis, observation, and focus group discussion, and were analyzed using appropriate quantitative and qualitative data analysis tools. The study contends that a socio-critical mathematical modelling project can develop capacities for student agency that drive observable behaviors of critical citizenship.

INTRODUCTION

It is with no doubt that we live in a volatile, uncertain, complex, and ambiguous (VUCA) world. Teaching, then, should aim at developing students as holistic beings fulfilling their truest potentials and helping them as citizens who are concerned about their well-being, and that of their communities and of the world. Mathematics teaching would mean providing students with learning opportunities that could help them concretize mathematical knowledge, and at the same time, help them become active, responsible, and engaged citizens ready for their future (Taguma et al, 2018). This could be possible by integrating mathematical modelling to mathematics classrooms. As a school of thought, mathematical modelling drives students to solve real world problems using mathematical tools that develop students' mathematical knowledge, enabling them to live as constructive, concerned and reflective citizens (Kaiser, 2020). Real-world problems are key to understand the surrounding world more critically, which is a mathematical modelling approach under a socio-critical perspective (Kaiser and Sriraman, 2006). This study seeks to investigate a mathematical modelling project through the socio-critical lens as a potential learning resource to develop agency for critical citizenship.

THEORETICAL FRAMES

This study rests on the theoretical perspectives of critical mathematics education by Skovsmose (1994), human agency by Bandura (2001), socio-critical mathematical modelling by Kaiser (2020), and transdisciplinary lesson and unit framework by O'Donnell (2021). Critical mathematics education addresses social development issues, views of mathematics education as a contributor towards critical citizenship (Skovsmose & Niss, 2008), and develops agency (Andersson & Norén, 2011). Agency (or human agency) is the ability of people to regulate and control their cognition, motivation, and behaviour through what they believe in about themselves, and which has four core features, namely: intentionality, forethought, self-regulation, and self-efficacy (Bandura, 2001). Intentionality is an individual's will to act, while forethought is the anticipation of outcomes (Code, 2020). Self-regulation refers to actions, processes and strategies an individual employs in achieving their goals (Code, 2020; Pintrich, 2000) and self-efficacy is a personal belief about what an individual

is capable of doing (Code, 2020). Agency can be developed in a learning environment that supports and motivates each student to nurture his or her passions, make connections between different learning experiences and opportunities, and design their own learning processes in collaboration with others (Taguma et al, 2018). This learning environment can make use of a mathematical modelling project exploring socio-ecological concerns as a learning resource. Such a project exhibits the socio-critical perspective of mathematical modelling, emphasizing the role of mathematics in society (Kaiser & Sriraman, 2006), and can be designed using the transdisciplinary lesson and unit framework. Transdisciplinarity makes use of STEM knowledge to understand socio-ecological issues, and where students' local communities are considered as learning laboratories (O'Donnell, 2021). While the promotion of critical thinking of students is emphasized in any mathematical discourse, reflexive discussions amongst students within the modelling process are seen as an indispensable part of the whole modelling process, and for the development of critical thinking (Kaiser & Sriraman, 2006).

METHODOLOGY

This study employed an explanatory sequential mixed methods research design, where the initial quantitative data were further explained by the qualitative data (Creswell & Creswell, 2017). The quantitative component assessed the students' levels of performance and levels of agency in the mathematical modelling project. The qualitative component investigated how students develop competencies for agency and critical citizenship while completing the project, based on their reflections of their experiences.

The mathematical modelling project was designed for the lesson "Functions and Their Graphs" taught in General Mathematics to senior high school students in the Philippines. It was developed using a transdisciplinary lesson and unit framework, with the aim of exploring solid waste management (SWM) issues in the students' local communities. The project involved 30 purposively chosen students who conducted a household survey on solid waste accumulation among the 389 households in an environmentally protected and tourist destination island. The actual survey data were used to describe the solid waste status of the community using linear regression and analyzed using jamovi (The jamovi Project, n.d.), which was completed by six groups of students in separate classrooms. The mathematical descriptions of the solid waste status were represented by linear equations in slope-intercept form, as mathematical models. This output served as the basis for the students' level of performance in doing the mathematical modelling project. Selected students presented their findings to the local officials. Students were then observed during the presentation. To determine the competencies of student agency and critical citizenship, a focus group discussion was conducted. Quantitative data were described using the frequency counts and percentages and analyzed using Kruskal-Wallis' test and Dwass-Steel-Critchlow-Fligner pairwise comparison tests. Qualitative data were analyzed using thematic analysis.

RESULTS AND DISCUSSION

The mathematical modelling task was assessed using the suggested features from the Guidelines for Assessment & Instruction in Mathematical Modelling Education (GAIMME) (Garfunkel & Montgomery, 2019). In this study, the features were modified into five categories, namely: 1) defining the modelling problem; 2) building the model, which includes making assumptions and

acknowledging limitations, defining variables and identifying parameters; 3) the quality and accuracy of mathematical solutions, comprising the use of meaningful mathematics, accessibility to the audience, and correct analysis and assessment of the model; and 4) writing style and organization. Since the designed mathematical modelling task addressed solid waste management issues in the research environment, the possible impact of the proposed SWM solution was added as another feature (5). Framing of the potential environmental solution is deemed to indicate how students actively engaged as change agents.

Students' Performance in Mathematical Modelling

Using the GAIMME's language (Garfunkel & Montgomery, 2019), the students showed satisfactory to ideal performance levels in mathematical modelling. All of them showed ideal performance when defining the modelling problem, writing, and organizing. Most of them reached a satisfactory performance level when building the model and providing a mathematical solution, while one group demonstrated an ideal performance level in these categories. Interestingly, most of the groups displayed an ideal performance in proposing SWM solutions, although there were some that reached only a satisfactory performance level.

The ideal performance level indicates that students can present a concise problem statement from the given task, rationalize the need for primary factors influencing the phenomena explored, and provide a readable glimpse into the mathematical methods used. It also means that students can clearly present a quality and accurate solution consistent with the given problem, present a viable and reliable math modelling solution, organize thoughts correctly, and present a detailed description and explanation of the proposed solid waste management solution.

The students' performance was further interpreted using the guidelines for assessment of the Philippines' Department of Education (2015). The entire class demonstrated a fairly satisfactory to outstanding over-all performance. In particular, Group 4 showed an outstanding performance, Group 6 very satisfactory, Group 1 and Group 3 satisfactory, and Group 2 and Group 5 fairly satisfactory. All these performance levels indicated a passing over-all performance in varying degrees.

Since the data were not normally distributed, nonparametric tests were further employed to determine whether the varying differences are statistically significant or not. Using the Kruskal-Wallis' test and as shown in Table 1, it was found that the levels of students' performance in defining the mathematical modelling problem and in writing and organizing their thoughts are not different at all. But there exists a statistically significant difference in the levels of students' performance in building the model, quality and accuracy of mathematical solutions, and the possible impact of the proposed solutions to help address solid waste management issues.

To ascertain which of the levels of students' performances in the aforementioned mathematical modelling features differ statistically, Dwass-Steel-Critchlow-Fligner's pairwise comparison tests were further performed. The result indicates that students with outstanding performance level in building the model and in terms of the quality and accuracy of mathematical solutions did perform better in the modelling project than those with very satisfactory, satisfactory, or fairly satisfactory performance levels. Similarly, students with outstanding, very satisfactory, or satisfactory performance in the task had more potential ideas or more viable proposed solutions to help address SWM issues in their local community than those with fairly satisfactory performance.

Features of Mathematical Modelling	χ^2	df	P	Interpretation
Define the Modelling Problem	NaN	3	NaN	-
Building the Model	29	3	< .001	Significant
Quality and Accuracy of Mathematical Solution	29	3	< .001	Significant
Writing Style and Organization	NaN	3	NaN	-
Possible Impact of the Proposed SWM Solution	29	3	< .001	Significant

Table 1: Comparison of students' performance levels on the features of mathematical modelling using Kruskal-Wallis' test

We argue that students with higher levels of performances in building the model and in the quality and accuracy of mathematical solutions tend to have the ability to propose a more impactful idea in helping mitigate solid waste management issues than those who fall behind the continuum. It is highly likely for students with high mathematical performances to think of feasible ways to solve solid waste management concerns. The more mathematical knowledge the students have, the more viable their ideas are in helping address solid waste management problems.

Development of Student Agency for Critical Citizenship

This study recognizes that the development of student agency can be manifested in the increasing levels of agency measured through a researcher-designed scale instrument. Students who obtained an outstanding performance level in the mathematical modelling project, initially perceived themselves to have high levels of intentionality, forethought, self-regulation, and self-efficacy capacities. These capacities scale up to very high levels after being exposed to the mathematical modelling project. This pattern is the same for students with very satisfactory and satisfactory performance levels. The students with a fairly satisfactory performance level were noted to have an increasing level of forethought and self-efficacy, but still a little lower than those who have satisfactory, very satisfactory, or outstanding performance levels. It was earlier noted that the students with outstanding, very satisfactory, or satisfactory performance have ideal performance in proposing SWM solutions. These students are seen to have demonstrated abilities to present a detailed description and explanation of the proposed solid waste management solution, which are manifestations of critical citizenship.

The above results indicated that students with better performance in the mathematical modelling project than the rest of the group tend to have a consistent increase in their levels of agency across the core features. We surmise that these students manifest good behaviors within the framework of critical citizenship. This means that the development of agentic capacities through the mathematical modelling project leads to developing behaviors for critical citizenship. Hence, integrating the socio-critical mathematical modelling project as a learning resource helped in developing agentic capacities vital to establishing behaviors as critical citizens.

From the conversation between the teacher-researcher and the students, the development of agentic capacities was observable. *Intentionality* was seen as students stated a desire to delve into the unknown, engage with novel situations, deal with things they have yet to discover, desire to do better in mathematics, seek collaboration, and work hand-in-hand with fellow learners. *Forethought* was observed as students set goals in learning mathematics to be able to solve bigger and more complex problems in the future. *Self-regulation* was evident as students were drawn to constant practice,

learning and discovery. *Self-efficacy* was manifested as students were able to construct meaning from practicing mathematical skills while seeing this practice as a steppingstone for further success. They understood that their failures lent knowledge to future engagements.

Mathematical Modelling Project as a Potential Learning Resource for Mathematics Education and the Socio-Ecological

We contend that the socio-critical mathematical modelling project used in this study creates a learning experience towards agency for critical citizenship. Students attained an acceptable level of mathematics performance, critically understood their surrounding world, developed agentic capacities towards learning mathematics. This observation is believed to be influenced by the kind of socio-ecological problem used in the mathematical modelling project. We then argue that the socio-ecological entanglement requires teachers to have capabilities for a new type of media literacy – the ability in designing mathematical tasks that attend to the socio-ecological, that is not necessarily limited to mathematical modelling. Such tasks make use of the students’ local community as a medium to make mathematics learning more relevant, more meaningful, and more engaging.

However, designing mathematical tasks that attend to the socio-ecological could be challenging for teachers. While we propose that teachers need to have the capabilities in designing mathematical tasks that attend to the socio-ecological, this poses a greater challenge on the task development itself. This raises concerns among teachers regarding which mathematical lesson must be integrated with which kind of socio-ecological problem. For instance, in our study, mathematical modelling is used to deepen students’ understanding of linear functions using solid waste management issues of the students’ local community. This implies that task development requires not just teachers’ mathematical knowledge, but also knowledge of transdisciplinary approaches, and a commitment to help students understand a community problem using mathematics. Transdisciplinarity helps teachers to be equipped with the capability to use students’ STEM knowledge in understanding socio-ecological issues using the community as a learning laboratory (O’Donnell, 2021). Another challenge we potentially see is on how tasks could be implemented without jeopardizing the authenticity of the assessment results. Authenticity of assessment can be challenging, especially on standardizing and effectively implementing the tasks (McArthur, 2023), and ensuring fair and consistent open assessments while allowing for flexibility and creativity (O’Neal, 2016).

Authenticity of assessments calls our attention to resource availability, faculty training, and institutional support (O’Neal, 2016). Looking into what textbooks can offer when used as resources for socio-ecological mathematics education, scholars have noted that textbooks face several limitations as they often present abstract mathematical concepts, which lack integration of real-world socio-ecological contexts (Pepin & Gueudet, 2020; Stylianides et al, 2017). Typically, these textbooks are designed to cover a standardized curriculum and are not easily adaptable to students with varying needs and interests (Pepin & Gueudet, 2020; Rezat et al, 2021), and primarily emphasize on mathematical procedures and technical skills, setting aside the broader social, ethical, and environmental dimensions (Pepin & Gueudet, 2020; Pepin & Kock, 2021). These textbooks often focus on the application of formulas, which may not adequately support the development of higher-order thinking skills vital for understanding and solving socio-ecological challenges (Pepin & Gueudet, 2020; Kaplar et al, 2022). Overall, while textbooks are an important learning resource, their traditional format and content can limit their effectiveness in socio-ecological concerns.

To work with local-global socio-ecological questions offers several vital implications for teachers, specifically in their teaching and curriculum design approaches. For one, *teachers need knowledge for transdisciplinary approaches*. While interdisciplinary understanding is important (Pepin & Gueudet, 2020), a transdisciplinary approach is much more useful for teachers to design learning tasks that will prepare students to thrive in the VUCA world. The transdisciplinary approach brings together many different disciplines in education, socio-emotional learning, and global citizenship education, with the idea that the complex global issues can be brought to a local level that will allow students to engage in local investigations and in local decision making and take some action. Doing these will facilitate students' understanding that local actions do have global impact (O'Donnell, 2021). Also, *teachers need an understanding to build or develop student agency*. Teachers must be supported with professional development programs in creating learning spaces that will help develop agentic capacities for students. When teachers are equipped with knowledge to develop student agency, students could be given with opportunities where they can showcase their abilities to regulate, control, and monitor their own learning (Bandura, 2001). These agentic capacities are all vital as they develop a sense of responsibility to participate in the world and, to influence people, events and circumstances for the better (Taguma, 2018), thus, helping students become critical citizens. Similarly, *teachers need to foster critical thinking and problem-solving skills in students*. Teachers need to create learning experiences that encourage students to analyze data, evaluate sources of information, and develop well-supported arguments (Pepin & Gueudet, 2020). This is mathematics education for critical citizenship (Skovsmose, 1994; Jazby, 2017), helping students become active and empowered members of society. Without mathematics, people will not be able to participate in democratic activities effectively; hence, mathematics teachers can empower students to be active and critical members of their societies (Skovsmose, 1994). As critical citizens, students would have the awareness of the current situations around them both environmental and social aspects, and would be able to contribute to the discussions addressing socio-ecological concerns. This means that mathematics teachers can utilize mathematics as an important element in students' social participation, and in the attainment of students' civic competence.

CONCLUSION

The study contends that integrating a socio-critical mathematical modelling project into classroom instruction can be a potential learning resource to develop student agentic capacities of intentionality, forethought, self-regulation, and self-efficacy. The development of these agentic capacities drives people to become critical citizens. The socio-critical perspective of mathematical modelling education can ignite observable behaviors of agency for critical citizenship.

Acknowledgements

The authors would like to thank the Department of Science and Technology – Science Education Institute (DOST-SEI) through the Capacity-Building Program in Science and Mathematics Educators (CBPSME) for funding the first author's graduate education; Surigao del Norte State University for allowing the first author on full-time study; Ateneo de Manila University for honing the research skills of the first author, and for granting ethics clearance through the Ateneo University Research Ethics Council with Protocol ID: SOSEREC_23_002; Schools Division Office of Siargao – Department of Education; Siargao National Science High School as the research environment; Siargao Island Protected Landscapes and Seascapes (SIPLAS), Local Government Unit of Dapa,

Surigao del Norte for allowing the researchers conduct this study. Special thanks is expressed to the Grade 12 students of Siargao National Science High School SY 2023-2024, who are the respondents of this study.

References

- Andersson, A., & Norén, E. (2011, February). Agency in mathematics education. In Pytlak, M.; Rowland, T.; Swoboda, E. (Eds.), *Proceedings from 7th conference for European research in Mathematics education – CERME VII* (pp. 1389-1398). Rzeszów: University mathematics of Rzeszów. <https://goto.now/ISNS4>
- Bandura, A. (2001). Social cognitive theory: An agentic perspective. *Annual review of psychology*, 52(1), 1-26. <https://doi.org/10.1146/annurev.psych.52.1.1>
- Bandura, A. (2008). Toward an agentic theory of the self. *Advances in Self Research*, 3, 15-49. <https://goto.now/o7UrK>
- Code, J. (2020, February). Agency for learning: Intention, motivation, self-efficacy and self-regulation. In *Frontiers in education* 5, Article 19. <https://goto.now/d0Kei>
- Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications. <https://goto.now/6o7b8>
- Department of Education (2015). *Policy guidelines on classroom assessment for the K to 12 basic education program*. <https://goto.now/zrRNh>
- Garfunkel, S., & Montgomery, M. (Eds.). (2019). *GAIMME—guidelines for assessment & instruction in mathematical modeling education*. Society for Industrial and Applied Mathematics. <https://epubs.siam.org/doi/abs/10.1137/1.9781611975741>
- Hanafin, J., Shevlin, M., Kenny, M., & McNeela, E. (2007). Including young people with disabilities: Assessment challenges in higher education. *Higher Education*, 54, 435–448. <https://link.springer.com/article/10.1007/s10734-006-9005-9>
- Jazby, D. (2017). Is Mathematics Education Worthy? From Mathematics for Critical Citizenship to Productivity Growth. *Mathematics Education Research Group of Australasia*. <https://files.eric.ed.gov/fulltext/ED589458.pdf>
- Kaiser, G., & Sriraman, B. (2006). A global survey of international perspectives on modelling in mathematics education. *Zdm*, 38, 302-310. <https://link.springer.com/article/10.1007/BF02652813>
- Kaiser, G. (2020). Mathematical modelling and applications in education. In Lerman, S. (Ed.), *Encyclopedia of mathematics education*, 553-561. Springer Cham. https://link.springer.com/referenceworkentry/10.1007/978-3-030-15789-0_101
- Kaplar, M., Radović, S., Veljković, K., Simić-Muller, K., & Marić, M. (2022). The influence of interactive learning materials on solving tasks that require different types of mathematical reasoning. *International journal of science and mathematics education*, 1-23. <https://link.springer.com/content/pdf/10.1007/s10763-021-10151-8.pdf>
- McArthur, J. (2023). Rethinking authentic assessment: work, well-being, and society. *Higher education*, 85(1), 85-101. <https://link.springer.com/content/pdf/10.1007/s10734-022-00822-y.pdf>
- O'Donnell, C. L. (2021). Discover, understand, act: a STEM education framework for empowering youth to address the global goals. *Diplomatic Courier: UN General Assembly (UNGA) Special Print Edition*. <https://goto.now/DhnPv>
- OECD. (2019). An OECD Learning Framework 2030. *The Future of Education and Labor*, 23-35. https://link.springer.com/chapter/10.1007/978-3-030-26068-2_3
- O'Neal, C. (2016). Beyond Authenticity: What Should We Value in Assessment in Professional Education?. *Assessing competence in professional performance across disciplines and professions*, 51-71. https://link.springer.com/chapter/10.1007/978-3-319-30064-1_4

- Pepin, B., & Gueudet, G. (2020). Curriculum resources and textbooks in mathematics education. *Encyclopedia of mathematics education*, 172-176. https://link.springer.com/referenceworkentry/10.1007/978-3-030-15789-0_40
- Pepin, B., & Kock, Z. J. (2021). Students' use of resources in a challenge-based learning context involving mathematics. *International Journal of Research in Undergraduate Mathematics Education*, 7(2), 306-327. <https://link.springer.com/article/10.1007/s40753-021-00136-x>
- Pintrich, P. R. (2000). An achievement goal theory perspective on issues in motivation terminology, theory, and research. *Contemporary educational psychology*, 25(1), 92-104. <https://www.sciencedirect.com/science/article/abs/pii/S0361476X99910172>
- Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM–Mathematics Education*, 53(6), 1189-1206. <https://link.springer.com/article/10.1007/s11858-021-01309-3>
- Skovsmose, O. (1994). Towards a critical mathematics education. *Educational studies in mathematics*, 27(1), 35-57. <https://link.springer.com/article/10.1007/BF01284527>
- Skovsmose, O., & Niss, M. (2008). Critical mathematics education for the future. In *ICME-10 Proceedings: Proceedings of the 10th International Congress on Mathematical Education*. IMFUFA, Roskilde University. <https://vbn.aau.dk/ws/portalfiles/portal/17034122/CriticalMathematicsEducation>
- Stylianides, G. J., Stylianides, A. J., & Weber, K. (2017). Research on the teaching and learning of proof: Taking stock and moving forward. *Compendium for research in mathematics education*, 237-266. <https://goto.now/ulvZZ>
- Taguma, M., Feron, E., & Lim, M. H. (2018). Future of education and skills 2030: Conceptual learning framework. *Organization of Economic Co-operation and Development*. <https://goto.now/y8pro>
- The jamovi project (2022). *jamovi*. (Version 2.3) [Computer Software]. Retrieved from <https://www.jamovi.org>.

MATHEMATICAL DRAMATIC CODIFICATIONS AS ECOPEdagogICAL RESOURCES

Katherina von Bülow

Simon Fraser University

Answering the call for mathematics education and the socio-ecological (Coles et al., 2024), this paper focuses not on curricular content focused on ecology per se, but on pedagogical resources that foster “relational knowing” (Boylan, 2017). Drawing on critical pedagogy, data feminism, ecopedagogy, and new materialist perspectives, I discuss a mathematics classroom activity that refuses to rely on creating or maintaining distances within and between actors. Ethics, emotions, and knowledges about the world are as integral to the activity as mathematics, the students’ bodies, and the space between them. Students’ reflections indicate that the activity fostered their awareness of themselves in relationship with others’ whole selves, with data and representational choices, and with a social justice issue. I argue that the design and enactment of such experiences of inter-dependence and inter-relatedness constitute an ecopedagogical approach to mathematics education.

INTRODUCTION

In mathematics education, we create, re-create, interpret, and consume mathematical representations of data pertaining to our lives, and likewise we are re-created, interpreted, and consumed by them. In this paper, data feminism, new materialist (de Freitas & Sinclair, 2014) and ecopedagogical perspectives dialogue with Paulo Freire’s (1970/2000) critical pedagogy in a classroom activity about wealth disparity. Students are challenged to graph data on Canadian household wealth (tellingly also termed *net worths*) using their own bodies in space, and to reflect on their perceptions, knowledge and feelings about the social issue, the activity, and their experiences of both. Previously (von Bülow, 2024), through a thematic analysis of students’ dialogue and reflections, I drew a parallel between a representation of data that refuses to act as a “technology of distance” (Porter, 1996, p. xxi) and students’ engagement both with the social issue as their own, and with themselves as subjects in a mathematics classroom. Here, I focus on how a physical, more visceral (d’Ignazio & Klein, 2020) representation of data finds its materiality in whole bodies in relationship, as each student’s graph cannot be completed without the others’ bodies; in parallel, it aspires to a “mutual construction of intersubjectivity” (de Freitas & Sinclair, 2014, p. 20), with each student’s role in the ‘space of net worths’ acquiring depth of meaning through the others’ relative position and the poignancy of the space in between. As discussed by Boylan (2017), “as well as the need for an ecologically informed curriculum, there is also a need for a critical relational pedagogy” (p. 1), since “current crises stem, at least in part, from the forms of our relationships with each other and with other beings in the world” (p. 2). I submit that the kind of activity I describe here, designed through Freire’s (1970/2000) pedagogy of liberation and given depth by data feminism (d’Ignazio & Klein, 2020) and new, inclusive materialism (de Freitas & Sinclair, 2014), is a resource for mathematics education because it is a *practice* for healing our relationality from alienation and disconnections from empathy, ethics, and our bodies.

THEORETICAL FRAMEWORK

Critical researchers and educators, aware of the relational entanglement of mathematics, school practices, and social, political and ecological concerns, grapple with the ways our educational practices work to position ourselves, students and society within this entanglement (e.g., Freire, 1970/2000; Coles et al., 2024). An ecopedagogical (Boylan, 2017) perspective considers the learning environment as always relational and adaptive, which coincides with Freire’s (1970/2000) pedagogy, in which it is students’ perceptions and knowledge about their relationships, with actors both inside and outside the classroom, that form a necessarily adaptive curriculum of inquiry. In *Pedagogy of the Oppressed*, Freire (1970/2000) develops a theory and method of liberating, challenge-posing education. In Freire’s method, educators first select elements of reality from students’ “background awareness” (p. 83), where they exist objectively but are not yet objects of cognition and action. These must be elements that are present in situations that involve *contradictions* in students’ lived experience. I understand contradictions as involving two or more aspects that are at some level irreconcilable; for example, an awareness of injustice, or harm, alongside a need, acceptance, or participation in that injustice. Each such situation is then “codified” into an object, image, or brief dramatization that “shows some of [the situation’s] constituent elements in interaction” (p. 105), in such a way that students will be able to recognize the situation, and to recognize themselves as subjects, together with other subjects, in the codification (p. 114). The codification presents “the theme only—no ‘solutions’!” (p. 122) to students, who are collectively *challenged* by it. Freire argues that, since students apprehend the challenge not as a theoretical question but within a context where the contradiction is interrelated with other issues, their responses tend to be critical and thus less alienated, and that in their responses they express their personal themes and views of the world. This reflexive response process, called *decodification*, stimulates shifts in “perception and knowledge” (p. 115). Students’ “*generative themes*” (p. 96, my italics), that is, questions and ideas that come from students, emerge, forming the curriculum for further investigation.

In a mathematics and social justice (MSJ) dramatic codification, students are presented with a challenge related to a social situation involving or modeled by mathematics (von Bülow, 2024). Students simulate the situation using a drama technique, such as role-play or frozen tableau, that is adapted to a classroom context. Thus, dramatic codifications centre elements that can otherwise be under-represented in mathematics classrooms, such as students’ bodies, movement, emotional expression, interaction, knowledge from lived experience, and awareness of the different spaces that we and mathematics can inhabit. When students collectively use their bodies to simulate mathematical models of social justice issues, they bring to the fore the materiality of mathematical concepts and the ways in which using different senses to represent, communicate, and experience these concepts *matters* to learning (de Freitas & Sinclair, 2014). I submit that MSJ dramatic codifications are opportunities for making our whole selves present within social contradictions while witnessing the other(s) being just as present. As such, they are *resources* for a mathematics education that seeks to participate in “the foundation of informed participatory and critical citizenship” (Coles et al., 2024, p. 13). Furthermore, as pedagogical practices, MSJ dramatic codifications strengthen the union of the socio-political with the affective and—through the centering of bodies and relational awareness—the ecological.

In the pilot study I present here, students are asked to represent data showing the distribution of wealth in households in Canada, but to do so using their bodies, as a *human graph*. Traditionally, students use points or bars to represent data even if it is about a social contradiction in which they themselves are involved; but such a representational choice means that the data is no longer “situated as a link in a chain of interaction between persons (to be understood and judged in communicative, social, and moral contexts)” (Barth, 1995, p. 65). As Porter (1996) points out, the language of quantification has historically been used to exert power over people, by creating a unified environment that objectifies people into social classes and supplants local cultures with methods that are validated as more systematic and rational (p. 77). Interacting with data about our wealth inequality at a distance, “as if the numbers spoke for themselves” (p. 92), works to minimize intimate knowledges, subjectivity, and solidarity. Refusing traditional choices of mathematical representations of data on paper or a board here means refusing to maintain or reinforce an alienating distance from our sense of (and the reality of our) belonging to a chain of interaction—an ecology. Just as codification/decodification is used by Freire (1970/2000) as a pedagogy that uses “limit situations”—which are literally situations that limit us—to challenge people to *be* more, likewise *refusal* “marks the point of a limit having been reached” (McGranahan, p. 320). To refuse to be limited by the use of mathematics as a technology of distance can be “generative and strategic, a deliberate move toward one thing, belief, practice, or community and away from another.” (McGranahan, p. 319). Such a refusal matters to critical mathematics, not least because, as Boylan (2017) finds in his analysis of scholarship on ecological selves, “ethical responses arise from embodied experience as relational beings” (p. 8). This understanding of the body as a resource for expressing our knowledge also underpins *data feminism* (d’Ignazio & Klein, 2020), which embraces emotion and embodiment as ways of knowing that foster accessibility and inclusion of much-needed but often invisibilized diverse perspectives and subjectivities. This has led to a re-creation of data visualization as “data visceralization” (p. 84), that is, as data representations that interact with our whole selves and refuse to reinforce false binaries and hierarchies, such as reason over emotion. D’Ignazio and Klein (2020) present a wide variety of examples supporting an ecopedagogical stance, showing that “when people make knowledge, they do so...from a situated, embodied location in the world[, and that] by pooling our standpoints...together, we can arrive at a richer and more robust understanding of the world” (p. 136).

In this human graph activity, pooling the stand-points of the students is what allows the graph to be created and gives meaning to each position. Challenging students to use their bodies and the space between them in a mathematical representation parallels the challenge to their feelings about the distances between us that are engendered by the social construct of wealth inequality. Additionally, the activity aims to encourage a shift in students’ perceptions of mathematics as something with which we feel the world, not just think it, and as relational, in that one can witness others feeling it and know that they witness one feeling it. Hence, I ask:

- Do students use language that indicates a collective learning experience during the activity?
- How do students view the use of the body as a resource in this mathematical representation?
- What actors does each student connect with their own reflections about the activity?

METHODOLOGY

Context and Description of the Activity

This study was conducted in a Simon Fraser University course called ‘Shape and Space’, for liberal arts students. Ethics was applied for and granted for this research. In class, the activity took approximately 45 minutes, and a questionnaire was given to students afterwards.

The activity began with a brief introduction that raised the issue of how things are distributed among people in Canada and, in particular, of how wealth is distributed. Students gave examples of different kinds of assets that could be included in the calculation of a person’s total wealth. I then recounted how I searched Statistics Canada for information on wealth distribution and found wealth data for population quintiles, in terms of households, that is, persons or groups who share a dwelling, including temporarily absent members. The table below was displayed, at first with the actual wealth numbers missing so that students could predict the values A-E.

Table 1: Distributions of Canadian household economic wealth [Source: Statistics Canada (2023)].

Canadian households ordered by wealth	Average wealth (net worth)
0 – 20%	A = -2,762
20 – 40%	B = 125,936
40 – 60%	C = 429,271
60 – 80%	D = 946,048
80 – 100%	E = 3,139,492

Before the actual values of A-E were revealed, students discussed where they would predict that average might fall, with respect to the letters A-E. After predictions were discussed, the numeric values A-E were given, and reactions and interpretations were shared. For instance, students discussed what a negative ‘net worth’ means. The average Canadian household wealth was revealed to be \$927,597.00 (Statistics Canada, 2023).

When asked what we might do with this data, graphing was suggested. I then asked, what shall we use on a graph; should it be little dots we call points? Maybe little wallets, or little people? This data is about us—we are the data points! Students were then challenged to each take on a role (i.e., a wealth category) and form groups so that they could physically graph the data. Groups set out to find room, inside or outside the classroom, to represent this situation using their own bodies in space, rather than using points or bars on paper or boards. Students simulated the situation by figuring out the relative distances at which to stand. The economic distances between people were symbolized and witnessed as spatial distance, and conversations ensued about how it feels, or must feel, to be in a specific position in that space of ‘net worths’. Once all the students were back in the classroom, one of the groups was invited to show their process and their human graph. The group spread out diagonally across the classroom, and the whole class was challenged to make sure that the spatial distances between group members really corresponded to the numeric gaps in the data. Rounding and estimating were employed, until the class was satisfied that the bodies were graphing the data. Students remarked on how far from the rest the person playing the role of the richest 20% of Canadian households stood, while the remaining four group members stood comparatively close to each other.

Data Collection and Analysis

Here, I analyze the reflective writing of students (referred to as S1-S22) about the human graph activity, described above, responding to the following prompts:

- What were your impressions, as your group worked to decide how far apart to stand in order to represent the differences in wealth?
- How would you compare graphing this data on the board/paper to making a human graph? What information (if any) is lost/gained in either case? What kind of data do you think is suitable for either case?
- In this activity as a whole, how did you feel, physically and emotionally? What (if anything) felt helpful, interesting, unpleasant, or ...?

I analyzed students' responses and found four themes: a move from I to we and from "just numbers" to something social in the activity; the presence of other human actors, as well as local issues, from outside the classroom, in the students' experience; the recruiting of non-human actors as well as ethical considerations, emotions, sensations and physical senses, in students' physical graph creation and explanations thereof; and students' conceptions of the location of mathematical representations within or without the body. For each theme, I give excerpts of student writing to illustrate the findings. Boldface was added to students' statements below in order to highlight theme keywords and flows.

Theme: Learning from I to we

Learning is described in several reflections as a move from I to we:

- S22: I didn't fully understand what was going on or what was being measured until **we** got into groups and physically acted it out.
- S8: I did think it was interesting at first how high the average was. Once **we** went into detail and talked about the richest it made so much sense.
- S9: At first, **we** were unsure of where to place each person, but after talking and working **together** more, we realized the depth to this activity.

Creating a physical collective graph took 'just numbers' to something social:

- S9: Simply seeing the **numbers on the board** doesn't have the same effect as a visual of the **social standings**.

Theme: Presence of other human actors from outside the classroom, as well as local issues

Students name "others" (family, anonymous, or well-known people), in explaining their understandings of wealth inequality and in asking questions about the issue.

- S18: I also started to think about where **my family and I** would land on this graph...This activity...enable[s] us to think about **what determines someone's wealth** and **what being wealthy really means to everyone**.
- S2: ...a humbling reminder of where I am, and likely will remain for the rest of my life. It's also nice to know that **many, many others are there with me**.
- S13: I was slightly frustrated on a personal level considering I most likely fall within the lower/middle half of group "B", and how much accumulated wealth/assets **people have in comparison to others**.
- S17: **What kind of people will be lost** (poorest people who are spending money on drugs, alcohols/poorest people who are spending money on their daily necessity)?

- S7: I considered **real life millionaires** such as Jeff bazos or Elon musk to help put into perspective how far apart I should stand from my classmates.
- S9: I felt very surprised throughout the activity because finding out how drastically different the two extremes were (0-20% and 80-100%) is heartbreaking to me. It is hard to know that **some people are living in complete poverty while others have an abundance of money**.

Some students also connect the activity with local issues, such as the local housing crisis. For example:

- S18: There are significant differences in wealth **here in Vancouver**.

Theme: Recruiting of non-human actors as well as emotions, sensations, physical senses, and ethical considerations as part of the activity

Paces, lengths along a line connecting their bodies, and the tiles on the balcony are recruited as units of measure for students. For some students, visual impact, and not just the relative distances, matters:

- S6: We used the **tiles on the balcony as units of measure** to make our estimates less approximated.
- S13: My group...physically created **our own line**...Everyone sort of stood equal **lengths apart** in a line, and only then did we begin adjusting via the data.
- S10: I was thinking **the extent of the wealth gap** being represented, **the available physical space**, and **the desired level of visual impact** since the classroom is not so big.

Emotions are clearly a part of the activity, as students share that they felt “surprised”, “shocked”, “unpleasant”, “disbelief”, “envy”, “saddened”, “disturbed”, “heartbroken”, “disappointed”, “jarred”, “frustrated”, “uncomfortable” and “sorrowful”.

Ethical considerations of the social issue and the mathematics (as in the image of the completed graph, the spatial gaps, data and numbers) are linked in several reflections:

- S6: I think that these **huge disparities** could be **easily rectified through respective taxing** based on income and pre-existing personal funds. I think that these **disturbing numbers** really showed me how much the **government protects the upper class and disregards the impoverished**.
- S13: I think it would be more interesting to **break down the data further**, not just people across Canada but being more specific with one’s ... **SES based upon one’s race/place of living/gender**...
- S19: **This imagery** exposes a **hierarchical structure** that often we tend to avoid, as it exists in our peripheral vision... And I suppose that **ignorance, conformity, or impunity continue to mark these distinctions, separating humans from humans**.

Theme: Locations of mathematical representations within and without bodies

To describe the human graph, students use emotions, different physical senses including memory, and a sense of being engaged ‘within’ the data; while in descriptions of graphing on a board or on paper they consider accuracy, expediency, and objectivity:

- *Human graphing*: tangible, impactful, engagement, tactile feel, realistic, easy to move, visual, easy to grasp, put it in perspective, effective, powerful, emotional impact, immediacy, visceral, drastic, emotional connection, eye opening, involved, thinking required, memorable; and

- *Graphing on board/paper*: objective, precision, overall adaptability, simple to do, accurate, exact, for complex variables, detail, useful.

Here are some related excerpts:

- S16: It is more **time efficient** to simply put up a slide of a graph to **get your point across**, but it is more **memorable** as a human graph.
- S10: Making a human graph...can be a more **visceral and impactful way to represent data**.
- S13: Emotionally something about **being physically present within** the content material is valuable to one's connection to the material.
- S3: The human graph can let you **visualize the actual differences**.
- S2: It's like the difference between **reading about the height** of the Eiffel Tower, and **standing at the base of it looking up**.

Discussion

The Data—capitalized to distinguish from the wealth data that the students graphed—from these twenty-two students suggests that this human graph activity functioned as a dramatic Freirean codification/decodification. The activity was also a relational learning experience, in that it “support[ed] a conception of being human which decentres the individualist sense of self” (Boylan, 2017, p. 9) and “challenge[d]...beliefs that mathematics is an abstract, disembodied and depersonalized discipline” (de Freitas & Sinclair, 2014, p. 14). Thus, such a relational learning experience constitutes a refusal of individualistic and depersonalized practices in mathematics education.

Firstly, the Data indicates a shift in students' ability to see themselves in relationship with others within the social issue of wealth disparity, and to perceive it as a contradiction in their lives and in society. As the activity challenged students to predict and then to compare their predictions to the actual data from the 2022 Canadian census, students gained awareness of their previous perceptions and knowledge about the issue of economic disparity. Generative themes emerged, in which students proposed investigations of the experience of the people behind the numbers, of intersectionality with other social and local issues, and of policies such as taxation. The Data shows students' awareness that they needed human and non-human others in order to make sense and to derive meaning from the activity. These shifts were accomplished by welcoming the bodies back into the data, that is, by affirming the body—and the ecology of the collective body—as a mathematical resource.

Secondly, students' use of language involving their whole selves, positioned *within* the data, may indicate shifts in traditional conceptions of mathematics as neutral, context-free and related only to thinking or cognition. The shift is towards relational knowledge, which “supports critical reflection by introducing value, ethics and affect” (Boylan, 2017, p. 4). The activity renders it clear that “the political and the material are inextricably entangled” and that affect is an “engine[] of mathematical activity” (de Freitas & Sinclair, 2014, p. 42). For example, students ponder the relative importance of effectiveness and precision, which are traditionally highly linked to mathematics, in contrast with qualities that enmesh themselves and the data, such as “visceral” or “emotional”. Students' actions and reflections show that they are grappling with the materiality of (school) mathematics in relationship with the collective (of whole selves, mathematics and social issues) capacity to affect and be affected (de Freitas & Sinclair, 2014).

CONCLUSION

In the human graph activity described here, students took on roles from data categories on wealth, and created a frozen tableau with their bodies to spatially represent the economic gaps. During the activity, mathematical objects were students' bodies, statistics were people's experience, and social concerns were emotions and voices in the classroom. Mathematics shifted from an abstract static discipline to one that is in relationship with the physical world and embodied (de Freitas & Sinclair, 2014). The data points, though flattened into apparently neutral positions on a table, are not equal; this fact, together with the fact that we, our bodies and the ways we relate to each other and the world, take part in this inequality, lie at the heart of the contradiction codified in the activity. The physical act of standing apart from each other and the witnessing thereof reverberated within and between students' bodies and their lived experiences; it is this inter-related collective that was recruited in the making of meaning. I submit that the activity thus functioned as a *practice* of relational knowing, which “points to awareness of our relationships with mathematics and with other actors” (Boylan, 2017, p. 4). This shift in awareness came through refusal, rather than passive acceptance, of traditional mathematics practices as “the given”. Refusal here is understood as a counter-story (Piacentini, 2024, p. 131), applied to what it means to represent data mathematically. The refusal to interact as if equality existed, or as if our bodies could be absent from the contradiction, opened possibilities for “redirecting levels of engagement” (McGranahan, 2016, p. 320) with each other. Thus, this experience of relating to each other within the space of net worths and the discomfort therein is not just for visibilizing oppression; ultimately, the aim is to connect with our inter-dependence and understand that systems of injustice involve all of us, mathematics, and the ways we relate in and to the world.

References

- Barth, F. (1995). Other knowledge and other ways of knowing. *Journal of Anthropological Research*, 51(1), 65–58.
- Boylan, M. (2017). Towards a mathematics education for ecological selves: Pedagogies for relational knowing and being. *Philosophy of Mathematics Education Journal*, 32.
- Coles, A., le Roux, K., Borba, M., Bose, A., Geiger, V., Gutiérrez, R., Makramalla, M., Solares-Rojas, A., Sinclair, N., Valero, P., Vistro-Yu, C., Leung, F.K.S., & Dorier, J-L. (2024). *ICMI Study 27 discussion document. Mathematics education and the socio-ecological*. ICMI Study 27.
- De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.
- D'Ignazio, C. & Klein, L. F. (2020). *Data feminism*. MIT Press.
- Freire, P. (2000). *Pedagogy of the oppressed*. Continuum. (Original work published 1970).
- McGranahan, C. (2016). Theorizing refusal: An introduction. *Cultural Anthropology*, 31(3), 319–325.
- Piacentini, T. (2024). *Developing a critical pedagogy of migration studies: Ethics, politics, and practice in the classroom*. Bristol University Press.
- Porter, T. (1996). *Trust in numbers: The pursuit of objectivity in science and public life*. Princeton University Press.
- Statistics Canada (2023). *Table 36-10-0660-01 Distributions of household economic accounts, wealth, by characteristic, Canada, quarterly (x 1,000,000)* [Data table].
- Von Bülow, K. (2024). Human graphs as mathematical dramatic codifications. In T. Evans, O. Marmur, J. Hunter, & G. Leach (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 193–200). PME.

REFLECTIONS ON ABSTRACTION IN A WORLD WHERE ATTACHMENT TO PLACE MATTERS

David Wagner¹, Annica Andersson², Sacha DeWolfe³, Julianne Gerbrandt¹, Hilja Lisa Huru⁴, Anita Movik Simensen⁴, Elisabeth Wirkola⁵

¹University of New Brunswick, ²University of South-Eastern Norway, ³Mount Allison University,
⁴UiT the Arctic University of Norway, ⁵Alta Ungdomsskole

As collaborators in research that listens and responds to the experiences of Indigenous and newly migrated mathematics students, we reflect on abstraction and detachment in the context of socio-ecological precarity. We situate our reflections in our experiences and find a common thread that recognizes tension among abstraction, place, and worlding.

ABSTRACTION AND DETACHMENT

In an earlier reflection, we drew on our experiences, especially in Indigenous contexts, to consider how detachment connects with abstraction (Huru, Andersson & Wagner, 2023). We described abstraction as a process that attaches an idea to an experience—e.g., the realization that a line carved into a bone can represent a real thing, or five such lines, or the symbol 5 referring to the five lines. But abstraction can also lead to detachment—e.g., when the symbol 5 is manipulated as a thing itself. We described abstraction as natural in mathematics and in language—e.g., your name is not you, it indexes you. We see abstraction as an action and thus as a move within a cultural context.

For Kven, Forest Finn, Mi'kmaq and other Indigenous people, attachment to traditions, stories, traditional artifacts and land are important to their identities (e.g., Huru, Räisänen & Simensen, 2018; Culligan, DeWolfe & Simensen, 2023). Such attachments may seem at odds with the detachment associated with abstraction. Rubel & Nicol (2020), in their promotion of place-based mathematics education identified a tension between abstraction and engagement with local contexts. Barwell (2013) described how abstractions can promote environmentally responsible action—e.g., to develop models that identify and predict climate trends. D'Ambrosio (2010) implored mathematics educators to acknowledge the role of mathematics in the destruction of ecosystems.

We reflect on abstraction and detachment in mathematics education alongside our hopes for sustainable futures for “all the relations” (an expression used by many Indigenous people in Canada to refer to all beings including the four-legged ones, the ones that fly, the ones that swim, the trees, the rocks, ...). As students, as teachers, as researchers, as Indigenous peoples, as settlers, we position our reflections as situated in our places, practices and histories to avoid jumping to synthesis (i.e., a premature abstraction of our reflections).

In our research projects in Canada and Norway focused on the experiences of Indigenous and newly migrated mathematics students, we expected to see students' subjectivities in their accounts of their experiences. Language and culture were mentioned, but none of the students made connections to the socio-ecological crises that are increasingly visible in our world. The absence is an indictment of mathematics education: they cannot imagine mathematics class addressing society's deepest concerns.

ANNICA: THE ABSENCE OF PLACE IN STUDENT REFLECTIONS

Genevieve Ryan’s poem (cited in Huru et al., 2023) deeply inspires my work as a teacher and researcher. Her words indicate feelings and experiences of detachment from mathematics. We know this feeling is common in mathematics classrooms (e.g., Andersson, Valero & Meaney, 2015).

In our Norway-based research project, I was part of many interviews with school leaders, in-service and becoming teachers, students, and community members. We asked about their wishes and hopes in the context of mathematics education. We invited them to think outside the box—if they had all the resources in the world, what would their dream mathematics education look like? Many becoming and in-service teachers wished for changes in the classroom environment—to make the teaching space more welcoming and creative—different furniture, colours and light, outdoors instead of indoors. It seems to me that they wanted to create physical spaces, often out in nature, where students would feel safe and creative to experience attachment with and in mathematics. A number of the students (c.f., Ryan’s poem), talked about detachment, being invisible and not understanding why they should learn this subject despite the fact that mathematics is talked about as being very important. They expressed abstraction as detachment from contexts familiar to them. What contexts would be familiar, and open up possibilities and spaces for attachment to mathematics?

The project aims to develop and evaluate strength-based pedagogies, building upon individuals’ strengths and assets and identifying past positive experiences. A challenge for us is to identify these pedagogies as the participants don’t talk about them—probably because they have not experienced them. For me, being Swedish and closely attached to nature and land—e.g., through *allemansrätten*—climate change is discussed often when I talk with youth and friends. Not surprisingly, it is a young Swedish woman, Greta Thunberg, who leads demonstrations and a global movement. Thunberg has related mathematics to her values and her rage. What could mathematics education be like if we took Greta Thunberg as its inspiration? Could critical mathematics education be part of this, if it is “driven by urgent, complex questions; is interdisciplinary; is politically active and engaged; is democratic; involves critique; and is reflexive and self-aware” (Andersson & Barwell, 2021 p. 7)?

JULIANNE: THE PURSUIT OF OBJECTIVITY

Some of my earliest memories of learning mathematics on the land that is now known as Canada, are of being seated at a desk in a classroom, pencil in hand, fluorescent lighting overhead—racing to finish sets of questions on paper, questions that had knowable and correct answers. It was important to me to find the right answer to each of these questions. I loved the comfort and security of school mathematics—a space where answers existed within a binary understanding of right and wrong, correct and incorrect. In this space, my younger self understood mathematics as a discipline offering tools to understand things objectively. It was a space where one could be good or bad at mathematics, where trivial and decontextualized calculations were intertwined with a developing personal identity. This space was controlled, contained, and competitive. School mathematics was learned in this space.

The concept of objectivity generally refers to being unbiased or impartial, and to work towards objectivity requires that we set aside our own perspectives so that personal values do not influence our judgements (Toole, 2022). I no longer believe in the possibility of objectivity in school mathematics. In my practice as an elementary mathematics teacher, I was not unbiased or impartial. I planned for my students’ learning, responded to their needs, and communicated their progress to

their families. I noticed which concepts caused difficulties and became curious about how I was teaching, which questions I was asking, and whose voices I was hearing. I discovered that teaching mathematics was deeply personal and unavoidably subjective. So how is it that my perception of school mathematics was so different when I experienced it as a student?

Now that I am an elementary mathematics teacher educator and a mathematics education researcher, I wonder about the lingering effects of school mathematics on the general public. I wonder whether people appeal to mathematics as a tool in the service of objectivity because they were taught that mathematics was value-free. I am reminded of the relationship between mathematics and objectivity when I reflect on the absence of sustainability in my course syllabus for student teachers. As I navigate the complexities of teaching mathematics concepts and mathematics pedagogy to future teachers of elementary mathematics, I continue to confront the impact of deeply rooted assumptions about the purpose and practice of school mathematics.

I offer a possible step towards a more sustainable approach to mathematics education. In elementary mathematics pedagogy, there is a practice of moving from concrete materials towards more symbolic representations of mathematical concepts. While this movement towards abstraction helps students to generalize, perhaps there is an opportunity to cycle between symbolic and concrete ways of learning, doing, and knowing mathematics. Rather than teaching content along a linear path, maybe a cyclical ebb and flow might remind us of relationships between subjective experiences and the yearning for objective understanding. Both are needed to engage mathematically with the world.

DAVE: PROBLEM SOLVING VS. MYSTERY

Motivated by experiences from my local political action, I reflect on how school mathematics (and my past work as a mathematics teacher) are complicit in leading our society to toxic relations and toxic ecologies. In an earlier meditation on subjectivity in mathematics education (Wagner, 2022), I lamented the poor understanding of mathematics among the people in my communities, even people with advanced education in STEM subjects. In that meditation, I called for positioning students as subject to mathematics, to experience the regulation and surveillance that mathematical structures inflict upon them. I called for mathematics teachers to provide such experiences for their students and to discuss these experiences with their students to better understand mathematics as a political force.

More recently, Gabriel Marcel's (1952/2009) distinction between *problems* and *mysteries* has helped me conceptualize subjectivity. The person engaging with a problem is not positioned as part of the problem. By contrast, "mystery is a place where I find myself (...) committed as a whole [person] in so far as I achieve a unity which (...), by its very definition, can never be grasped in itself, grasped as something apart from me" (pp. 40–41). Marcel did not write about mathematics, but I see most/all school mathematics under his rubric of problem. Even the work on sustainable futures I find in mathematics education research seems to be mostly problem-based in Marcel's distinction. When students are investigating and measuring socio- and/or ecological phenomena in their contexts it is problem-based unless they are doing mathematics to which they themselves are subject. For example, if students are given the responsibility to calculate/decide how much of a resource (maybe paper or water) to use in a year, it would be mystery—they use mathematics to assess their own participation in the (eco)system and to inform their decisions for action in that system. Responsibility and

subjectivity are key, with subjectivity including awareness of the impact of their experiences in their choices and including the kind of subjectivity that means being held subject to the decisions.

Marcel's (1952/2009) distinction is part of his reflections on abstraction: he distinguished between abstraction and the spirit of abstraction. He recognized the necessity of abstraction and saw it necessary for action because action requires distillation, reduction, or assessment of a situation. But the spirit of abstraction gives primacy to the abstractions and results in violence. My meditation on subjectivity, aided by Marcel, leads me to realize that the violences that underpin socio-ecological precarity are connected to the way we humans conceptualize others (our relations) in the world. I see mathematics classrooms as a good place to work on navigating between abstraction and the spirit of abstraction.

HILJA: LANGUAGE

My reflection starts with my first visit to Finnskogen (an area of the Forest Finns). The place is called Abborhøgda (Perch Hill) in Norwegian language and Yöperinmaki in Forest Finn language. The Norwegian name seemed strange as there is no lake nearby, and when we arrived we were greeted by butterflies in all sizes and colours. Exploring this mystery, I concluded that the name is from *moth*, which in Forest Finn language is something like *night butterfly*. My conclusion may be true or not true, but we see this with Norweganized names of Kven and Sami areas too: Probably *yöper* sounds like *abbor* (perch).

My work on language and culture in teaching and learning mathematics started with the realization, and hope, that reforming how mathematics is taught in schools has the potential to save languages and that language and culture can enhance students' understanding in mathematics (Huru, Räisänen & Simensen, 2018). Exploring words such as the example of *Yöperinmaki* is abstraction: students figuring out the parts of the word *yö-peri-n-maki* can be compared with finding the x , y and constants in a word problem. A place-based approach also explores with figuring ourselves out (c.f., Ryan's poem). Connecting it all to nature—surrounded by butterflies, hearing the water beneath the ice in the spring, feeling the wind and hearing the waves through using all the senses—intensifies learning experiences. The place-based aspect of sustainability, ecology and traditional knowledge and wisdom in nature added to the framework from Huru, Räisänen and Simensen (2018) would expand and strengthen teaching and learning mathematics.

With stable seasonal cycles, connection with nature allows us to know when to find these experiences. Organizing teaching and learning around the timing of nature could radically resist the “efficiencies” of modern schooling and the rigidity of counting hours and common core elements (the surveillance described by Sacha). Immersed learning experience—diving into language and nature, feeling and keeping the senses open, connecting with all beings, wondering with curiosity and learning the wisdom and experiences of those that have been here before us—will hopefully strengthen the wish to pass this experience and knowledge on to new generations. This has the potential to promote relational understanding in mathematics (c.f., Mellin-Olsen, 1975) adding in “all the relations.”

I see some possible futures, depending on if we manage to save the balance of the ecosystems from shifting too much. Night Butterfly Hill may be filled with butterflies for future visitors; or the place name could be the only reminder that there used to be butterflies here. If we continue to extinguish

these old, Indigenous languages and with them the ancient knowledge, no one will hear the sound of our people, and looking out from Perch Hill one will think that there must have been a lake here once.

ELISABETH: VALUES FROM HOME

As I am from Finnmark, a region characterized by strong Kven and Sami culture and traditional ecological knowledge, I reflect on how abstraction in mathematics education can affect students' connection to both their identity and environment. I am of Sami and Kven heritage, which deeply influences my understanding and appreciation of the natural world. Through my upbringing and professional experience, I have seen how abstraction in education can create a distance from the concrete, everyday experiences that are crucial for understanding and valuing our environment. Huru, Andersson and Wagner (2023) have gone further to explain how even attempts at integrating culture can cause disconnection and argued that teachers need to be cautious and intentional in their efforts.

In Finnmark, sustainability is not just a modern concept, but a way of life passed down through generations. By integrating this understanding into mathematics education, we could make abstract concepts like sustainability more concrete—e.g., mathematics problems calculating sustainable resource management, such as the optimal use of grazing areas for reindeer herding. I see great value in bringing context back into mathematics education, especially through the integration of ecological and sustainability thinking. This not only strengthens students' connection to their environment and culture but also promotes a deeper understanding of sustainable principles. By reflecting on these perspectives, I hope to contribute to a richer, more connected mathematics education that resonates with all students, especially those from minority and Indigenous groups.

Detachment in popular culture is something encouraged when you must leave something behind, often something that is toxic—something that does not serve you, even though you are emotionally attached to it. Detachment is for self-growth. In mathematics and in educational contexts, this detachment seems to suggest that culture, what students know from home, is toxic—it must be left behind. Students learn that their experience and culture do not serve them in the field of mathematics. They must detach.

Culture forms identity. By detaching in a subject, you also have to detach from the culture whose values have shaped you, thus abandoning your identity in your culture. One could argue that a child will start to form a new identity to mathematical values, abandoning their own. When you are nagged to put the mathematics in your culture on display (ethnomathematics), it is like asking someone to open old wounds from your detachment from something you loved but was not serving you.

As Huru, Andersson, and Wagner (2023) said, mathematics is not entirely without culture, and culture will always exist in mathematics. So, is it possible to see similar values in both cultures (both the Indigenous and the mathematical) through finding common values? One approach might be what Kimmerer (2013) did in science, namely asking, “What is our need?” and find how we can honour an inner calling, or an important value inherited from home.

ANITA: DETACHMENT IN INTRA-POSITIONS

I am inspired by a reflexive text about abstraction, attachment and detachment by Huru, Andersson and Wagner (2023, p. 11):

The purpose of mathematical conversations should not be to win an argument or to promote one's myths. In mathematics and in the mathematics classroom the tradition is to explore possibilities, preferably together, discuss until we agree, find a solution, a truth or at least a step towards that. Social activities are real because they have real people interacting with each other in a particular context and community.

My reading of the article about abstraction, attachment, and detachment is that conversation has a powerful place in the heart of mathematics education. I consider communication important, but also very difficult. Especially when the aim is to discuss abstract phenomena and to agree. Perhaps it will be helpful for the reader to follow my reflections if I describe my background.

I have difficulties defining my ethnic identity as my family is a mix of Norwegian, Finnish, Sami and Kven people. Several of my ancestors have lost their mother tongue, leading to breaks in the chain of communication within the family. To exemplify this, my father's parents did not speak Norwegian well enough to have abstract and philosophical conversations in Norwegian (which is the language I speak). My father's mother spoke Finnish and his father spoke Kven, Finnish and Sami.

I define myself as both a sheep farmer and a mathematics teacher educator. As a sheep farmer, I am living a traditional life, following natural cycles—the cycles of the seasons, the day, and nature itself. The lambs are born when the grass sprouts, and the sheep are collected in the mountains before the winter comes. I celebrate every new lamb, and I grieve the loss of those who did not make it. The interaction is fine-tuned to achieve sustainable sheep farming, a balance between human activity and nature. As a teacher educator, I observe that teacher educators are showing a growing interest in sustainable development and education (e.g., Tesfamichael & Enge, 2024). Sustainable development is also addressed In the Norwegian Core Curriculum: “Sustainable development as an interdisciplinary topic in school shall help the pupils understand basic dilemmas and developments in society and how they can be dealt with.” (Ministry of Education and Research, 2017, p. 16)

When reading this curriculum, I get a feeling of farming and mathematics education for sustainability as intertwined. Taking care of the life on Earth and caring for future generations are fundamentals in both positions. Despite this common ground, I sometimes feel that my positions as teacher educator and farmer are detached. I struggle to come to an agreement in my intra-communication. Often I feel that my teacher educator position talks about sustainability from a *knowledge* perspective and my farmer position talks from a *knowing* perspective.

SACHA: EMBODIED KNOWLEDGE

I always felt unsettled in classrooms as a learner and educator, and wondered how that space was honouring place. Learning and teaching felt disconnected and disembodied. Later in my professional life, I managed to break from institutional barriers and unleash my desires as an educator which led to fostering the desires of students and communities. This proved meaningful for them and carried a deliberate honouring of affective knowledge gained from our immersion in place.

What is a neighborhood for? This is a critical question when thinking about best practice in education. In subjects like mathematics, offered mainly in institutions, there is a focus on regulatory practice which may dislocate students from lived realities and require them to become disembodied. Renga (2017) noted, “The immersive experience of community participation constricts the heart's yearning, thereby tethering who we are and are becoming to what we love” (p. 272). The neighborhood is a place that welcomes the heart's yearning with knowledge that can be gained only in that place.

Lippard (1997) described relationship with place and with people as equally important: “Space defines landscape, where space combined with memory defines place” (p. 9). Severing the connection between place memory, you remove yourself from the storied knowledge, the embodied knowledge that we gain through our relationship with place. The concern is that by offering learning only in a classroom, educators may be disconnecting students from this vastly important form of knowledge. Are students lacking connection to issues of sustainability because they are removed from place?

Kimmerer (2013), writes aesthetically about relationship to land, plants, animals and how knowing their names creates an intimate relationship, one where we are never alone. She imagines about “species loneliness” which is a “deep, unnamed sadness stemming from estrangement from the rest of Creation, from the loss of relationship” (pp. 208–209). Educators should strive to foster this relationship in their teaching so that students are embedded in place, land, and relationships; an environment where “subjects” become an extension of that relationship. Low and Altman (1992), wrote that “One of the hallmarks of place attachment that appears consistently in most analyses is that affect, emotion and feeling are central to the concept” (p. 4). These elements are often left out of conversations in the classroom and replaced with efficiency, test scores and surveillance. Grappling with concepts like this led to the realization that learning is feeling, and place is vastly important to relationships, and relationships are pivotal to connection.

Offering courses in communities has provided me a more nuanced analysis wherein I could observe students in place, connecting with land and people and fostering relationships through learning. Interestingly, in the place-themed courses I have offered, land has never been a dominant theme—drawing from hundreds of conversations with students and First Nation community members. The conversations focused on the importance of relationships. This points to a very real issue of understanding place in the context of how students are experiencing it. If I stand at the front of a classroom and lecture about how to take better care of Mother Earth or how to solve a mathematical problem, but never honour stories and experiences of place and the embodied knowledge that lives there, then I am doing a disservice to students. Reflecting on why community members may not have mentioned land as a focus of fostering place, I eventually realized that when community members said that relationships needed to be fostered they were talking about place, land, and learning.

RESPONSE: RESPONSIBLE RELATIONAL MATHEMATICS

Our reflections position situated experience as central to learning. Anita, Dave and Sacha position community engagement as the work of living responsibly. Annica, Hilja, Elisabeth and Julianne yearn for classroom spaces that connect with place and sensory experience, and recognize that school math provides alternative sensory experiences that detach students from their community spaces. Some of us have Indigenous roots in the lands claimed by Canada and Norway, which may impact our reflections, but we see the call for community-centered mathematics as not exclusively Indigenous. Instead of foregrounding resources in mathematics education, we advocate relations—mathematics as part of the way humans live responsibly in relation to community (needs). Positioning land, other beings, or objects as resources somehow permits humans to exploit and do violence to them. De Freitas (2024) suggested the idea of worlding instead of modeling in mathematics. She described modeling as reductive—subtracting experience, sense and place from ideas—and described worlding as building and elaborating. We see opportunities for engaging students as responsible actors in their spaces through world-making interplay between abstraction and connecting with the places students

occupy and imagine. In this way, learning can be seen as a process of becoming, which Wenger (1998) describes for an individual, but we see as becoming in relation: “Learning [is...] a process of becoming (...) a certain person. (...) We accumulate skills and information, not in the abstract (...) but in the service of an identity, [which] can become a source of meaningfulness and of personal and social energy. (...) learning entails both a process and a place” (p. 215).

Acknowledgements

This research is part of projects funded by Canada’s Social Sciences and Humanities Research Council (David Wagner, principal investigator (PI)), and the Research Council of Norway (Annica Andersson, PI).

References

- Andersson, A. & Barwell, R. (2021). Applying critical mathematics education: An introduction. In A. Andersson & R. Barwell (Eds.) *Applying critical mathematics education* (pp. 1–23). Brill.
- Barwell, R. (2013). The role of mathematics in shaping our world. In A. Coles, R. Barwell, T. Cotton, J. Winter, & L. Brown (Eds.) *Teaching mathematics as if the planet matters* (pp. 3–15). Routledge.
- Culligan, K., DeWolfe, Simensen, A. (2023). Challenges, opportunities, and key questions in research for mathematics in Indigenous and migrational (MIM) contexts through a language-focused lens. *Journal of Mathematics and Culture*, 17(3), 145–161.
- D’Ambrosio, U. (2010). Mathematics education and survival with dignity. In H. Alrø, O. Ravn, & P. Valero (Eds.), *Critical mathematics education: Past, present and future* (pp. 51–63). Sense.
- De Freitas, E. (2024). Xenomathematics. *Canadian Mathematics Education Study Group*, Québec, Canada.
- Huru, H., Andersson, A. & Wagner, D. (2023). “...but you cannot buy the sunset”: Abstraction, attachment and detachment. *For the Learning of Mathematics*, 43(3), 7–12.
- Huru, H.L., Räisänen, A.-K. & Simensen, A.M. (2018). Culturally based mathematics tasks: A framework for designing tasks from traditional Kven artefacts and knowledge. *NOMAD: Nordic Studies in Mathematics Education*, 23(3–4), 123–142.
- Kimmerer, R. W. (2015). *Braiding sweetgrass*. Milkweed Editions
- Lippard, L. R. (1997). *The lure of the local: Senses of place in a multicentered society*. New Press.
- Low, S. & Altman, I. (1992). *Place attachment: A conceptual inquiry*. Springer.
- Marcel, G. (1952/2009). *Man against mass society* (9th edition). St. Augustines Press.
- Mellin-Olsen, S. (1975). *Instrumentalisme som pedagogisk begrep. Belyst ved undersøkelser av matematikkundervisning* [Instrumentalism as a pedagogical concept. Exemplified with investigations of mathematics teaching]. Universitetet I Bergen.
- Ministry of Education and Research. (2017). Core curriculum—values and principles for primary and training. Laid down by Royal decree. The National curriculum for the Knowledge Promotion 2020. Retrieved from <https://www.regjeringen.no/contentassets/53d21ea2bc3a4202b86b83cfe82da93e/core-curriculum.pdf>
- Renga, I. (2017). Unpacking a liturgical framing of desire for the purposes of educational research. *American Educational Studies Association*, 53(3), 263–284. <https://url.za.m.mimecastprotect.com/s/tlKTC76y5XtAKLO8Kf8fJHo6BvK>
- Rubel, L. H., & Nicol, C. (2020). The power of place: Spatializing critical mathematics education. *Mathematical Thinking and Learning*, 22(3), 173–194. https://url.za.m.mimecastprotect.com/s/zIpTC8qz5ES609720s1hyHyG_yr
- Tesfamicael, S., & Enge, O. (2024). Revitalizing sustainability in mathematics education: The case of the new Norwegian curriculum. *Education Sciences*, 14(2), 174.
- Toole, B. (2022). Objectivity. *The Philosopher*, 110(2), 35–39.
- Wagner, D. (2022). Subject to mathematics. *For the Learning of Mathematics*, 42(1), 35–39.
- Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge University Press.

THEME D – MATHEMATICS EDUCATION FUTURES

A KIND OF MATHEMATICS THAT HELPS ME TELL AND HEAR STORIES

Yasmine Abtahi

University of South Eastern Norway

“The universe is made up of stories, not atoms.” –Muriel Rukeyser

Research shows that including stories in mathematics classroom activities enriches the student’s learning experience by making lessons more engaging and relatable. Storytelling helps students develop a more nuanced understanding of social and cultural issues. In this paper, I explore the power of storytelling as to how the inclusion of stories in mathematics classrooms, can highlight relationality as an epistemology, and mathematics as a system knowledge. This is important because understanding relationality allows us to better connect issues values of social and ecological justice to the learning and teaching of mathematics. I conclude by an example and my reflections on it.

STORYTELLING IN MATHEMATICS EDUCATION

Egan (1988) claims that stories are not just some casual entertainments; it reflects a basic and powerful form in which we make sense of the world and experience. Similarly, Blyth and Aslanian (2022) explain that, it is through stories that our understanding of ourselves in relation to the world, – of what it means to be a human, and a part of more-than-human – works are developed. The ideas of storytelling have been used in mathematics education from different angles such as the teaching and learning of mathematics and in teacher education programs. Teaching and learning mathematics through stories have multiple positive effects, including promoting the retention of learning (Gulten & Gulten, 2004), having students remain interested and engaged in mathematical activities for a longer period (Sertsoz and Temur, 2017) and increasing students’ motivation for the learning of mathematics (Albool, 2012). Balakrishnan (2008) noted that stories facilitate generalizing mathematical ideas, presenting mathematical ideas from a different perspective, as they help students look for links between story elements and real life. Balakrishnan elaborates that storytelling is a way for students to discover connections with everyday life and a way to naturally associate them with the mathematics that they learn at school. He finally concluded that stories should be generated carefully enough to enhance students’ creativity. Stories have also been used in mathematics classrooms to help students think about the challenges of the world around them. To explore 13–15 year-old students’ capacity to apply mathematics knowledge and skills in the challenging real world, Albano and Pierri (2016) focus on the exploitation of digital storytelling. They explained that digital storytelling in mathematics classrooms can improve the student’s capabilities to face and solve such challenges.

Altintas (2018) explores storytelling within the context of mathematics teacher education. To gain experiences through the story generation process, the study reported on how primary mathematics preservice teachers created stories for teaching middle school students’ mathematics. They further evaluated the use of stories in mathematics teaching. They showed that mathematics teaching through stories “raises interest in topics and provides clarity, promotes love for mathematics and imagination,

fosters the reinforcement and retention of mathematics topics, facilitates establishing connections with everyday life, fosters student engagement, can be used as a tool of role-playing and as a technological support, promotes students' mathematics achievement, facilitates understanding abstract and difficult mathematical concepts, and offers such benefits as imagination and retention" (p. 17). The participating students also reported that the story-generation process helped them to gain empathy and creativity. Walters, et al. (2016) also explored how digital storytelling helped pre-service teachers in critical thinking in mathematics. Their results indicated that the construction of digital stories aimed at increasing student engagement, motivation and knowledge.

The common thread between many of these studies is the usefulness of telling stories in teaching and learning mathematics since such inclusion helps to motivate students, keep them engaged and help them make broader meaning in relation to the world around them. From these perspectives, the stories have the ability to situate the mathematical concepts in histories, cultures, socially or personally. Hence, humanizing not only context in which they happen, but also mathematics itself. I think, it is through this process of situating and humanizing that issues of social and ecological justice are brought to the forefront, in the teaching and learning of mathematics. In the following, I present interrelational epistemology and the epistemological power of stories in situating and humanizing the system of knowledge that is called mathematics and its teaching and learning.

THE POWER OF TELLING AND HEARING STORIES: ETHICS, EMPATHY, HOPE

As I previously stated, my ethical stance is rooted in a simple place. Saadi, 750 years ago:

Humans are members of a whole	In creation, one essence and soul.
If one member is afflicted with pain	Other members uneasy will remain

These four lines tell me we are selves-in-relation; we need empathy in understanding the pain of others; and we need to mobilize actions, when other members are uncomfortable. This is the ethical place from which I think and I hope to act. I have placed a special emphasis on the "Humans are members of a whole" (selves-in-relation) to describe **interrelational epistemology**, as a way of becoming to know things, that considers learners as selves-in-relations rather than isolated individuals. Within this epistemological view, **knowledge** is an understanding of one's position in webs of inter-relationality. Different relationships can give rise to different knowledges, therefore, knowledges change over time, as relations evolve. And finally, **mathematics will be** a system of knowledge that makes us aware of our positions in relation to all that we are in relation to and interdependent with; the trees, bees, ants, peoples, reindeer, and oxygens (see Abtahi 2022 for more details). The following web of nodes and lines (Figure 1) could be a visual representation of this epistemological structure, at two levels: the representing the learners and representing the knowledges. There lines and nodes, show the concept of relationality, in showing how one is -in-relation to many things and how knowledge is fluid and interconnected and how it might change as relations move (evolve).



Figure 9 Representation of interrelationality

Now, where does story fit in this structure? And why it is important? As mentioned earlier, stories help us make sense of the world and our experiences. Additionally, they deepen our understanding of ourselves in relation to others. Let's assume that Figure 2, is a second snapshot of Figure 1, at a point in time, after a story has been told. When a story is voiced



Figure 10 Representation of interrelationality in time, after a story

and heard, then we have different understanding of ourselves, in relations and different and different sense of the world. Therefore, Figure 2, now has other lines and the nodes are not the same size and are different colors. Within the structure of interrelational epistemology, these are the ways in which stories hold power:

- A) Telling and hearing stories can serve as a way of becoming to know which considers the learner not as an individual but as a self who is in relations (interrelational epistemology).
- B) Telling and hearing stories can serve in understanding of one's position in webs of interrelationality (knowledge)
- C) Telling and hearing stories through different systems of knowledge (mathematic) makes us aware of our positions in relation to, and our responsibilities towards, the trees, bees, ants, peoples, reindeers, and oxygens that we are in relation to and interdependent with.

Let me tell a factual story.

UN World Food Programme (U.N. WFP) declares that global hunger is one of the world's most solvable problem (2023). However, in recent years, the number of extremely hungry people has more than doubled. Estimated in 2023, 828 million people on the planet were hungry – that's 2.5 times more than the entire population of America. Of those 828 million, 345 million are experiencing the most severe levels of hunger including starvation (U.N. WFP, 2023). This is due to change of climate causing extreme weather, the socioeconomic impacts of the COVID-19 pandemic, causing rise in the

food costs, and conflicts. July of 2021, U.N. WFP said that it would take an estimated \$40 billion each year to end world hunger by 2030.

To end world hunger by 2030	\$40 billion yearly	\$360 billion eradicates world's hunger
-----------------------------	---------------------	-----------------------------------------

On the other hand, Stockholm International Peace Research Institute (SIPRI, 2024) reports that the world military expenditure in 2023 was \$2443 billion. This is an increase of 6.8 per cent from 2022. The following is the breakdown of some numbers and some interesting facts:

Military expenditure in 2023

Stories of interrelationship

The world \$2443 billion	increase of 6.8% from 2022	That is \$165.4 billion
Russia's \$109 billion	This is 57% rise since 2014	That is \$62 billion
Israel \$27.5 billion	24% increase since 2022	That is \$6.5 billion
NATO \$1341 billion	US holds 68% of total NATO military spending	That is \$916 billion
China \$296 billion	Directing much of this money goes to boost the combat readiness of the People's Liberation Army	

As an example of interrelational epistemology, this is a story, if shared in mathematics classrooms could promote knowing in which individual learners are seen as part of a broader web of relationships. For example, assuming that our current students are future citizens who vote, this knowledge could give them ethical awareness to demand more information and action from the political parties. The knowledge of mathematics in this story makes us aware of our positions in relation to, and our responsibilities towards, whatever we are in relation to, in the case of this story eradicating the world's hunger. By simply doing the mathematics, we can see that the yearly expense of \$2443 billion could have eradicated world's hunger 6.78 times. We can also see that only the increase in expenditure could achieve 5 years towards eradicated world's hunger. This is important, because in 1953, Eisenhower, US president and Commander of Allied Forces in WWII said: "Every gun that is made, every warship launched, every rocket fired, signifies in the final sense a theft from those who hunger and are not fed, those who are cold and are not clothed." It is now 2024.

NONE-CONCLUDING THOUGHTS

In this text, I explored the power of telling and hearing stories. Research has shown that including stories in mathematics classrooms is useful because it helps motivate students, keep them engaged and help them make broader meaning in relation to the world around them. This paper elaborated on the power of stories; that can help us understand mathematics not just as groups of fragmented

knowledge but also understand the positions of relationality the mathematical knowledge puts us within. I finish with a message of hope. Hope, for Rorty, is more than a state of mind and a goal in action. Rather, he understands hope as a narrative, ‘a story that serves as a promise or reason for expecting a better future’. The example of Gutstein and Peterson, above, can serve as a story of hope. That is to help our students understand that if governments reduce excessive military spending and instead reallocate those resources toward more meaningful initiatives, we can actually make significant progress toward achieving the Millennium Development Goals (MDGs). By shifting priorities in this way, it becomes clear that a better future is not only possible but within reach.

References

- Abtahi, Y. (2022). What if I was harmful? My experience of teaching of the dominant mathematics. *Educational Studies in Mathematics Journal*.108. 149–165.
- Albool, R. (2012). The effect of utilizing storytelling strategy in teaching mathematics on grade four students’ achievement and motivation towards learning mathematics. *International Conference The Future of Education* 2nd Edition. Retrieved from <https://fada.birzeit.edu/bitstream/20.500.11889/2451/1/8466.pdf>
- Altintas, E. (2018). Analyzing Students' Views about Mathematics Teaching through Stories and Story Generation Process. *Educational Research and Reviews*, 13(7), 249-259.
- Balakrishnan, C. (2008). Teaching secondary school mathematics through storytelling. (Unpublished master dissertation). Simon Fraser University, Canada. Retrieved from <http://www.peterliljedahl.com/wpcontent/uploads/Thesis-Chandra-Balakrishnan.pdf>
- Blyth, C. & Aslanian, T. K. (Eds.). (2022). *Children and the power of stories: Posthuman and autoethnographic perspectives in early childhood education*. Springer Nature.
- Gulten, D. C. & Gulten, I. (2004). *Binom acilimi ogretimine farkli bir yaklasim*. *Ilkogretim-Online*, 3(2): 60-66, Retrieved from <http://ilkogretimonline.org.tr>
- Gutstein, E., & Peterson, B. (Eds.). (2005). *Rethinking mathematics: Teaching social justice by the numbers*. Rethinking Schools.
- Sertsoz, A. Temur, O. D. (2017). 6 yas cocuklarina oykulestirme yontemi ile verilen matematik egitiminin cocuklarin matematik basarilarina etkisinin incelenmesi. *Acad. J. Educ. Sci.* 1(1): 1-20.
- U.N. WFP (2023). What Would It Cost to Feed the Entire World? Retrieved from <https://www.wfpusa.org/articles/how-much-would-it-cost-to-end-world-hunger/>
- SIPRI (2024). Global military spending surges amid war, rising tensions and insecurity. Retrieved from <https://www.sipri.org/media/press-release/2024/global-military-spending-surges-amid-war-rising-tensions-and-insecurity>
- Walters, L. M., Green, M. R., Goldsby, D., Walters, T. N., & Wang, L. (2016). Teaching Pre-service Teachers to Make Digital Stories that Explain Complex Mathematical Concepts in a Real-World Context: The "Math-eo" Project, Creating "Cool New Tools". *International Journal for Technology in Mathematics Education*, 23(4).

MATHEMATICAL AI-MODELING IN THE SOCIO-ECOLOGICAL: FUTURING CRITICAL MATHEMATICS EDUCATION

Christian H. Andersson¹, Paola Valero²

¹Malmö University, ²Stockholm University

Mathematical AI-models are a force in socio-ecological crises that mathematics education cannot ignore. Through a dialogue, we explore how the capacity and procedures in AI-mathematical models alter the power of mathematical modeling in society. We address the entanglement between AI-models, the economics of the ongoing fourth industrial revolution, and the socio-ecological. We also explore the implications for mathematics education. Three aspects of mathematical modeling are suggested to be of specific importance for how AI-models pose risks in the socio-ecological: transparency, scale, and the relationship between model and the phenomena that is being modeled. We propose making the examination of mathematical AI-models a central part of future imaginations of a socio-ecological matheracy.

PROLOGUE: AI IN THE SOCIO-ECOLOGICAL

The entanglements of AI and the socio-ecological are many. On the one hand, it is possible to argue that AI can make even more powerful the multiple functions of mathematical models in knowing about, managing and monitoring many of the technologies that support social and environmental changes towards a more sustainable direction. Smarter, and more efficient data managing and processing on all aspects of the environment and society using AI can help to such an end. On the other hand, it is also possible to argue that the very same AI is associated with a worsening of current socio-ecological crises. While the modeling in AI presents new enlarged technological promises, it also poses augmented risks. In this, AI is no different from any other technology of the past. Yet, AI may do both at a larger extent: capacities of prediction go hand in hand with lessening of human agency; increased computing power uses electricity and hardware that contribute to larger pollution and CO₂ emissions; and the efficiency of access to complex information and creative potential displaces human workforce and threatens some human qualifications to become obsolete.

How are these double gestures significant for mathematics education in the socio-ecological? Paraphrasing the two key questions for this ICMI Study: What does the mathematical AI-modeling in the socio-ecological mean for mathematics education? Inversely, what can math education mean for mathematical AI-modeling in the socio-ecological? Grounded on our current research, namely, Andersson's PhD project on discourse and power in mathematics education related to learning about societal aspects of AI (Andersson, 2021) and Valero's discussions on power in mathematics education in a time of climate change (2023), we explore these questions through the discussion of three interrelated topics: the changes in mathematical modeling and their connection to power; the connections among AI-modeling, the use and management of data, and the advancement of current forms of capitalism; and possible imaginations for a renewed critical mathematics education (CME) in the time of the socio-ecological with AI-models. Given that we are exploring "what is not yet the case" (Skovsmose & Borba, 2004), our critical reasoning and imagination unfolds in a dialogue between us, to make explicit how our different, yet related, research positioning comes together.

CHANGES IN MATHEMATICAL MODELING AND POWER

Christian: It seems to me that mathematics education research has a lot to offer in terms of theories and ethical perspectives on mathematical modeling. But it also seems to me that the new machine learning modeling techniques – AI – are fundamentally different from traditional modeling, and that new theoretical perspectives may be needed, especially in relation to the agency of the human, the role of the machines, and their effects on social justice (Andersson, 2021). And of course, all of these in relation to how they play in mathematics education (Andersson, et al., 2024). For instance, in traditional mathematical modeling following old protocols on scientific methods, a human formulated a hypothesis. Then criteria for rejecting the hypothesis (or null hypothesis) were formulated as a statistical test. No data was collected prior to this, since that was believed to risk objectivity. If the hypothesis survived the test, the credibility of the hypothesis increased. The tests involved a lot of data; so, the tests were run on computers. In these processes, humans generated ideas for models, and computers worked as gatekeepers for those ideas; gatekeepers in the sense that they had a deciding role in filtering out which presumptive models would be allowed to be considered as valid. In contrast, in AI-modelling it seems to me that it is the other way around, especially with representation learning where the computers find complex patterns in the data and use these patterns as building blocks for models. There is little need for humans to formulate any hypothesis; training data can directly be fed to the computer. The computer runs algorithms to identify patterns —or representations— in the data and generates a model based on these patterns (Goodfellow et al., 2016). A human monitors the process and evaluates if the model is sufficiently trained. If human judgment infers that the model is not up to standard, then it is re-trained until it is. So, in a sense, now machines generate ideas and humans are the gatekeepers deciding what is valid. I think we need a new language for this shift in roles and what people and machines can do, since it fundamentally redefines mathematical modelling.

In addition, we probably need to foreground the ethics of human judgment in the gatekeeping of models. Data industry is primarily dominated by certain perspectives – white and male – and by economic interests. There is also a risk that the gatekeeping function will fail due to lack of awareness of possible algorithmic bias towards minorities (D’Ignazio & Klein, 2020). What do you think about new mathematical modeling techniques in relation to power and math education?

Paola: First, it is important to acknowledge that mathematics as modeling tools have been central for critical mathematics education, as Skovsmose (1994) pointed already 30 years ago. Skovsmose examined the critical role of mathematics as part of the many models present in techno-scientific formatted societies. At that time, the digital revolution that we have today was probably starting, yet computational power to process large quantities of data, distributed globally in the Internet was not yet there. Many researchers have seen mathematical modeling both as an important mathematical competence to acquire, but also as an activity to be critically examined (e.g., Jablonka & Gellert, 2011). However, you pointing to how, in the recent expansion of AI technologies, there are onto-ethico-epistemological differences to consider when we think the relationship between humans and the mathematical models that enable AI; and what this may mean for the aspirations of connecting mathematical modeling and mathematics education in a time of socio-ecological crises. So, Christian, is this difference connected to the role that humans play in operating with models, to the models —mathematically speaking—, or to the processing capacity of the machines and algorithms that make the models run, even generating the sense that their agency becomes more and more independent of

human agency? Is it that AI models are in themselves different from other types of previous models that mathematics educators have before brought into mathematics teaching and learning processes?

C: For your second question, I would say yes. AI models are different for several reasons. First, they are more potent in the sense that we can now build models that were impossible before. Second, the methodology of building these models follow new epistemological paths. Third, and following from their previously unmatched capabilities, their impact on societies, ecologies, and sustainability is far greater. For your first question, I would say that it is both. The training of AI models can now go far beyond human imagination. As mentioned, the technology is no longer limited to which hypothesis humans can imagine. This changes the agency of humans, machines and algorithms. However, the machines cannot do this on their own. They still depend on human involvement. Humans still make overarching design decisions and retain the important gatekeeping function. There is some sort of symbiotic relationship, where humans and machines are co-dependent. It is the role of the humans to make ethical decisions. Skovsmose's work is very useful here. He describes meta-knowledge in relation to critically assessing models (1994). He also describes and critiques the banality of mathematical expertise for sometimes lacking the consideration of social and ethical aspects (2020).

In short, the *mathematical aspects* of the new AI-modeling move more and more towards the machines. But that movement foregrounds the importance of ethical awareness and responsibility of (human) mathematical expertise. It would be a simplification to say that the machines are doing the mathematics, and humans the ethics. Rather, the roles of machines and humans are entangled and so are mathematical and ethical reasoning. For instance, in my PhD study, pre-service teachers display more mature reasoning if they synthesize mathematical and ethical reasoning when discussing Big Data applications (Andersson & Register, 2023). To prepare teachers to teach in an AI-dominated world, the tradition of CME needs to be carried on in connecting models to ethics and the socio-ecological. But also investigating what the human-machine entanglement brings to that tradition.

AI MODELING, DATA AND ECONOMY

P: An argument for working with mathematical modeling in mathematics education is that an important aspect of *matheracy* — mathematical literacy as the critical competence of developing a sense of the formatting power of mathematics in society (Skovsmose, 1994). Even if how and to what mathematics education is thought to respond to changes in time, recognizing and taking a stance towards mathematical modeling has been important (e.g., Geiger & Schmid, 2024). The 'critical stance' has also studied how mathematics and mathematics education are central to particular types of economic, capitalist relationships (e.g., Pais, 2014). In this view, it may not be enough to see through the potential risks generated with/through mathematical modeling. The inseparability of mathematical modeling from the production of capitalist surplus value and exploitation are also central element of critique. How do you see this relationship?

C: I think this relationship is important for critical research. Characteristics of economic systems impact the socio-ecological, such as capitalism's dream of endless economic growth, or the Marxist view on nature as an infinite source for humans to control and produce rather than to care of.

P: Not only is mathematics linked to capitalist exploitation; the latter is at the center of socio-ecological crises. We can't talk about the latter without the former, and without considering the characteristics of the production of AI-models...

C: The current data driven economy is centered on training AI-models and selling their output. Algorithms need for data seems to have rewired the economic system, where providing data is becoming a currency. Data transactions are being done between companies, and that pays for consumers' access to online services such as social media, cloud services, etc. Varoufakis (2023) calls this new economy as *techno-feudalism* or *cloud-capitalism*. He points to a new configuration of existing and new economic relationships highly mediated by data intensive, AI-driven technologies. This economic system is central to the ongoing fourth industrial revolution, which poses both promises and challenges for humans and non-humans, all of which cannot be ignored when defining and addressing current socio-ecological crises. How do you see the entanglement of the data driven economy, mathematics and new challenges for the socio-ecological?

P: I think we need an example. Straehler-Pohl (2017) discussed the challenges that processes of de/mathematization pose to a critical matheracy, in relation to current forms of digital technologies. He is concerned with the ideological effect of de/mathematisation with technology in a capitalist configuration. He defines mathematization as the process in which mathematics and the world become entangled and undistinguishable, and where a high degree of expertise to recognize the mathematization is at stake. Even though we know that technologies require a lot of mathematizing done by some engineers or experts, the it is hidden to the users. Since experts are behind, the dialectics of de/mathematization generates a sense that mathematized realities and objects are —almost— infallible, certain. Connecting to our discussion, we could further say that the sense of certainty is augmented as we are aware that AI de/mathematization not only builds on big data, but also has been designed to constantly revise and fill in gaps of information, to produce —with a strong sense of agency— more and more precise and accurate results. Additionally, de/mathematization occurring within and through a wide range of digital, AI-based technologies brings closer de/mathematization and capitalization, as Varoufakis argues.

C: Jablonka (2017) also critically points out that discourses among the experts on the new mathematical technologies may attribute too much agency to machines, and thereby risk to conceal the responsibility of the experts themselves. Examining the ideology operating among experts and through processes of de/mathematization is important. But there is one more layer to unearth here. AI-algorithms can learn patterns that exist in the training data, but that no one has explicitly set AI to learn. For instance, stereotypes. A well-known example here is the young woman who gave instructions to an AI to make her photo look more professional and the AI provided a new picture in which she turned white (Bhaimiya, 2023). If an AI-model is trained on huge amounts of data, e.g. scraped from the internet, it may capture the stereotypical perspectives portrayed in that data. Huge training data sets can reflect what is said, written and done online collectively by humans. For instance, there might be a correlation in the training data between 'being white' and being portrayed as 'professional'. Not because this is fair, but because that is unfortunately what society looks like. This is an issue for AI companies. They do not want their AI to reproduce racist stereotypes, so they try to remove this kind of learning from training. But it can be difficult since structural racism is a very strong societal pattern, and AI-algorithms are prone to learn such patterns. Something similar can happen with sexism and other mechanisms of difference in society. Furthermore, such injustices may unfortunately be reproduced in how we all speak and act, and that includes actions by members of minoritized groups themselves. There is not necessarily a clear oppressor-oppressed dichotomy here, rather these injustices risk being amplified by AI-models.

Coming back to the economy, there might be two modes for capitalist ideology to operate in AI-technologies. First, we have that tech companies use AI to make profit. Aiming for profit may format how their experts act, as we have talked about. But second, what if notions of capitalism are also ingrained in the training data for AI-models? Just as stereotypes are? Given the global dominance of capitalism, it is likely that training data from the internet also amplify positive discourses on capitalism. But in contrast to the stereotypes on people, there is no uproar about AI learning such positive discourses on capitalism. Thus, it is not seen as an issue by the tech companies. Which means that AI-models are left free to learn and through their output reproduce a capitalist world view.

P: In other words, cultural and economic scripts are being “algorithmized” into the AI-models, and then can even more easily enter new arenas and be reproduced. It becomes even more difficult to break the existing logic, the values and behaviors that have produced the socio-ecological crises. And how do we address this in research?

C: Across the studies in my PhD project, I am starting to see a pattern that may relate to that. Both students and pre-service teachers can easily see that their digital traces can say a lot about them, especially if a mathematical model is used on the data. For instance, those models only need as few as a 100 data points on what you have liked on Facebook to predict several of your characteristics. This includes potentially sensitive information such as self-identified gender, sexual orientation, political views, and religion (Kosinski et al., 2013), all of which are considered sensitive personal data protected by new data regulations. You would perhaps hesitate to share this information with people you do not trust, and you would perhaps not post publicly online. However, your digital traces still reveal this information to whoever gets your data and applies AI modeling techniques on it. Students and pre-service teachers realize that (unintentionally) providing your data creates vulnerability. Data sharing can be an arena for preying on vulnerabilities, where mathematical models can be used to identify things to exploit and scam (O’Niel, 2016). The pattern that I am seeing is that the realization often stops there: They can see these issues; sometimes they get appalled by them. But rather soon they return to ‘I still want to use my phone’ or ‘I need to connect to my friends’. It is as if they start seeing a *system* that has its ‘issues’, but that they can’t dismantle or modulate. So they have no other option than to be complicit, surrender and enjoy.

P: This makes me think of how AI is indeed part of the forms of subjectivity that are being formed now. I have argued that the mathematical subjectivity that modern forms of mathematics education promote risk continuing the aggravation of socio-ecological crises (Valero, 2023). But I had not considered how current technologies and their entanglement with capitalism plays in, and what this means for a political view of math education in times of climate change. Checkmate Christian!

IMAGINING CME IN THE AI-RULED SOCIO-ECOLOGICAL

P: AI is currently bursting everywhere, and all educators, particularly math educators, are being compelled to connect AI to the mathematics curriculum. Some years ago, for example, the Swedish Agency of Education made “programming” a compulsory part of mathematics, very much notched by industry. Connected to it, arguments for the need of expanding digitalization added (Player-Koro, 2012) a focus on the use of digital tools in mathematics education. With that, also algorithmic awareness or computational thinking was formulated as new desired competencies for all students. There are many interconnected optimistic interests that push for making the whole palette of current

digital technologies a part of the mathematics curriculum. One often sees that the technological optimism —and its connected economic interests— tend to dominate agendas in mathematics education. But, if there are so many serious worries and concerns such as the ones we have discussed, the question is: What does this configuration imply for the possibilities of a CME?

C: CME may provide ideas about ways to go beyond that point where students and pre-service teachers stopped in the studies that I have been involved in. A language to formulate a critique with, a language that offers possibilities for new future imaginations. What if we start to ‘follow the money’, i.e., *the data*? Could that be a point of departure for critical mathematical education in this age? And not only “following the data” as such, but rather one could follow what the receiver of the data can do with the data with means of AI technology.

P: I hate this question, but I have to pose it: What is then the ‘mathematical’ in this configuration?

C: Understanding the mathematical patterns that constitute the AI-models, and connecting them to the other agencies in a “critical zone”, to follow your proposal (Valero, 2023) that draws on Latour (2018) about possible new forms of mathematical subjectivation in the time of climate change. Pilot studies have shown that mathematical aspects of machine learning are accessible in mathematics education for elementary school (Araya, 2007), and for upper secondary school, including ethical aspects of the technology (Register, 2024). Ethical aspects include discussing contextual variables which the model may neglect but which has an effect. Awareness of the specificity of AI-models, its characteristics and its ethical implications is a new component of a critical matheracy in the socio-ecological. But that would lead to a messy landscape with entanglements between data, mathematical models, predictions, human and non-human actors. What theories could guide us?

P: The current socio-ecological crises are an entanglement of environmental, social and political situations and trends. This is what we have learned from Latour’s (2018) call to rethink in a relational way “climate change”. AI-mathematical modeling has seemingly a new type of agency of extreme relevance in the technological configuration of the moment: All these are entangled in generating, let us say, the socio-technological-ecological crises!

C: AI-modeling can then be thought of in a Latourian way: the agency of models is in a network. Models always contain simplifications, but fewer simplifications are now necessary since computing power is less limiting when building models. Models can act back on the phenomena that they are a model of. For example, apps that predict traffic jams can influence decisions for people’s driving, which in turn affect the traffic jam situation. Such feedback mechanisms between a model and the phenomenon it models are much faster now. Both the decreased amount of simplification and the faster feedback mechanisms would imply that AI-models may have the potential to bring the model closer to the phenomena it models. So that is one connection in the network that is made tighter. Also, the connection between the experts and the models is less tight now. The models are turning into ‘black boxes’ to a higher degree, also for experts. In conjunction with the changing role of the experts, from building models based on their own conjectured hypotheses to more passively judging and validating models that machines have produced, the characteristics of models turning into ‘black boxes’ changes the agency of the models.

P: To sum up our conversation: *Transparency* is of importance, not only for people to critically assess the models especially because experts may have less answers to give here: how they were built, on

which data, and how they work. Another reconfiguration that has taken place is the cost–efficiency ratio of the models, and therefore their *scale*. How many people are affected, in how many ways, and how often can be related to the socio-ecological in various ways. AI-modeling is many ways similar to traditional modeling, both mathematically and in relation to the socio-ecological, so existing theoretical perspectives in mathematics education research can serve as a solid foundation to stand on. But some things have changed; *closeness to the phenomena* that is being modeled, increased *scale* and less *transparency*. Now how does that relate to math education?

C: Mathematics education is touched by this in two ways: AI-based technologies enter the space of mathematics education (teaching *with* AI), and mathematical competencies are tightly connected to the production of AI-models (teaching *about* AI). A critical mathematics education perspective would have to open up for specific understandings of both the use and production of such types of mathematical models. Yet another part of the reconfiguration relates to power and how *resistance* towards the current configuration may be formed. We can work to research and bring to practice “what is not the case yet”, as Skovsmose and Borba (2004) reminded us 20 years ago! The challenges of a future critical mathematics education in the socio-ecological need to engage seriously the pitfalls and possibilities of AI-models as part of a socio-ecological math literacy.

EPILOGUE: CME IN THE SOCIO-ECOLOGICAL

What does our dialogue lead us to conclude? Based on existing literature in CME and mathematical modeling, we have argued that mathematics education has the potential of helping us critically understand the technologies that partially produce multiple socio-ecological crises. Technologies such as AI are not only a formatting power in society and the environment through mathematical modeling. They are also entangled with the very capitalist economic and social fabric that it contributes to build with sometimes productive and also destructive consequences. This means that CME to become a powerful theoretical stance to think about the connections of mathematics education and technology in the socio-ecological needs to develop a serious exploration and conceptualization of new emerging AI-models and technologies. For this, it is important to continue advancing new theoretical understanding of AI and mathematics education in the future.

Acknowledgements

This research is part of the graduate school RelMaS: Relevancing Mathematics and Science Education (VR 2019-03679), funded by the Swedish Research Council.

References

- Andersson, C. H. (2021). A critical gaze on new digital technology: Answers from mathematics education? In D. Kolloche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (pp. 129-132). Tredition.
- Andersson, C. H., Andersson, J., & Ljung, P. (2024). Mathematical understanding of training data and AI: a lesson plan for critical thinking and ethical perspectives. In L. B. Boistrup & B. Di Paola (Eds.). *Proceedings / Actes CIEAEM 74: Mathematics and practices: Actions for futures, Vol. 13*, s. 681-685.
- Andersson, C. H., Register, J. T. (2023). An examination of pre-service mathematics teachers’ ethical reasoning in big data with considerations of access to data, *The Journal of Mathematical Behavior, Volume 70*, Article 101029. <https://doi.org/10.1016/j.jmathb.2022.101029>
- Araya R. (2007). What is inside this box? Look at these other opened boxes for clues. Paper presented in *Fifth Conference of the European Society for Research in Mathematics Education*.

- Bhaimiya, S. (2023, August 1). An Asian MIT student asked AI to turn an image of her into a professional headshot. It made her white, with lighter skin and blue eyes. *Business insider*. <https://www.businessinsider.com/student-uses-playground-ai-for-professional-headshot-turned-white-2023-8#post-headline>
- D'Ignazio, C., & Klein, L.F. (2020). *Data feminism*. The MIT Press.
- Geiger, V., & Schmid, M. (2024). A critical turn in numeracy education and practice. *Frontiers in Education*, 9. <https://doi.org/10.3389/educ.2024.1363566>
- Goodfellow, I., Bengio, Y., & Courville, A. (2016). *Deep Learning*. MIT Press.
- Jablonka, E., & Gellert, U. (2011). Equity concerns about mathematical modelling. In B. Atweh, M. Graven, W. Secada, & P. Valero (Eds.), *Mapping equity and quality in mathematics education* (pp. 223-236). Springer.
- Jablonka, E. (2017). Mathematisation in environments of Big Data—"implicit mathematics" revisited. In *Proceedings from CIEAEM* (Vol. 69, pp. 43-51).
- Kosinski, M., Stillwell, D., & Graepel, T. (2013). Private traits and attributes are predictable from digital records of human behavior. In K. Wachter (Ed.), *Proceedings of the National Academy of Sciences 110*(15) (pp. 5802-5805). PNAS
- Latour, B. (2018). *Down to earth: politics in the new climatic regime*. Polity press.
- O'Neil, C. (2016). *Weapons of math destruction: How big data increases inequality and threatens democracy*. Penguin.
- Pais, A. (2014). Economy: The absent centre of mathematics education. *ZDM*, 46(7), 1–9.
- Player-Koro, C. (2012). Hype, hope and ICT in teacher education: a Bernsteinian perspective. *Learning, Media and Technology*, 38(1), 26–40. <https://doi.org/10.1080/17439884.2011.637503>
- Register, J., T. (2024). Designing for ethical mathematics consciousness: a synthesis of findings from a design based research project exploring how educators may design for students' ethical reasoning in data science. *Philosophy of mathematics Education journal* 41, 1-39.
- Straehler-Pohl, H. (2017). De|mathematisation and Ideology in Times of Capitalism: Recovering Critical Distance. In H. Straehler-Pohl, N. Bohlmann, & A. Pais (Eds.), *The Disorder of Mathematics Education: Challenging the Sociopolitical Dimensions of Research* (pp. 35-52). Springer International Publishing.
- Skovsmose, O. (1994). *Towards a Philosophy of Critical Mathematics Education*. Springer-Science.
- Skovsmose, O., & Borba, M. C. (2004). Research methodology and critical mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 207-226). Kluwer.
- Skovsmose, O., (2020). Banality of mathematical expertise. *ZDM Mathematics Education*, 52, 1187–1197.
- Valero, P. (2023). Mathematical subjectivation: Death sentence to the planet or chances for a terrestrial life. In M. Ayalon et al. (Eds.), *Proceedings of the 46th PME* (Vol. 1, pp. 53-68). Univ. of Haifa.
- Varoufakis, Y. (2023). *Technofeudalism: What killed capitalism*. The Bodley Head Ltd.

ENHANCING MIDDLE SCHOOL STUDENTS' MATH ENGAGEMENT AND SOCIO-ECOLOGICAL AWARENESS THROUGH PROJECT-BASED LEARNING

Mrinal Jyoti Baruah¹, Kumari Shalini¹, Arindam Bose², Saurabh Singh¹

¹Mantra Social Services Bengaluru, ²Tata Institute of Social Sciences (TISS)

This paper proposes an initiative to integrate Project-Based Learning (PBL) into middle school education for Grades 6 to 8 in Bihar, India, a region facing significant educational challenges. The initiative aims to enhance student engagement, foster critical thinking, and promote socio-ecological awareness by embedding socio-ecological contexts into the mathematics curriculum. Grounded in a socio-ecological framework, this approach addresses real-world challenges through PBL, emphasizing the interconnection between social and ecological systems. Unlike traditional PBL, which emphasizes hands-on learning through real-world applications, this initiative focuses on deepening students' understanding of how mathematical concepts intersect with societal and environmental issues. The paper explores the potential of integrating socio-ecological perspectives into PBL to provide a more meaningful and contextually relevant education. It also examines how this framework can transform educational practices in marginalized communities, aligning with the study's goal of improving academic outcomes and real-world problem-solving skills.

THE CONTEXT

Addressing students' cognitive development and socio-ecological awareness becomes imperative as the demand for effective mathematics instruction intensifies within STEM fields. This paper explores a project aimed at integrating Project-Based Learning (PBL) into middle school education for Grades 6 to 8 across Bihar, India—a state noted for significant educational challenges and performance gaps. The intervention is designed to enhance student engagement, critical thinking, and socio-economic awareness by embedding socio-ecological contexts into the mathematics curriculum. Bihar faces systemic educational challenges, including resource constraints and diverse student needs, contributing to its students' consistent lag behind national averages in mathematics proficiency (NAS, 2021). This aligns with research indicating that contextualized learning can improve engagement and outcomes in underperforming regions (Hiebert & Grouws, 2007; Boaler, 2016). Middle school is a pivotal stage for foundational mathematical learning and skill development. Integrating real-world applications and socio-ecological perspectives into mathematics education not only addresses academic performance but also tackles context-specific challenges (Frykholm & Glasson, 2005). By drawing on the principles of PBL and socio-ecological frameworks, this project aims to enhance students' understanding and relevance of mathematical concepts (Krajcik & Blumenfeld, 2006). The paper will detail the conceptual foundations of this integration and its potential impact on students' mathematical understanding and socio-ecological awareness. This initiative supports Sustainable Development Goal (SDG) 4, emphasizing inclusive and equitable quality education, and demonstrates how contextually adapted PBL strategies can transform mathematics education in marginalized communities.

LITERATURE REVIEW

Contemporary approaches to mathematics education increasingly emphasize pedagogical strategies that foster profound understanding and active engagement among students. Project-Based Learning (PBL) represents a transformative instructional practice where students engage in authentic, real-world problems that require the application of mathematical concepts within meaningful contexts (Savery & Duffy, 1996; Hmelo-Silver, 2004). This approach aligns with constructivist theories advocating for student-centered learning and the active construction of knowledge (Vygotsky, 1978), which are essential for addressing the complex socio-ecological challenges faced by today's students. Recent studies highlight diverse applications and impacts of PBL in mathematics education. Smith et al. (2021) demonstrate that PBL enhances students' conceptual understanding and problem-solving skills through complex, contextualized mathematical problems. Jones and Brown (2022) further illustrate PBL's positive influence on students' ability to transfer mathematical knowledge and develop favorable attitudes towards mathematics. These findings underscore PBL's potential to cultivate critical thinking and socio-ecological awareness, aligning with the study's focus on creating educational interventions that integrate socio-ecological contexts into mathematics education. The integration of socio-ecological perspectives into educational practices has been increasingly advocated by scholars.

Alf Coles (2023) discusses the importance of embedding socio-ecological thinking in mathematics education, arguing that it prepares students to tackle real-world challenges by understanding the interconnectedness of social and ecological systems. This perspective is crucial for the study's theme, as it aligns with the intervention's goal of integrating socio-ecological contexts into PBL to address real-world issues effectively. Theoretical frameworks supporting PBL, such as cognitive load theory, suggest that embedding learning in meaningful socio-ecological contexts reduces extraneous cognitive load (Sweller, 1988). Socio-cultural perspectives emphasize collaborative learning environments in PBL, fostering socio-emotional skills alongside academic achievement (Vygotsky, 1978). By integrating socio-ecological contexts into PBL, students are encouraged to develop a deeper understanding of their environment and the role of mathematics in addressing ecological challenges. This connection is vital for the study, which aims to explore how socio-ecological integration in PBL can improve educational outcomes and contribute to sustainable futures. Comparatively, PBL outperforms traditional methods by promoting active engagement, critical thinking, and the application of mathematical concepts in socio-ecological contexts (Barrows, 1996; Hung, 2008; Polya, 1957). Effective implementation requires structured support such as comprehensive teacher training, well-designed tasks, and opportunities for reflective practice (Hmelo-Silver et al., 2007; Walker et al., 2004). Recent innovations, including digital tools and adaptive platforms, enhance scalability and effectiveness in addressing educational challenges like improving mathematics proficiency and fostering 21st-century skills (Gijbels et al., 2020). PBL's alignment with socio-ecological goals is evident in its capacity to connect mathematical learning with real-world environmental issues. For example, students might work on projects related to sustainable agriculture, water conservation, or renewable energy, thereby gaining insights into ecological sustainability while developing their mathematical skills. These projects support the development of a socio-ecological consciousness, preparing students to contribute to sustainable, plural futures. This focus on integrating socio-ecological themes into PBL directly supports the study's objective to enhance mathematics education and socio-ecological awareness in diverse settings. Implications for

policy and practice are significant, informing curriculum development, teacher training, and educational leadership practices globally. By embedding socio-ecological perspectives into mathematics education through PBL, this approach advances educational theory and practice, aligning with Sustainable Development Goal 4 (SDG 4) of ensuring inclusive and equitable quality education for all.

PROJECT OVERVIEW

Our initiative represents a collaborative effort between Knowledge Partner Education Above All (EAA) and Mantra4Change to develop and contextualize 18 Project-Based Learning (PBL) projects tailored for Grades 6 to 8, rooted in current educational research and best practices. Drawing upon literature emphasizing the efficacy of PBL in enhancing student engagement and deepening conceptual understanding (Hmelo-Silver, 2004; Thomas, 2000), each project undergoes a meticulous three-step vetting and correction process to ensure alignment with local curriculum standards and educational goals. These projects (see Figures 1, 2) are strategically designed to integrate with existing chapters, covering 3 to 4 concepts per project, and are framed around a central, open-ended question to stimulate inquiry and critical thinking (Blumenfeld et al., 1991; Hung, 2008).

जानवरों के लिए आश्रय! (वर्ग 6)

विवरण	विद्यार्थी अपने आस पास के जानवरों की देखभाल के लिए एवं उनके आश्रयों के लिए मॉडल डिजाइन कर उसे बनाएंगे। वे ऐसा करने के लिए भिन्न और दशमलव से संबंधित गणितीय प्रक्रियाओं की अवधारणाओं का उपयोग करेंगे। वे आश्रय चलाने की लागत और उसमें रहने वाले जानवरों द्वारा आवश्यक भोजन की मात्रा की भी गणना करेंगे।
प्रमुख सवाल	हम अपने इलाके में जानवरों की देखभाल के लिए एक आश्रय या रहने का स्थान कैसे डिजाइन कर सकते हैं?
पाठ (संख्या एवं नाम)	7. भिन्न 8. दशमलव
कुल आवश्यक समय	कक्षा में 5 दिनों के लिए प्रतिदिन 40 मिनट घर पर 4 दिनों तक प्रतिदिन 10-20 मिनट
आवश्यक संसाधन	कागज या चार्ट पेपर, कागज, पेंसिल/पेन, रंग
अधिगम प्रतिफल	LO06MT10: अपने दैनिक जीवन में मुद्रा, यजन, लम्बाई, तापमान आदि पर आधारित भिन्न एवं दशमलव भिन्न का उपयोग करते हैं। LO06MT11: भिन्न एवं दशमलव भिन्न आधारित जोड़- घटाव से सम्बंधित दैनिक जीवन की समस्याओं को हल करते हैं। भिन्नो को सम (उचित भिन्न), विषम (अनुचित भिन्न) और मिश्रित भिन्नो में बदल कर तुलना करते हैं एवं पहचान करते हैं कि भिन्न a/b में, a बराबर भागों का प्रतिनिधित्व करता है जबकि ($b \neq 0$) संपूर्ण का प्रतिनिधित्व करता है। SDG4 (21वीं सदी कौशल): आश्रय का मॉडल बनाने के लिए अपनी टीमों के साथ सहयोग करने के लिए अपने संचार कौशल का उपयोग करते हैं।
शिक्षक / शिक्षिकाओं के लिए सुझाव	1. सभी शिक्षक / शिक्षिकाएं कक्षा शुरू होने के पहले प्रत्येक दिन की सामग्री और प्लान देख लें। 2. जिस दिन की सामग्री स्कूल में उपलब्ध ना हो, उसे बच्चों को एक दिन पहले बताएं ताकि वे अगले दिन सामग्री लेकर आये। 3. पहला दिन शुरू करने से पहले सभी बच्चों को समूहों में बांट दें जिससे वे समूह कार्य, प्रयोग या अन्य कोई गतिविधि समूह में ही करें। ध्यान दें कि समूह हमेशा मिश्रित एवं समावेशी (जिसमें सभी लिंग एवं सभी स्तर के बच्चे) हों। 4. शिक्षक / शिक्षिका, पाठ योजना में दिए गए QR कोड को स्कैन कर वीडियो या आलेख कक्षा से पहले देख लें।

Figure 1: Screenshot of the Project - "जानवरों के लिए आश्रय! (वर्ग 6)" [shelter for animals]

SHELTER FOR ANIMALS! (GRADE 6)

Description	Students will design and build models of shelters and care for the animals in their neighborhood. They will use mathematical concepts related to fractions and decimals to do this. They will also calculate the cost of running the shelter and the amount of food needed by the animals living there.
leading questions	How can we design a shelter or living space to care for animals in our area?
Chapter (numbers and names)	7. Fractions 8. Decimals
Total Time Required	40 minutes a day for 5 days in class 10-20 minutes a day for 4 days at home
Required Resources	Paper or chart paper, Paper, Pencil/Pen, Colours etc.
Learning Outcomes	LO06MT10: Use fractions and decimals in daily life to measure currency, weight, height, temperature etc. LO06MT11: Solve daily life problems related to addition and subtraction based on fractions and decimals. Compare fractions by converting them into even (proper fractions), odd (improper fractions) and mixed fractions and recognize that in the fraction a/b , a represents equal parts while ($b \neq 0$) represents the whole. SDG4 (21st Century Skills): Use their communication skills to collaborate with their teams to build a model of shelter.
Tips for Teachers / Teachers	1. All teachers should see the material and plan for each day before the class starts. 2. If the material for a particular day is not available in the school, inform the children a day in advance so that they can bring the material the next day. 3. Before starting the first day, divide all the children into groups so that they can do group work, experiments or any other activity in groups. Keep in mind that the groups should always be mixed and inclusive (which should include children of all genders and all levels). 4. The teacher should scan the QR code given in the lesson plan and watch the video or article before the class.

Figure 2: Translation screenshot, Project "जानवरों के लिए आश्रय! (वर्ग 6)"- Shelter for animals! (Gr 6)

For instance, the project "जानवरों के लिए आश्रय! (वर्ग 6)/ Shelter for animals! (Grade 6)" is one of six projects designed for Grade 6 students. In this project, students are challenged to design shelters for local animals, integrating mathematical concepts with environmental stewardship. The central question, "हम अपने इलाके में जानवरों की देखभाल के लिए एक आश्रय या रहने का स्थान कैसे डिजाइन कर सकते हैं?/ How can we design a shelter or living space to care for animals in our area?", promotes critical thinking and environmental awareness, aligning with Sustainable Development Goal 4 (SDG 4 - Quality Education) to encourage students' meaningful community contributions. Specific learning outcomes (LO06MT10, LO06MT11, *see* Picture 2) mapped to Bihar SCERT books ensure transparency and educational efficacy. This project is one example among the 18 total projects, six for each grade from 6 to 8. Each project is meticulously crafted to cover essential concepts and integrate socio-ecological themes relevant to the students' educational context.

Guided by educational theories emphasizing cooperative learning (Slavin, 1995; Hickey & Zuiker, 2005), teachers will facilitate diverse group dynamics and effective communication. Students will culminate their experience by presenting their designs and reflections, fostering community engagement. Structured over five days, each project begins with a comprehensive overview and learning outcomes explicitly aligned with thematic objectives identified through extensive research and stakeholder engagement. Detailed daily plans provide teachers with clear guidelines for effective facilitation, emphasizing communication strategies that promote inclusivity and collaborative

learning experiences (Hickey & Zuiker, 2005; Krajcik et al., 1998). Supporting materials, accessible via embedded links and QR codes, ensure easy access to supplementary resources that enrich student learning experiences and support differentiated instruction (Thomas, 2019). The effectiveness of the 'Shelter for Animals' project will be assessed through student presentations, peer reviews, and feedback collected via QR-coded Google Forms. This evaluation process ensures that the project meets its learning objectives and provides insights for ongoing refinement. Emphasizing practical application and contextualization, students are encouraged to apply learned concepts to real-world scenarios, promoting a deeper understanding of mathematical principles (Barron et al., 1998; Kolodner, 1997). Group activities are integral to each project, fostering peer interaction and collaborative problem-solving skills essential for 21st-century learning (Johnson & Johnson, 1999; Slavin, 1995). At the conclusion of each project, students showcase their insights and outputs, fostering a culture of sharing and reflection within their classrooms and communities. By engaging students in meaningful, real-world problem-solving, the 'Shelter for Animals' project is expected to enhance student motivation and foster a sense of accomplishment. The collaborative nature of the project aims to promote teamwork and socio-emotional development and prepare students for future academic and life challenges. Continuous improvement is facilitated through structured feedback mechanisms, where teachers collect student input via QR-coded Google Forms, ensuring ongoing refinement and effectiveness of the PBL approach (Savery, 2006; Helle et al., 2006). By integrating robust educational research with practical implementation strategies, our initiative aims to enhance access to quality education and to also empower teachers and students alike in adopting innovative pedagogical practices.

THE IMPLEMENTATION STRATEGY

The implementation of Project-Based Learning (PBL) in Bihar's middle schools represents a comprehensive strategy to address educational challenges, promote student engagement, and integrate real-world applications within a socio-ecological framework. Aligned with Theme D1—enhancing students' understanding and engagement in mathematics through innovative pedagogies—the initiative is tailored to meet the needs of Bihar's diverse communities and educational contexts. Central to this approach is the development and contextualization of 18 PBL projects across Grades 6 to 8, focusing on integrating mathematics with local socio-ecological issues such as environmental stewardship, community development, and sustainable practices. These projects aim to cultivate critical thinking, problem-solving, and collaborative skills while ensuring that students apply mathematical concepts to real-world challenges. The implementation begins with leadership conferences and training workshops for District Institutes of Education and Training (DIETs) and district/block leadership teams. These sessions equip educational leaders with the skills needed to introduce, manage, and sustain PBL across Bihar's schools, while fostering an environment of co-creation with teachers and community stakeholders. This collaborative approach ensures that PBL projects are aligned with the socio-economic realities of Bihar's diverse communities, integrating practical, hands-on learning opportunities that resonate with local contexts. To ensure comprehensive reach and engagement, webinars and live sessions on platforms such as YouTube will be launched, providing accessible venues for introducing and reviewing PBL initiatives. These virtual forums serve as collaborative spaces for teachers, educational leaders, and community members to exchange best practices, address challenges, and celebrate the program's milestones. These webinars will emphasize the role of PBL in empowering students to contribute meaningfully to their communities, fostering a

deeper connection between classroom learning and socio-ecological awareness. Complementing these efforts, observational assessments, surveys, and feedback forms will be systematically deployed to evaluate the impact of PBL on student learning outcomes, teacher engagement, and overall program efficacy. Data-driven insights will guide continuous refinement of the PBL projects, ensuring they remain responsive to local educational and socio-ecological needs. The initiative will also focus on enhancing hands-on learning through the establishment of five STEM labs within DIETs, promoting innovation and experiential learning in science, technology, engineering, and mathematics. These labs will provide students with the resources and opportunities to explore STEM concepts through project-based approaches, reinforcing the importance of environmental stewardship, technological problem-solving, and community-focused innovation.

Throughout the implementation, challenges such as logistical constraints, variability in teacher preparedness, and community engagement hurdles are anticipated. To address these issues, a robust support system involving regular field visits, leadership support teams, and continuous professional development for teachers will be established. These efforts will be complemented by the proactive involvement of community stakeholders, ensuring that the PBL projects are sustainable and culturally relevant. Incorporating stakeholder engagement strategies remains critical to the success of the PBL initiative. Empowering DIET teams, state/district/block leaders, and local educators will ensure a shared responsibility for the program's success. Toolkits and resources will be distributed to assist in PBL implementation, and a focus on inclusivity in STEM labs will encourage participation from diverse groups of students. In conclusion, the implementation strategy for the PBL initiative represents a comprehensive, multi-tiered approach that fosters innovation in middle school education, promotes socio-ecological awareness, and enhances STEM education in Bihar. By incorporating real-world applications, hands-on learning experiences, and community engagement, this initiative not only enriches classroom learning but also empowers students to become critical thinkers and problem solvers within their communities.

PBL IN ACTION: INSIGHTS AND CHALLENGES

This study, framed within a socio-ecological context, explores the potential of Project-Based Learning (PBL) to enhance middle school students' mathematical understanding and socio-ecological awareness. Although the intervention is ongoing and final findings are not yet available, preliminary observations and theoretical frameworks offer valuable insights into the anticipated outcomes and challenges associated with this educational approach. Integrating PBL into school mathematics curriculum is expected to significantly impact students' socio-ecological awareness. By engaging students in projects that address real-world problems, such as, local environmental challenges and community health issues, the intervention aims to contextualize mathematical concepts within meaningful and relevant everyday contexts. This approach draws on Coles' (2023), who emphasizes the importance of connecting academic content to socio-ecological issues to foster a sense of responsibility and agency among students. The expectation is that students will develop a deeper appreciation for their local environment and community, thereby enhancing their commitment to sustainable practices. PBL is also anticipated to improve student engagement and learning outcomes by promoting active, hands-on learning. Previous research suggests that PBL can lead to deeper conceptual understanding and retention of mathematical concepts. Active involvement in solving real-world problems is expected to make learning more engaging and motivating for students. This

aligns with what the current literature proposes, say, the benefits of student-centred inquiry-based learning approaches in fostering academic achievement and building critical thinking skills (see, Barron & Darling-Hammond, 2008). Effective implementation of PBL requires comprehensive professional development for teachers. Preliminary observations indicate that teacher training workshops have been crucial in equipping educators with the skills and confidence needed to facilitate PBL effectively. These workshops emphasize the integration of socio-ecological themes into the curriculum, preparing teachers to adopt a more facilitative role and support students' creative problem-solving processes. The ongoing professional development adapts Barron and Darling-Hammond's approach (2008), underscoring the importance of continuous support for teachers in transitioning to innovative teaching methodologies.

The intervention also seeks to strengthen the connection between students and their local communities by involving them in projects that address local socio-ecological issues. This approach is informed by the principles of place-based education, as advocated by Gruenewald (2003), which emphasizes the importance of connecting students to their immediate environments. The anticipated outcome is that students will not only enhance their mathematical skills but also develop a stronger sense of community and environmental stewardship. Implementing PBL in a resource-constrained setting like Bihar presents several challenges. Limited access to materials and technology, varying levels of teacher preparedness, and potential resistance to innovative teaching methods are significant obstacles. Addressing these challenges requires careful planning, resource allocation, and ongoing support for both teachers and students. In response, specific resource management strategies, such as leveraging local materials and using online resources where possible, have been introduced to mitigate these challenges. Furthermore, ensuring that the intervention is inclusive and adaptable to diverse socio-economic and geographical contexts within Bihar is crucial for its success and scalability. As the intervention progresses, ongoing monitoring and evaluation will be essential to assess its impact on students' mathematical understanding and socio-ecological awareness. Future research should focus on collecting and analyzing both quantitative and qualitative data, such as teacher feedback, classroom observations, and student performance metrics, to determine the effectiveness of PBL in achieving the desired educational outcomes. Additionally, studies should explore the long-term effects of PBL on students' academic and socio-ecological development, as well as its scalability in different educational contexts. While the final outcomes of this ongoing intervention are yet to be determined, the integration of Project-Based Learning within a socio-ecological framework holds promise for enhancing middle school students' mathematical understanding and socio-ecological awareness in Bihar. The anticipated benefits, supported by theoretical frameworks and preliminary observations, highlight the potential of PBL to create more engaging, relevant, and impactful educational experiences. As the intervention continues, careful monitoring, support, and adaptation will be essential to realize its full potential and inform future educational practices and policies.

CONCLUSION AND FUTURE DIRECTIONS

This study highlights the transformative potential of Project-Based Learning (PBL) within a socio-ecological framework to improve mathematical understanding and socio-ecological awareness among middle school students in Bihar. By linking mathematical concepts with real-world issues, PBL engages students deeply and fosters critical thinking and responsible citizenship. Successful

implementation relies on effective teacher training, adequate resources, and strong community involvement, especially in resource-limited contexts like Bihar. While preliminary results are promising, challenges such as varying teacher preparedness and resource constraints need to be addressed through targeted strategies and ongoing support. Ongoing monitoring and evaluation will be essential for assessing the long-term impact of PBL on student outcomes and socio-ecological awareness. This research provides valuable insights for scaling PBL initiatives in similar contexts and contributes to the broader discussion on innovative, student-centred pedagogies.

References

- Barrows, H. S. (1996). Problem-based learning in medicine and beyond: A brief overview. *New Directions for Teaching and Learning*, 1996(68), 3-12.
- Coles, A. (2023). Towards a socio-ecological practice of mathematics teacher education. *Advances de Investigación en Educación Matemática*. Advance online publication. <https://doi.org/10.35763/aiem23.5489>
- Gijbels, D., Van de Watering, G., Dochy, F., & Van den Bossche, P. (2020). The development of the learning environment and students' engagement in higher education: A systematic review. *Educational Research Review*, 30, Article 100326. <https://doi.org/10.1016/j.edurev.2020.100326>
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Information Age Publishing.
- Hmelo-Silver, C. E. (2004). Problem-based learning: What and how do students learn?. *Educational Psychology Review*, 16(3), 235-266.
- Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist*, 42(2), 99-107.
- Hung, W. (2008). The 9-step problem design process for problem-based learning: Application of the 3C3R model. *Educational Research Review*, 3(1), 27-38.
- Jones, A., & Brown, C. (2022). The impact of project-based learning on mathematics achievement in middle school students. *Journal of Educational Psychology*, Advance online publication. <https://doi.org/10.1037/edu0000662>
- Ministry of Human Resource Development [MHRD]. (2021). *National Achievement Survey (NAS) 2021*. Government of India. Retrieved from https://ncert.nic.in/pdf/nas/2021/nas_report_2021_final.pdf
- Polya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton Univ Press.
- Savery, J. R., & Duffy, T. M. (1996). Problem-based learning: An instructional model and its constructivist framework. *Educational Technology*, 36(3), 31-38.
- Smith, B., Johnson, L., & Hill, P. (2021). Impact of project-based learning on middle school students' mathematics performance: A meta-analysis. *Review of Educational Research*, 91(4), 599-622.
- Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, 12(2), 257-285.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
- Walker, A., Leary, H., Hmelo-Silver, C. E., & Ertmer, P. A. (2004). *Essential readings in problem-based learning: Exploring and extending the legacy of Howard S. Barrows*. Purdue University Press.

THEORETICAL AND METHODOLOGICAL ASPECTS OF TRANS-DISCIPLINARY EDUCATION: MATHEMATICS AND THE SOCIO-ECOLOGICAL

Lisa Björklund Boistrup^{1,2}, Magnus Ödmo¹, Ann-Sofi Røj-Lindberg², Mats Braskén²

¹Malmö University, ²Åbo Akademi University Vasa

We present an exploration of how to move towards trans-disciplinary teaching and research, including mathematics education, related to the socio-ecological. We draw on three fields, and pinpoint relevant notions, which we then combine in a proposal of theoretical and methodological positionings for our future potential research. The notions that we address are: not avoiding complexities, critical mathematics education, wicked problems, relational mathematics, epistemic aspects such as praxis and logos, and sustainability.

THE RELEVANCE OF EXPLORING ASPECTS OF THE ROLES OF MATHEMATICS IN TRANSDISCIPLINARY EDUCATION

In this paper we explore theoretical and methodological aspects of the roles of mathematics in transdisciplinary education (elaborated on by Chronaki, 2000) which connects to current socio-ecological challenges (e.g. Boylan & Coles, 2017). In our exploration, we want to avoid a binary take on how mathematics may be related to other knowledge areas (Gutiérrez, 2019). We aim to embrace complexities and ethical aspects, which for instance concerns epistemic (in)justice, in the sense of what knowing is counted, and in what ways (Nieminen & Lahdenperä, 2024). We will return to these perspectives in the concluding discussion.

When defining transdisciplinary education, we draw on Mård and Klausen (2024) and Klein (2017):

In the literature, the approaches of multi-, inter-, and transdisciplinary teaching are distinguished by their degree of integration between subjects and/or the degree to which subjects as such are transcended or ignored. Traditionally, multidisciplinary teaching is understood as an approach with a low degree of integration, while inter- and transdisciplinary teaching refer to more integrated teaching (Klein, 2017). (Mård & Klausen, 2024, p. 8)

Following the quote above, we acknowledge that there may be shifts in the degree of integration, but by using transdisciplinary teaching we mean integration in all four dimensions addressed by Beane (1997; see also Røj-Lindberg, Hartivk & Ingves, 2022): (1) Integration of experiences, (2) social integration, (3) Integration of knowledge, and (4) Integration when constructing curriculum. In the following section, we draw on previous research, including research carried out by us authors in the contexts of Finland and Sweden. These studies are, in different ways, transdisciplinary. Drawing on these studies, we, in the subsequent section, address future potential ways for researchers and teachers to engage in teaching and research where two or more knowledge areas are inter-connected, specifically in the contexts of socio-ecological challenges. One essential notion here is how transdisciplinary work in education may pave the way for addressing democracy (Beane, 1997).

THREE FIELDS IN MATHEMATICS EDUCATION RESEARCH TO BUILD ON

For our exploration, we here present three fields of research that we build on, Mathematics education and climate change; Relational mathematics for cross-curricular teaching; and Praxis and logos when

mathematics is connected to other knowledge areas. For each field, we (a) present the field in general terms, (b) present our own research within the field, with a focus on theoretical and methodological aspects, and (c) summarize what we bring with us from this research for our exploration.

Mathematics education and climate change

One relevant field for this paper is the growing field in mathematics education which investigates critical mathematics education (e.g. Skovsmose, 2023) and climate change, suggesting that complex issues such as climate change bring forth an awareness of the roles mathematics may get in education on climate change (Steffensen et al., 2021). A consequence is how it is relevant for teachers to be aware of the formatting power of mathematics, which points to how “[a]t times, mathematics provides a formatting of such crises by shaping how we see and react to them” (Skovsmose, 2023, p. ix). While the study by Steffensen et al. was empirical, Hauge et al. (2017), similar to what we do in this paper, took a theoretical approach when they developed a framework bringing forward three categories that support critical reflection when mathematics is adopted for discussions on climate change: climate change as a vehicle, climate change as critique, and climate change as content.

Ödmo with colleagues has investigated critical mathematics in relation to climate change, with a particular interest in statistics and mathematics teacher education, in Sweden. Here is investigated both the teacher educator perspective (Ödmo et al., 2023a) and the perspectives of pre-service teachers (Ödmo et al., 2023b). In these studies, the authors aimed at bringing forth the complexities when teaching critical mathematics, particularly statistics, in the context of climate change. Critical mathematics education was coordinated with actants and controversies, working with the data through Latour (e.g. 2005). In Ödmo et al. (2023b), networks of actants were construed for each of the interviewed pre-service teacher. One example is shown in Figure 1.

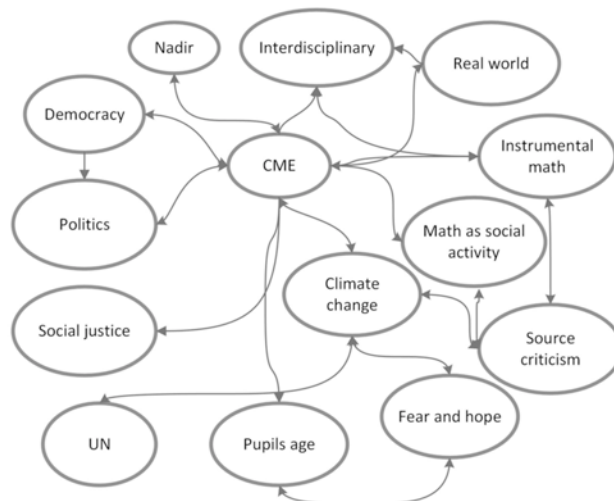


Figure 1: Example of a pre-service teacher's actant network (Figure 5 in Ödmo et al., 2023b)

The analyses of the data illuminate how both the teacher's and pre-service teachers' experiences are being shaped by actants such as the curriculum, social justice, democracy, and source critique, among others. The data from the student in Figure 1, reflected a large number of actants, where many are connected to CME. Moreover, there are several complex connections across and between the actants. This pre-service teacher took, to some extent, a political perspective, connecting democracy and

social justice to CME. Figure 1 illuminates one example of the complexities of mathematics education in relation to climate change.

Ödmo et al. address the relevance of that teacher education should invite active discussions of the complexity created when a CME perspective is required. Such a move “would allow for a critical approach to critical mathematics education itself that could prepare student teachers to navigate, instead of ignoring or opposing, such complexity” (Ödmo et al., 2023b, p. 1221).

There are several notions that we bring with us from this field, and the relevance for the socio-ecological is clear. One notion is to *not avoid complexities* when working in transdisciplinary projects. Furthermore, we want to address the notion of *critical mathematics education* following Skovsmose (e.g. 2023), where learning mathematics includes not only to learn “pure mathematics”, and to apply mathematical methods and concepts, but also to staying critical to how mathematics is adopted in society, for instance when matters of climate change are visualized through diagrams.

Relational mathematics for cross-curricular teaching

When it comes to cross-curricular teaching, which here is taken as a subfield of transdisciplinary education, there is an ongoing research project which has a specific focus on what this may entail. The project is located in the context of Finland, where it is required through the national curriculum, that students encounter cross-curricular teaching as part of their education. In the project, an overarching model of cross-curricular teaching is developed (Figure 2).

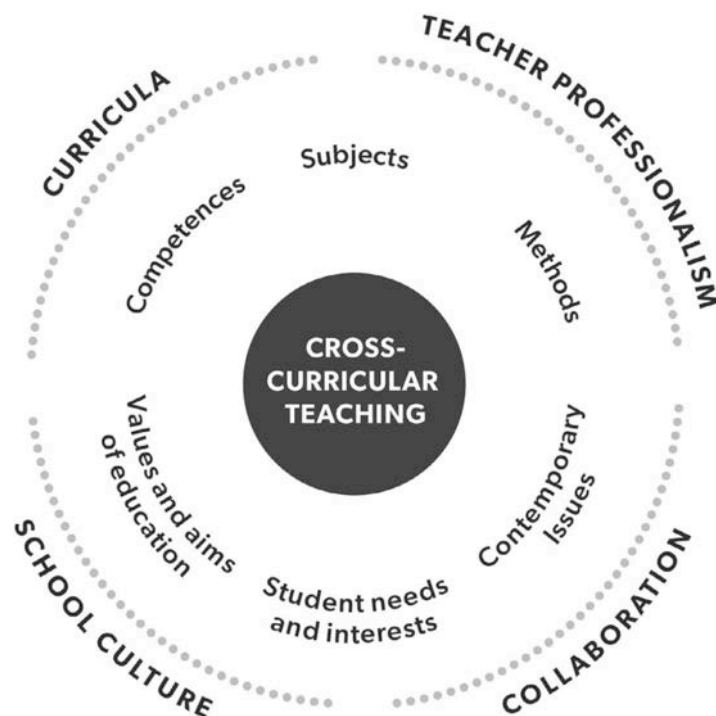


Figure 2: A didactic model for crosscurricular teaching (Mård & Hilli, 2024, p. 47)

The model in Figure 2 has cross-curricular teaching as its main focus, while connecting the world of school with different agents (e.g. curriculum, teachers and students), perspectives (e.g. values, aims and contemporary issues,) and practices (e.g. teaching and collaboration).

While the model in Figure 2 is general, three authors, Røj-Lindberg et al., in the same volume address mathematics education in particular, vis-à-vis cross-curricular teaching (see also Røj-Lindberg et al.,

2022). In their chapter, Røj-Lindberg et al. argue that there are unique challenges to how mathematics is taught and learned within a cross- and trans-curricular setting. These challenges are linked to mathematics often viewed as having a strict hierarchical structure and to perceiving learning in mathematics as acquiring a set of techniques that can be applied outside of mathematics. Røj-Lindberg et al. label this view as instrumental, and stress how such a view makes mathematics difficult to integrate with other subjects. The authors emphasise an alternative view on mathematics teaching and learning, relational, which allows for seeing the value of integration differently:

Adopting a relational view on mathematics means accounting for the intra-mathematical relations and mathematical competences as well as for the relations between people within and outside school and between these people and mathematics. (p. 108)

What is of particular interest for this paper, is when the authors address how *trans-disciplinary* (or cross-curricular) teaching is expected to prepare the students for grappling with grand societal challenges, often called *wicked problems*. This is highly relevant for mathematics in relation to the socio-ecological. This is a notion that we bring with us from this research field. Moreover, we will draw on the notion of mathematics education as *relational*.

Praxis and logos when mathematics is connected to other knowledge areas

In one research area, epistemic aspects of mathematics in relation to other knowledge areas are investigated. Some researchers are particularly interested in mathematics in vocational education (see e.g. Bakker, 2014). There is an interest in capturing not only mathematics that is clearly part of vocational activities (e.g. calculations), but also more implicit and integrated mathematical aspects (FitzSimons & Boistrup, 2017). Within this sub-field, there is an interest in power relations between mathematics and other knowledge areas, such as vocational knowing. This is expressed by Boistrup and Hällback (2021) as to “challenge the dichotomous conception where mathematics is taken as the theoretical knowing as opposed to the vocational knowing which is taken as purely practical” (p. 313). In this way, we here mainly connect to the *socio* in socio-ecological. Having stated that, we see potential for, in the future, to also address wicked problems, and hence, more clearly connect to the socio-ecological.

Here we particularly draw on those who engage in analysis through praxeology. Praxeology is part of the anthropology of the didactics (ATD) (Chevallard, 2006) and was developed mainly for mathematics education. Over time, it has also been expanded for analysis in relation to other knowledge areas. One example is Ladage et al. (2019), who investigated didactic work in a museum. Another example is Castela (2015) who addressed the opportunities of adopting ATD in research on mathematics in connection to other knowledge areas, such as dressmaking.

In short, praxeology is a model for addressing epistemic aspects, where a knowledge domain (e.g., mathematics) is viewed as a practice, constituted by institution-based actions. Praxis and logos are two interconnected parts of praxeology, where praxis concerns the know-how and logos concerns the know-why. With this framework, Johansson et al. (2024), in a study carried out in Sweden, gained access to ways of investigating mathematics and vocational knowledge from an assumption that both mathematics and vocational knowing include aspects in line with praxis and logos. This means that mathematics is not exclusively taken as the theoretical teaching content (logos) applied into the practical teaching vocational teaching content. Rather, quite often mathematics will contribute with praxis, and the vocational knowing with logos.

The vocational education in which Johansson et al. conducted their study was the education of electricians. In Figure 3 an example of a praxeology of an electrician assignment is provided. This assignment serves as an example of how to address epistemic aspect, which in this example has relevance for sustainable use of material. The assignment in this case was “Estimating how much cable when it is rolled”. The task is to find out how much cable there is on a cable reel. The students might roll out the cable and measure and then roll it back in, leading to wear and tear on the cable, and the cable might not be usable. Using the diameter of the reel, the circumference of one turn can be calculated and then multiplied by the number of turns on the reel. This way the students may arrive at an approximate value of how much cable is left on the reel. The background to the task is to save resources and to use cable that is available rather than purchasing new cable.

	Electrician knowledge		Mathematics		
PRAXIS (know-how)		To estimate cable on a roll, and then choose the best roll			
			Round up the calculation of cable and choose longer than calculated		Calculate circumference and layers
LOGOS (know-why)	To calculate is a way to not make the cable dirty, and to save time.	Saving material, and money for customer.			
		How much cable to bring to a site.			Knowing in geometry including understanding concepts

Figure 3: Praxeology of “Estimating how much cable when it is rolled” (Reduced version, drawing on Figure 5 in Johansson et al., 2024)

In the analytical model (constructed by Boistrup & Hällback, 2021), which was the basis for the analysis reflected in Figure 3, there is a continuum between vocational knowing (left) and mathematical knowing (right). This means that the middle column reflects knowing which is equally considered as vocational and mathematical. For the details of the analysis, we refer to Johansson et al. (2024). What we bring with us, is how all kinds of *epistemic aspects*, relevant for this assignment were aimed to be captured. It is clear that the vocational students needed to engage in *praxis* (upper half of Figure 3) and *logos* (lower half of Figure 3) for both mathematical and vocational knowing, and also for integrated knowing. Also, we can see how there are aspects of *sustainability* part of this example, where relevant use of mathematical and vocational knowing reduces the material used.

FUTURE TRANS-DISCIPLINARY EDUCATION AND RESEARCH FOR MATHEMATICS AND THE SOCIO-ECOLOGICAL

What we aim at, for some of our future research and collaboration with teachers, is to draw on notions from the three presented fields, when we work out our theoretical and methodological positionings. For our discussion here, we use an idea of a project, where we connect mathematics education with a, for us, new knowledge area, social studies, and in relation to the socio-ecological.

The notions that we address here, and that we have mentioned in the previous section are: not avoid complexities, critical mathematics education, trans-disciplinary teaching, wicked problems, relational mathematics, epistemic aspects such as praxis and logos, and sustainability.

What we want to inquire together with mathematics teachers and teachers in social studies is the relevance of *transdisciplinary teaching* for addressing *wicked problems* of today, and in the future. Our main interest is in *epistemic aspects* of how the knowledge areas of mathematics and social studies may be connected. Here we want to draw on *critical mathematics education*, not least in relation to how mathematics may be adopted in society. Included here is to problematize and elaborate on how mathematics may be part of social studies, including when mathematics might not be the most relevant perspective to draw on. Specifically, we want to address *wicked problems*, such as matters concerning *sustainability*, that the students may suggest. The chosen problems will have socio-ecological relevance. To dig deeper into epistemic aspects, we see that the model in Figure 3, with *praxis* and *logos* is a way forward, where social studies is at one end of the continuum and mathematics at the other. As we see it, the three aspects of mathematics in *CME* are part of such a model, where it is clear that the applications of mathematics need to be critically reflected on. It is also clear that mathematics is taken as *relational*. Furthermore, such a model invites into an inquiry, which does *not avoid complexities*, at least the ones that are related to *epistemic aspects*.

In our three fields, presented in the previous section, there are other notions that we also might engage with, in our future research. For instance, Figure 1 may constitute inspiration for future research on actants and connections between them (following Latour, 2005), also concerning mathematics and social studies in collaboration. Figure 2 may serve as a model for addressing other aspects than mainly epistemic, including notions highly relevant to teachers. Even though, we have used an example of mathematics education in collaboration with social studies education here, we see our contribution as relevant for other trans-disciplinary studies with relevance to mathematics and the socio-ecological.

CONCLUDING DISCUSSION

We end this paper by shortly discussing our contribution in relation to the literature we drew on initially. Chronaki (2000) inquired how mathematics teachers encounter the complexity of coordinating linkages across disciplinary areas in thematic work. This is much in line with the transdisciplinary approach suggested in this paper. Similar to Chronaki, we are interested, not only in the thematic aspects, but also in embedded mathematics. Our hope is that the model we present can be a help to counter challenges in addressing such epistemic aspects. Ethical aspects are addressed by Boylan & Coles (2017), and also by e.g. Ödmo et al. (2023). Even if we did not articulate the ethical aspects of our contributions in this paper, we do acknowledge that much of what we address is possible to understand in relation to ethics. For instance, epistemic justice (Nieminen & Lahdenperä, 2024) concerns ethics in relation to identifying “epistemic injustice in which students

are not recognised as knowers because barriers are set to their ways of knowing and displaying knowledge”, p. 302). Our hope is that this paper will contribute to a nuanced understanding of what mathematics in trans-disciplinary education may be about, in order to reduce such barriers. As we see it, an understanding of mathematics as relations, and praxis and logos to be flexible within and between knowledge areas, are providing opportunities to go beyond the often taken-for-granted. Finally, we want to address the non-binary take we aimed at, with inspiration from Gutiérrez (2019). Even if Gutiérrez addresses indigenous mathematics in relation to western mathematics, we see some potential parallel in how we have tried to avoid a strict either-or, i.e. a binary take on mathematics in trans-disciplinary teaching and research, in relation to the socio-ecological.

Acknowledgements

The production of the paper was funded by Åbo Academi University Vasa, through Lisa’s guest professorship. Secondly, it was funded by the Swedish Science Council, through Magnus’s PhD studies in the research school Relevancing Mathematics and Science education. Thirdly, it was funded by Svenska kulturfonden (Swedish culture fund) by Ann-Sofi’s and Mats’s participation in “Överdid - ämnesövergripande didaktik” (Project on cross-curricular teaching).

References

- Bakker, A. (2014). Characterising and developing vocational mathematics knowledge. *Educational Studies in Mathematics*, 86, 151-156.
- Beane, J. A. (1997). *Curriculum integration: Designing the core of democratic education*. Tr Coll Press.
- Boistrup, L., & Hällback, M. (2021). Mathematics in vocational education: An epistemic framework. In D. Kollosche (Ed.), *Exploring new ways to connect: Proceedings of the Eleventh International Mathematics Education and Society Conference* (pp. 311-320). Tredition.
- Boylan, M., & Coles, A. T. (2017). Is another mathematics education possible?: An introduction to a Special issue on mathematics education and the living world. *Philosophy of mathematics education journal*, 32. <http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome32/index.html>
- Castela, C. (2015). When praxeologies move from an institution to another one: The transpositive effects. In *Proceedings of the 23rd Annual meeting of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 6-19). SAARMSTE.
- Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the 4th Conference of the European Society for Research in Mathematics Education* (pp.21-30). CERME 4.
- Chronaki, A. (2000). Teaching maths through theme-based resources: Pedagogic style, ‘theme’ and ‘maths’ in lessons. *Educational Studies in Mathematics*, 42, 141–163.
- FitzSimons, G., & Boistrup, L. B. (2017). In the workplace mathematics does not announce itself: Towards overcoming the hiatus between mathematics education and work. *Educational Studies in Mathematics*, 95(3), 329-349.
- Gutiérrez, R. (2019). Mathematx: Towards a way of being. In J. Subramanian (Ed.), *Proceedings of the 10th International Conference of Mathematics Education and Society*. MES.
- Hauge, K. H.; Götze, P.; Hansen, R.; Steffensen, L. (2017). Categories of critical mathematics based reflections on climate change. In A. Chronaki (Ed.), *Proceedings of the 9th International Conference of Mathematics Education and Society* (pp. 522–532). MES.
- Johansson, T., Boistrup, L. B., & Dahl, J. (2024). Patterns of connections between mathematics and electrician knowledge: Praxeology in vocational education. In *Proceedings of CIEAEM74*, 15-19 August 2023. Malmö

University. *Quaderni di Ricerca in Didattica*, 13, 641-650.
https://sites.unipa.it/grim/quaderno_2024_numspecc_13.htm

- Klausen, S. H. & Mård, N. (Eds.). (2024). *Developing a didactic framework across and beyond school subjects*. Routledge.
- Klein, J. T. (2017). Typologies of interdisciplinarity. The boundary work of definition. In R. Frodeman, J. T. Klein, & R. C. S. Pacheo (Eds.), *The Oxford handbook of interdisciplinarity* (2nd ed., pp. 21–34). Oxford University Press. <https://doi.org/10.1093/oxfordhb/9780198733522.013.3>
- Ladage, C., Achiam, M., & Marandino, M. (2019). Research on ATD outside mathematics. In M. Bosch, Y. Chevallard, F. J. García, & J. Monaghan (Eds.), *Working with the Anthropological theory of the didactic in mathematics education: A comprehensive casebook* (pp. 12–30). Routledge.
- Latour, B. (2005). *Reassembling the social: An introduction to actor-network-theory*. OUP.
- Mård, N., & Hilli, C. (2024). Crosscurricular teacher collaboration actualizing teacher professionalism: Revising a didactic model. In S. H. Klausen, N. Mård (Eds.), *Developing a didactic framework across and beyond school subjects* (pp. 47-58). Routledge.
- Mård, N., & Klausen, S. H. (2024). Speaking and thinking about crosscurricular teaching: Terms, concepts, and conceptions. In S. H. Klausen, N. Mård (Eds.), *Developing a didactic framework across and beyond school subjects* (pp. 7-18). Routledge.
- Nieminen, J. H., & Lahdenperä, J. (2024). Assessment and epistemic (in) justice: how assessment produces knowledge and knowers. *Teaching in Higher Education*, 29(1), 300-317.
- Ödmo, M., Boistrup, L. B., & Chronaki, A. (2023a). A teacher education statistics course encounters climate change and critical mathematics education: Thinking about controversies. In R. Marcone, P. Linardi, R. Milani, J.P. A. de Paulo; A. Moura Queiroz, & M. T. da Silva (Eds.), *Proceedings of the 12th International Conference of Mathematics Education and Society* (pp. 1307-1321). MES.
- Ödmo, M., Chronaki, A., & Boistrup, L. B. (2023b). Bringing critical mathematics education and Actor–Network Theory to a statistics course in mathematics teacher education: Actants for articulating complexity in student teachers’ foregrounds. *Education Sciences*, 13(12), 1201.
- Røj-Lindberg, A., Braskén, M., & Berts, K. (2024). Mathematics beyond and across the curriculum. In S.H. Klausen, N. Mård (Eds.), *Developing a didactic framework across and beyond school subjects* (pp. 106-118). Routledge.
- Røj-Lindberg, A. S., Hartvik, J., & Ingves, M. (2022). En fallstudie om finlandssvenska lärares insikter från ett utvecklingsprojekt. *Nordic Studies in Education*, 42(2), 176-193.
- Skovsmose, O. (2023). *Critical mathematics education*. Springer.
- Steffensen, L., Herheim, R., & Rangnes, T. E. (2021). The mathematical formatting of how climate change is perceived: Teachers’ reflection and practice. In A. Andersson & R. Barwell (Eds.), *Applying critical mathematics education* (pp. 185-209). Brill.

ETHNO-SUSTAINABILITY USING MATHEMATICS: MAPPING OUT A FUTURE LANDSCAPE FOR MATHEMATICS EDUCATION

Arindam Bose¹, Catherine P. Vistro-Yu²

¹Tata Institute of Social Sciences (TISS), ²Ateneo de Manila University

This paper offers a future landscape for mathematics education and proposes funds of knowledge as a theoretical lens through which mathematics education can be re-oriented. It begins with an acknowledgement of the severe climatic conditions that our planet and the whole of humanity have been experiencing in the last few decades. We briefly discuss the concept of funds of knowledge in rationalising the need for the socio-ecological in mathematics education. We put forward the concept of ethno-sustainability which mathematics education can draw from, to guide us into a new world of learning mathematics. We cite studies from India and the Philippines where the local communities' voices and sustainable practices become a rich source of knowledge in addressing issues of loss, wastage, and degradation. Our data show evidence of ethno-methodologies which children in the communities employ and which have elements of ethno-sustainability within routine everyday practices of minimum wastage, optimization, and judicious decision making.

INTRODUCTION

Imagine a world that is teeming with life -- rich, fresh, abundant, and where there is no limit to growth and progress. Is it a world that is truly possible? Can mathematics help create such a world? At the outset, there is a growing belief that technology and artificial intelligence using mathematical knowledge can make many things possible. As a foundational knowledge for science, technology, and economics, mathematics education offers limitless possibilities that we can imagine creating for all of humanity. Unfortunately, there is a serious precarity at this juncture for both our planet Earth and for human existence, where degradation at several levels arising from devastating wars, increasing inequity and joblessness, growing poverty, environmental fallouts, severe climate change, and moral downhill are co-creating conditions which are blowing up out of proportion and going beyond human control. This fast-changing reality arises in the backdrop of the fact that our resources are not limitless or unlimited and that there is a limit to which the Earth can bear the consequences of the degradation (Roe et al., 2018). Therefore, there is a need to examine our human practices in managing resources and consuming goods to preserve life and draw from informal knowledge systems which allow people to interact with their immediate environment focusing on preservation, conservation, and usefulness of the socially shared resources.

THE CONTEXT AND THE PROBLEM

The recent decades have been marked with unprecedented environmental changes in all parts of the world. For example, extreme weather conditions have been reported to occur at odd times in a year and are happening frequently. The changing scenario of global climate and its pattern across the world has impacted most of the world's population directly or indirectly. Peoples' sufferings have increased manifolds but more importantly, there appears to be a lack of knowledge base and clarity as to how this global crisis can be tackled and addressed. This is despite several inter and multi-national deliberations at various levels and despite several governmental and non-governmental agencies

taking up the matter for quite many years now. The changing climatic condition is just one among the many crises currently being faced globally as discussed above in the Introduction. Climate change is something that we do not have any total control of, and human practices exacerbate its effects our planet and its people. Unlimited human consumption is a major culprit. Increased productivity of goods seems to have encouraged this crisis. Unlimited consumption leads to uncontrolled wastage. Waste, when unmanaged, pollutes the environment and contributes heavily to biodiversity loss, land degradation, and serious health conditions for all living beings.

Interestingly however, the communities that live at the margins or the ones that are disadvantaged and written off by the urban, elite, modern, and self-proclaimed developed communities, often have their own ways of tackling these crises using insights from their inherent and community-based knowledge resource and cultural practices which anthropologists often refer to as funds of knowledge. There are many examples of funds of knowledge which have uncovered multi-layered cognition in practice prevalent in different communities of practice (see, Lave & Wenger, 1998) such as guided participation (see, Rogoff, 1990); different forms of community's work practice, apprenticeship and hand skills (Bose & Farsani, 2023) and various forms of ethnomathematical practices (D'Ambrosio, 1985; Lipka, 1991; Greer & Mukhopadhyay, 2015; Saxe, 1992). These works demonstrate practices which are differently aligned compared to the knowledge base and practice which are often considered as mainstream and popularly practised as one collective in most parts of the world. This paper wishes to explore a few examples of sustainability practices and use of indigenous methods among cross country communities in the South Asian and Southeast Asian contexts which help the communities conserve and optimally use limited resources or constantly indulge in minimum waste that collectively build a community practice we term as ethno-sustainable. We present examples of these ethno-sustainability practices and argue how these ideas can influence and re-purpose future landscapes of mathematics education.

FUNDS OF KNOWLEDGE AS A THEORETICAL LENS

Funds of knowledge as a community's resource pool emerges from people's life experiences and is available to the members of the group which could be households, communities, or neighbourhoods. It guides our understanding of the nature and extent of knowledge gained in out-of-school contexts, and therefore also available within the community of the classroom. Such knowledge is closely tied to practice and used in specific contexts. Funds of knowledge emphasise the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (Gonzalez et al., 2005, p. 40).

This perspective also opens possibilities of teachers drawing on such funds of knowledge and relating them to the work of the classroom (Moll et al., 1992). In his review of four studies that were conducted in different countries and contexts, Moll cites how researchers re-orient the notion of funds of knowledge to address issues that are particularly suitable to their contexts (Moll, 2019). Funds of knowledge as a sociocultural approach allows us to document and utilise knowledge resources of communities and represent them based on their strengths, effectively challenging the dominant orientations that fail to capture the unique contexts of the communities. Funds of knowledge gives us a lens to look at community's insights and cultural wisdom which often have a socioecological angle since these insights offer a balancing act with the environmental concerns.

WHY SOCIO-ECOLOGICAL AND MATHEMATICS AND WHY NOW?

We adopt Roe et al.'s (2018) definition of sustainability as “the ability of a social, economic, or ecological system to meet the needs of the present generation without compromising the ability of future generations to meet their own needs. A process or practice is sustainable to the extent that it contributes to the sustainability of the social, economic, or ecological systems in that it is embedded” (p. vii). Mathematics education is a socio-ecological system that begs to ask questions of sustainability. Like van Hoesve and Alpár (2024), we seek new sustainable approaches to address challenges that have recently confronted all of humanity. According to the 2022 Intergovernmental Panel on Climate Change, these challenges, all related to climate change, include “biodiversity loss, overall unsustainable consumption of natural resources, land and ecosystem degradation, rapid urbanisation, human demographic shifts, social and economic inequalities and a pandemic” (van Hoesve & Alpár, 2024, p. 157). Innovative solutions will be needed to arrest all forms of degradation, loss, and wastage. Therefore, we argue that all of humanity must make a concerted effort to address environment-related problems brought about by climate change. We, mathematics educators, are especially enjoined to add our voices to the many solutions being offered.

ETHNO-SUSTAINABILITY: WHAT IS IT AND WHY TALK ABOUT IT?

It is now becoming globally acceptable that we human beings cannot remain aloof and disconnected from the precarities which have emerged due to the growing environmental degradation leading to hazardous living conditions (see Roe, deforest & Jamshidi, 2018 and others). On the other hand, sociocultural studies on human cognitive development have shown different aspects of cognitive practices and notions of development derived from interpersonal interactions within communities and semiotic mediations among the members of these communities which had not yet been influenced by the Western form of schooling (Lave, 1988; Lave & Wenger, 1991). Such communities and those living in low-income urban settlements often possess inherent funds of knowledge consisting of their community's cultural insights and wisdom (see Bose, 2015). Such funds of knowledge contain wisdom and insights about survival and about tackling immediate natural hurdles. These insights can vary from the range of medicinal values of resources or plants around them to providing social and emotional support to each other (for example, guided participation in collective tasks) to agricultural insights and so on. There are inherent knowledge resources and insights within these practice which also contain wisdom about sustainability in the face of various forms of precarity. It is these notions of sustainability which we are referring to as ethno-sustainability. These are culturally embedded inherent insights within the community that tackle natural challenges arising from environmental degradation, climate change and widespread diseases and adopt life-means through sustainable methods. In our growing precarious living conditions, it is increasingly imperative to be aware of such ethno-methodologies and use them in preparing our next generation ready enough to tackle the challenges.

DOCUMENTED SUSTAINABILITY EFFORTS

In the Philippines, efforts to minimise waste, repurpose unwanted materials, or to recycle used products abound even before the presidential proclamation was issued in 2014 declaring January of each year as Zero Waste month to help Filipinos remember (Carretero, 2021). Environment groups such as ‘Break Free From Plastic’ (BFFP) that had been working toward zero waste for a plastic-free

future, The Ecowaste Coalition, Mother Earth Foundation, and War on Waste Dumaguete that have been working with communities and the local governments in the provinces of Cavite in the south, Pampanga in Central Luzon, and Negros Oriental and Siquijor in the Visayas, have been providing training programs for the communities to implement zero-waste systems. For many of these groups, the first step is to reduce if not eliminate single-use plastics.

India has been leading the way in mitigating the disastrous effects of climate change and promoting sustainable practices through policy, dialogue, and adoption of energy-efficient measures (Pandey, 2017). It has been cited as one of the least wasteful economies by the World Economic Forum. Pandey (2017) reports that India's culture, philosophy and values consider sustainability as a core principle and that re-use and re-cycle are generally promoted in India for people to adopt. Recently, the Govt. of India unveiled a list of 75 practices that promote a climate-friendly lifestyle (Habbu & Dixit, 2023) and categorized in seven groups: energy saving, water saving, reduced use of single-use plastic, sustainable food systems, waste reduction, healthy lifestyle and e-waste handling. This pronouncement emphasises adoption of a circular economy, described as cyclic in nature, directing the economy towards reuse, as opposed to waste when a product is no longer used (Habbu & Dixit, 2023) and these practices are part of the current cleanliness mission (*swachhata* actions).

EVIDENCE FROM OUR STUDIES

This paper draws inputs from studies done in India and in the Philippines. An ongoing study for a dissertation project by Villamor (2024) in the Philippines and supervised by the second author drew from the local youth their collective wisdom in proposing solutions to manage their province's solid waste problems. The province, Surigao del Norte is where Siargao Islands is found, a recognised tourism destination in the country (Department of Tourism, n.d.) and a protected area under the category of protected landscape and seascape declared by the President of the Philippines (Proclamation No. 902, s. 1996), pursuant to Republic Act No. 7586 or the National Integrated Protected Areas System (Philippine Congress, 1992). Being a protected area, Siargao Islands must be managed well to sustain its biodiversity and remain protected against environmental degradation. Like any tourist destination in the world, Siargao Islands is vulnerable to "high waste generation with a low rate of waste collection" (Villamor, 2022, p. 54) and disposal.

A major concern by the people of Siargao is to find sustainable solutions to manage solid waste aside from building landfill areas. In preparation for the main study, Villamor engaged Grade 11 students in a public school in Siargao who were tasked to survey their communities on the kinds of solid waste being generated in their homes and schools. The results of this survey became the springboard for designing a lesson on mathematical functions and graphs using mathematical modelling as an approach for Grade 12 students. The mathematical modelling task was an application problem on volumes of prisms, which became the stimulus for serious discussions on mitigating solid waste (Villamor & Vistro-Yu, 2024). The study employed a mixed-method research design using the advocacy and participatory worldview. The students' levels of performance in completing the mathematical modelling task comprised the quantitative data. The qualitative data obtained through observations and focus group discussion consisted of how students framed solutions in mitigating solid waste issues in school. Part of the research study was student participants presenting their findings and results of the modelling tasks first to fellow students and teachers in their school community, and then to representatives of the Local Government. This study shows that the student-

participants' experiences of learning mathematics concepts referencing the real-world problem of solid waste management become the foundational sources of local knowledge and eventual wisdom in dealing with sustainability issues. It is this shared knowledge of the people who know their circumstances and care for their communities that we seek to add to our funds of knowledge in addressing issues of sustainability for the future of our planet. We ask how mathematics education in the future can help create grounds for building these insights for achieving and maintaining socio-ecological which is not just crucial but also brings in sensitivity towards our environment and in understanding ways to address environmental concerns.

The study done in India came with the aim of exploring middle graders' out-of-school mathematical knowledge and these middle graders came from an economically active low-income settlement and had exposure to various income generating work practices dispersed in micro enterprises in the neighbourhood. Additionally, the researcher's (first author) prolonged connection with the community living in this settlement following an ethnographic approach revealed the community's knowledge resource and wisdom in the form of funds of knowledge. The data is drawn from observations of the community, interaction with community elders and children, and semi-structured interviews with randomly selected six graders from two governmental schools located in the settlement (see, Bose, 2015). The aspects of funds of knowledge of the community which we discuss in this paper include ideas about minimising waste and recycling the leftover produce or raw materials, optimising resources which are often than not, limited and scarcely available, and decision making. We argue in this paper that these insights and wisdom offer ideas which entail ideas of sustainability, conservation and preservation, i.e., in other words, ethno-sustainability.

Recycling leftover resources

Waste collection for the garment and plastic recycling work are independent tasks which were valorised by the community and many children from the settlement were engaged in this work. Recycling is a micro enterprise, and the low-income settlement has large networks of establishments that engage in recycling various kinds of materials drawing in a sizeable population from the locality and providing them with livelihood. Armaan (sixth grader, 13-year-old boy) explained that personal contacts help in getting *chindhi* (leftover garment pieces) and in getting a good price for the collected material. The garment recycling work is valued since the community's funds of knowledge valorises "creation from waste" and "minimising waste". This work entails collection of leftover cloth pieces of varying sizes from the garment stitching workshops. These cloth pieces (called "*chindhi*") are collected for recycled garment manufacturing as well as for other purposes, such as, for use as stuffing in sofas, chair cushions, quilts or mattresses, and so on. *Chindhi* are measured not by size but by their weight. The collection is then sorted according to the colour and quality of the material, texture, type and sold at different rates. Armaan has been involved in this work since he was a fourth grader and would do this once a week along with many other young children from the locality. Such work skills and other hand skills are valued in the community and these practices are seen as necessary as part of growing up for the children.

Judicious decision making

In addition to waste recycling work, Armaan has to make quick decisions while closing a deal of what price the leftover cloth pieces were to be sold. He handled several variables such as, size, colour,

texture, type of cloth and time taken which considers while quoting the selling price - all calculations done mentally. Such multi-variable mental calculations are commonly practiced in the community and valorised. Below is an excerpt from an interaction between the researcher (first author) and Armaan. It indicates Armaan's sorting and estimation skills and mental calculation of the selling prices based on cloth colour and texture. "S" and "R" refer to student and researcher (first author) and numbers on the left represent transcript line numbers.

- 364 S sabse zyada to bahut pehle kamaya tha, chhe sau chalis rupaya [maximum earning was long back, six hundred forty rupees]
- 365 R kitna kilo tha usme? [How many kilos were in it?]
- 366 S panchanve kilo[Ninety-five kilo]
- 367 R panchanve kilo ka chhe sau chalis rupaya? Kaise? [Six hundred forty rupees for ninety five kilo? How?]
- 368 S ...white alag, lal alag kar liya, white becha tha bees rupaya kilo, aur lal becha tha aath rupaya kilo aur
 chindhi becha tha (char rupaya kilo) [...white and red separated, sold white for twenty rupees a kilo, and sold red for eight rupees a kilo and sold *chindhi* (four rupees a kilo)]

Such practices involve quick and judicious decision making since children like Armaan are quick in arriving at a figure to quote for selling the collected *chindhi* for a price which they think is an optimal quote by estimating the weight to carry, time taken, purchase capacity of the buyer, contemporary rates and possible price-quotes of similar other groups. Such tasks involve decision making prowess on the go and adds to children's autonomy and their training towards recycling and earning but also, implicitly reinforces practices around the socio-ecological.

Resource optimization

Community's funds of knowledge often emphasise on optimal and meaningful use of resources. It is a skill which is ingrained in every child's mind. We argue that such skills are valuable from a socio-ecological perspective too, as such lifelong practices help the community members learn techniques in minimising waste and surviving on few resources. Armaan offered another perspective of optimisation which emerged from his requirement of optimising his income. His explanations showed use of proportional reasoning to make decisions about how much to collect of the white variety and the coloured variety. He was aware that collecting the white variety fetched him five times more than the ordinary or smaller pieces of *chindhi* (20:4), while the red variety brought only double the amount of the smaller *chindhi* (8:4). Optimality of time, traverse and weight to carry are considered when Armaan does *chindhi* collection work. *Chindhi* collectors earmark workshops where they are most likely to get a handsome quantity and know how quickly they can fix a deal before other groups can drop in. One must keep in mind the weight that can be carried easily and yet can fetch a good amount. Embedded here is a reflection of the community practice of the socio-ecological where a collective attempt is made to minimise waste and enhance waste management. Community children learn this skill and more of such use of optimisation knowledge in routine everyday practices. These are ethno-sustainability practices which communities employ and children living in these communities have access to or take part in these practices right from an early age. Examples of these practices or

engagement in them offer ways to inculcate concerns for our environment among school students through mathematics learning. Tasks designed around such practices can help in reinforcing these community based funds of knowledge and wisdom by bringing onboard socio-ecological values.

WHAT ARE WE ABLE TO SEE, SAY OR DO?

With our growing concerns for global climate change as mentioned in the beginning, comes broader environmental degradation and rapid depletion of natural resources. Poor countries and poor continents are often discounted for the long-term casualties which Nixon refers to as “slow violence” and further argues these countries are “discounted as cultures possessing environmental practices and concerns of their own” (Nixon, 2011, p.2). We argue that environmental degradation is a different kind of ‘slow violence’ which human beings are subjecting the Earth to. As Nixon says, this violence is “neither spectacular nor instantaneous, but rather incremental and accretive, its calamitous repercussions playing out across a range of temporal scales” and which remains relatively invisible (p.2). The more visible forms of slow violence are climate change, the thawing cryosphere, toxic drift, biomagnification, deforestation, the radioactive aftermaths of wars, acidifying oceans and other environmental catastrophes (Nixon, 2011). Even to make sense of these forms of slow violence and comprehend ways to arrest its onslaught, the need is to remodel and use mathematics for creating a socio-ecological balance by re-evaluating the priorities of mathematics education and its practice (Coles, Solares-Rojas & le Roux, 2024). Such efforts call for extending support to our teachers by making them capable with resources to reach this goal.

There is a great demand for building capacities for environmental sustainability. What our analysis and community’s funds of knowledge enable us to realise is that in the present times even environmental sustainability may not be enough. Rather, what is further required is environmental restoration and environmental regeneration and insights for ways to achieve these. Drawing from the data given above, we see evidence of ethno-methodologies which children in the community employ and which have elements of ethno-sustainability within routine everyday practices of minimum wastage, optimization, and judicious decision making. These examples invite us to think how mathematics education can be reoriented and equip itself to allow for our learners the use of community’s funds of knowledge towards restoration and regeneration. We are compelled to scrutinise and pick out how school mathematics content can draw from ethno-methodologies that help our learners develop awareness of and eventually inform practice to address the socio-ecological concerns. The examples of ethnosustainability practices of the community discussed above offer insights for us to trace future landscapes of mathematics education and to reframe mathematics for the future, along with the hope of furthering towards sustainable practices and at the same time equipping learners to face the changing futures for an inclusive liveable world.

References

- Bose, A. (2015). *Work, knowledge and identity: Implications for school learning of out-of-school mathematical knowledge*. Unpublished PhD dissertation thesis. Mumbai: Homi Bhabha Centre for Science Education, TIFR.
- Bose, A. & Farsani, D. (2023). Making visible “the invisible”: can mathematics embedded in work practices enable critical questioning? *Journal of Mathematical Behavior*, 70, 101028. <https://doi.org/10.1016/j.jmathb.2022.101028>

- Carretero, G. M. (2021, January 31). An effective zero-waste approach for PH. *Philippine Daily Inquirer*. <https://opinion.inquirer.net/137384/an-effective-zero-waste-approach-for-ph>
- Coles, A., Solares-Rojas, A. & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165–183.
- D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics*, 5(1), 44-48.
- Department of Tourism. (n.d.). *DOT celebrates Siargao inclusion in Time Magazine as one of the world's greatest places of 2021*. Retrieved from https://beta.tourism.gov.ph/news_and_updates/dot-celebrates-siargao-inclusion-in-time-magazine-as-one-of-the-worlds-greatest-places-of-2021/
- González, N., Moll, L. C., & Amanti, C. (Eds.). (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Lawrence Erlbaum.
- Greer, B., & Mukhopadhyay, S. (2015). Honoring diversity in intercultural mathematics education. *Intercultural Education*, 26(4), 261-265.
- Habbu, V. & Dixit, V. (2023). India's initiative towards sustainable environment. *The Times of India*. <https://timesofindia.indiatimes.com/blogs/voices/indias-initiative-towards-sustainable-environment/>
- Lave, J. (1988). *Cognition in Practice: Mind, mathematics and culture in everyday life*. Cambridge University Press.
- Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge University Press.
- Lipka, J. (1991). Toward a culturally based pedagogy: A case study of one Yupi'k Eskimo Teacher. *Anthropology & Education Quarterly*, 22(3), 203-223.
- Moll, L. C. (2019). Elaborating funds of knowledge: Community-oriented practices in international contexts. *Literacy Research: Theory, Method, and Practice*, 68(1), 130-138.
- Moll, L. C., Amanti, C., Neff, D., & Gonzalez, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory into practice*, 31(2), 132-141.
- Nixon, R. (2011). *Slow violence and the environmentalism of the poor*. Harvard University Press.
- Pandey, A. (2017, October 2). What India can teach the world about sustainability. *World Economic Forum*. <https://www.weforum.org/agenda/2017/10/what-india-can-teach-the-world-about-sustainability/>
- Roe, J., deForest, R., & Jamshidi, S. (2018). *Mathematics for sustainability texts for quantitative critical thinking*. Springer.
- Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. Oxford Univ Press.
- Saxe, G. B. (1992). Studying Children's Learning in Context: Problems and Prospects. *The Journal of Learning Sciences*, 2(2), 215-234.
- van Hoeve, M. & Alpár, G. (2024). Open maths meets sustainability. A model for integrating sustainability and mindset into mathematics education. In M. Doorman, E. Schäfer, E., & K. Maaß (Eds.), *STEM & Open Schooling for Sustainability Education: Proceedings of the 4th 'Educating the Educators' Conference*, 157-164. WTM-Verlag Münster.
- Villamor, J. J. B. (2022). *Using mathematical modeling to build students' agency for critical citizenship*. Unpublished manuscript. Ateneo de Manila University.
- Villamor, J. J. B. & Vistro-Yu, C. P. (2024). Mathematical modelling for critical citizenship. In M. Doorman, E. Schäfer, E., & K. Maaß (Eds.), *STEM & Open Schooling for Sustainability Education: Proceedings of the 4th 'Educating the Educators' Conference* (pp. 148-156). WTM-Verlag Münster.

TERRAPOLIS STORIES: WEAVING IDEAS ABOUT DIGITAL TECHNOLOGY AND THE SOCIO-ECOLOGICAL

Lucely Figueroa-Suárez¹ Anna Chronaki¹, Ulrika Ryan¹

¹Malmö University

In this paper we weaved Terrapolis stories with Molly, Maria, an AI chatbot, Rosi Braidotti, Donna Haraway amongst others. The aim is to locate ideas and questions in relation to the paradoxical advantages and disadvantages of new digital technologies that have permeated mathematics education. To do that, we used Donna Haraway's tentacular thinking to think together new ideas through other ideas in an affirmative way. We also acknowledge that we, as researchers, are entangled in this study and that we are part and produced the explored phenomenon. Molly and Maria were participants of an interview that was conducted in a Swedish school with 8 grade students. New entangled ideas were also weaved with responses generated from our AI chatbot.

LIVING TERRAPOLIS AND MATHEMATICS EDUCATION

Our current spacetimes realize the expansive growth of digital technologies (i.e. the 4th Industrial Revolution) and the severe extinction of biodiverse species (i.e. the 6th extinction) alongside ecological catastrophes due to irresponsible human activity on Earth with irreversible effects. As this convergence denotes austere socio-ecological crisis, Braidotti (2019) argues for the need to decenter the human and embrace critically how post-humanity permeates technological, social and environmental questions. She is not alone in this task but aligns with other feminist science scholars such as Sandra Harding, Donna Haraway and Karen Barad who ask similar questions. Mathematics education as field of theory, research and didactic praxis seems, by and large, to prioritize technological development (e.g. through mathematical modelling, algorithmic design, machine learning and AI tools) without questioning its assumed neutrality and, at times, irresponsible or ethical contribution to socio-ecological crisis (Yasukawa, 2002). In a similar vein, educational policy in many countries, including Sweden where this study is based, include programming and computational thinking within the compulsory mathematics curriculum (Figueroa-Suárez & Ryan, 2023; Vinnervik, 2020). Some argue that computer programming (Tissenbaum et al., 2019) and AI awareness should be taught at school because it is relevant to citizenship (Andersson & Register, 2023). As such, we think it is important to inquire how we, as researchers, along with students and scholars who strive to create a critical and affirmative distance could think together on how these ideas become weaved and what critical questions they allow us to note or what new spaces they allow us to open about how to live our life on Earth.

Terrapolis is a way to describe the complex world we live in. 'Terrapolis', a term coined by science philosopher Donna Haraway, combines the Latin 'terra' for earth, with the Greek 'polis' for city or citizens. Haraway (2016) elucidates the mystery of living in the place where all critters live, the mystery of living within the convergence, in the n-dimensional niche space that she calls Terrapolis. Terrapolis is Haraway's speculative fabulation of an earth where humans, non-humans, technologies and knowledge coexist enacting practices for critters to be able to flourish and die well in our planet. It is on our planet earth, where we have provoked mutations that endanger both human and non-human lives. Hence, Haraway (2016, p. 44) invites us to think with her by claiming: "*There is only*

one real mystery at stake, here; it is the answer we, meaning those who belong to this history, may be able to create as we face the consequences of what we have provoked". We morph this mystery in this paper as the question: *What ethical and onto-epistemic Terrapolis stories do we weave in relation to the advantages and disadvantages of digital technological development?* The convergence between revolution and extinction has created uncertainty and anxiety and to a certain extent drives us to co-produce situated and response-able answers as critters who belong to this history.

We think with one fragment of the collected data that is perhaps part of the unexpected emergences in research data that in a traditional research process tend to be forgotten (MacLure, 2013). MacLure (2013) acknowledges the capacity for wonder that dwell within the relation of data-researcher. Enacting tentacular thinking through the practice of string figures of Haraway (2016) with this fragment allows us to animate the capacity for wonder of data and to be open onto the new and unexpected. We use excerpts from an interview with two 13 years old students to think with tentacularity and co-produce situated and response-able answers to the Terrapolis stories that are being told in relation to digital technological development and socio-ecological issues. This interview was done in grade 8 in an international school in Sweden. We carry out tentacular thinking by thinking with fragments from student's accounts about the use of technology in our lives. By presenting and excerpt from an interview we think with Haraway (2016), Barad (2007), Braidotti (2019) amongst others and the interview fragments to co-produce answers, ideas and open questions in relation to our position within the paradoxical convergence between revolution and extinction. The present paper is organized by first discussing the theoretical concepts of string figures and entanglements and then presenting and analyzing a story of Terrapolis in this frame.

STRING FIGURES AND ENTANGLEMENTS

Through the practice of string figures as "*a series of interlaced trails*", Haraway (2016) highlights the interconnectedness across living beings, other-than-human beings and things. Practicing games of string figures is about enlacing, interconnect, relaying and receiving stories. These games are about holding still and passing on ideas that matter to us to connect with other ideas that can create important statements. String figure in its literal meaning is the thread game for which two hands are needed to pass on, tangle and relay, for example, the Navajo string figure called "Coyote Running Opposite Ways" (see figure in: Haraway, 2016, p. 14). Through the metaphor of string figures, Donna Haraway explains the collective and tentacular way in which the Terrapolis story needs to be told and how through tentacular thinking is possible to tell these stories. Tentacles are tools to enact string figures games and create entanglements of collective thoughts that can coproduce open and unpredictable stories that have the capability of unexpected change.

In our case, tentacular thinking is a collective practice in which participants are the students Molly and Maria with their accounts, ourselves as researchers with our attempts to analyze and discuss, the AI chatbot with its algorithmic account and strangeness companionship and the scholars cited or quoted here. Murriss (2022) explains the use of accounts and quotes become material pieces that talk for the students and authors in their absence. Certainly, this is an ethical moment and we, as researchers and authors of this paper, carry responsibility in this creative endeavor, however, we try to unfold this process. Our tentacular thinking connects to Barad (2007) agential realism in the sense of our entanglement as researchers, producing and being part of the explored phenomenon. We are analysts and discussants within the conversation; we hold still and rethink ideas critically and

affirmatively. As researchers we are not only entangled with the phenomenon that we are exploring, but we are also affecting the very fate of the research process in in/determinate ways. Data-researcher entanglement (MacLure, 2013) matters in the co-production of meaningful statements. Within this entanglement we co-produce new connections to think with other connections. Therefore, we emphasize the use of the word co-production, because it is a production of different actors in different times and spaces.

TENTACULAR THINKING WITH STUDENTS

We use the following episode to enact our work on string figures. These string figures work comes from a Swedish school context. From this fragment we weave string figures and co-produce ideas within our own Terrapolis story in relation to socio-ecological issues and the use of digital technology. This episode is part of Lucely's PhD project in which she is investigating how computer programming is taught and learnt in mathematics classrooms. Lucely conducted stimulated recall interviews (Sanchez & Grimshaw, 2019) after a programming activity that was done during a mathematics lesson. Although the interview aimed to inquire about students interests and perceptions on computer programming learning at school, the discussion touched upon wider issues of digital technology and AI in our times. Below we present the excerpt that was co-produced during the interview (excerpt from interview, 2023/12/20).

Researcher: How do you feel about digital technology? I mean, do you think there are good or bad ways to use technology?

Molly: It has pros and cons, one of the pros is, they can technology like it every day, like things like, if they have to take the bus in the morning or whatever you take the bus. You use your bus card on your phone, you just, you know, to see when the bus is coming can also, we do like our all our work is done on the computer. So, I think it's more efficient to have technology, but I mean there's also cons that some people might feel like AI.. with the students cheating and copying or whatever, that could be like a con to like technology.

Maria: How, like as technology advances, I think we probably do need it more in our daily life than we used to back then because, like we're both. So like, like technologies in our everyday life and it's advancing as well, so like if you have, for example, there are a lot of workers who have AI taking their job or AI replacing what they work in and they are forced to find other jobs, so I think as technology advances, I think there will be like a time where AI might take over most of the jobs and most of the economy. But they're also like pros to it where it does help us like emmm, taking the bus, calling one another, using it in workspaces and in school. Yeah, that's pretty much it, because there are pros and cons to every like aspect of technology.

Molly: Another like pros is like, making cars, like electric cars are all like made by technology, and I mean that's like very important in today's world or even solar panels, that's good for like the environment that's made by technology.

Researcher: And then for you, the use of technology could help with solving to some extent environmental problems.

Molly and Maria: Yeah (and assenting)

Molly and Maria discuss the role of digital technologies in our ways of living. They, as students in Sweden are, like many of us entangled in string figures with the technological apparatus that are within our grasp. In Molly's words: "*it is more efficient to have technology*". Taking the bus in the morning and being able to see when the bus is coming could be a way to say that technology render us, as humans, capable to be more efficient and so to not waste our time by, for example, waiting too much for the bus and be able to be on time at school. However, our technologies are made by natural resources extraction that have provoked environmental crisis, in addition, the growth of digital technologies supposedly have reinforced inequities by making rich people even richer and by concentrating wealth in small groups of people (Alarco Tosoni, 2023; Varoufakis, 2024).

According to Molly, technology like AI could make us cheat or copy at school. Maria weaves this idea about AI, by saying that AI could take their job or replace what people work in. Maria said: "*I think there will be like a time where AI might take over most of the jobs and most of the economy*". According to these accounts technology is and will be affecting/benefiting people in Terrapolis. Since the industrial revolution, many jobs have been transformed or as Maria says machines (AI) have been *replacing people in their jobs* and separating the workers from the means of production. This could be problematic for the modern economy that needs people to have money in order to sustain the economy. However, could this replacement of people be a positive/negative from some workers' point of view? Don't these changes open/close new job possibilities? Maria's account resembles Terrapolis as in Haraway (2016, p. 11)'s words *Terrapolis makes space for unexpected companions (like AI), is open, indeterminate and polytemporal*.

Finally, Molly mentions how the technology could be good for the environment by making electric cars and solar panels. This is important in today's world or in today's Terrapolis. Technology is making humans capable, but it is also to some extent, rendering Terrapolis capable of counteracting the crisis. At the same time, we as humans are rendering technology capable of doing its job. New ideas made from our string figure game help us to think about how to respond to this paradox and to think about the wonders and horrors of the advanced technological developments in a more tentacular and less binary way (Haraway, 2016).

TENTACULAR THINKING WITH AI COMPANION

Including, nowadays, a Copilot AI chatbot seems not only unavoidable as this technology becomes more and more accessible for generating reviews and conversations, but also desirable as it allows to enter, explore and experiment with curiosity and with ideas. We are bringing here part of our interaction with AI chatbot. Despite that we want to affirm AI chatbot we are committed to remain critical of our interaction.

Our prompt: Thank you. I think that Molly and Maria are problematizing the complexity of digital technology, seeing both pros and cons while you seem to focus on the pros of digital technology. Do you want to comment on that?

AI chatbot: You make a great point! Molly and Maria indeed highlight both the positive and negative aspects of digital technology, which is essential for a balanced discussion. My response did focus more on the positive aspects, particularly on how to address the digital divide and promote equitable access. To add to the complexity, it's important to acknowledge the cons as well. For instance, the rapid advancement of technology can lead to job displacement, as Maria mentioned, and

issues like data privacy and cybersecurity threats are significant concerns. Additionally, over-reliance on technology can impact mental health and social interactions. Balancing these pros and cons is crucial. While technology can offer incredible benefits, we must also be mindful of its potential drawbacks and work towards mitigating them. This balanced approach ensures that we can harness the power of technology while addressing its challenges responsibly. What are your thoughts on how we can better balance these pros and cons in our daily lives?

As is noted from the excerpt, at the beginning of the conversation, the AI chatbot was focused on the positive aspects of digital technology like for example, how technology could enhance education using learning platforms and digital resources that can make education more accessible to people around the world. It also mentioned how to address the digital divide and promote equitable access. It seems that the AI chatbot takes for granted that digital tools are empowering and advantageous. Is that always the case? Digital tools are beneficial for all? We became intrigued about how AI chatbot could shape our minds or is it trying to manipulate us? Could we talk about the paradoxical advantages / disadvantages of new digital technologies with the materialized technological apparatus that is AI? We tried to counteract positive aspects as is noted in our prompt, then its responses overlapped with the issue that Maria mentioned about job displacement. The AI chatbot also mentioned issues of data privacy and cybersecurity. We agree on the need to be mindful of its potential drawbacks, but is it possible to mitigate this? Since AI machinery is going full throttle. What is it that we are really trying to mitigate? It would be important to know, as mathematics educators, who have the power behind the machine learning models that are trained through algorithms and how the information is labeled. It would be also important to know how the information we are reading from the AI could be biased.

TENTACULAR THINKING WITH CRITICAL SCHOLARS

Braidotti (2019) argues that despite digital technology present us with the risks, we must strive for creating affirmative but also critical relation. Our research interest for computer programming and computational thinking in the context of mathematics education and schooling, brings this work close to a critical reading of posthuman knowledge and its role in Terrapolis as Haraway coins the term. The Terrapolis equation (Haraway, 2016, p. 11) which is connected to the idea of solving environmental issues by means of technology is an idea that we use to think other ideas.

Here, we can think, for instance, of mathematics in action (Skovsmose & Niss, 2008) that brings us programming that has been the base of most of the new digital technologies, including machine learning and AI. Mathematics in action, then have contributed to climate change by enabling the extraction of natural resources in the name of societal progress. Paradoxically, mathematics in action enacted by digital technologies can also contribute to solutions and to some extent to solve environmental problems. According to Skovsmose and Niss (2008), mathematics in action has enabled technological developments that brings both, wonders and horrors. Through this idea we think about the paradox or in Skovsmose and Niss (2008) words, we can think about the aporia that is a difficulty that remains open and can shake our taken-for-granted assumptions about, for example, mathematics education as the basis for societal progress. How to respond to this paradox? This resonates with Chronaki (2024)'s insights about moving beyond the binary produced by the polarized dialectic amongst “*mathematics for all*” and “*no to mathematics*”. Coproducing tentacular reflections on the teaching of mathematics and weaving Terrapolis stories allows us to question and produce a spectrum of im/possibilities to mathematics education and consequently to mathematics in

action that could move beyond the binary discourses. This movement among a wider spectrum could also enable reflections about how to co-produce ethically and situated responses. We have trust in Braidotti (2019)'s approach to the earth histories in this posthuman world. We, the inhabitants of this planet, humans, non-humans and other-than-humans are in this together, but we are not one and the same. "We" own a collective existence, but we are different and have different roles within our shared earth. (Braidotti, 2019)'s quote: "*we are in this together, but we are not one and the same*", aligns with (Haraway, 2016)'s insights about tentacular thinking and response-ability and with Barad (2007)'s agential realism that emphasizes the entanglement of living beings within the planet earth and the distribution of responsibility among us. Responsibility and compassion to who have suffered most of the consequences of climate crisis. And to not ignore the complexity of this paradox from our privileged positions. Responsibility to not blame entire populations for causing most of the ecological disasters or to not put big burdens on our students and responsibility of reading the numbers and statistics with a critical and affirmative view. Responsibility to be critical by being able to question what is assumed or taken-for granted if we are to use AI in our classrooms. This paradox resonates with (Skovsmose & Niss, 2008, p. 14), who say that: "an aporetic situation can destroy foundations", and specifically of mathematical knowledge. (Skovsmose & Niss, 2008, p. 14) resorts to Michel Foucault: "*we have to move from one discourse into another*". We would add that we have to learn how to move from one discourse into another and to be able to be critical and question our own ideological assumptions. It would be important to recognize not only the wonders and the horrors but the wide spectrum of im/possibilities that digital technologies have brought to our lives. This is something that we also recognized in the episode with Maria and Molly, but also in their entanglements with the AI companion. Regarding data from the interview with students, we found that students talk about this issue of technology and take response-ability from their positions within Terrapolis by being aware of the different pros and cons of technological developments and the use of technology. For example Molly said: "*I think it's more efficient to have technology, but I mean there's also cons that some people might feel like AI.. with the students cheating and copying or whatever, that could be like a con to like technology*". Molly acknowledged how technological development has made a more efficient life, however, people could be worried about the fact that these technologies could make students cheat or copy other people's work. We take the relay from Molly and think with Braidotti (2019) about the tech revolution and the accelerated technological development that make us be more efficient, which, apparently Molly considered it as an asset. Braidotti (2019) emphasizes how the fourth industrial revolution has promoted labor exhaustion due to the advanced capitalism and accelerated development. Thus, we are facing the efficiency/exhaustion quandary. Does Terrapolis need us to be more efficient? How much efficiency is needed? How much do we have to be exhausted from our jobs? How much work is enough? How much homework do students need?

Molly is apparently responding positively about the use of technology and its impact on the efficiency of the world. Moreover, she is talking about the issue of cheating that could be related to a lack of responsible conduct at school. This co-produces the questions: how could we use technology responsibly? How could we, as teachers, include AI responsibly in our mathematics classrooms? These are open questions that are co-generated through this analysis which is done by thinking with Braidotti (2019), Haraway (2016) and Molly's ideas to think another ideas.

In another fragment Maria says: *“technologies are in our everyday life and it's advancing as well, so like if you have, for example, there a lot of workers who have AI taking their job or AI replacing what they work in and they are forced to find other jobs, so I think as technology advances, I think there will be like a time where AI might take over most of the jobs and most of the economy. But they're also like pros to it where it does help us like emmm, taking the bus, calling one another, using it in workspaces and in school”*. Maria is weaving the conversation with Molly, and she was apparently supporting Molly's statement about efficiency by mentioning that technology is in our everyday life, and it is advancing. Technology is part of the world, and its advances are inevitable. Technology also facilitates our life, since we use it in workspaces and school. However, the issue of AI taking the jobs of people emerges from the conversation. To what extent is it a problem that AI replaces people in their jobs? How could we live a just life with this replacement?

Finally, Molly says: *“another like pros is like, making cars, like electric cars are all like made by technology, and I mean that's like very important in today's world or even solar panels, that's good for like the environment that's made by technology.”* Technology is helping to solve environmental issues, but at the same time it has been damaging the earth. Taking this relay from Molly and thinking with Haraway (2016) enable us the question of “how to stay with this trouble? How to be responsible about it? How to create answers to this as we face the consequences of what we have provoked and think of it in an affirmative and yet critical way? The idea that technology could support efforts to find solutions to environmental issues emanated from Lucely's initial thoughts and a fragment of text co-constituted by Molly, this idea affected us and enabled our tentacular thinking. The idea emerged from our entangled relation with data, students concerns and also our concerns about environmental issues. This fragment was in MacLure (2013) words, an intensity, a relational affect that emerged from the intra-acting distributed agencies in that particular time and space. Molly, Maria, Karen Barad, Donna Haraway, Rosi Braidotti, and we amongst others have made possible this modest tentacular thinking and made connections that matter, we, through our texts, were and are open onto wonders and more questions within our Terrapolis stories.

CONCLUDING REMARKS

In this paper we used ideas from different authors and students accounts to create other ideas that matter. Together with students and authors we weaved Terrapilos stories in which answers and questions emanated. The main aim was to locate ideas and questions in relation to the paradoxical advantages and disadvantages of new digital technologies that are ubiquitous to mathematics education. We think with Karen Barad, Donna Haraway, Rosi Braidotti, Molly, Maria, and an AI chatbot amongst others to enact our tentacular thinking and take the relay from different perspectives to weave Terrapolis stories. Practicing string figures has made it possible to think with tentacularity to create new ideas from other relevant ideas and to be able to tell Terrapolis stories. Ideas like the time efficiency in our lives, the responsible way in which we could use digital technologies such as AI chatbots in our lives and classrooms, emerged from our tentacular thinking. These are questions that remain open, to enable more conversations and to weave other ideas. We also acknowledge the importance of our collective existence and thoughts, but also are aware of the different roles we have as students, researchers, AI, and scholars and the ability to respond with response-ability from each one role. Students' thoughts are important to think other ideas that could enable critical and affirmative discussion about digital technological development. Advantages and disadvantages of

technological development are part of Terrapolis stories. Moreover, mathematics in action has enabled wonders as well as horrors, this is an aporia that also remains open and that can influence mathematics classrooms. Going beyond the development/progress discourse could trigger a wider spectrum of im/possibilities for the teaching and the learning of mathematics. The use of technology to solve the very same problems that were created by it is a paradox and it is a serious idea to think with. Weaving Terrapolis stories in relation to this convergence has the potential to foster our capacity to think. The open questions from this analysis are formulated to think with the readers and to weave stories together within Terrapolis. Now the relay is in your hands.

References

- Alarco Tosoni, G. (2023). Concentration of wealth and income in postpandemic Latin America: Measurement, results, and perspectives. *Max Planck Institute for the Study of Societies (MPIfG)*, 25(1), 14-21. <https://www.econstor.eu/handle/10419/280757>
- Andersson, C. H., & Register, J. T. (2023). An examination of pre-service mathematics teachers' ethical reasoning in big data with considerations of access to data. *The Journal of Mathematical Behavior*, 70, 101029. <https://doi.org/https://doi.org/10.1016/j.jmathb.2022.101029>
- Barad, K. (2007). *Meeting the universe halfway: Quantum physics and the entanglement of matter and meaning*. duke university Press.
- Braidotti, R. (2019). *Posthuman Knowledge*. Wiley. <https://books.google.se/books?id=nOAlvgEACAAJ>
- Chronaki, A. (2024). The Masters: Speculative Assemblages of Mathematics Education as Enclosures and Commons with Ursula K. Le Guin's Feminist Utopian Anarchism. In *Proceedings of the 14th International Congress on Mathematical Education* (pp. 119-149). https://doi.org/10.1142/9789811287183_0009
- Figueroa-Suárez, L., & Ryan, U. (2023). Discourses in Colombian and Swedish Syllabuses: Is there room for action competence by means of STEM integrated computer programming? *Prometeica - Revista De Filosofía Y Ciencias*, 27, 613-622. <https://doi.org/https://doi.org/10.34024/prometeica.2023.27.15356>
- Haraway, D. J. (2016). *Staying with the Trouble Making Kin in the Chthulucene*. Duke University Press. <https://doi.org/10.2307/j.ctv11cw25q>
- MacLure, M. (2013). The Wonder of Data. *Cultural Studies ↔ Critical Methodologies*, 13(4), 228-232. <https://doi.org/10.1177/1532708613487863>
- Murris, K. (2022). *Karen Barad as educator: Agential realism and education* (Vol. 710). Springer.
- Sanchez, H. S., & Grimshaw, T. (2019). Stimulated recall. In J. McKinley & H. Rose (Eds.), *The Routledge handbook of research methods in applied linguistics* (pp. 312-323). Routledge.
- Skovsmose, O., & Niss, M. (2008). Critical mathematics education for the future. ICME-10 Proceedings: Proceedings of the 10th International Congress on Mathematical Education,
- Tissenbaum, M., Sheldon, J., & Abelson, H. (2019). From computational thinking to computational action. *Communications of the ACM*, 62(3), 34-36. <https://doi.org/10.1145/3265747>
- Varoufakis, Y. (2024). *Technofeudalism: What killed capitalism*. Melville House.
- Vinnervik, P. (2020). Implementing programming in school mathematics and technology: teachers' intrinsic and extrinsic challenges. *International journal of technology and design education*, 1-30. <https://doi.org/10.1007/s10798-020-09602-0>
- Yasukawa, K. (2002). Mathematics and technological literacy. Third International Conference: Mathematics Education & Society,

UTILIZING THE INHERENT MATHEMATICAL KNOWLEDGE SYSTEMS OF INDIGENOUS COMMUNITIES IN ADDRESSING SOCIETAL AND ECOLOGICAL ISSUES FOR SUSTAINABLE FUTURE

Frankie Fran^{1,2}, Catherine Vistro-Yu¹

¹Ateneo de Manila University, ²Romblon State University

Indigenous knowledge systems and practices play a crucial role in solving the present societal and ecological issues. They are critical elements in attaining not only a sustainable future but also a responsive mathematics education. In this paper, we hope to discuss the value of inherent indigenous mathematical knowledge systems in addressing socio-ecological issues. We discuss the significance of understanding the importance of these knowledge systems not only to the community where it belongs but to other members and stakeholders of society to achieve a sustainable future. This study also presents significant research trajectories that may be pursued, particularly in terms of environmental ethics, indigenous knowledge systems, mathematics education, and sustainability.

INTRODUCTION

Learning mathematics through indigenous knowledge systems, practices, and activities has demonstrated potentialities in developing a deeper understanding of concepts and principles of knowing and doing mathematics. By utilizing the socio-ecological lens, stronger connections may be linked between these indigenous knowledge systems and mainstream mathematics education. As pointed out by Senanayake (2006), indigenous knowledge covers a wide array of sociocultural practices and activities that are vital in sustaining a society and their environment, and a community's inherent mathematical knowledge system is one of these. Similarly, Inaotombi and Mahanta (2019) highlighted that aboriginal communities who are deeply immersed in and exposed to their environment showcase rich traditional knowledge on how to perceive, predict, and adapt to their environment especially during disasters and other related risks which they have learned from their fishing practices that may also be linked to their capacity to measure and calculate the amount of risk that they may be exposed to. This inherent skill of estimating and climate change adaptation is crucial in creating a more resilient community. It was also reiterated by Mbah and Fonchingong (2019) that indigenous knowledge and ways of knowing is vital in attaining environment and societal sustainability. Hence, one may pursue research in these fields and build a "socio-ecologically-minded" society. Indigenous knowledge is viewed as a critical element in developing "effective, equitable, and meaningful strategies" to deal with the pressing socio-ecological issues (Parsons, Nalau, & Fisher, 2017, p.1). They further noted that to create a meaningful understanding of these knowledge systems, a conceptualization of environmental ethics is highly needed. Therefore, to create a more sustainable future, mathematics education researchers should not do away with exploring the indigenous communities' and marginalized sectors' ways of knowing and doing mathematics. Conducting in-depth documentation, observation, and analyses of these sociocultural practices may provide an avenue for teachers and students alike to clearly understand and give value to these significant knowledge systems, a direction that this present study is trying to explore.

The Sibuyan Mangyan Tagabukid and their Environment

As an indigenous community, the *Sibuyan Mangyan Tagabukid* largely depends on the environment where they live. Hence, everything that they do including their sociocultural practices and activities intertwines with it. This includes their inherent mathematical knowledge systems as well as their unique ways of knowing and understanding mathematics. In this paper, we aim to present the link between the inherent mathematical knowledge systems and the environment and discuss its importance to the community including its relevance to the future landscape of mathematics education.

Theoretical Framework

This research was anchored on Freudenthal Institute’s Realistic Mathematics Education (RME) where real-life situations can be utilized as starting points in creating meaningful learning experience by utilizing their contexts. In RME, opportunities are given to learners to reinvent their own understanding of the mathematical content and processes through horizontal and vertical mathematization (Van den Heuvel-Panhuizen & Drijvers, 2014). Since the study involved the observation and documentation of sociocultural practices and activities of *Sibuyan Mangyan Tagabukid*, this theoretical construct is essential in understanding how to carefully select significant realistic experiences that can be used as sources in developing mathematical concepts and processes which they can utilize and apply in “less context specific” learning situations in the future.

METHODOLOGY

This study utilized a qualitative research design through ethnographic fieldwork. In this research, the inherent mathematical knowledge systems of *Sibuyan Mangyan Tagabukid* were documented and observed. It was conducted in the municipality of San Fernando and Cajidiocan in Romblon and was participated in by the community leaders, tribal elders and members of the Indigenous Peoples community. This research undertaking was given consent by the community through a Tribal Resolution and Memorandum of Agreement as required by the existing issuances of the National Commission on Indigenous Peoples (NCIP). This consent was formalized through the issuance of the Certificate of Compliance (Certification Precondition) by the NCIP. In gathering the data, unstructured interviews, participant observation, document review analyses, and field notes were utilized. Thematic analysis by Braun and Clarke (2020) was used in analyzing the data gathered through key informant interviews.

RESULTS AND DISCUSSION

Based on the key informant interview, participant observation and fieldwork, the following results were gathered, as presented:

Ways of Knowing and Doing Mathematics of the *Sibuyan Mangyan Tagabukid*

Most members of the *Sibuyan Mangyan Tagabukid* are farmers. Hence, they spend most of their time preparing their fields, harvesting their yields, and selling what they have harvested. In time, they were able to strategize and create mechanisms in doing their tasks. As they prepare the fields, they ensure that the distance between plants is accurately and equally distributed. For them, doing this provides more yields. But this differs depending on the kind of crop that they are to plant. To do this, they make use of vines and sticks initially measured based on the length of both arms outstretched, or

dupa. They then use these vines and sticks as bases for future length measurements. Even today, they are utilizing these measurement strategies, and this evidently gives higher crop yields.

For selling their yields, such as bulks of bananas, counting them one by one would be tiring and time consuming. Hence, the members of the community sort them into groups of 100, and use a pebble for each group of 100, for ease of counting. Most of these indigenous peoples have not been given an opportunity to study; hence, they only have a basic knowledge of Western mathematics. By doing this systematic way of counting, they were able to count their harvest and sell to the buyers. So, if there were five pebbles, they know that the total harvest is 500 bananas. If there is an excess in their products, they give it as a bonus. Even in this counting practice, the *Sibuyan Mangyan Tagabukid* still utilize their environment and associate it with the sociocultural activity that they are doing.

Inherent mathematical knowledge systems were also found to be evident in the fishing activities and practices of the *Sibuyan Mangyan Tagabukid*. These were demonstrated by those who chose to live in the coastal communities and decided to leave their homes in the upland areas. Not all members of this indigenous community have access or financial capacity to purchase their own boats and other fishing tools. For this reason, fisher folks borrow boats and rent from those who have. In exchange and in payment for renting the boats, they give a portion of their catch to the owner of the boats. This is where product sharing practices were observed from the members of the community. They follow a system of dividing the catch into four equal partitions. Of these four partitions, one portion will be given to the boat owner while the remaining partitions will be divided equally by those who worked. This does not only encapsulate an inherent mathematical knowledge system, but it also shows the cultural values of the community which is centered on equality and impartiality.

They ensure that these practices are also taught to their children. Though most of the younger generation are given the opportunity to pursue their studies in formal schools, they are still encouraged to help their parents, especially during weekends. This gives their parents the opportunity to train them on how to strategize in their planting and fishing practices and ensure that it is still being sustained. By utilizing Realistic Mathematics Education as a theoretical lens, these sociocultural practices and activities can be used as concrete and real-life examples to gain deeper understanding of the concepts being explored and discussed. These inherent mathematical knowledge systems can be contextualized and used in solving real world problems by engaging students to reflect on their own informal ways and techniques of knowing and doing mathematics. With exposure to and familiarity with these sociocultural practices, the learners will also become more involved not only in solving these problems but also in doing reflective activities that would help them better understand how to generalize and formalize the solutions that they have made. After learning and reflecting on how their own ways of mathematizing be generalized and formalized, they can now apply it in working with more abstract concepts of mathematics.

Utilizing Inherent Indigenous Knowledge Systems in Solving Societal and Ecological Problems

The *Sibuyan Mangyan Tagabukid* gives high importance to their environment as observed during the researchers' participant observation and fieldwork. For them, their sociocultural practices and activities are attached to the environment where they live. But these are currently being challenged by external forces that try to destroy their land. This issue is associated with the present mining

exploration that affect some of the ancestral domains of the indigenous community. In January 2023, residents of Sibuyan Island, Romblon, including the indigenous peoples, were alarmed when they found that a mining company conducted a public scoping in one of its barangays. The activity is a requirement by the Philippine Environmental Impact Statement System for their proposed mining exploration activities. This was strongly opposed by the residents, indigenous peoples, and environmentalists, highlighting the damaging effects this might pose to the ecological system of Sibuyan. In response to this socio-ecological issue, concerned citizens together with various anti-mining groups formed a human barricade to stop the mining trucks containing nickel ores.



Figure 1: Sibuyan Island residents forming a human barricade to stop mining trucks (Galicha, 2023)

In connection with this, the Department of Environment and Natural Resources ordered the mining company to temporarily stop its operations due to its alleged violations of the environmental issuances and regulations. This was after they have conducted an inspection and found that the mining company does not have an environmental compliance certificate for the mining related activity (Gozum, 2023a). Likewise, Philippine Senator Risa Hontiveros filed a Resolution in the Philippine Senate to direct the appropriate Senate Committee to conduct an inquiry regarding the metallic mining activities in Sibuyan Island, Romblon, with emphasis on the importance of conserving the island's flora and fauna and to prevent ecological devastations (Hontiveros, 2023). The Senator personally visited the site and saw first-hand the effects of the scoping activity. Furthermore, the Supreme Court of the Philippines issued a *Writ of Kalikasan* against the Mines and Geosciences Bureau, Department of Environment and Natural Resources, and the mining company involved (Supreme Court, 2023). A *Writ of Kalikasan* is a legal remedy that provides protection to one's constitutional right to a healthy and balance ecology (Supreme Court, 2010).

These lands do not only represent their physical environment but also the spiritual and cultural bonds that connect them to their heritage. As a result, the preservation of their inherent knowledge systems is being challenged and they have been struggling to protect and conserve their primary source of learning and knowledge. Recently, the *Sibuyan Mangyan Tagabukid* issued a strong opposition to these societal and political challenges through a Tribal Resolution asserting their rights as an indigenous community and protectors of their mathematical knowledge systems. According to Gozum (2023b), a Philippine News Writer of Rappler, twenty-two (22) members of the *Manughusay Kamaguyangan*, composed of the community elders and tribal leaders of the *Sibuyan Mangyan*

Tagabukid denounced the memorandum of cooperation for a mining exploration signed by one (1) indigenous person who signed purportedly on behalf of the *Sibuyan Mangyan Tagabukid*, a person who is not authorized to represent the whole indigenous community, according to the article. This denouncement demonstrates the communities' strong understanding of solidarity in dealing with societal and political acts that trample on their integrity and dignity as a community. It has been pointed out in earlier discussions that their inherent mathematical knowledge systems highlighted social equality and fairness as important moral values, as demonstrated in their product sharing activities. With a clear understanding that these inherent mathematical knowledge systems are crucial in securing a sustainable future, this strengthened their assertions of what is right for the community and the rest of the people of Sibuyan.

Environmental Ethics, Indigenous Knowledge Systems, and Mathematics Education

Environmental ethics is defined by the Stanford Encyclopedia of Philosophy as a discipline that delved into the moral relationship of human beings with the environment (Brennan & Lo, 2021). It particularly positions indigenous knowledge systems as one of the key elements in creating solutions to modern-day societal and ecological issues. Additionally, it presents the ethics in addressing socio-ecological concerns particularly in terms of sustainability and climate change. As initially mentioned in this paper, the need for the conceptualization of environmental ethics is necessary to address the pressing socio-ecological issues specifically those being faced by the marginalized sectors, in this case, the *Sibuyan Mangyan Tagabukid*. One of the issues raised in the study's results is the *Sibuyan Mangyan Tagabukid's* stand on the proposed mining exploration that will consequently affect their ancestral domains. The indigenous community's moral relationship with their environment is significant not only as a sociocultural practice but also as an essential element in making their decisions in these socio-ecological issues. This means that in dealing with critical matters concerning their environment, such as mining activities, sociopolitical functions of this indigenous community are also demonstrated. Through the years, indigenous knowledge and other forms of traditional ways of knowing and doing had shown sustainable practices that are deemed to be essential in environmental conservation and management. Hence, *Sibuyan Mangyan Tagabukid's* inherent mathematical knowledge systems particularly on counting, measuring, and product sharing may be utilized in managing their resources, conserving biodiversity, and boosting community resilience. As demonstrated in their product sharing practices, it can be noted that great importance is given to fairness, equality, and equity. It shows the high regard of *Sibuyan Mangyan Tagabukid* for their sociocultural values and demonstrates how their inherent mathematical knowledge systems on equality and equity are embedded in their day-to-day activities. The concepts of fairness and equity are two of the primary reasons why the members of this indigenous community petitioned and denounced the actions made by one of their own as mentioned in the previous discussion. This independent act of one of their community members directly violated the principle of fairness and representativity. They value their ancestral domains and their moral relationships with their environment including the non-human entities in the community. Hence, in deciding on important matters such as this, the collective decision of the indigenous community must be considered.

This course of action does not only affect the present generation of *Sibuyan Mangyan Tagabukid*. More importantly, this action sets a strong example to the younger generation on how to make a stand as a community and how to position themselves in making significant decisions especially when

their environment, non-human entities included, are being exploited. This will also prepare and equip their future generations in facing the uncertainties through the promotion and preservation of their inherent mathematical knowledge systems to achieve sustainability.

Mathematics Education Future for *Sibuyan Mangyan Tagabukid*

The sociocultural practices and activities of *Sibuyan Mangyan Tagabukid* were observed to be deeply associated with their environment. With their inherent practices, a mathematical knowledge system emerged showing their innate way of knowing and understanding their surroundings. These knowledge systems were not learned and taught in formal schools. Unaware of its potential contributions to sustainable mathematics education future, these Indigenous Peoples were able to create a systematized form of communicating ideas through their own ways of doing mathematics. These inherent mathematical knowledge systems revolve around their day-to-day routines especially when it comes to their primary line of work. Since they have been constantly doing these tasks, they are able to strategize and create mechanisms that help make their tasks and activities easier. Thus, despite their little exposure to education due to limited access, they are still able to organize and systematize the way they do things. This demonstrates the creativity and versatility of indigenous communities in creating unique ways of meaningfully understanding their realities.

Mathematical knowledge systems generated from their sociocultural practices opened the doors of understanding mathematics from a context-based perspective. Some of these inherent mathematical knowledge systems also included product sharing, work arrangements, age and gender role structures, and measurement (Fran & Vistro-Yu, 2023). For instance, product sharing practices for harvesting their yields called as *inupat* clearly articulates that portions of the harvest will be shared by the tenant and the landlord in a systematic way of product sharing. Since these practices are part of their daily routines, utilizing them as starting points in understanding deeper concepts of mathematics is beneficial to the community, especially to its younger generations who are also exposed to such practices. Learning would therefore no longer be confined in the way Western mathematics is taught but, in a manner, relevant to their own identities. Therefore, mathematics education's primary aim is to ensure that the indigenous communities' culture and environment are protected and preserved to be able to utilize as source of knowledge and understanding of their respective identities. For instance, language played a vital role in the transmission of knowledge through oral traditions which includes all inherent mathematical knowledge systems. It is not our intention to establish an indigenous community that is stagnant and unresponsive to change. We want them to make progress by utilizing these inherent mathematical knowledge systems to help them develop and prosper while ensuring that their lands, culture, language, and other indigenous knowledge systems and practices are preserved and protected. Thus, we dream for the members of the *Sibuyan Mangyan Tagabukid* to have a life that cultivates their inherent mathematical knowledge systems and keeps their cultural identities known and recognized.

It can, thus, be inferred that utilizing the inherent mathematical knowledge systems of indigenous communities can contribute to more meaningful learning experiences inside and outside the classroom. Exposing students to activities and tasks associated with their real-life experiences and context can provide a more realistic view of understanding the concepts of mathematics such as counting, measurement, and fractions. Therefore, curriculum developers and educators should not only develop instructional materials that discuss the abstract concepts of formal mathematics but more

importantly, context-based lessons and learning opportunities should also be designed to attain the most favorable outcomes and develop the full potential of the learners. When appropriately used in designing the school curricula, educational institutions may provide an engaging and immersive learning experience to their learners since they will most likely establish a strong connection with the lesson being discussed. Utilizing these kinds of learning opportunities will enhance the school curricula's responsiveness to the unique and individual needs of our learners, especially for those who are coming from marginalized sectors of our society. Hence, instructional materials that highlight the connection of these inherent mathematical knowledge systems with the socio-ecological may be designed and, thus, develop a culturally responsive mathematics education. The development of instructional materials relevant to their knowledge will not only aid educators in teaching the learners with the necessary knowledge and skills but more importantly, these indigenous knowledge systems will also be preserved and protected that may be used by future generations in their community and beyond.

CONCLUSIONS

With the findings drawn from this study, it can be concluded that indigenous knowledge has the potential in creating a more sustainable future. The sociocultural practices and activities observed from the community can be utilized to provide more concrete teaching and learning activities to better understand the mathematics around us particularly in addressing the socio-ecological issues surrounding the community and even beyond. By utilizing Realistic Mathematics Education (RME) as a theoretical lens, counting, measuring, and product sharing sociocultural practices and activities of the *Sibuyan Mangyan Tagabukid* may be used to broaden their understanding towards mathematics and mathematics education. These newly found and documented inherent mathematical knowledge systems can be utilized to contextualize the various essential concepts of mathematics. These can also allow learners to reinvent and strategize in solving mathematical problems which they can also utilize in addressing societal issues and concerns. These knowledge systems may then contribute to preparing future ready citizens and learners who have the capacity to understand and solve socio-ecological problems. By utilizing these inherent mathematical knowledge systems, indigenous communities, such as *Sibuyan Mangyan Tagabukid*, and including the other members of society, may continually engage in these sociocultural practices as a guide to build a more resilient and adaptive community. It can also be noted that indigenous ways of knowing and doing mathematics may provide a better avenue for learners to engage themselves in learning and understanding relevant mathematics concepts. Hence, culturally responsive curriculum reforms and development of contextualized learning lessons and activities are vital in attaining a more inclusive learning environment.

Acknowledgements

The authors would like to thank the Ateneo de Manila University, Romblon State University, Department of Science and Technology – Science Education Institute, National Commission on Indigenous Peoples, RSU Research, Extension, Development and Innovation Office, and the Sibuyan Mangyan Tagabukid.

Ethical Statement

This study was approved by the University Research Ethics Committee of the Ateneo de Manila University with Protocol ID: AdMUREC_22_131. This research complied with the requirements for securing Free and Prior Informed Consent from the Sibuyan Mangyan Tagabukid through the National Commission on Indigenous Peoples with COA Control No. CP RIVB-ROM-2023-08-088.

References

- Braun, V., & Clarke, V. (2020). One size fits all? What counts as quality practice in (reflexive) thematic analysis? *Qualitative Research in Psychology*, 18(3), 328–352. <https://doi.org/10.1080/14780887.2020.1769238>
- Brennan, A., & Lo, N. (2021). Environmental ethics. In Zalta, E. & Nodelman, U. (eds) *Stanford Encyclopedia of Philosophy*. Department of Philosophy, Stanford University.
- Fran, F.A. & Vistro-Yu, C.P. (2023). Mathematical knowledge structures of socio-cultural communities: The case of *Sibuyan Mangyan Tagabukid*. *Proceedings of the ICMI Symposium on Mathematics Education and the Socio-Ecological*. International Commission on Mathematical Instruction.
- Galicha, R. (2023). *Sibuyan Island residents forming a human barricade to stop mining trucks*.
- Gozum, I. (2023a, February 6). Suspension of Altai mining ops in Sibuyan welcomed, but ‘fight far from over’. *Rappler: Philippine News*. <https://www.rappler.com/philippines/denr-temporary-suspension-mining-operations-sibuyan-island-residents/>
- Gozum, I. (2023b, May 24). Mining is destructive but legal, scientific – Altai CEO. *Rappler: Philippine News*. <https://www.rappler.com/philippines/altai-ceo-cascolan-says-mining-destructive-but-legal-scientific/>
- Hontiveros, R. (2023). *Resolution direction the appropriate senate committee to conduct an inquiry, in aid of legislation, into the nickel and metallic mining activities in Sibuyan Island, Romblon, with the end in view of conserving the island’s endemic flora and fauna, defending its coastal communities from long term ecological devastation, and protecting the residents from violent incursions of mining companies*. Philippine Senate.
- Inaotombi, S., & Mahanta, P. C. (2018). Pathways of socio-ecological resilience to climate change for fisheries through indigenous knowledge. *Human and Ecological Risk Assessment: An International Journal*, 25(8), 2032–2044. <https://doi.org/10.1080/10807039.2018.1482197>
- Mbah, M., & Fonchingong, C. (2019). Curating indigenous knowledge and practices for sustainable development: possibilities for a socio-ecologically-minded university. *Sustainability*, 11(15), 4244. <https://doi.org/10.3390/su11154244>
- Parsons, M., Nalau, J., & Fisher, K. (2017). Alternative perspectives on sustainability: Indigenous knowledge and methodologies. *Challenges in Sustainability*, 5(1), 7-14. <https://doi.org/10.12924/cis2017.05010007>
- Senanayake, S. G. J. N. (2006). Indigenous knowledge as a key to sustainable development. *Journal of Agricultural Sciences - Sri Lanka*, 2(1), 87-94. <https://doi.org/10.4038/jas.v2i1.8117>
- Supreme Court (2010). *Rules of procedure for environmental cases*. Supreme Court.
- Supreme Court (2023). *Writ of Kalikasan against Mines and Geosciences Bureau, Department of Environment and Natural Resources, and Altai Mining Corporation*.
- Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic mathematics education. In Lerman, S. (eds) *Encyclopedia of Mathematics Education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_170

INTEGRATING EARLY ALGEBRA AND THE SOCIO-ECOLOGICAL

Jenny Fred, Paola Valero, Hendrik Van Steenbrugge

Stockholm University

This paper is an invitation to imagine new futures for mathematics education in the context of early algebraic thinking in the socio-ecological. The issue at hand is to re-imagine the concerns of why, what and how to teach central areas of the school mathematics curriculum. In this paper, the research revolves around Jenny Fred's current PhD work, in which a research team consisting of in-service teachers, pre-service teachers, teacher educators and researchers have been working on the idea of what we call algebraic wicked problems (AWP) to simultaneously integrate early algebra and the socio-ecological into students' work (Fred et al., 2024b). We highlight five pedagogical features of AWP's and the research team's work in constituting a critical work zone integrating early algebra and the socio-ecological. We conclude the paper with ideas to further theorize the possibilities of integrating early algebra and the socio-ecological in significant ways.

INTRODUCTION

As we all, even young children, are confronted with the multiple challenges of sustainability and climate change, it is necessary to make explicit connections between mathematics education and the current entanglement of societal and environmental problems. Coles (2022) emphasizes that what has been called the *socio-ecological* “points to a desire to think about human and non-human together and a key belief [...] is that such a perspective is necessary for a mathematics education that is relevant to the future”. (p. 212). The argument that young generations need to “build tools, options, and paths to prepare [...] to confront” (Solares-Rojas et al., 2022, p. 204) the challenges posed in socio-ecological crises, is a call for mathematics educators to examine the different areas of the school curriculum and find new, significant ways to connect the contents of the curriculum and the socio-ecological. In particular, we are interested in early algebra as the area of school mathematics education that initiates and prepares children to what is argued to be one of the main components of the curriculum: algebra (e.g., Kieran, 2006). The question at stake here is the ways in which early algebra and its emphasis on the development of small children's reasoning on patterns and mathematical structures (e.g., Cai & Knuth, 2011) can be linked to the socio-ecological.

Such a connection, we argue, can be done in at least three distinct ways. A first way is to follow the traditional argument to justify the centrality of algebra in general: Since algebra is a fundamental element for successful mathematical learning as children progress through schooling, solid algebra teaching and learning within itself is the best mathematics educators can strive for. A good foundation in algebra will support all mathematics learning, and this in turn will offer people the possibility of using mathematical knowledge to solve different types of problems, among which are those posed by the socio-ecological. In other words, this first approach is what we have called the argument of *algebra as logic and psychological powerful knowledge* (Fred et al., 2022), or the option that doing the best early algebra focusing on the characteristics of algebraic reasoning in itself is the best form of connecting to the possibility of solving socio-ecological problems in the future.

A second way could be to connect the contents and forms of reasoning that are desirable to develop in children by *contextualizing problems in real-world situations of sustainability, climate change or environmental impact*. This would mean that, instead of figuring out, for example, the general rule for a number pattern without context or formulating relationships among the age of siblings, word problems could use situations and data related to CO₂ emissions in different countries or the recycling of food to express relations and patterns among quantities. This is the well-known tendency in mathematics education to bring areas of real-life to the attention of children through word problems, as pedagogical inscription devices to promote a change not only in cognition, but also in children’s morality and behavior —e.g., see Yolcu & Kirschgasler, 2024, for the case of Black children to become good citizens. This second way has already been problematized, since often in this type of word problems the context is not important, and the problem-solving activity —although probably leading to desirable mathematics learning— does not invite children to go “behind the data” and engage in ethically reflective dialogue to discuss possible underlying structures/values of the data and the situation itself (e.g., Boylan & Coles, 2017; Barwell, 2013).

A third way that we have explored is what we could name *the integration of early algebra and the socio-ecological*. The current predicaments of sustainability and climate change and their multiple environmental, social and economic dimensions invite us to push the boundaries of the significance of early algebra for new forms of critical citizenship. Introducing the idea of integrating the socio-ecological into practices of teaching and learning early algebra, the socio-ecological is not only considered as a context of the problem. The context is rather seen as an integrated part of the students’ work with problems where the algebra and the socio-ecological are co-constitutive. This way contributes to a consideration of early algebra as cultural and sociological powerful knowledge (Fred et al., 2022), a gap in early-algebra research yet to be tackled.

To discuss this third way and its potential to imagine new futures for early algebra in the socio-ecological, we start by presenting the research project which has led us to develop this third way. We will briefly highlight the projects’ main characteristics and main results. We will then focus on what we see can help us think about the integration of early algebra and the socio-ecological, in an attempt to bring forward considerations to re-imagine the concerns of why, what and how to teach central areas of the school mathematics curriculum. We conclude by pointing to ideas on what we envision as further directions for this third way.

A CRITICAL ZONE FOR WORKING WITH ALGEBRAIC WICKED PROBLEMS

This research revolves around Jenny Fred’s current PhD work (Fred, forthcoming). In a first stage of the PhD (Fred, 2019), Jenny explored aspects of teaching that create the conditions for students in grades F–3 to engage in algebraic work. In this first stage, Davydov’s (2008) premise that carefully designed teaching can support children’s exploration of underlying mathematical structures, and as a result the emergence of algebraic thinking oriented the work. Jenny studied students’ qualitatively different ways of seeing pattern generalization to identify critical aspects in their learning (Fred & Björklund Boistrup, 2017). She also collaborated with in-service teachers to bring the principles of learning activities (Davydov, 2008) to establish and sustain algebraic work in the classroom (Fred, 2019). This first stage provided a solid idea on what may characterize productive early algebraic work in the classroom.

In the second stage of the PhD, Jenny enlarged her interest to explore in which ways productive early algebraic work does not only equip children with logical and psychological powerful knowledge, but also and simultaneously can make possible for those forms of knowing and reasoning to be culturally and sociologically powerful (Fred et al., 2022). The enlargement meant bringing in conversation the field of early algebra with the field of critical mathematics education (e.g., Skovsmose 1994) and the recent concern with mathematics education for critical citizenship in the time of climate change and socio-ecological crises. An important guiding idea was what Skovsmose and Borba (2004) have called *critical research* or the research exploration of “what is not the case—but what could be a possibility” (p. 211). Jenny organized a systematic collaborative work to imagine possible new situations. Jenny built a research team consisting of four grade 1–3 teachers working in Swedish elementary schools, four pre-service teachers enrolled in Swedish (mathematics) teacher education, and three researchers (the three authors of this paper) from Stockholm University. Jenny and the team of in-service and pre-service teachers engaged in collaboration through one school year, during which 10 research group meetings were closely documented and studied. The set of documentation of these meetings is the main core of data for the project. Besides, Jenny, Hendrik and Paola have collaborated in the analysis and discussion of the data material.

There are two interconnected processes that we want to highlight here. One is the research team’s imagining of how early algebraic work can relate to social, political and environmental issues of relevance for grade 1-3 students’ critical engagement. This has led to making together what we call *algebraic wicked problems* (Fred et al., 2024b). Another process is the work itself in the research team and how it constituted what we call a *critical zone*.

Algebraic wicked problems

Algebraic wicked problems (AWPs) have provided us a focus in studying the third way to connect early algebra and the socio-ecological. In studying what is not yet the case, we developed our thinking about AWPs. Whereas initially we loosely thought of AWPs as a pedagogical tool to facilitate moving in the critical and sociological terrains of early algebra, we have gradually come to a more principled understanding of central features of AWPs. Indeed, we have come to characterize AWPs by means of the following five key features:

- AWPs involve powerful algebraic ideas sociological speaking

- AWPs mobilize a commitment with sustainability in its intellectual, ethical, and practical aspects

- AWPs have inbuilt dilemmas and value conflicts

- AWPs involve schematic, psychological and sociological tools as materializers

- Teachers’ actions promote students’ engagement, exploration and awareness

These pedagogical features have allowed us to further conceptualize the integration of early algebra and the socio-ecological. We come back to this point later. But first we delve into the research group’s critical work that has led to thinking of AWPs as pedagogical tools and the conceptualization of AWPs.

The research team’s moving in a critical zone

Latour’s (2018) idea of the *critical zone* has helped to conceptualize the research team’s working with AWPs. According to Gleason (2019, p. 2018), the critical zone is an arrangement “where debate and conflict are necessary, where interpretive battles are waged, fueled by investments and interests, and

where scientists must learn to innovate by forging new alliances between human and non-human agents, rather than speaking for the natural world”. We see here connections with Coles’ (2022) argument to connect the human and non-human in thinking about the socio-ecological and future mathematics education. In other words, the connection between human and non-human has helped to explore the simultaneity of early algebra and the socio-ecological.

The research group’s movements during the one-year process of trying to imagine and experiment with AWP and the connections taking place in these movements can be seen as working towards a critical work zone in which in-service teachers, pre-service teachers and a researcher-teacher educator (Jenny Fred) get the opportunity to generate and appropriate knowledge as active agents, as the AWP are formed. Thus, the work in the critical zone consists of multiple connections that emerge between the actors consisting of persons (human agents) and algebraic aspects and socially relevant issues (non-human agents). Furthermore, these connections move over time in the collaboration (Fred et al., 2024a). Our developed methodological approach focused on the forged connections between the persons and ideas during the course of research group meetings. The methodology allowed us to trace the work in the critical zone of working with AWP during the course of one school year. For instance, the research group’s (in- and pre-service) teachers had to move away from an initial focus on students’ learning of algebra, on learning algebraic concepts and linking these concepts to one another (Fred et al., 2022, see also Cai & Knuth, 2011, Kieran, 2018) to instead moving to focus on cultural and sociological interpretations of algebraic work concretely tied to daily classroom practice.

THE SIMULTANEITY OF ALGEBRA AND THE SOCIO-ECOLOGICAL

So far, developing AWP in a critical zone of collaboration has led us to identify five pedagogical features that we see are connected to the potential of AWP to integrate early algebra and the socio-ecological. First, the pedagogical feature *powerful algebraic ideas sociological speaking* allows us to think about algebra as a resource or tool for critically evaluating models such as for example diagrams, graphs of different kinds of socio-ecological concerns. The integration of algebraic ideas and the socio-ecological becomes realized, when algebra is used as an analytical resource, tool or facilitator, for concrete critical thinking and action, when exploring, interpreting, analyzing and reasoning about the underlying structures of, and possible corresponding relationships between, given values or quantities of data. This kind of work can also enable students to become aware of the limitations of such models, which only visualize certain “things” and leave others unnoticed (Fred, et al., 2022; Fred et al., 2023; 2024b). Second, the *commitment with sustainability in its intellectual, ethical, and practical aspects* (see Öhman & Sund’s (2021) didactic model of sustainability) allows us to think of sustainability not only as a context, as in the case of the school tradition of word problems, that surrounds the problem (Fred et al., 2023). Instead, the different commitment aspects of sustainability can help us to think of the context in a broader sense and thus enable a simultaneous integration of algebra and the socio-ecological. The intellectual aspect relates to the selection of subject content on sustainability. In making this selection, it could be essential to consider that the subject content is engaging and relevant to students’ lives. In other words, it may be important that the content chosen is dependent on the age of the students, their geographical location and their current concerns about sustainability issues. It could be a topic that is frequently addressed in the media or in the local communities. For example, in Sweden “Lilla Aktuellt”, a television news program for children which students often watch at school, may be a source of information. Related

sustainability news topics could then become a focus to work around. Another illustrative example related to intellectual aspects is the carbon footprint table (Figure 1), constructed and used in our try-outs with students in a classroom context.













You get a carbon footprint budget with the value of two orange rods.	
TV-game 	Carbon footprint: 1 blue rod 
Cell phone 	Carbon footprint: 1 black rod 
Loll-doll 	Carbon footprint: 1 yellow rod 
Widget toy 	Carbon footprint: 1 yellow rod 
Scooter 	Carbon footprint: 1 green rod 
Lego 	Carbon footprint: 1 white rod 

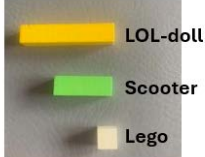
Figure 1: A carbon footprint table

The table lists things/toys and their carbon footprints values expressed in terms of Cuisenaire rods. When asked to choose what they wanted to buy with a budget of two orange rods, students started to discuss the given values in terms of durability of different materials (Fred et al., 2023). The intellectual aspect could here be understood as the knowledge of durability of different materials. The integration of algebra and socio-ecological consists here in the sense of exploring, analyzing and reflecting on the underlying structures of the given values to establish and reflect on a consideration for sustainability in terms of the durability of different materials to assign values to the things'/toys' (Fred et al., 2024b). The emotional aspect addresses the choice of data in the model of reality, which prompts students to interact with one another and challenges them to engage in critical reflection and discussion. The aim for students is to become emotionally involved, which may challenge some of their assumptions about how they or their family live, which then also connects to an ethical dimension that requires careful handling and respect. The ethical could be put in relation to, on the one hand, the desire to buy a toy one likes and, on the other, the responsible consumption of the environmental resources on Earth. One related example that came up in the research team's reflection is when children started to discuss the desire for having a new mobile phone in contrast to keeping the used/refurbished one with respect to the consumption of the environmental resources on Earth (Fred et al., 2023). In this situation, the research team imagined a not yet tried-out problem situation, where students would be invited to reason about three fictional persons and their budgets, where the value of a mobile phone should have different values depending on the person's environmental choices, e.g., if one prefers to buy a new or a used/refurbished telephone. This was suggested by the in-service and pre-service teachers as a possibility to invite children to explore, interpret and reason about how the use of Planet Earth's resources relates to emotional/ethical aspects in the different budgets. The practical aspect focuses on how the subject content and the choice of data can prompt students to act. The actions undertaken in students' work are not limited to practical tasks such as sorting waste or establishing an environmental group. Rather, they encompass a broader range of activities, including discussions, reflection, and the formulation of new ideas (Öhman & Sund, 2021).

Third, the pedagogical feature of *having inbuilt dilemmas and value conflicts* takes its departure in Davydov's (2008) notion of contradiction and allows us to think of how to challenge the students to explore, interpret, analyze and reason algebraically about the underlying structures of, and possible

corresponding relationships between, given values or quantities of data. That is, the dilemma is a built-in incompatible condition consisting of some kind of value conflict between two incompatible values. The value conflict invites students to explore, analyze and reflect on corresponding relationships between quantitative data in a model of reality. In our exploration of the classroom situations, we recognized that there emerged dilemmas as students worked algebraically to establish connections between quantities, objects and their economic and carbon footprint values. The dilemmas were related to the built-in incompatible condition causing value conflicts, which made it possible for algebraic thinking to unfold and connect significantly to sustainability.

Fourth, the pedagogical features, *schematic, psychological and sociological tools as materializers*, can facilitate students' argumentation, reasoning, and exploration of underlying structures and values that are of a socio-ecological nature. In the try-outs, the use of C-rods as representing the values of objects seemed to allow the students to argue and reason about corresponding relationships analytically, by using a visible and concrete artefact. The C-rods mobilized the conversations. More specifically, they were using the artefact when reasoning about the value conflict between the toys' and things' economic and environmental values, as illustrated below:

<p>Student 4: That little lol-doll can't be much more than Lego, can it?</p> <p>Student 5: And then the scooter... Scooters are big...</p> <p>Student 6: But you have a scooter for a really long time... And there are a lot of other things than just plastic.</p>	
----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------

Fifth, the pedagogical feature *teachers' actions promote students' engagement, exploration and awareness* allows us to think of how the teacher sometimes needs to guide the students' awareness to the in-built dilemmas, the incompatible conditions, or the value conflict by actions such as questions or statements. In other words, the teachers' actions become crucial to enable the integration between aspects of algebra and the socio-ecological. In one of the try-outs, a teacher's question "Does the table feel fair?" together with the value conflict seemed to invite the students to explore, analyze and reflect on corresponding relationships between the things'/toys' values in the table and a consideration for sustainability concerning the durability of different materials. In another instance unfolded by other questions, had no challenge. The value conflict that was realized concerned the quantities and their correspondence in value with no integration of socio-ecological:

Student 1: What is one [emphasizes is and one] Lego?

Teacher: This is probably a box of Lego. Not just a piece but... What do you say? What do you think?

Student 2: One Lego bag?

Teacher: One Lego bag? And...

Student 1: I think it is one Lego set.

Teacher: Yes, one Lego set. But not the biggest.

The actions of the teacher are of great importance, as they facilitate the students' awareness of both the algebraic and socio-ecological aspects of the subject matter. Furthermore, they encourage the students to persevere in their endeavors, rather than merely attempting to identify a single optimal solution, which they typically are used to.

DISCUSSION AND QUESTIONS FOR THE FUTURE

After developing our initial thinking about what AWP's may be, we conclude that the five pedagogical features highlighted above are a way to make visible the elements that come together in the attempt of designing teaching where both early algebra and the socio-ecological are equally significant and become integrated. Towards the future, there emerges the question of how to better conceptualize AWP's, their characteristics and the types of activities and relationships that may take place among different actors. Following Latour (2018), we have started to pay attention not only to the human actors involved (e.g., the teacher, students) but also to the non-human actors (e.g., algebra, socio-ecological problems, models, Cuisenaire rods, etc.) that become entangled as an AWP situation unfolds. If AWP's is to become an idea for early algebra teaching and learning in the socio-ecological, we would not want it to be reduced to or become conflated with a nice, new world problem. Rather, AWP can be complex and unpredictable situations where these multiple actors mobilize, enter in relationship and generate new possible agencies together. In other words, they are dynamical spaces for action, a network of human and non-human actors gaining agency to make possible the integration of algebra and the socio-ecological (see e.g., Fred et al, 2024a).

We would also like to conceptualize algebra and what may be early algebraic work as potentially dynamic agents. In contrast to the view of algebra as a “dead” content of the school curriculum, the finding of underlying structures (relationships, variables and constants, and the patterns among them) come to life as a potential actor in exploring the multiple value dilemmas in socio-ecological predicaments. This type of dynamics was clearly visible in the example presented above. The challenge of conceiving of early algebra and what it can do as a force in AWP is an important point towards advancing the dominant view that has been established in the field concerning algebra as powerful logical and psychological tools. Furthermore, we also want to initiate a discussion about the notion of critical zone as a possible methodological approach within mathematics education research that allows the imagination of new knowledge, curricula and pedagogies through the interconnection of the human and the non-human in future mathematics education practices. In other words, to create new cross-contextual spaces for imagining new relations of greater mutuality across multiple scales of space, time and concepts.

Acknowledgements

This research is part of the graduate school RelMaS: Relevancing Mathematics and Science Education (VR 2019-03679), funded by the Swedish Research Council.

References

- Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16.
- Boylan, M., & Coles, A. (2017). Is another mathematics education possible? An introduction to a special issue on “mathematics education and the living world: Responses to ecological crisis”. *Philosophy of Mathematics Education Journal*, 32, 1-17.
- Cai, J. & Knuth, E. (Eds.). (2011). *Early algebraization: A global dialogue from multiple perspectives*. Springer.
- Coles, A. (2022). Towards A Socio-Ecological Perspective Within Mathematics Education. *Proceedings of the British Society for Research into Learning Mathematics*, 42(3), 1–6.

- Coles, A., & Ahn, A. (2022). Developing algebraic activity through conjecturing about relationships. *ZDM: Mathematics Education*, 54(6), 1229–1241.
- Davydov, V. V. (2008). *Problems of developmental instruction: a theoretical and experimental psychological study*. Nova Science Publishers, Inc.
- Fred, J. (2019). *Att etablera och upprätthålla ett algebraiskt arbete i årskurs 2 och 3: En undervisningsutvecklande studie med matematiska mönster som innehåll*. [Licentiate thesis, Stockholm University].
- Fred, J., & Boistrup, L. B. (2017). Students' qualitatively different ways of experiencing pattern generalization. *Quaderni di Ricerca in Didattica" QRDM (Mathematics)*, 27(2), 155–162.
- Fred, J., Valero, P., van Steenbrugge, H. Powerful algebraic ideas: Early algebraic thinking and active citizenship. *Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*, 489–496.
- Fred, J., Valero, P., & Van Steenbrugge, H. (2023). Algebraic Wicked Problems: Bridging the gap between early school algebra and socially relevant issues?. *Thirteen Congress of the European society for Research in Mathematics Education (CERME13)*, 536–543.
- Fred, J., Valero, P., & Van Steenbrugge, H. (2024a). Tracing connections in working toward a critical zone of making algebraic wicked problems. *Quaderni di Ricerca in Didattica" QRDM (Mathematics)*, 13, 127–136).
- Fred, J., Van Steenbrugge, H., & Valero, P. (2024b). Connecting early algebra and sustainability through dilemmas in algebraic wicked problems. *The 15th International Congress on Mathematical Education (ICME15)*.
- Gleason, T. (2019). Towards a terrestrial education: a commentary on Bruno Latour's "Down to Earth". *Environmental Education Research*, 25(6), 977–986.
- Kieran, C. (2006). Research on the learning and teaching of algebra: A broadening of sources of meaning. In *Handbook of research on the psychology of mathematics education* (pp. 11-49). Brill.
- Kieran, C. (Ed.). (2018). *Teaching and learning algebraic thinking with 5- to 12-year-olds. The global evolution of an emerging field of research and practice*. Springer International Publishing.
- Latour, B. (2018). *Down to Earth: Politics in the new climatic regime*. John Wiley & Sons.
- Öhman, J., & Sund, L. (2021). A didactic model of sustainability commitment. *Sustainability*, 13(6), 3083.
- Skovsmose, O. (1994). Towards a critical mathematics education. *Educational Studies in Mathematics*, 27(1), 35–57.
- Skovsmose, O., & Borba, M. (2004). Research methodology and critical mathematical education. In P. Valero & R. Zevenberger (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*, (Vol. 35, pp. 207–226). Kluwer Academic Publisher.
- Solares-Rojas, A., Arellano-Aguilar, O., García González, M. M., López-Vargas, M. del R., Coles, A., & Méndez Serrano, A. (2022). Mathematics education and social-environmental crises: an interdisciplinary proposal for didactic innovation with rural communities in Mexico. *Research in Mathematics Education*, 24(2), 202–223.
- Yolcu, A., Kirchgasser, K.L. (2024) Social (justice) mathematics: racializing effects of ordering pedagogies and their inherited regimes of truth. *Educational Studies in Mathematics*, 116(3), 351–370.

SOME RESPONSE-ABILITIES WITH MATHEMATICS EDUCATION IN THE CARE AND PROTECTION OF MOTHER EARTH

Carolina Higueta-Ramírez¹, Marcos Yule-Yatacué², Luis Yimer-Pavi³, Roger Miarka⁴, Paola Valero⁵

¹ São Paulo State University, ² Plan de Vida Proyecto Nasa,
³ Institución Educativa El Sesteadero, ³ São Paulo State University,
⁴ Stockholm University

*In this document we present some experiences with children in the care and protection of Mother Earth in different parts of the world, and we deploy a proposal with Nasa children from Toribío, Cauca (Colombia), between four and five years old, in the context of implementing a policy of resistance and re-existence: *Uma kiwe kwekwe atxh tul umn* (weaving the shelter of the body of *Uma kiwe*, Mother Earth). This experience invites us to think-with mathematics and school mathematics-possibilities to deploy such politics. This is done in dialogue with authors of the Nasa people, knowledgeable of their cosmivision and their struggles, in a relational way with the approaches of potshumanism authors who have in common the concern for ways of life that are not restricted to the human but seek to learn with the earth.*

INTRODUCTION

The objective of this text is to make visible the responsibilities that children, together with the Nasa people in Toribío, Colombia, are proposing for the care and protection of Mother Earth (*Uma kiwe*). These people live in the midst of an upsurge of violence by illegal groups that seek to dominate the land to expand a drug trafficking economy. In contrast, the Nasa community struggles to re-establish connections of reciprocity and care with *Uma kiwe* in order to resist and re-exist in the midst of such violence.

The responses proposed by children through photographs and dialogues challenge the official mathematics curriculum. This curriculum is in a process of transition from an education oriented by the Colombian State with the curricular policy documents (Curricular Guidelines, 2006; Competency Standards, 2006), towards the idea of *Piiywejxa* (learning, make learn and learning from ancestral wisdom). This idea has been felt and thought by the Nasa people and has implications to think differently about mathematics in the education of children. These ideas have been proposed in the *Ipkat* or Tulpas of legislation, which are the spaces where the Nasa people have discussed and articulated a policy of resistance and re-existence: *Uma kiwe kwekwe atxh tul umn* (Weaving the coat of the body of *Uma kiwe*). The *Ipkat* also raised the need to “gather experiences from other cultures to take [their own³⁰] education to another level” (*Ipkat* Notes, *Sekshab Kiwe*, 6.12.2022).

In response to this desire of the community, we first present five experiences that took place in different parts of the world, which focused on working with children between the ages of four and five, as well as ways of caring for different beings that inhabit the earth. Then we unfold the context and the process in which Nasa children propose respons-abilities to wrap the fabric of *Uma Kiwe's*

³⁰ See Parra, A., & Valero, P. (2021).

body. Finally, we dare to formulate some elements of a school mathematics that, together with children, have ways of reciprocal relationship with all the beings of the earth. These elements are inspired by the questions of the children themselves, by the criticisms made by the Nasa community to the official school mathematics taught so far in the territory and by the dialogue with authors who have in common the concern for forms of life that are not restricted to the human and the interest in learning with the earth.

EXPERIENCES IN THE CARE AND PROTECTION OF MOTHER EARTH

Below we present common elements from five research studies (Nxumalo & Montes, 2023; Giorza & Murriss, 2021; Postila, 2019; Nxumalo & Pacini-Ketchabaw, 2017; Mikulan & Sinclair, 2017) that we approached to address the task proposed in the Ipkat of recognizing experiences from other cultures to think about our own Nasa education. These common elements will be exemplified with elements of the aforementioned works to evidence the commitment of children in their concern and care for the earth. This allows highlighting the responsibilities in the ways in which diverse environmental problems were faced and the answers given to them from concrete contexts and with situated knowledge (Haraway, 2019). In addition, the role of school knowledge such as biology, environmental education and mathematics in such responsibilities is evident.

A first common element was the critique of how children are conceived in educational policies, schools and other institutions, and their role in caring for diverse forms of life. Nxumalo and Pacini-Ketchabaw (2017) question the separation that seems to be made between children and the rest of the world with the typical environmental problems of the Anthropocene that we are facing. A character of innocence, purity and protection is attributed to children that seems to prevent them from acting in the face of certain situations. Similarly, Giorza and Murriss (2021) state that “Children tend not to be listened to or taken seriously as knowledgeable, because they are children and, therefore (it is believed), cannot claim to have knowledge [...] They are assumed to be (still) developing, innocent, fragile, immature, irrational and so on. The child, situated predominantly as a consumer of knowledge, not as a producer, is epistemically denied on the basis of a certain ontology” (pp. 7-8).

A second element raises a critique of how the school and other institutions propose the teaching and learning of biology, environmental education and mathematics in relation to the environment, to the earth, to the climate crisis based on a curricular organization that continues to prioritize the human being (Mikulan and Sinclair, 2017). The learning of categories and knowledge is organized hierarchically in relation to the earth. That is, a binary and non-relational separation is generated between children and entities such as tree, insects, water and knowledge (Giorza & Murriss, 2021; Postila, 2019; Nxumalo & Pacini-Ketchabaw, 2017).

A third element is the use of photographs and videos taken by children as a way of approaching the world they see and feel to help discuss the problems they face in their immediate context. In Giorza and Murriss (2021) children became co-researchers by taking photographs and videos. By capturing with their eyes and their skin they show how close they are to the land and made visible what seemed unnoticed or of little importance to adults, for example, the felling of trees. Postila (2019) states, “During the sessions we took photographs or movies on a digital tablet; for example, Oscar, one of the 4-year-olds who participated most of the time through his snapshots, has been an important contributor to this article” (p. 224). Giorza & Murriss (2021) conclude regarding photographs and

videos that “[...] posthuman visual research practices are an important step toward early years education that is concerned with multispecies survival in the Anthropocene.” (p. 4)

A fourth element refers us to an epistemological-methodological perspective, compost (also known as posthumanism), based on three fundamental principles. First, the entanglements between human and non-human, or more-than-human, what Haraway (1988) calls companion species, multispecies. Second, the realization that it is necessary to continue with the problem in the face of the complexity of the environmental and climate change issues we face and the urgent need for respons-abilities to that present. And a last principle related to the valuation of a situated knowledge that includes knowledge of indigenous peoples or local communities to face and respond to different problems and that differs from absolute or universal ideas of knowledge. Tsing's (2015) work helps to highlight the need to look at nature and the potentiality of the unpredictable encounters that occur between children and nature in order to act.

A fifth common point points out the respons-abilities of children with the earth, among these are: systematic observation of beings (trees, water, insects) in local territories, monitoring and conversations based on what was seen, relational thinking that recognizes the beings that are there and those that were there (Postila, 2019), the search for alternatives (other habitats) to minimize harmful effects to beings such as insects, and attempting to restore damaged territories and reconstruct histories of colonized places in the company of teachers, the sciences and indigenous epistemologies (Giorza & Murriss, 2021; Nxumalo & Montes, 2023).

A sixth and final common point is the need to create new methods or pedagogies: water as a method (Postila, 2019), a pedagogy of the common world (Nxumalo & Pacini-Ketchabaw (2017) and mathematical pedagogy in the Anthropocene (Mikulan & Sinclair, 2017). The first two proposals bet on a relationality between humans and non-humans and are based on the questions that children have or the knowledge (including spiritualities) of indigenous peoples for the survival of different beings. A pedagogy of mathematics starts from the need to abandon the very human exceptionalism. It is a sign of the impossibility of continuing with mathematics that curricularly have been proposed from an organization of concepts in a linear and progressive way according to assumptions about the development of the child. In an attempt to overcome this, the authors propose a stratigraphic pedagogy that involves different layers of depth. For this, they point out that it is necessary to go to the history of mathematics and science fiction, and to speculative stories to think of mathematics in which concepts are not completely formed, but are virtual and real.

We find in these commonalities a syntony with the ways in which the politics of resistance and re-existence of the Nasa people challenge the current official curriculum and the mathematics curriculum for preschool children. We value these proposals as ways to confront in specific places forms of care and protection of the land that deserve to be recognized and serve as inspiration to value the possibilities that children have to act. The Nasa people have much to learn from dialoguing with these investigations. Likewise, it is important to say that these experiences call for both dialogue and the creation of pedagogies of the Nasa people.

AN EXPERIENCE FROM THE NASA PEOPLE

The Nasa people (or “Paeces” in historical terms) are the third most populous indigenous group in Colombia, with 243,176 people and preceded, by the Wayuu and Senú peoples (DANE, 2018). The

Nasa claim to come from a spiritual family in which they are daughters and sons of the water. They are located mainly in the departments of Huila and Cauca. However, they are currently creating community in different places of the Colombian territory, due to colonization processes that have led them to displacement and the struggle for land in the places they have historically occupied (Tierradentro).

To the northeast of the department of Cauca is the municipality of Toribío, whose name in the Nasa Yuwe language is *Sekshab Kiwe*, which means “territory where the navel of the sun is”. 95% of the population of Toribío is assumed to be Nasa, out of a total of 32,673 inhabitants (Yule & Vitonás, 2019a). This territory has historically been affected by the armed conflict, that is, by the presence of illegal groups that try to impose on the community practices and ways of being and doing that are contrary to the spirit of care and protection of life that the Nasa cosmovision has tried to defend. Currently there is an increase in violence, expressed in the murder of leaders defending the territory, recruitment of children and youth by illegal groups, and promotion of illegal monoculture practices. These “disharmonies”, as they are called by the Nasa people, affect and challenge the spiritual, physical and knowledge accompaniment of children, conceived by the Nasa as “seeds” (*Fxiw*).

Uma kiwe kwekwe atxh tul umn: a politics of resistance and re-existence

Since colonization, the Nasa people have been constantly challenged, in different times and spaces, to rebuild modes of community organization that allow them to continue to exist. The power of the state or of different illegal groups seems to impose itself with force, sometimes fracturing the people themselves. Nevertheless, the Nasa manage to continue to exist based on a demand for rights, but also thanks to the strategy and the struggle to resist.

The most recent proposal of the Nasa people: *Uma kiwe kwekwe atxh tul umn* (weaving the coat of the body of *Uma kiwe*)³¹, is a policy of resistance and re-existence that has been being built from the *Ipkat* since 2019 when the Nasa people, following the struggles of their ancestors: Cacica Gaitana, Juan Tama, Quintín Lame, the platform of struggle of the Regional Indigenous Council of Cauca (CRIC) and the zonal mandates (First Congress of Jambaló, Cauca in 2002; and the Third Congress of Tóez, Caloto in 2017) a mandate that guides the path of the Nasa people is raised. It is a proposal to:

[...] root in Nasanwe'sx (nasa family) the care, medicine, harmony and protection of Uma Kiwe (Madre Tierra) understood as a living being that comprises all the beings that inhabit it and kiwe (climb space), Naa Kiwe (middle space), Kiwe Dxiiju (bottom space). From the nasa cosmovision, Nasanwe'sx (nasa family) is an integral and continuous part of Uma Kiwe (Madre Tierra) in the different spaces where it is located, Nasanwe'sx (nasa family) does not own of Uma Kiwe (Madre Tierra), Nasanwe'sx (la familia nasa) belongs to Uma Kiwe (Madre Tierra); Therefore, destroying, exploiting or bleeding Uma Kiwe (Madre Tierra) is destroying ourselves. (Republic of Colombia, Official Gazette, 19.06.2024, p. 58)

The previous one clearly shows from the cosmovision of the community a policy of resistance that is both conceived as an integral part and continuity of *Uma kiwe*, as opposed to a conception of domination by human beings over the land. And it is also a policy of re-existence because it means that having the shelter of an *Uma kiwe* is the possibility of continuing to be in Nasas. The notion of politics is understood here in the senses. The first is how to “group, gather and feel, the different

³¹ To learn more about this proposal see: Official Gazette, year CLX No. 52,792 of Wednesday, June 19, 2024, pp. 58-65, from the Republic of Colombia.

views, thoughts and group them as one solo” (Marcos Yule, May 24, 2023, virtual meeting). And according to *Dxi'j pakwen*, this is the policy that “is about guiding people, about making paths, about making reality, about experiencing” (Taurino Cometa, conversation, 2.12.2022).

This proposal unfolds into the idea of *Piiywejxa*. The deep critique of uprooted, colonial, anthropocentric and fragmentary education demands another type of responses, other responsibilities, some of which were written in the *Ipkat* themselves, as we recover the continuation: “Education is not only forward, follow them guidance from a spiritual family: *Uma* (abuela, water, mother) and *Tay* (abuelo, sun). It is a natural order [...] There are things to come back from the modern world, but we have to know what”. (Notes *Ipkat, Sekshab Kiwe*, 30.08.2022). “*Piiywejxa* must guarantee that we form ourselves in completeness from our vision and western vision with the mandate to shelter the fabric of the Mother Earth”. (Notes *Ipkat, Sekshab Kiwe*, 10.10.2022).

The previous ideas were planted by participants in the *Ipkat* that were mostly adults, only in the presence of some young people and, also, *Kaapiya'jnaswe'sx* (teachers who accompany the maturation process of children, young people in institutional and own spaces). However, as the sign of women and leaders in *Ipkat*: “There is a lack of care for the family, but there is a lack of the presence of children, of children, and the feeling of children and children. Today we need an empowered woman and girl. There was a lack of light for children and children to experience these *Ipkat*.” (Notes *Ipkat, Sekshab Kiwe*, 19.09.2022). What I said for the leader's problem was with the force of a policy that, even though it considers children, given that it recognizes what could be important for them, maintains the idea that it is only adults: priests, mothers, *Kaaapaya'we' sx* those who propose ways of caring for Mother Earth. As a result, *Ipkat* is unaware of the voice, feeling and thinking of children in the creation and capacity to propose and materialize this policy. This is the capacity for agency of children to shelter *Uma Kiwe's* body.

Working on a proposal with Nasa children to build shelter for the body of a kiwe

This experience has been spent with Nasa children between four and five years at the El Sesteadero Educational Institution, located in Toribío. With the intention of knowing what worries them and seeing what they have to decide and live a policy of creating the shelter of *Uma Kiwe's* body, we decided to give them cameras so that they can record their world and think about the photographs. Children took photographs between October 31st and December 1st, 2022 in the school space or in family and community spaces where they participated. The following photographs (Figure 1, 2, 3) invite us to think about the interests and concerns that children have for the beings that inhabit the earth and their relationship with these. The photographs were selected by children when we invited them to talk about an exhibition of more than 20 photographs taken by them. Furthermore, these same photographs were also thought of in encounters with *Kaapiya'jnaswe'sx* of the primary education of the same educational institution and with research groups in which we participate.



Source: research file, authored by children,
3.10.2022

Figure 1: Wooden-children



Source: research archive, authored by children,
11/22/2022.

Figure 2: Earth-children



Source: research archive, authored by children, 22.11.2022.

Figure 3: Climate-children

These are the questions asked by the children after seeing the photographs printed and asking them what they would ask the beings that are present in the photographs?:

With-wood: Why are they there?

With-earth: What is under the earth?

With-sun: Of the sun (*Sek* in Nasa Yuwe language): Why does [the sun] dry the bushes? Why is [the sun] hot? Why is it getting so hot? How can the sun be hot in space? (Children, conversation, 03.13.2024)

In most of the photographs non-human life forms or bugs appear as Haraway (2019) would say. For the Nasa people they are beings that are part of their spiritual and earthly family. Likewise, the photographs reveal the closeness that children have with *Uma kiwe* and the beings that inhabit her. However, the school and the mathematics class do not consider these interests, since their teaching continues to be structured according to the ways in which these were learned by the *Kaapiya'jnaswe'sx* in colonization processes, which are not far from common practices in other places in the world for these ages. This is a teaching focused on arithmetic, reduced to the presentation of symbols and counting numbers up to 10, as well as the one-to-one correspondence between objects and symbols. The core of mathematical thinking in the Nasa people tries to follow the guidelines of a national curriculum that ignores the problems and challenges that this people are facing, mainly that of the defense and protection of the territory.

Hence, the need to think about other mathematics that decenters the human aspect from its training proposal, exclusivity in the abstract and individual, linear learning for the future. In this sense, we have thought together with the Nasa people, with the photographs proposed by children, with various

authors (e.g., Haraway, 2019; Fasheh, 2021; Le Guin, 2022; Tsing et al., 2024) the following points that we consider urgent to deploy:

A mathematics learning proposal based on a relational (or companion species, Haraway, 2019) and reciprocal perspective. Although in the field of mathematics education we can recognize perspectives such as ethnomathematics and critical mathematics education to discuss issues related to environmental problems, these proposals seem to follow a logic of starting from the recognition of environmental problems for the learning of mathematical concepts. In coherence with the people Nasa and Haraway (2019), there is a need for a shift from thinking “from” to “a thinking-with that is reciprocal.”

A look at the local, at what is close, at what happens with the beings that live or arrive at the places where we live, this is what the Nasa children showed us in their photographs. These photographs following Tsing et al. (2024) are an invitation to look at the effects of an uneven, fragmented Anthropocene. It is about the possibility of observing and describing, together with mathematics, the effects and transformations of specific places in which children live. It is about co-creating knowledge to “find ways to limit its [unequal Anthropocene] force that destroys the world” (Tsing et al., 2024, p.13).

A community perspective in the teaching and learning of mathematics, since some Nasa leaders have stated that until now mathematics has been taught to favor individual thinking and against *Uma kiwe*. In this same logic, Fasheh (2021) says regarding a learning that flourishes in the land, the cultural, the communal-community and in the heart, in the soul. The photographs also invite us to think about the common, here the questions that children posed in relation to felling, the growth of trees and the thermal sensation.

Learning processes linked to the present of the Nasa people, a present in which a pedagogy of feeling is necessary. It is a proposal that reveals the superstition of teaching and learning mathematics for the future, since children need to know and act in the here and now to continue existing, but also linked to a Nasa spirituality.

Acknowledgments

To preschool children (2022) and *Kaapiya'jnaswe'sx* from the El Sesteadearo Educational Institution, who participated as peer researchers. To the Nasa people of Toribío for allowing us to think with them. To the Ministry of Science, Technology and Innovation of Colombia for financing the doctoral studies of Carolina Higueta Ramírez.

References

- Bajo el sol. (16 de noviembre de 2021). Bajo el sol 3: munir fasheh + asunción molinos gordo. [Archivo de Vídeo]. Youtube. <https://www.youtube.com/watch?v=YsT4oDCvc38>
- Departamento Administrativo Nacional de Estadística-DANE. (2018). Comunicado de prensa Entrega de resultados población Indígena Censo Nacional de Población y Vivienda CNPV-2018, Disponible en: <https://www.caroycuervo.gov.co/documentos/imagenes/comunicado-grupos-etnicos-2019.pdf>
- Fobzu (Friends of Birzeit University). (17 de diciembre de 2019). Munir Fasheh on radical approaches to learning [Archivo de Vídeo]. Youtube. https://www.youtube.com/watch?v=djbj-kts_04

- Giorza, T. & Murriss, K. (2021). 'Seeing' with/in the world: becoming-little. *Childhood & philosophy*, 17, 1-23.
- Haraway, D. (2019). *Seguir con el problema. Generar parentesco en el Chthuluceno* (3rd ed.). Consonni.
- Haraway, D. J. (1988). Situated knowledges: The science question in feminism and the privilege of partial perspective. *Feminist Studies*, 14(3), 575-599.
- Le Guin, U. (2022). *El nombre del mundo es bosque* (1nd ed.). Editorial Planeta Colombia.
- Le Guin, U. (2021). *Los desposeídos*. Editorial Planeta Colombia.
- Mikulan, P. & Sinclair, N. (2017). Thinking mathematics pedagogy stratigraphically in the anthropocene. *Philosophy of Mathematics Education Journal*, (32), 1-13.
- Ministerio de Educación Nacional de Colombia.(2006). *Estándares básicos de competencias en lenguaje, matemáticas, ciencias y ciudadanas. Guía sobre los que los estudiantes deben saber y hacer con lo que aprenden*. MEN. https://www.mineducacion.gov.co/1621/articulos-340021_recurso_1.pdf
- Ministerio de Educación Nacional de Colombia. (1998). *Lineamientos curriculares de matemáticas*. MEN. https://www.mineducacion.gov.co/1621/articulos-89869_archivo_pdf9.pdf.
- Municipio de Toribío, Proyecto Nasa & Cecidic. (2015). El camino del corazón. Modelo pedagógico del plan de vida del proyecto nasa (Documento de circulación interna).
- Nxumalo, F. & Pacini-ketchabaw, V. (2017). 'Staying with the trouble' in child-insect- educator common worlds. *Environmental Education Research*, 23(10), 1414–1426
- Nxumalo, F. & Montes, V. (2023). Encountering creative climate change pedagogies: Cartographic interruptions. *Research in Education*, 117(1), 42–57.
- Parra, A., & Valero, P. (2021). Propio as a decolonising tool for mathematics education. In A. Andersson & R. Barwell (Eds.), *Applying critical mathematics education* (pp. 71-99). Brill. <https://brill.com/display/book/9789004465800/BP000012.xml>
- Postila, T. (2019). Water as Method: Explorations of Locally Situated Environmental Issues Together with Preschool Children. *Australian Journal of Environmental Education*, 35(3), 222-229. <https://doi.org/10.1017/ae.2019.27>
- República de Colombia. (19 de junio de 2024). Asociación de cabildos indígenas de Toribío Tacueypo y San Francisco. Mandato-Ikahnxi eç de 2023 Khabu fxizehnxí phaz ephkabh eba tekxh. Diario oficial, 58-65.
- Tsing, A., Deger, J., Keleman, A. & Zhou, F. (2024). *Field guide to the patchy antropocene. The new nature*. Stanford University Press.
- Tsing AL (2015) *The Mushroom at the End of the World: on the possibility of life in capitalist ruins*. Princeton University Press.
- Yule, M. & Vitonás, C. (2019a). *Pees kupx fxi'zenxi "La metamorfosis de la vida"* (4nd ed.). Neehnwe'sx vxu Behkiwe-Toribío, Cauca.

ABENG FOR CLIMATE DISARMAMENT, REPARATIONS & MULTISPECIES' FLOURISHING

Steven Khan

Brock University

I introduce, use, and encourage exploration of the concept of Abeng as a signal of the need for increased and clearer organization of mathematics' and mathematics education's agenda in relation to responsibility and responsiveness to the existential threat posed by the diversity of socio-ecological challenges faced by our generation and those yet to come. I speculatively consider the role of climate disarmament and reparations movements as part of the key policy agenda and infrastructure necessary to finance the transitions to cultures of peace and multispecies' flourishing. I suggest that (some) mathematics educators must become more directly engaged with developing/influencing green swan policies/practices at various pedagogical pivots of power (family, school, councils, municipal, national, international, multi-national etc.) i.e. across the loci of teaching, research, and public service. I connect my work to decolonial and anti-colonial movements particularly those associated with scholars from Canada and the Caribbean whose foci might be framed as anti-transactional, anti-supremacist, creolized ways of knowing, doing and being.

INTRODUCTION

Earthkin know what is happening. Earthkin know they must change, are changing. Earthkin feel in in their bodies and bones (Gumbs, 2020). This is a work in progress. It is a speculation and provocation on the potential role of mathematics and mathematics education in facing many socio-ecological challenges (not just climate change) and those yet to come. In order to open up the target conceptual domain (Kövecses, 2020) of mathematics education and the socio-ecological, I draw from the source domain of maroon theory & experience (James, 2002), in particular the concept of Abeng.

ABENG

Abeng is a Ghanian word for an animal's horn. In the West Indies the blowing of the horn called enslaved peoples to the plantation fields and also served as a form of communication among Maroon armies (Cliff, 1984/1995) in engaging in freedom raids and rebellions. Hill (2007) also recounts its deliberate choice as the name of a Jamaican newspaper and its emphasis on thought, action and organization as structurally coupled, with organization serving as mediator and pedagogical pivot (Ellsworth, 2005) for radical change. My intent in deploying this concept from a non-traditional source domain is to foreground the need for strategic organized responses to the challenges and phenomena that fall under the umbrella term of 'socio-ecological' and to bring to awareness to what Walcott (2021) has called the long emancipation and the necessity of continuing to work towards the many freedoms-yet-to-come (Walcott, 2021) the majority of which we are unlikely to be present to see.

THE GOAL IS MULTISPECIES' FLOURISHING

In previous work I have elaborated a socio-ecological framing of mathematics for multispecies' flourishing (Khan, 2020; Khan et al., 2022; Khan et al., 2023) which stands in opposition to the

underlying plantation logics of many world systems and which decenters but does not devalue human beings in relation to their multispecies kin. The gist of this work is that there cannot be human flourishing without attention to the fate of our multispecies' partners on this planet and that a plantation-plot infrastructure, while allowing for emergence, creative expression and novelty, continue to replicate the very conditions it seeks to transform, compounding coeval catastrophes. Attention to land, language, lore, logic, living and learning while being oriented by values that acknowledge and promote needs for survival (security), transcendence, dignity, belonging, and challenge form the counter-point to the plantation-plot infrastructure.

THE CLIMATE OF WAR AND CULTURES OF PEACE

The many visible conflicts currently taking place between/among nations and communities across the planet and their tragic impacts on human and multispecies' lives as well as the less visible ones that take place to enable and support such conflict (eg. mineral mining) contribute to socio-ecological, humanitarian and multispecies crises. In returning to the work of D'Ambrosio (2001) and discovering the work of Einstein (1988) and several others in mathematics education on peace (eg. Wagner, 2002). I extend my ongoing work by arguing that intentionally fostering cultures of peace (peacebuilding and diplomatic work) is a necessary strategy to move towards multispecies flourishing. This is aligned with the survival, transcendence, dignity and challenge aspects of the framework.

CLIMATE DISARMAMENT

The term 'climate disarmament' was brought to our attention via Daniele Taurino's (2021) work and it has become a critical missing piece in our understanding of what needs to be done and where our limited human energies need to be focused. The short thesis is that the micro and macro revolutions needed to move towards more sustainable human futures and multispecies' flourishing can only be financed by reallocating the money and resources that are currently used to make war and which exacerbate the impacts of socio-ecological crises upon the most vulnerable and the voiceless. Claussen (2021) points out that the carbon and other material resource footprints are unaccounted for in the major climate agreements, that militarization of the climate crisis is counterproductive and that such rearmament redirects financial resources that are necessary for funding the transition.

Taurino offers several concrete proposals for climate disarmament. These include the converting of military spending to social programs; investment in educational programs in nonviolence; demilitarizing borders and financing research in peace studies. A different but related set of arguments is offered by Klein (2020) where he calls for a "wartime approach" to the climate emergency. By this he means the leadership, policy, financing and public/social engineering that was essential to international, national, and community cooperation efforts.

Climate disarmament is a necessary critical aspect of changing the global vulnerability system (Khan, 2014). If our field of mathematics education requires another 'turn' I would argue it is a turn to peace and diplomacy to reduce the vulnerability, insecurity, precarity of both humans and non-humans – Earthkin – so that survival with dignity and the possibility of flourishing remains open and increasing probable. But our field must also reckon with a long and unfinished history of suffering and exploitation, the plunder of peoples, places and pathways (futures) in the making of the concept of 'property.' The connection between these two sections is around the costs of energy to perform work/labour (eg. nuclear v human v animal v renewable (read v as set notation rather than vs) in

creating capital and property and the need for armaments and a military-industry-prison complex to secure, defend, and continuously extend capital and property.

CAPITALISM AND SLAVERY AND SOCIO-ECOLOGICAL CRISIS

The labor of enslaved peoples provided the energy in the pre-Industrial period that birthed the modern and postmodern world (Nikiforuk, 2012; Williams, 1964/2021). An estimate of a plantation’s carbon footprint is provided by Herschthal and Brooke (2024) based on the detailed diary of an 18th century tobacco plantation owner and slaveholder. They argue and provide evidence that prior to the European (and later American) Industrial Revolution agricultural land use and soil emissions contributed the most to human’s carbon footprint. In particular they argue that British Atlantic colonial plantations (of rice, tobacco and sugar) reliant on enslaved labour and changing land use practices to maximise profitability produced an average emissions footprint 10 times as large as the average New England family farm of the same time period. Blau and Blau (2020) advance a similar thesis.

Slave labor continues into the present day contributing to some of the most environmentally destructive industries (mining, forestry, manufacturing). Bales and Sovacool (2021) argue that the activities of the estimated 40-46 million enslaved people generate the third largest carbon dioxide emissions – more than 4 times as much as the United Kingdom and Canada. A key reason for this they argue is that, “the average acquisition cost of a person is now only about \$90...when the cost of a human being falls to this level, they cease to be capital purchase items and become disposable inputs into economic processes or patterns of environmental rapaciousness” (p.2). They note that slavery is a threat multiplier for climate change and vice-versa which also interacts with social and political threats such as corruption, discrimination and poverty. In the context of this ICMI study we draw attention to the way robotics and machine learning are reducing the ongoing cost and value of physical and intellectual and emotional labor across a variety of sectors.

Walcott’s (2021a) long emancipation foregrounds the fact that human freedom, Black freedom in particular, is an ongoing process and not an event which depends on overturning centuries of white supremacist and colonial mindsets towards other humans and land as conceivable as property (Walcott, 2021b) requiring protection or expansion through the use of violence, force, and today weapons of mass destruction.

It is at this point that we return to the abeng and the maroons – groups of free people – formerly enslaved, indigenous AND Europeans (mainly sailors) cast away when no longer useful due to age or injury – able to trouble colonial and plantation power through thought, action and significantly organization. As Beckles (2023) sates, “every generation of enslaved people revolted.” Ferdinand (2022) offers that the figure of the maroon, “points to ecological practice as a condition for emancipation” (p.148) but notes that maroon ecologies still fail to overthrow or change the slave-making colonial order of the plantationocene. However, it is the multiple refusals – “the refusals of inequality, of humiliation, and destruction” (p.158) – that point to the future in which the freedoms yet-to-come-will-have-come and from which we must begin.

REPARATIONS

Financing the transition to cultures of peace and multispecies’ flourishing beginning with climate disarmament must turn also to a consideration of reparations (Araujo, 2017, Coates, 2014, Táíwò, 2022). The perspective adopted by Táíwò (2022), that “reparation is a constructive project” (p.4) is

the one adopted here. In the modern context of ongoing climate catastrophe, he argues that the reparations project must address head on issues of climate justice for which risk and vulnerability are unevenly distributed due to the legacies of empire and capitalism.

Bales and Sovacool (2021) although not connected to the reparations movement offer proposals that are consonant with such movements through their emphasis on climate justice. They propose that freeing the 40 million plus enslaved peoples would cost about \$20 billion over a 2 decade period, or about the cost for a 4,800 megawatt nuclear power plant. These individuals would then need to be reintegrated into social systems and following Ferdinand (2022) could become deliberately engaged in more sustainable land use practices, possibly linked to the provision of carbon credits.

The turn to a serious consideration of reparations in a context of climate disarmament and simultaneous liberation/emancipation of the third largest source of carbon emissions is a necessary one. Conceptually, it involves a deep folding back to first acknowledge discomfiting aspects of our histories and our field's complicities and a motive for organizing to take deliberate action in multiple spheres. The extension of this work to forced incarcerations of other species is also necessary (Gruen & Marceau, 2022).

REHEARSING THE IMPLICATIONS

This is the speculative part of the work. From the Abeng there is an urgent need to organize ourselves as a field to take on socio-ecological issues at multiple scales of influence. This is the area perhaps where we have fallen far short compared to others. Some members of our field must become more directly involved with developing and/or influencing policies and practices at the various pedagogical pivots of power where they operate. This ranges from the home to schools to local government, national government, international organizations, social media and work with other sectors (non-profit and for-profit). Now is the time for solidarity (Hunt-Hendrix & Taylor, 2024). In particular we need to consider how we develop green swan policies (Bolton et al, 2020), i.e. policies that address and mitigate the economic and existential risks faced by the climate emergency by taking them seriously not as some future probability but as already present reality for the most vulnerable and near certainty for the majority of Earthkin.

Continuing this speculative thread the challenge here is the current limited political power of 'green' parties and the democratic values that underpin modern political systems. This is not a nod to rising authoritarianism and fascism but an acknowledgement that the mathematics of electoral and political systems in WEIRD societies more frequently tips towards a singular majority (rather than including the plurality of minority) opinion. The question of influence and reputation economies is suggestive that leadership in this area may not necessarily come from within the ranks of mathematics education researchers but perhaps from popularizers and social media influencers. And so perhaps it is to working with this group that we might align some of our attention. Though this does not come without risk as well as the current motivation and direction within influencer culture is often driven by 'memecry' and gaming the algorithms.

Considering even a more narrowly focused emphasis on the common worlds of mathematics education at the K-12 school level and curriculum what are the implications. As I have rehearsed elsewhere, there may come a time when the current enterprise of Mathematics Education as a specific signifier comes to an end and becomes subsumed under a more inclusive framework (Isaac Asimov's

depiction of psychohistory in the Foundation series of novels and adaptation for example where history, sociology, psychology, politics and mathematics are one discipline). While this might be terrifying for us to consider or even imagine, it is part of the possibility landscape however unlikely in the present moment. That being said there is much mathematics beyond the existing content of K-12 curricula that might become a focus. Ideas from Dynamical systems, Networks and Topology (Knots for example) lend themselves to foregrounding the importance of agent participation in influencing the future and of connections. Though perhaps what is needed here are powerful stories, apocryphal and mythological that take root in the imagination (for eg. Edward Doolittle’s (2018) retelling of the Bridges of Konigsberg starting from a different value-system is exemplary in this regard). And so I am left with the thought that we continue to need new stories and new storytellers. I am hopeful that we might find a different story to tell than, for example, that of Gauss’ mathematical precocity in opening up the worlds of arithmetic, series, and analysis.

In terms of thinking through reparations, here my work at this time is extremely speculative and requires engagement with historians, economists, philosophers and policy makers. Given mathematics’ (and mathematics education’s) ongoing relationship with the technological-military-industrial-plantation complex and the direct and indirect links with climate change as well as the historical inertia with respect to reparations most considerations ultimately must confront what Andrews (2024) calls the paradox of reparations campaigns, “that they leave intact a system founded on White supremacy and the exploitation of the Global South [and] therefore impossible for reparations to be realized without bringing an end to the current political and economic system [i.e.]...the need for revolution” (p.1). Whether such revolution(s) can find purchase and simultaneously remain committed to non-violent action is an open question and one that is under constant tension.

Some of us will have to follow the example of maroon refusals and community building outside of the regular structures. Many of us will have to share space and time with members of other movements, not all of whose values and strategies we may agree with (eg. violent vs non-violent) in a spirit of organizing for effective action. At the core though of such collectives should be at least common values of anti-transactional, anti-supremacist, creolized (Glissant, 2020) ways of knowing, doing and being with other humans and with our multispecies partners on this planet.

I write this noting the difficulty and committing myself to this challenge for myself as well to become a better local and global citizen at home and in my many communities, including mathematics education.

References

- Andrews, K. (2024). The knife is still in our backs: Reparations washing and the limits of reparatory justice campaigns. *Development and Change* 0(0), 1-12. <https://doi.org/10.1111/dech.12848>
- Araujo, A. L. (2017). *Reparations for slavery and the slave trade: A transnational and comparative history*. Bloomsbury Academic.
- Bales, K. & Sovacool, B. K. (2021). From forests to factories. How modern slavery deepens the crisis of climate change. *Energy Research and Social Science*, 77. <https://doi.org/10.1016/j.erss.2021.102096>
- Beckles, H. (2023). The case for reparations in the Caribbean and who should pay. *The Dial* (3) (April 25). <https://www.thedial.world/issue-3/caribbean-reparations-payment-slavery-barbados>

- Blau, R. & Blau, J. (2020). *Climate chaos and its origins in slavery and capitalism*. Anthem Press.
- Bolton, P., Desres, M., Pereira da Silva, L. A., Samama, F & Svartzman, R. (2020). *The green swan. Central banking and financial stability in the age of climate change*. Bank for International Settlements, Banque de France. <https://www.bis.org/publ/othp31.htm>
- Claussen, A. (2021). Global disarmament to protect the climate. *Medicine, Conflict and Survival*, 37(3), 177–183. <https://doi.org/10.1080/13623699.2021.1960466>
- Cliff, M. (1984/1995). *Abeng (Ābēng)*. Plume.
- Coates, T. N. (2014). The case for reparations. *The Atlantic* (June).
<https://www.theatlantic.com/magazine/archive/2014/06/the-case-for-reparations/361631/>
- D’Ambrosio, U. (2001). Mathematics and Peace: A reflection on the basis of Western civilization. *Leonardo*, 34(4), 327–332. <http://www.jstor.org/stable/1577157>
- Doolittle, E. (2018). Off the grid. In S. Gerofsky (Ed.) *Contemporary Environmental and Mathematics Education Modelling Using New Geometric Approaches* (pp.101-121). Springer.
- Einstein, A. (1988). *Einstein on peace*. Random House Publishing
- Ellsworth, E. (2004). *Places of learning. Media architecture, pedagogy*. Routledge.
- Ferdinand, M. (2022). *Decolonial Ecology: Thinking from the Caribbean world*. Polity.
- Glissant, E. (2020). *Treatise on the whole world*. C. Britton (Trans). Liverpool University Press.
- Gruen, L. & Marceau, J. (2022). *Carceral logics. Human incarceration and animal captivity*. Cambridge University Press. <https://doi.org/10.1017/9781108919210>
- Gumbs, A. P. (2020). *Dub. Finding ceremony*. Duke University Press.
- Herschthal, E & Brooke, J. L. (2024). The plantation carbon complex: Slavery and the origins of climate change in the early modern British Atlantic. *The William and Mary Quarterly*, 81(2), 255-306. <https://doi.org/10.1353/wmq.2024.a925934>
- Hill, R. A. (2007). From New World to Abeng: George Beckford and the Horn of Black Power in Jamaica, 1968-1970. *Small Axe*, 24, 1-16
- Hunt-Hendriz, L. & Taylr, A. (2024). *Solidarity. The past, present and future of a world-changing idea*. Pantheon.
- James, C. (2002). *The maroon narrative: Caribbean literatures in English, across boundaries, ethnicities and centuries*. Heinemann.
- Khanm S. K. (2014). Fear and cheating in Atlanta: Evidence for the vulnerability thesis. *Transnational Curriculum Inquiry*, 11(1), 1-12. <https://doi.org/10.14288/tci.v11i1.184212>
- Khan, S. K. (2020). After the M in STEM: Towards Multispecies’ Flourishing. *Canadian Journal of Science, Mathematics and Technology Education*, 20, 230-245. <https://doi.org/10.1007/s42330-020-00089-4>
- Khan, S., La France, S & Tram, H. T. T. (2022). After plantations’ precarities: curating math-thematic curriculum plots in initial teacher education for multispecies’ flourishing and a freedom yet to come. *Research in Mathematics Education*, 24(2), 170-186 <https://doi.org/10.1080/14794802.2022.2090421>
- Khan, S., Bowen, G. M & Karrow, D. (2023). Abeng for multispecies’ flourishing. *Journal of the American Association for the Advancement of Curriculum Studies*, 15(2), 35 pp. <https://doi.org/10.14288/jaaacs.v15i2.197502>
- Klein, S. (2020). *A good war: Mobilizing Canada for the climate emergency*. ECW Press.
- Kövecses Z. (2020). *Extended Conceptual Metaphor Theory*. Cambridge University Press.

- Nikiforuk, A. (2012). *The energy of slaves: Oil and the new servitude*. Greystone Books.
- Táíwò, O. O. (2022). *Reconsidering reparations*. Oxford University Press.
- Taurino, D. (2021). Climate disarmament: how nonviolence can resist the militarisation of the climate crisis. War Resisters' International <https://wri-irg.org/en/story/2021/climate-disarmament-how-nonviolence-can-resist-militarisation-climate-crisis>
- Wagner, D. (2002). Teaching mathematics for peace. *Connections*, 26(2), 9-12.
- Walcott, R. (2021a). *The long emancipation: Moving toward Black freedom*. Duke University Press.
- Walcott, R. (2021b). *On property: policing, prisons and the call for abolition*. Biblioasis.
- Williams, E. (1964/2021). *Capitalism and Slavery (3e)*. The University of North Carolina Press.

NETWORKING ETHNOMATHEMATICS, EMBODIED COGNITION-NEW MATERIALISM, AND MATHEMATICAL MODELING TO REIMAGINE PROFESSIONAL DEVELOPMENT

Jessi Lajos¹, Jessica Benally², Sindura Kularajan¹

¹Utah State University, ²University of California Berkeley

In conjunction with the Utah State University Science, Technology Engineering, Arts and Mathematics (STEAM) Expo, faculty, graduate students, collaborators and teachers in rural Utah districts came together for a cultural-exchange community event. We facilitated a two-session professional development workshop for mathematics educators as part of this event. The purpose of our workshop was to address the need for teacher supports in rural areas of Utah, respond to Coles et al.'s (2024) calls to make “gestures” towards socio-ecological practices in mathematics education, and to promote epistemic pluralism—multiple ways of knowing and doing mathematics. The professional development activities we implemented were inspired by frameworks in Coles et al. (2024) that have been aligned with socio-ecological perspectives of mathematics education. These frameworks are ethnomathematics (EM), embodied cognition-new materialism (ECNM), and mathematical modeling (MM). After introducing these frameworks, we present workshop activities, and responses from participants’ feedback. Participants’ feedback reveals a conflict between what participants valued in the activities and their perception of what constitutes learning mathematics.

INTRODUCTION AND THEORETICAL BACKGROUND

There is a need to offer community building professional development (PD) events that bring together mathematics teachers in rural areas and university educators; and that are contextualized according to the cultures of local sites (Bundock, 2023; Vaughn & de Beer, 2020). We focus our efforts on rural areas of Utah located in the southwest United States of America. The United States Department of Agriculture (USDA) defines rural as settlements with less than 2,000 housing units and 5,000 residents that have a limited social and economic integration. According to the Utah Rural Schools Association (URSA) it is estimated that 15% of Utah’s students reside in rural areas spreading across the state, yet rural areas have difficulty in hiring or maintaining teachers with specialization in rural areas. URSA recommends key actions to combat these issues. One such action is to develop pre-service and in-service PD opportunities. And two, to strengthen relationships among rural schools and institutions of higher education. Within the mathematics education research community Coles et al. (2024) have brought together a body of work to make a case towards “entangling” the social and ecological. That is, to transform and view our classrooms as adaptive systems in which “social constructs” such as language/culture, “bodies; in-animate things; ecologies; space”, and “the symbolic (including mathematics)” are integrated (p. 171).

The purpose of our present work is two-fold. First, we address the need for community building PD events that encourage relationships among rural teachers and university faculty. Second, we respond to Coles et al.’s (2024) call to make “gestures” towards socio-ecological practices in mathematics education under Theme D. Mathematics Education Futures in which we view futures as “imagining new stories” and ways of teaching that counter the “present” ways of curriculum standardization. The “gesture” we wish to make and explore is a professional development workshop inspired by

frameworks in Coles et al. (2024) that have been aligned with socio-ecological practices in mathematics education. These frameworks are ethnomathematics (EM), embodied cognition-new materialism (ECNM), and mathematical modeling (MM). In particular, we address the following questions: How can professional development workshops for mathematics' educators be reimagined through networking socio-ecological frameworks (EM, ECNM, MM) that promote epistemic pluralism—multiple ways of knowing and doing mathematics? What barriers are signaled by educators' feedback in incorporating similar workshop activities in their classrooms?

Ethnomathematics

D'Ambrosio (1985) defined Ethnomathematics as “the mathematics practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on” (p. 45). It can be considered as epistemological approach to mathematics education that centers Indigenous philosophies and values (Barton, 2008; Borba, 1990). Cajete (1994) provides a framework for Indigenous Education and lists following principles (p. 28-30):

- A sacred view of Nature permeates its foundational process of teaching and learning.
- It recognizes and incorporates the principle of cycles within cycles (there are deeper levels of meaning to be found in every learning/teaching process).
- It uses story as a way to root a perspective that unfolds through the special use of language.

Orientation and pathways are central concepts in indigenous education. For instance, the winds and sun are symbols for orientation in terms of cardinal directions and animate guides. These symbols are integral to pathways of past ancestors and a present spiritual journey. We now turn to embodied cognition as a complementary frame to indigenous education principals given that embodied cognition theory asserts that language and meanings are derived from sensorimotor activity (Nathan, 2021; Arzarello & Robutti, 2008), indigenous language is action-verb based (Barton, 2008), and an epistemological alignment of subject-subject relationships when learners are not dissociated from mathematical objects but rather become them (Bang & Vossoughi, 2016; de Freitas & Sinclair, 2014).

Embodied Cognition and New Materialism

An embodied approach to mathematical instruction positions socio-material interactions and body as playing a key role in cognition (Nathan, 2021; Vaughn & de Beer, 2020). Such interactions lead to emergent multi-modal models, that entangle sensory information, feelings, learners' collective body formations, movement, gestures, speech, and tools, which drive the direction and cadence of classroom lessons (Kelton & Ma, 2018). An embodied approach to teaching and learning mathematics has been described as a counterforce to the dominant “culture-free, unemotional, universal, decontextualized, disembodied, asocial...”, assimilatory knowledge for profit approach to mathematics instruction (Cajete, 1994; Lakoff & Núñez, 2013, p. 23). Lakoff and Núñez (2013) advocated for mathematics education to move away from the dominant western tradition and “...towards views that consider the mind as situated in context, grounded in experience, shaped by culture, and dependent on the peculiarities of human embodiment... (p. 23).” Instructional environments that activate the sensorimotor system also enable students to develop their own language and meanings for the mathematics via grounding metaphors (Lakoff & Núñez, 2013). Grounding metaphors represent mappings between a physically based resource domain and a

mathematical domain. For example, instruction that situates arithmetic as motion supports learners in developing metaphors such as “arithmetic operations are acts of moving along the path” and “equations are routes to the same location” (p. 37). In Lakoff and Núñez’s work learners are given the status of mathematical agents—an “idealized actor in the source domain of a metaphor characterizing some aspect of mathematics” (p. 33) via connections to their identities or experiences (Adiredja & Zandieh, 2020). However, learners are not yet given the status as creators or living constituents of the mathematics itself. Thus, it is important to develop instructional environments in which mathematical opportunities are rich and learners are able to create their own mathematical language for familiar and new co-created concepts.

Thus, we adopt a materialist approach within the embodied approach to mathematics education referred to as “new” or “inclusive materialism” (Barad, 2007; Châtelet, 2009; de Freitas & Sinclair, 2014). “Inclusive materialism” asserts that a body defined as an “assemblage of human and non-human components” moving through space, not necessarily constrained by physical laws, plays a central role not only in illustrating and communicating mathematical concepts, but are intrinsic to the creation and materiality of mathematical concepts (de Freitas & Sinclair, 2014, p. 25, 42). Inclusive materialism aligns with a relational ontology in which mathematical concepts and their properties are not fixed and do not exist a priori. Rather, interactions among humans and non-human materials such as an apparatus determines properties and concepts which are always in flux (Barad, 2007). Mathematics is neither prescribed nor bounded. Mathematics and the learner do not exist apart from one another, but rather the mathematics is born, lives, and evolves indefinitely through bodily-kinesthetic interactions, the virtual (Châtelet, 2009), and a blending of semiotic resources (Arzarello & Robutti, 2008) such as gestures, verbiage, inscriptions, diagrams, manipulatives.

Mathematical Modeling

Mathematical Modeling (hereafter: modeling) is the process of representing a real-world situation in mathematical terms. The product of this process is a mathematical model that has the ability to make predictions or provide more insights regarding the real-world phenomena of interest. The process of modeling is typically considered to be non-linear and iterative in nature, consisting of multiple subprocesses. Examples of these subprocesses include identify the problem to be solved, making assumptions to simplify the situation, mathematize the simplified situation, work mathematically to get the results, validate the final and developing models, analyze and interpret the results in terms of real-world constraints, and report the results (e.g., Garfunkel & Montgomery, 2016; Blum & Leiss, 2007; Anhalt & Cortez, 2016).

Modeling has been used by educators and researchers to promote conceptual development of mathematical concepts (e.g., Ellis, et al., 2016), increase interest and self-efficacy in mathematics (e.g., Czoher et al., 2019), and address socio-critical, social-justice, and environmental issues (e.g., Jung & Magiera, 2023). These skills are attainable through selecting modeling tasks that have the capacity to be relevant, authentic, culturally responsive, and experientially real to the learner. For example, the Flint water Task (Aguirre et al., 2019), an open-ended modeling task related to the water crisis in Flint Michigan, USA in 2014, has pedagogical implications to motivate modelers to engage in critical discussions about societal issues while promoting utility of mathematics to solve real-world issues. Mathematical modeling can then be a fertile learning environment for synergizing culturally relevant pedagogy, social-justice oriented teaching, and conceptual development.

TEACHER PROFESSIONAL DEVELOPMENT REIMAGINED

We now present our efforts in reimagining teacher PD by detailing how we drew from the above theoretical frameworks to facilitate workshop activities as part of a four-day cultural-exchange teacher PD event in rural Utah. This event coincided with the USU Steam Expo on the Blanding campus in which participants also volunteered by running math booths and sessions for school children. PD participants included 12 USU Mathematics and Statistics faculty from Logan, Blanding, Eastern, and Uintah Basin campuses, 1 Biology faculty, 1 USU academic advisor, and 13 USU students. The 34 in-service participating teachers were from the San Juan, Uintah, and Grand County school districts and teach at the elementary and highschool level. The PD event included two two-hour teacher workshop sessions that integrated EM, ECNM, and MM along with cultural talks in a ceremonial Hogan, lunch and dinner gatherings, a trail walk at the Nations of the Four Corners Cultural Site, a service project, and a tour of the Monument Valley Navajo Tribal Park guided by a Navajo elder with music and dancing at the eagle amphitheater rock formation (Fig. 1).



Figure 1: Community dance around the fire in Monument Valley Navajo Tribal Park

The authors' met regularly and had conversations regarding how we could collaborate and use our research training in EM, EC, and MM to inform workshop activities. We leveraged these three theoretical perspectives and what we learned from one another to inform our approach to educator PD akin to the way our teaching practices pull from a complex web of multiple perspectives (e.g., theoretical, mentoring educators, students, family, identity, etc.). From these conversations and our initial implementations, elements of a design-heuristic emerged. One such element was to intentionally foster shared experiences that were specific to the local culture/context ahead of the workshop sessions that could be built-upon during the sessions. For instance, we scheduled talks in the Hogan and a hike of the Four Corners' Region located by the Blanding Campus picnic site ahead of session 2. Another element was to provide opportunities to use the natural environment, artefacts, and bodies/subject-subject relationships as semiotic resources. During session 2, talks in Hogan, and cultural sites encountered on the four corner's hike to define direction and track spatial orientation by looking to the Hogan's door as a point of reference.

Next, we present teacher workshop activities that were implemented at this event. The first session involved constructions of hand-held and life-size icosahedrons (Abrahamson & Rosenbaum, 2016) along with a demo of Náhookos Bi'ka' constellation constructions with STARR (Benally, in press). We expand on the activities—modeling an optimal hike and exploring cycles in nature which were implemented in the second session. These activities highlight the synergistic power among the socio-ecological frameworks (EM, ECNM, & MM) along with teachers, students, and faculty who brought the mathematics to life through an exchange of culture, space, and experiential knowledge. A variety

of essential concepts in number, school to abstract algebra, geometry, and trigonometry could be generated through activities but were not explicitly revealed by workshop facilitators in order to foster an environment open to the creation of mathematical practices and concepts rather than the reproduction of a preconceived mathematical script (de Freitas & Sinclair, 2014). In addition, these activities were intentionally situated in cultural contexts that participants had experienced as part of the event such as the trail walk at the Four Corners cultural site.

Session 2 Activity 1: Modeling an Optimal Hike

The first activity of session 2 was a modeling task based on a hike the participants took after lunch. After lunch, the participants set out on a hike around the nations of the four corners which consists of multiple trails and monuments the belong to the Navajo culture. The participants were asked to collect any data that they thought was relevant (e.g., time, number of steps, bodily senses, emotions) while hiking. After the hike, participants were given the following modeling task:

Imagine that a hiker with only 45 minutes to experience the monuments of the Four Corners region, asks for your advice on the ultimate hiking route. You are tasked with designing the most optimal hiking route that showcases the essence of the four corners nation while maximizing the time on the trail. Clearly state your simplifying assumptions and justifications.

It was up to participants to determine what data was important to them and what optimal means—infusing new materialism into mathematical modeling activities—where the participants had to develop their own criterion for mathematization. Optimal route was operationalized in a variety of ways among the participant groups: higher number of monuments visited, scenic route, route that allowed for a good workout, and route that was relaxing while also facilitated the learning of the Navajo culture. For example, the group that operationalized optimal route as the route that allowed for a good workout, developed a model for the number of calories burned during the 45 minutes, considering the pace of walk and the distance travelled.

Session 2 Activity 2: The Lunar Turtle Calendar and Cycles in Nature

The second engaged participants in an instructional approach that combined principles from Ethnomathematics and Embodied Cognition-New Materialism. The activity began with cultural centering to fuel questions that were situated in the context of the 13-month Lunar and Navajo Calendar, followed by embodied play to experience cyclical patterns, and ended with the formulation of conjectures via participants language. As part of the cultural centering, participants watched a video of a Navajo elder describe the 13-months of the Navajo calendar in which months named according to nature, the maturing of life, and traditions such as the shoe games. Next, an excerpt from the book to was read to connect to the Turtle's back as a Native American symbol for the 13-month and 28-day calendar system. Participants conversed about their favorite seasons and their connections to the lunar calendar. Start-up questions were provided to exemplify how one could pose questions that were situated in a culturally relevant context. For instance, "imagine it is the month of the shoe games and you observe a new moon. You begin recording the moon every 5 days, how long will it take until you see a new moon again?" The concepts of orientation and adding was defined according to the cultural practice of moving clockwise with the sun upon entering the ceremonial hogan. Facilitators also encouraged participants to develop their own questions involving cycles in nature. During the embodied play phase with hand-held and life-size materials, as displayed in Figure 2 (right), were made available. Finally, participants formulated, explored, and shared-out collaborative

conjectures that emerged from their assemblages. For example, one group situated their conjecture in the context of growing cycles.



Figure 2: Co-building icosahedron (left), modeling optimal path (center), calendar activity (right)

TEACHER FEEDBACK: TENSIONS WITH WHAT PARTICIPANTS VALUE AND STANDARDS OF DOING/LEARNING MATH

A survey with Likert and open-ended questions was administered following the two-day teacher workshop to gather feedback from participants. We denote responses RS#A# for responses to session/activity # and overall responses not linked to a specific activity as RO. A prominent theme that emerged from the responses was the tension between what tools/activities/ways of thinking participants valued during the workshop and “what we call doing/learning math”. Participants valued embodied semiotic resources such as plastic counters and life-size turtle shell remarking “big is fun!”. They were also surprised by but appreciated how gestures/physical action were important to attend to and connections to the 13-month Native American and lunar calendar. Participants also valued their agency to choose how to collect data and to define their own criteria for an optimal hiking path.

Survey Question: Can you point out some moments that advanced your perceptions of the problem situation?

RS2A2: Using the plastic counters and strings to connect and explore multiples of numbers and lengths of cycles to complete the circuit.

RS2A2: I really needed to use the large turtle shell model to understand the cycles.

RS2A1: The collecting of data was interesting. I always thought of collecting data as a chore, but with the creative freedom given it felt fun.

Participants also described value in constructing their own models versus the use of ready-made models of the mathematics

Survey Question: In your opinion, what is the added value of construction compared to working with ready-made models?

RO1: They are fun to see the progress. Though in practical implementation one must consider the time constraints, costs, and most importantly classroom management make ready-made models appealing.

RO2: Super important the use of imagination.

Even though, participants valued embodiment and the free-thinking workshop activities, they tended to regard these activities as standing in conflict with their practices.

Survey Question: In your opinion, is the activity presented in this workshop challenging what we call doing/learning math? In what way?

- RO1: Our experiences were messy, without specific outcomes defined. The thinking was open to creativity.
- RO2: As a teacher, I am not judged by how well my students learn the math, but that they can show their proficiency on a test. So, this switch in learning comes with trepidation. I can also get in trouble by deviating from the district prescribed curriculum. So, this method challenges a lot of the practical math teaching/learning I do.
- RO3: Not giving specific lesson objectives, [formal] vocabulary, or expected outcomes. It's difficult because these are things when we are observed or evaluated on are standard and without them, we are labeled as "less effective."

Discussion and Future Directions

Our findings indicated that assessment procedures in the K-12 system, requirements of standard and fixed content coverage, and emphasis on formal vocabulary/objectives up front, are in conflict with the non-deterministic, organic unscripted and incomplete nature of materialist inspired professional development (PD) activities (de Freitas & Sinclair, 2014). In linking back to the theme of futures for mathematics education, the feedback from teachers provides some initial pieces of stories that counter the "present" ways of standardization and doing mathematics. One such piece is that our socio-ecological approach to PD led to "messy" experiences "open to creativity". On the other hand, feedback such as "this method challenges a lot of the practical math teaching/learning I do" and evaluations of teaching effectiveness relies heavily on fixed mathematical content objectives, also signals barriers with "possible sustainable futures" in socio-ecological inspired pedagogies. Ongoing directions are to continue to explore the integration of ethnomathematics, new materialism, and mathematical modeling into PD educator workshops at Blanding and Uintah sites of Utah. We plan to gather additional examples from participants and feedback more systematically upon a revision and future implementations of workshop activities. We hope to share more on the benefits along with conflicts that arise when EM, EC-NM, and MM frameworks are used together to inform mathematics education PD and promote socio-ecological pedagogies. We wonder how policy makers and curriculum developers would respond to such PD and feedback from educators.

Acknowledgements

We thank the USU department of Mathematics and Statistics for their funding support. Thank you to all the volunteers and participants who brought life to this event.

References

- Abrahamson, D., & Rosenbaum, L. F. (2016). *Embodied Icosahedron Activities*. EMIC Working Group, PME-NA. <http://edrl.berkeley.edu/>
- Adiredja, A. P., & Zandieh, M. (2020). The lived experience of linear algebra: A counter-story about women of color in mathematics. *Educational Studies in Mathematics*, 104(2), 239-260.
- Aguirre, J. M., Anhalt, C. O., Cortez, R., Turner, E. E., & Simic-Muller, K. (2019). Engaging teachers in the powerful combination of mathematical modeling and social justice: The Flint water task. *Mathematics Teacher Educator*, 7(2), 7-26.
- Anhalt, C. O., & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation. *Journal of Mathematics Teacher Education*, 19(6), 523-545.
- Arzarello, F., & Robutti, O. (2008). Framing the embodied mind approach within a multimodal paradigm. In *Handbook of international research in mathematics education* (pp. 716-745). Routledge.

- Bang, M., & Vossoughi, S. (2016). Participatory design research and educational justice: Studying learning and relations within social change making. *Cognition and instruction*, 34(3), 173-193.
- Barad, K. 2007. *Meeting the Universe Halfway: Quantum Physics and the Entanglement of Matter and Meaning*. Durham, NC: Duke University Press. <https://doi.org/10.2307/j.ctv12101zq>
- Barton, B. (2008). *The language of mathematics: Telling mathematical tales*. Springer.
- Benally, J. (in press). Co-constructing Náhookos Bi'ka' constellation with STARR [Demo]. In "Inclusive happiness". *Proceedings of the annual meeting of ACM SIG Interaction Design & Children 2024*. Delft, The Netherlands.
- Blum, W., & Leiß, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, S. Khan (Eds.), *Mathematical modelling* (pp. 222–231). Elsevier.
- Borba, M. C. (1990). Ethnomathematics and education. *For the learning of mathematics*, 10(1), 39-43.
- Bundock, K., Rolf, K., Hornberger, A., & Halliday, C. (2023). Improving Access to General Education via Co-Teaching in Secondary Mathematics Classrooms: An Evaluation of Utah's Professional Development Initiative. *Rural Special Education Quarterly*, 42(2), 78-93.
- Cajete, G. (1994). *Look to the mountain: An ecology of indigenous education*. Durango, Co: Kivaki Press.
- Châtelet, G. 2009. *Figuring Space: Philosophy, Mathematics and Physics*. New York, NY: Springer.
- Coles, A., Solares-Rojas, A., & le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116, 165-183.
- Czocher, J. A., Melhuish, K., & Kandasamy, S. S. (2020). Building mathematics self-efficacy of STEM undergraduates through mathematical modelling. *International Journal of Mathematical Education in Science and Technology*, 51(6), 807-834.
- D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics*, 5(1), 44-48.
- de Freitas, E., and N. Sinclair (2014). *Mathematics and the Body: Material Entanglements in the Classroom*. Cambridge University Press. <https://doi.org/10.1017/CBO9781139600378>.
- Ellis, A. B., Ozgur, Z., Kulow, T., Dogan, M. F., & Amidon, J. (2016). An exponential growth learning trajectory: Students' emerging understanding of exponential growth through covariation. *Mathematical thinking and learning*, 18(3), 151-181.
- Garfunkel, S., & Montgomery, M. (Eds.). (2019). *GAIMME—Guidelines for Assessment & Instruction in Mathematical Modeling Education*. Society for Industrial and Applied Mathematics.
- Jung, H., & Magiera, M. T. (2023). *Connecting mathematical modeling and social justice through problem posing*. *Mathematical thinking and learning*, 25(2), 232-251.
- Kelton, M. L., & Ma, J. Y. (2018). Reconfiguring mathematical settings and activity through multi-party, whole-body collaboration. *Educational Studies in Mathematics*, 98, 177-196.
- Lakoff, G., & Núñez, R. E. (2013). The metaphorical structure of mathematics: Sketching out cognitive foundations for a mind-based mathematics. In L. D. English (Eds.), *Mathematical Reasoning* (pp. 21-85). Routledge.
- Nathan, M. J. (2021). *Foundations of embodied learning: A paradigm for education*. Routledge.
- Vaughn, M. S., & de Beer, J. (2020). Contextualising science and mathematics teacher professional development in rural areas. *Perspectives in Education*, 38(2), 213-226.

RECIPROCITY, FEEDBACK LOOPS, SYSTEM DYNAMICS AND THE EDUCATION REFORM IN MEXICO

Quitze Morales Meléndez¹, Ramiro Carrillo-Catalan²,
Marcela Santillán Nieto²

¹Universidad Pedagógica Nacional, ²CONAHCYT-Universidad Pedagógica Nacional

Inclusion, critical thinking and critical interculturality are principal articulation axes in the current education reform in Mexico aiming towards recognition and dialogues between indigenous ways of knowledge and scientific thinking, where mathematical ways of thinking are included. Project-based, problem solving and other active learning methodologies are the main teaching approaches proposed. This poses great challenges for teachers in secondary education (9th to 11th grade) who were hired for their disciplinary knowledge. In this paper we review the experiences from three cases that may shed light on how mathematics education and mathematical modeling could address this challenge: wetland restoration project in Mexico City, inductive intercultural method implementation in Oaxaca and Chiapas and mathematics teacher training programs in Universidad Pedagógica Nacional of Mexico.

MEXICAN CURRENT EDUCATION REFORM

The new curricular framework of the current education reform in Mexico focuses on human rights and critical education with the participation of indigenous and afro Mexican people, civil rights organizations, the Mexican women institute, teachers, scholars and artists among others. For the articulation of this framework four elements are proposed: an integrated curriculum, professional teachers' autonomy, the community as the integration core of educational processes and the human right to education. It recognizes teachers' knowledge developed in their praxis and calls for a collective curricular design.

The main value promoted in the so-called New Mexican School (NEM by its initials in Spanish) is human dignity and calls for responsible relationships among people, knowledge, environment, society, technology and so on, that are constructed in schools and outside of them (SEP, 2022). It recognizes the diversity of identities present in legal subjects such as children, young people, elders, women, indigenous peoples, afro Mexicans, migrants, refugees, sexual and civil organizations interested in sexual and generic diversities among others. The human right to life is extended to care, respect and protection of live organisms and natural life cycles. In this sense and taking a critical perspective, education is viewed as a key right for access to other rights. The idea of community as an integration core is founded on three main reasons: a) people from different communities with their languages, cultures, habits, identities, affections and expectations interact in school and school should not be considered as an isolated entity outside of this reality, b) *knowledges*, values and relations that are considered in the curriculum can only be truly integrated in daily life within the framework of the community where students live, including their schools c) using local reality to problematize the curriculum gives rise to the idea of a dynamically pertinent curriculum. The idea of community also introduces a historical perspective that enables questioning the transmission of knowledge and values from one generation to the next in a living dialogue.

On the other hand in the last 25 years and with the entry of Mexico to the OECD, standardized measurement of children's knowledge was started. This grew until it became annual and census-based and this led to a standardized evaluation policy for teachers. The removal of these policies began in the last six years with the curricular reform, and the president elect has declared that it will be replaced by a teacher training program. This decision might favor a deep change in school learning in search of social justice. In particular, this is an opportunity to rethink teacher training programs on mathematics education to promote new tools to model socio-ecological relationships and strengthen the use of mathematics as a tool of social justice.

The curriculum structure of NEM

The curriculum structure in NEM is guided by critical citizenship ideas. Citizens are expected to recognize and value both cultural and identity diversity. This includes the diverse ways of knowledge and knowing. In NEM, humankind is considered as part of nature and each citizen or member of a community is expected to participate in respectful ways of dialogue with a critical perspective. In this framework seven articulation axes reflecting the main aspects of citizenship in a democratic and diverse society are considered. These axes are used to connect and relate disciplines with teaching, learning and local reality. We identify three of them as main principles namely, inclusion, critical thinking and critical interculturality. The fourth, gender equality is a special case of inclusion. By taking a historical perspective, the description of the healthy life axis exposes domination within the ways in which health is conceived and accounts for alternative ways of conceiving caring. Alphabetization and artistic experiences are general strategies for developing sensitivity, recognition and valuing otherness. Through these axes, gender, social class, sexuality, ethnic origin and the ways of knowing are exposed as historical constructs used by control and domination structures. On the other hand, interculturality refers to subjects, communities and identities that by interacting, dialoguing and questioning each other, produce different realities in a framework of asymmetric relationships affecting health and the environment. Concepts such as the intersectionality of inequality and exclusion are used in this description. There is a call for ethics in dialogue and taking into account others' realities involving values such as recognition, respect and reciprocity. Contents in NEM are organized in four knowledge domains: ethics, nature and societies; humanity and community; languages, where linguistic diversity is recognized; and knowledges and scientific thinking, where mathematical contents are included. This last domain includes indigenous ways of knowing, and according to the critical interculturality approach, should address the ways in which the sciences, and mathematics, have been used for epistemic domination. Project-based learning is proposed as the main teaching approach and this is reflected in textbooks about community, school and classroom projects that offer some examples concerning waste production, water consumption, environmental issues and so on.

TRANSDISCIPLINARITY AS A GREAT CHALLENGE FOR TEACHERS

Implementing an integrated curriculum, particularly in secondary education requires collaborative efforts among teachers from diverse disciplinary backgrounds, each with distinct languages, methods, ways of knowing and values.

In the authors' experience, patterns of domination are often reproduced in contexts where teachers are more accustomed to job competition than academic collaboration and a pedagogical praxis oriented

by a previous curriculum with a strong Eurocentric approach. Even academic collaboration between mathematics teachers, mathematics education specialists, and mathematicians around teaching praxis can require years of interaction to agree on language, recognize diverse ways of knowing, and build shared knowledge.

The separation of knowledge into distinct disciplines is part of Western thought, tracing back to the philosophical ideas of Descartes. This worldview conceives human beings as separated from nature, which is then viewed as an object to be dominated and exploited (Grosfoguel, 2019). During colonial processes, the people of the Americas have been considered as part of exploitable nature, their ways of life and associated knowledge have been often invalidated, yet exploited by the dominant culture, including present day scientific and artistic communities, through processes of appropriation without proper recognition or restitution (Mgbeoji, 2006). Scientific methods are not only a tool for knowledge validation but also a tool for invalidation between different disciplines. For the purposes of this discussion the authors include mathematical proof and rigor in these methods. In the future of mathematics education as viewed through the current Mexican reform, ethical relations with communities' knowledges should be established. So, students' and communities' activities are to be discussed by them, in their own terms, and mathematical content and thinking might offer an external view of this reality whenever there is consent for such an intercultural process.

Project-based learning as a teaching approach in secondary education requires collaboration between teachers from different subject areas, and other holders of specialized theoretical or practical knowledge, who support students with the realization of the project in a given school within a given community. This calls for a transdisciplinary approach, but the tools for such a challenge still need to be developed by school communities at large.

THREE CASES RELATING MATHEMATICS EDUCATION AND SOCIO-ECOLOGICAL CONCERNS

In Mexico City, recent efforts have been made to offer textbooks under the project-based learning approach in secondary education. One example of these is the study of the very popular axolotl species (SECTI, 2021). This is an endangered endemic species from Mexico City's Xochimilco lake zone drawing worldwide attention for some time now. Related to the concept of endemic species, in this school project there are two key questions, namely: why is the species endangered? and why is it important to preserve it from extinction? These questions give reasons to question and address the effects of our current ways of living in urban areas: water pollution, lost habitats, introduction of foreign species for commercial purposes, species extraction and climate change. Related key mathematical contents are species' population, survival rate, population density, rate of change and dispersion. All of the above provide elements to consider mathematical modeling of the situation. Nevertheless, in the textbook a mathematical model is conceived only as a set of equations made for prediction purposes. Methods for observation and finding causal relationships are also considered.

Wetland restoration project and rediscovery of chinampas

While going back to the history of the *ambystoma mexicanum*, known colloquially as axolotl, one finds out that this is a flagship species for a big research and restoration project that started from an ecological perspective and was then addressed with a socio-ecological approach. Under this research approach, the feedback loop idea is a key concept for the phenomena analyzed. Perturbation loop

representations describe patterns in the dynamics of socio-ecological systems that affect equilibrium and that are related to usual urban problems, such as poor water quality and biodiversity loss. Restoration loops describe dynamics tending to recover socio-ecological equilibrium. A main finding in this research project was the fact that the missing link in the restoration loop was related to local peoples' way of life. This discovery led to the rediscovery and acknowledgment of traditional ways of life developed by local cultures before Spanish colonization, known as culture of the chinampas. Although this way of life can preserve the socio-ecological local equilibrium, it might be in conflict with dominating market culture in urban areas (Zambrano et al., 2020).

Reciprocity principle and the Intercultural Inductive Method

A research group working with teachers in indigenous communities in Oaxaca and Chiapas, Mexico, and in other Central and South American countries have identified common organizing life principles of these communities (Gasché, 2005). They have horizontal relations between household units and between functional adults from these units. They have various abilities, and autonomy to decide what kind of labor they will do on a daily basis. On the other hand, specialization does exist and is oriented towards talent, but it does not produce hierarchies and does not exclude from knowledge related to other activities.

The *solidarity* principle appears in three main ways in human relations: collaborative labor (called *tequio* in Mexico or *minga* in Peru), sharing goods (specially food) and joint festivities. Individuals gain leadership through many positions of service in the citizenship structure of the community, which also implies knowledge about all the activities in the community.

For people in these communities, “society” encompasses society and nature, and there is awareness of the effects of human activities in the environment. Individuals with more knowledge and sensibility to detect loss of equilibrium gain authority and leadership through others' esteem. They guide in restoration work for the recovery of equilibrium. One can see a common principle of *reciprocity* that guides how indigenous people relate to other beings (neither necessarily human nor living), that requires communication, sensibility and/or awareness of their *needs*.

In the educational realm, the research group developed a method for addressing communities' knowledges with respect and acknowledgment. In the so-called Intercultural Inductive Method (IIM) teachers, who are typically public servants and who do not initially belong to these communities, position themselves as learners of local ways of living and thinking (Nigh & Bertely, 2018). In this process, teachers explain local knowledges in their own terms and using their own experience. This experience is often explained in terms of Western concepts, ways of thinking and even values. But the positioning as learners allows them to appreciate local culture and even to be allowed to participate. When mathematical content and thinking appear in this approach, they might be questioned and problematized, which has great consequences for mathematics education.

An example of these intercultural processes, which is related to the prior discussion on the wetland restoration project, is the *milpa educativa* (educational maizefield) project (Sartorello, 2021). The *milpa* can be interpreted in Western terms as a way of producing crops for human benefits, but it is also described as a by-design micro agro-ecosystem in which there are three main crops: maize (*Zea mays L.*), beans (*Phaseolus spp.*), and squashes (*Cucurbita spp.*), among other species that differ from

region to region (Salazar-Barrientos et al., 2016). This has been often analyzed from a nutritional point of view (Zizumbo-Villarreal et al., 2012).

From an agroecological perspective, the function of squashes is similar to that of the pesticide glyphosate, preventing weed growth, but without the toxicity or adverse effects on biodiversity or gut microbiota. In contrast, squashes are a rich source of fiber in the human diet and maintain moisture in the soil. The value of this knowledges plays a key role in the feedback loop of wetland restoration analyzed in the first case.

Mathematics education and mathematics teachers' training programs in Universidad Pedagógica Nacional in Oaxaca, México

The authors belong to a research and mathematics teacher training group in Oaxaca, Mexico, founded by the indigenous Ayüuk professor Isaías Aldaz soon after the foundation of the Universidad Pedagógica Nacional in the late 70's. In those early years, the research aims were very similar to that of the ethnomathematics research program: recognizing knowledges from indigenous communities related to mathematics (de Bengoechea Olguín et al., 2016). The oral number systems of various indigenous communities are the most visible and actively used knowledge of these communities and have therefore been one of the main research topics (de Bengoechea Olguín et al., 2020). Nowadays, the research group also has other interests in mathematics education and in pure and applied mathematics.

In the teacher training program for primary and preschool indigenous school education, teachers analyze human activities using Bishop's universal activities (Bishop, 1991), and relate them with mathematics content in the curriculum. Secondary education teacher training program has been dedicated to developing mathematical content knowledge for teaching (MKT), to identifying situations where contents may arise, and to studying specific approaches related to task design as implementation (Kieran et al., 2015).

In indigenous communities in the state of Oaxaca project-based learning is actively practiced and promoted by the local section of the teachers' union. Their experience was considered in current education reform described above and generalized and adapted for secondary education. Teachers' knowledge in Oaxaca's primary and preschool education has been transdisciplinary by nature, prior to the current reform, because they teach contents from all disciplines considered in the curriculum and often address project-related subjects from the point of view of various disciplines in order to relate them to content considered in the curriculum.

A way of developing project-based learning in secondary education would be to engage in collective project design with the leadership of primary and preschool teachers with such experience. The collective design would also require teachers in secondary education, who were hired for their disciplinary knowledge, to engage in horizontal dialogues. Collaborative project design efforts have been made in the past in some school systems in Oaxaca, in indigenous communities, but eventually mathematics and natural science teachers had separated from project design and had been mostly dedicated to preparing students for undergraduate education, which is still separated by disciplines.

GRAPHS AS MATHEMATICS MODELS FOR THE SOCIO-ECOLOGICAL & MATHEMATICS EDUCATION

Mathematical thinking relies on means for representing a perceived reality. These models, or representations, grow into representation systems that can be mathematized to become mathematical models. The same representation system may be mathematized into different mathematical models. Mathematicians often have a more restrictive view of mathematical modeling, such as the one mentioned at the beginning of the previous section, or the one from formal model theory. Every representation or model is an abstraction and should not be confused with reality. Instead, it allows us to focus on some aspects of this reality. The models most often used for addressing socio-ecological problems come from graph theory (Gignoux et al., 2017), as they allow us to represent complex systems of relations between subjects (people, animals, cells, proteins, data and so on). The three cases discussed above pose questions about relationships *among* people (the social systems), *with* nature (the socio-ecological systems) and *within* nature (the ecological systems).

Graphs for representing extractive and reciprocity principles in the axolotl project

A common principle that can be identified in Western thinking and ways of living is the extractive principle: dominant subjects are driven by individual needs and establish extractive relationships to satisfy them. Thus, other subjects are, in this perspective, either a means to extraction or a resource source. As opposed to this, in the aforementioned *reciprocity principle*: subjects are aware of their own needs, and become aware of the needs of others through sensible analysis or other means, such as horizontal communication, seeking equilibrium. Such principles can also be found in feminist or gender/sexual diversity collectives as various kinds of relational ethics. This can be represented by graphs with directed arrows as in the examples below (Figure 1&2).

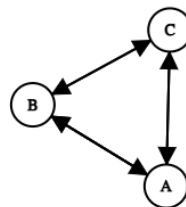
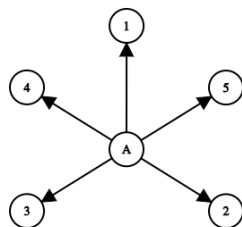


Figure 1: Extractive subject Figure 2: Reciprocal relations

A perturbation loop, such as the one mentioned before is extraction driven. In a graph representation, there would be a dominant subject constructing the urbanization (driven by a need for living space) and extracting water (for health needs), but passing problems to local subjects, such as polluted water (affecting their health needs), resulting in bad crops not satisfying market conditions and leading to lower income and migration to other urban spaces (affecting their needs, such as living space). This does not consider how local species are affected, it only accounts for human needs.

On the other hand, in the restoration loop there are sensible subjects aware of the effects of polluting water; these subjects use waterways adjacent to their milpas as axolotl refuges, care for water quality and produce healthy crops that are appreciated in the market. Subjects represented in a graph would include people, bodies of water, land and axolotls, among others.

Relationships in these graphs might include the following: people protect land from erosion and water from pollution, and need water and land for crop production and living space; bodies of water give

shelter to axolotls, and need to be kept, so extraction should be regulated and water returned should be purified; land yields crops and filters water, and needs vegetation and humidity; axolotls are highly dependent on the care of people, but their health is a good indicator of water quality.

Graphs for representing teacher communities

Here are some questions that can be addressed using graphs: what do relationships among teachers look like in the present educational system in Mexico? What are the dynamics opposing collaboration in school project design? What are the feedback loops for restoring? What should collaborative teacher communities look like? When one is constructing communities for the professional development of teachers one has to address such questions and develop means for decision making.

Collaborative mathematics teachers' communities working under the Japanese Lesson Study (JLS) system are well known and give a starting point (Isoda et al. (Eds), 2007). One key aspect shared by JLS, teacher communities in China described in MKT research (Ma, 2010) and the experience of our colleague Juan Martínez, is the existence of a stable school collective where knowledge is developed and shared, and where less experienced teachers learn from others and gain experience over time. In this way, if an experienced teacher retires, experience and knowledge developed by them has a good chance to be preserved for further questioning and development.

As opposed to this, in typical non-collaborative teacher environments in Mexico, individual teachers receive support by some specialists once or twice a year. They might develop their own knowledge and improve their teaching, but this experience is likely to be lost after retirement.

In order to create collectives meeting the needs of current education reform in secondary education, a missing link between teachers with specialization in various disciplines, is a teacher with experience in dealing with a subject from various perspectives as the teachers with experience in project-based learning. Such teachers would lead the process of school project design. Teachers with specialized knowledge would then have to identify pertinent content and aid other colleagues in understanding the relevance of such content. This would lead to shared language and knowledge in the collective. Some important issues here are perceived hierarchies between knowledges, differences in language, ways of thinking, methods and even values.

Acknowledgements

This work was supported by IIXM CONAHCYT grants 1522 & 3786. The authors thank Beatriz Luna and Marcelino Ramírez for fruitful discussions on mathematical modeling using graph theory.

References

- Bishop, A. (1991). *Mathematical enculturation: A cultural perspective on mathematics education* (Vol. 6). Springer Science & Business Media.
- de Bengoechea Olguín, N., González Agustín, T., Martínez Castañeda, J., & Ramírez Santiago, M. (2016). Avatares de un grupo de largo aliento en educación matemática en Oaxaca. *Entre maestr@s*, 16(57), 112-117.
- de Bengoechea Olguín, N., Ignacio-Velasco, Y., González Agustín, T., & Matus-García, A. (2020). Análisis aritmético de algunas Numeraciones Zapotecanas. *Journal of Mathematics and Culture*, 14(1), 114-137.
- Gasché, J. (2005). Hacia una propuesta curricular intercultural en un mundo global. *Revista Interamericana de Educación de Adultos*, 1(27), 177-200.

- Gignoux, J., Chérel, G., Davies, I. D., Flint, S. R., & Lateltin, E. (2017). Emergence and complex systems: The contribution of dynamic graph theory. *Ecological Complexity*, 31, 34-49.
- Grosfoguel, R. (2019). Epistemic Extractivism: A Dialogue with Alberto Acosta, Leanne Betasamosake Simpson, and Silvia Rivera Cusicanqui. En *Knowledges born in the struggle: Constructing the Epistemologies of the Global South* (pp. 203-218). Routledge.
- Isoda et al. (Eds). (2007). *Japanese Lesson Study in mathematics. Its impact, diversity and potential for education improvement*. (M. Isoda, M. Stephens, Y. Ohara, & T. Miyakawa, Eds.). World Scientific.
- Kieran, C., Doorman, M., Burkhardt, H., Ding, L., Jones, K., Koichu, B., Morselli, F., Simon, M., Stephan, M., Swan, M., Dooren, W. V., Clarke, D., Dolev, L., Kuntze, S., Pepin, B., Quaresma, M., Ruiz-higueras, L., Vamvakoussi, X., Verschaffel, L., ... Zaslavsky, O. (2015). Chapter 2: Frameworks and Principles for Task Design. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education an ICMI study 22. New ICMI Study Series* (pp. 19-81). Springer.
- Ma, L. (2010). Knowing and teaching elementary mathematics. Teachers' Understanding of Fundamental Mathematics in China and the United States. *American Educator. Taylor & Francis Group.*, 5, 169.
- Mgbeoji, I. (2006). *Syracuse Science and Technology Law Reporter. Global Biopiracy: Patents, Plants, And Indigenous Knowledge*. University of British Columbia Press.
- Nigh, R., & Bertely, M. (2018). Indigenous Knowledge and Education in Chiapas, Mexico: An Intercultural Method. *Diálogos Sobre Educación*, 9(16).
- Salazar-Barrientos, L. de L., Magaña-Magaña, M. Á., Aguilar-Jiménez, A. N., & Ricalde-Pérez, F. (2016). Factores socioeconómicos asociados al aprovechamiento de la agrobiodiversidad de la milpa en Yucatán. Socioeconomic factors associated to agrobiodiversity of the milpa in Yucatan. *Ecosistemas y recursos agropecuarios*, 9(391-400).
- Sartorello, S. (2021). Entramados socionaturales comunitarios para el buen vivir. *Revista Mexicana de Investigación Educativa (RMIE)*, 26(88), 283-309.
- SECTI. (2021). *¿Por qué hay pérdida de especies en la Ciudad de México? Proyecto 3*. Secretaria de Educación, Ciencia, Tecnología e Innovación. Gobierno de la Ciudad de México.
- SEP. (2022). *Programa de estudios. Nueva Escuela Mexicana*. Secretaria de Educación Pública.
- Zambrano, L., Rivas, M. I., Uriel-Sumano, C., Rojas-Villaseñor, R., Rubio, M., Mena, H., Vázquez-Mendoza, D. L., & Tovar-Garza, A. (2020). Adapting Wetland Restoration Practices in Urban Areas: Perspectives from Xochimilco in Mexico City. *Ecological Restoration*, 38(2), 114-123.
- Zizumbo-Villarreal, D., Flores-Silva, A., & Colunga-García Marín, P. (2012). The Archaic Diet in Mesoamerica: Incentive for Milpa Development and Species Domestication. *Economic Botany*, 66(4), 328-343.

EMPIRICAL DIFFERENTIATION OF STUDENT COMPETENCIES IN ESD

Hans-Stefan Siller¹, Katrin Vorhölter², Janina Just¹, Anna Orschulik³, Catharina Zieriacks³

¹University of Würzburg,² Technical University of Braunschweig,³ University of Hamburg

In Germany, teachers in mathematics lessons are required to enable students to recognize and understand local and global economic, social and environmental challenges, to evaluate possible solutions and to develop the ability to distinguish between options for action at local and global level as part of Education for Sustainable Development (ESD). The related competencies consist of three categories: recognizing, evaluating, and acting, which are further divided into 11 sub-categories. In addition to these normative, interdisciplinary competencies, it was theoretically derived for the individual subjects how these competencies can be demonstrated in the respective subject lessons. However, there is a lack of empirical evidence to confirm the extent to which these competencies can be applied in general and in mathematics lessons specifically, or whether they need to be further expanded through inductive approaches, which is therefore the aim of this paper.

INTRODUCTION

In response to global economic, social, and ecological changes and challenges, the United Nations has agreed to 17 Sustainable Development Goals (SDGs) and committed to their implementation. These goals are aimed at maintaining, developing, and restoring livable conditions for all people. This means we face local challenges in a global context, many of which can only be solved through global cooperation. It is therefore essential for students to be enabled to recognize and understand these challenges, evaluate potential solutions, and develop the ability to distinguish between options for action at both local and global levels. In subject lessons, students should therefore be given opportunities to learn how different subject disciplines can contribute to finding solutions for economic, social, and ecological challenges, both locally and globally (Coles et al., 2024). In this article we show which competencies students show in relation to ESD when they are confronted with appropriate problem situations in mathematics education.

Education for sustainable development and the socio-ecological

In Germany, the implementation of corresponding content in subject lessons is legitimized by the KMK (2024) recommendation on ESD in schools. The KMK defined eleven normative competencies that apply to all school subjects and can be divided into three domains:

Recognizing involves the targeted knowledge acquisition and is further subdivided into information gathering and processing, the recognizing of diversity, analysis of global change, and differentiating of levels of action.

Evaluating includes critical reflection and recognition of different values. It comprises the ability to change perspectives and empathize, form a critical opinion, and assess of development measures.

Acting aims to foster both the capacity and the willingness to act or communicate options for action. It encompasses solidarity and shared responsibility, understanding and resolving conflicts, the ability to act within the context of global change, participation, and co-determination.

It should be noted that these three domains represent desirable behaviors that should be acquired in German classrooms. They are complementary components that do not necessarily build on one another. The descriptions of the three competence areas illustrate the connection between what is understood as ESD in this framework and the aspects of socio-ecological practice (Coles et al., 2024): students should be enabled to perceive their surrounding environment, to see in connection between local and global as well as the entanglement between socio and ecological issues. They should further learn to evaluate different values and weigh them up against each other, always supported by perspective-taking and empathy. Finally, they should develop options for action to contribute to a sustainable future for the world. To achieve this, lessons should present problems that highlight the necessity of SD and empower students to develop their own attitudes without overwhelming or patronizing them (UNESCO & MGIEP, 2017).

ESD competencies in and for mathematics education

To implement these competencies in subject-specific lessons, they were expanded upon and outlined in subject-specific recommendations for both lower secondary level (Reiss et al., 2016) and upper secondary level (Siller et al., 2025) in Germany. Thereby the authors demonstrate the direct connection to mathematic knowledge and skills within the national curriculum. Consequently, the derivation of subject-specific sub-competencies in ESD is theoretically grounded in prior requirements and anchored in the curriculum.

In order for students to acquire relevant subject-specific ESD competencies in the classroom, it is necessary to provide them with tasks that foster the development of these competencies. We illustrate this demand by two tasks: one that focuses on social issues (poverty), and another that centers around ecological concerns (climate change). However, both aspects are addressed to some extent in each task. The goal of the first task is to highlight the complexity underlying seemingly simple but significant questions such as "Who is actually considered poor?" or "What factors interrelate with poverty?". In the first part, criteria for assessing poverty are developed and discussed, drawing for example on mathematical models used by organizations like the UN, to measure poverty. The aim is to critically examine the advantages, quality and limitations of these models to establish a connection with socio-cultural diversity. In the second part, the mathematical concepts of causality and correlation are employed to explore the consequences and root causes of social-political aspects in relation to poverty, such as health, are discussed. The second task focuses on the CO₂-Budget in relation to climate action. Despite overwhelming scientific consensus on the existence and human causes of climate change (CC), there is still widespread denial of this issue in societies. To enable students to form fact-based opinions and highlight the need for action, they first analyze graphical representations of greenhouse gas emissions and temperature anomalies. Students are then encouraged to engage in critical discussions about whether this data alone can prove anthropogenic climate change. However, since the data only establishes correlation and not causality, additional information is provided to the students to serve as support for causal relationship of climate change and greenhouse gas emissions, and therefore anthropogenic climate change. Using this knowledge, students should develop a model to determine the remaining CO₂ budget necessary to meet the 1.5°C limit. The scientific original datasets from are made available to students for this purpose.

Both tasks illustrate how to challenge students to critically evaluate the proposed solutions, learn to communicate their own solutions based on facts, and thereby recognize the importance of

mathematics as a tool for solving problems. Furthermore, it demonstrates how students can be challenged to identify appropriate mathematical methods solve the problems themselves. This enables them to recognize the potential for obtaining and processing such data, as well as to assess and evaluate different mathematical models as Skovsmose (1994) has already requested in the context of critical mathematics education.

RESEARCH QUESTION

There has been criticism of the versatility and lack of clarity in defining and operationalizing ESD competencies, as well as limited empirical evidence (Redman et al., 2020; Costa et al., 2024). As presented, in Germany core competencies were formulated, which consist of three areas and are divided into 11 sub-competencies. In addition, subject-specific derivations were made that are theoretically sound and curricular linked, but not empirically validated. The aim of this study is therefore to find answers to the following questions:

To what extent can the normative competencies on which all school subjects are based, or the theoretically derived and curriculum-related subject-specific competencies be empirically confirmed?

To what extent can and must they be extended inductively?

By answering these questions, we show which competencies students show in relation to ESD when they are confronted with appropriate problem situations.

METHOD

The study presented is based on a data set consisting of 6x4 videotaped groups, of two to four students, working on assignments completed by students in grades 9-11. These assignments encompass six different reality-based tasks in an ESD context. The tasks include various contexts and objectives, with video length varying between 45 and 180 minutes. This article focuses on two tasks.

The videos were analyzed using thematic qualitative content analysis (Kuckartz & Rädiker, 2023). The coding guide was created by first using the normative core competencies and then expanding them inductively. Furthermore, the coding of the student activities was enhanced by differentiating between activities that involve the "application of mathematics" and activities that do not include it. In this context, "application of mathematics" referred to activities and conversations that incorporated mathematical process-related activities in the sense of the process-related competencies. No distinction was made between different levels or activities, such as mathematical modelling or mathematical communication. Merely using of mathematical terms, that are also used in everyday language, without identifying mathematical activities, was not sufficient to categorize this activity as an "application of mathematics". When coding, no evaluation of student processing and conversations, such as the accuracy of content, was conducted.

The videos were each coded consensually by two people (Kuckartz & Rädiker, 2023). In case of discrepancies, agreement was reached based on the coding guide. The coding system, which represents the outcome of this study, will be presented in the following section.

RESULTS

In the analysis of the 24 videos of the 6 different objectives, 12 distinct sub-categories were identified and reconstructed. These sub-categories can be classified into to the normative domains of

recognizing, evaluating, and acting. They are presented below and supported by examples from the tasks, with a focus on the area of recognizing. We highlight the categories that were empirically verified in their original description in the following figures in *italic* and **bold**.

Recognizing

Four sub-categories were identified for the area of recognizing (see Figure 1). The first concern is the gathering of information, which can be further subdivided: extracting information from the task-related material provided, researching data primarily using the internet, and making assumptions that are necessary for their work. Scenes categorized where students engaged in these activities, either by referencing mathematical concepts and procedures or without any involvement of mathematics.

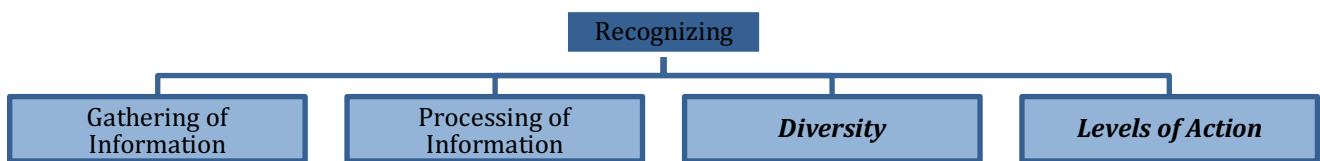


Figure 1: Reconstructed categories of the section of Recognizing

The following conversation is an example for *gathering of information* of provided material. The students describe the graphical representation of temperature anomalies:

S1: In graph 1, you can see on the x-axis [Side conversations] In graph 1 you can see the global temperature rise in degrees Celsius

S2: Progression

S1: No, the temperature rise.

S2: Huh? Why increase?

S1: Increase [graph labeling]

S2: Yes, of course, but [...]

S1: Like increase, that means it got more.

S2: Over the years

S1: Then let's see what the lowest point is.

S2: We have to read that off [...not understandable]

S1: In 1905, it was even colder.

S1: Yes, that's right, look! -0.5 here

S1 and S2 look at the graph

S1: Ah yes, there's an average line at 0, that's where it started. That's the normal temperature and the warming is upwards, and the cooling is downwards

The second category concerns information processing. This involves students to *recognize their own ability and need to act* and to technically analyze the information. In the following there are two examples for the second sub-category provided. In the first one students analyze available information without application of mathematics:

S: Poverty rate affects life expectancy because you don't have money for a good house.

In the second example students work with an Excel table provided and analyze the effects of the monthly income level and limitations of the model to measure poverty application of mathematics.

S: But in itself, just because one person earns more does not mean that there is more money in circulation, i.e. it cannot be the case that one person earns more and the other earns less.

The third category relates to students recognizing the diversity in our world. On one hand, this can also be done without regress to mathematics.

S: It's not rich - but look how it is elsewhere in Namibia.

However, communication in this category can also be based on mathematical concepts:

S: What effect does income have on the models? Especially with the median, because there or with the mean values it is shifted, so with model 1 it pushes up further and further, so despite a high standard, you are then poor.

The fourth category involves recognizing different levels of action, as a student works out in the task:

S: People also have to fight their own way through. [...] If you really want something, you get something, but the risk is still higher.

Evaluating

Five sub-categories were reconstructed for the area of evaluation (see Fig 2). The first sub-category, change of perspective and empathy includes places where students become aware of, recognize and reflect upon their own and others' value orientations and how they impact their way of life.

Students talk about what it would be like to live below the poverty line:

S1: That's nothing. The maximum income you can have to still be considered poor - €59.72 a month

S2: That's nothing!

S1: That's nothing, you can't afford anything with that in Germany, especially in Germany.

S2: You can eat in Germany. You get food.

S3: Yes, but how much? Not for the whole month.

S1: And you have nothing for the apartment

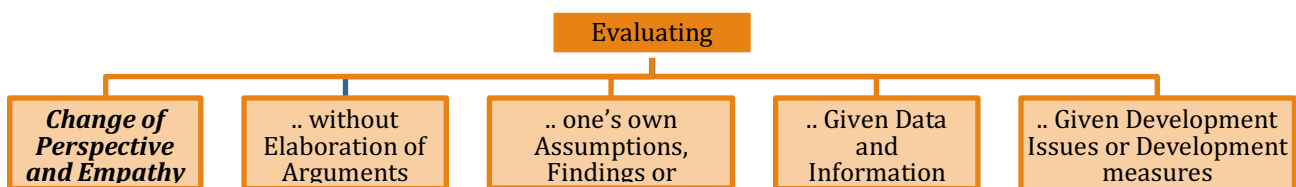


Figure 2: Reconstructed categories of the section of Evaluating

The second sub-category, *evaluating without elaboration of arguments*, refers to instances where students express their opinion without further justification. This distinguishes the sub-category from the following three categories that are differentiated regarding the varying aspects of evaluation. Therefore, it was distinguished whether the students evaluate (partial) aspects of *one's own assumptions, findings, or models* or assessment of *given data and information* and the measures derived as well as *given development issues or development measures*.

The examining or verifying of the students own assumptions, findings or models can be initiated by a teacher (L) or by the group alone. In the following examples the students reflect their own results regarding the relation of greenhouse gas emissions and temperature rise as a part of an intervention:

S3: Causality - Yes

L: Why?

S3: Well, because suddenly, we googled it and there was an excerpt about cars [L: Okay] Well, you can see that the curve roughly rises from 1930 onwards

S2: That's when the first cars were already on the road

S3: And there's a huge increase from 1970 onwards and more cars were being driven then, so a few more people could afford cars

S2: And I think lignite-fired power plants came along too

L: So now you're looking for reasons to explain this increase, right?

S2: Yes, so it's actually connected, the whole thing promotes the rise in temperature, because they both coincide.

L: Hm, yes. Read through the task again. Can the statement be derived from the data, your data are the two graphs here.

S3: Oh, then not.

The following discussion about the three models provided illustrates the final sub-category within the context of poverty:

S1: I just think that with this model, if 10% are poor, then 10% are always poor. So you can, I don't know, even if they might not live so badly and can afford a lot. [S2: hmm (agreeing)]

S1: I don't know, if they can go away at Christmas and buy birthday presents, they're still classed as poor, even though they might earn €1500 net. [S2: Yes; S1: So]

S2: Yes, it's just not necessarily related to the situation.

S1: Yes, and in my opinion the first model is better.

S2: Better and more related to people

S1: And realistic

S2: Exactly and also probably looked at a bit, um, do they have the standard you want?

S1: that you want, that you need to live and in your case it's simply based on figures, so the absolute figures are based on figures that are perhaps not realistic in every country. Because inflation in some countries, you can earn very little and still live well. [S2: Yes]

Acting

In the domain acting three categories were reconstructed: personal solidarity and joint responsibility, communication and conflict resolution, and participation and co-creation.

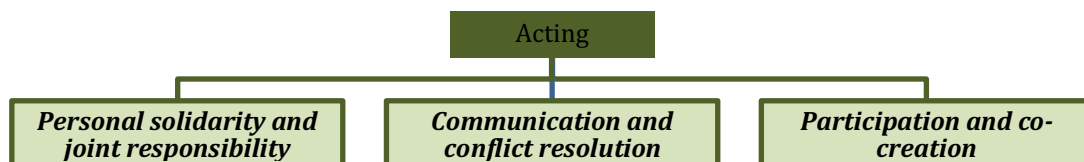


Figure 3: Reconstructed categories of the section of Acting

The first sub-category for example, involves the responsibility to verify existing statements and the realisation of the need to act. This is illustrated in the following conversation in the context poverty:

S1: So, everyone gets more money

S2: No, only the poor get more citizen's income. [...]

S2: In any case, we have to change something here.

The second emphasizes mathematics as a neutral language for presenting factual information without emotions. This is illustrated in the following conversation where students talk about reasons why mathematical models are useful:

S3: Yes, well - you have some kind of guideline to define poverty - and probably also how you want to define it for yourself [...]

S2: What else?

S1: Benefits - so in itself it's an illustration, so for the political and social scientists

The third sub-category underscores the need to establish a foundation for decision making and taking action. In the context of mathematics lessons it is particularly important and beneficial for students to actively participate in these activities, which are based on mathematical procedures and concepts, but can also and should not be limited to mathematics, as the following conversation where students discuss measures to fight poverty illustrates:

S3: The point is simply that laws need to be expanded [S2: hmm]

S3: Security must be provided and people in developing countries must be included in the development process.

S2: And above all, people should no longer be - it's almost racist, the way some people think about it. So the image of people in poorer countries/classes has to change. [S3: Yes.]

COMPARISON OF ORIGINAL AND EMPIRICAL RECONSTRUCTED CATEGORIES

When comparing the empirically reconstructed categories with the normatively established and theoretically derived categories, six categories highlighted in the figures above have been empirically verified. However, some categories needed reconstruction due to a lack of empirical differentiation, being too broad, or diminishing important information, particularly in the realm of mathematics. Additionally, certain categories were further subdivided. In the domain of recognizing, the category gathering information and processing information was reconstructed as two distinguishable categories of the same name. They were subdivided to highlight approaches to information search and processing. In this regard, the category of the ability to act in response to global change was partially incorporated into the information processing category. In the domain of evaluation, critical reflection, opinion, and assessment of development measures could not be empirically differentiated. Consequently, they were initially summarized and then differentiated inductively based on evaluated aspects, as well as rating without justification. In the third domain, acting, three categories have been empirically verified. The aforementioned category of the ability to act in global change was partially assigned to information processing in the domain of recognizing, but it was also partially assigned to the category of using mathematical concepts to verify existing statements according to subject-specific differentiations

OUTLOOK

As shown, the empirical analysis of 24 work processes for six different tasks confirmed some categories and differentiated others. The analysis showed that ESD competencies can be initiated and realized in subject lessons. At the same time, it was shown that the competencies were not equally developed in all tasks and in all groups of students. The next step is therefore to analyze the extent to which specific contexts and tasks as well as support by the teacher lead to students being encouraged to apply certain ESD competencies. The study created an empirical basis for further research, for

example for the development of test instruments. In addition, the results can contribute to the creation of learning environments and the development of special teaching training programs.

References

- Coles, A., Solares-Rojas, A., & Le Roux, K. (2024). Socio-ecological gestures of mathematics education. *Educational Studies in Mathematics*, 116(2), 165–183. <https://doi.org/10.1007/s10649-024-10318-4>
- Costa, J., Alscher, P. & Thums, K. Global competences and education for sustainable development. A bibliometric analysis to situate the OECD global competences in the scientific discourse. *Z Erziehungswiss* (2024). <https://doi.org/10.1007/s11618-024-01220-z>
- KMK (2024). *Empfehlung der Ständigen Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland (KMK) zur „Bildung für nachhaltige Entwicklung in der Schule*. [Recommendation of the Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of Germany (KMK) on "Education for sustainable development at school"] https://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/2024/2024_06_13-BNE-Empfehlung.pdf
- Kuckartz, U. & Rädiker, S. (2023). *Qualitative content analysis: Methods, practice and software*. SAGE.
- Reiss, K., Ufer, S., Ulm, V. & Wienholz, G. (2016). Mathematik – fachdidaktischer Teil [Mathematics - subject didactic part]. In KMK / BMZ (Eds.), *Orientierungsrahmen für den Lernbereich globale Entwicklung im Rahmen einer Bildung für nachhaltige Entwicklung* [Orientation framework for global development in the context of education for sustainable development] (pp. 300-308). Cornelsen.
- Redman, A., Wiek, A. & Barth, M. (2020). Current practice of assessing students' sustainability competencies: a review of tools. *Sustainability Science*, 16(1), 117-135. <https://doi.org/10.1007/s11625-020-00855-1>
- Skovsmose, O. (1994). Towards a Critical Mathematics Education. *Educational Studies in Mathematics*, 27(1), 35–57. <https://www.jstor.org/stable/3482665>
- Siller, H.-S., Vorhölter, K., Oldenburg, R., Schneider, K., Wagener, M. & Warmeling, A. (2025). Mathematik [Mathematics]. In KMK, BMZ & Engagement Global (eds.), *Orientierungsrahmen für den Lernbereich Globale Entwicklung im Rahmen einer Bildung für nachhaltige Entwicklung für die Gymnasiale Oberstufe (OR GOS)*. [Orientation framework for global development education in the context of education for sustainable development for upper level]
- UNESCO & MGIEP. (2017). *Textbooks for sustainable development*. UNESCO.

A SOCIO-ECOLOGICAL STEM TASK: RE-ATTACHING PRESCHOOLERS TO NATURE

Elizabeth Suazo-Flores, Christine McGrail, Amber Adgerson

University of North Dakota

Schools have few spaces to have experiences with the living world and opportunities to represent them using STEM practices. We are an interdisciplinary group of teacher educators that partnered with preschool educators to plan and teach socio-ecological STEM tasks. Socio-ecological STEM tasks engage learners in 1) attending to the living world (e.g., earthworms, soil, plants, birds) and 2) using STEM practices, such as visuospatial reasoning to represent their experiences with the living world. In this proposal, we report on one task we created and piloted thanks to our collaboration. Preliminary analysis suggests that the co-creation of tasks and preschoolers' experiences with them cultivated a learning environment for all. We claim that envisioning new futures for mathematics and STEM education requires attention to the living world and relationships with different stakeholders.

MATHEMATICS EDUCATION AND THE SOCIO-ECOLOGICAL

The effects of climate change are being experienced globally. As a response, researchers are calling to shift from individualistic, standards-based, perspectives on teaching and learning to relationality, environmental consciousness, and attachment to nature within socio-ecological systems (e.g., Boylan & Coles 2017; Vossoughi et al., 2023). There are calls to stop seeing the living world (Boylan & Coles, 2017) as a “fixed background” (e.g., Coles, 2023, p. 28). This shift from individualistic and human-centered perspectives challenges educators to prepare school learners to consider the living world as essential for its existence and human beings' existence (e.g., Khan, 2020). We drew from socio-ecological approaches to teaching and learning to de-center the individual to center on relationships; relationships among people, and between people and the living world (Vossoughi et al., 2023).

Locally, countries such as the U.K. and U.S. are being impacted by the increased use of social media and technological devices. Covid-19 increased the use of technological devices and made families skeptical about letting their children get dirty or play with others (Haidt, 2024). Thus, the increased use of technological devices and families' reticent attitudes towards being outside and playing with others is de-attaching children to nature as they are experiencing less embodied social interactions.

Researchers in mathematics education are calling to replace mathematics in STEM with an M referring to multispecies flourishing (Khan, 2020; Khan & Armstrong, 2019). In this study, we took up such a challenge and started envisioning a new future for STEM education by thinking of curriculum development in relationship with educators and learners and with a focus on considering the living world (i.e., earthworms, soil, plants, trees) as needed for itself and human existence. Our study also answers calls to report on how interdisciplinary collaborations happen (Bakker et al., 2021; Cai et al., 2020) as we considered it necessary to collaborate with educators and colleagues from other disciplines to create and teach a curriculum embedded in socio-ecological perspectives (Vossoughi et al., 2023). This research is part of a larger project documenting a curriculum collaboration between

a childcare center and a group of STEM teacher educators at the University of North Dakota in the United States. We report on the creation and piloting of one activity we called, *Is the Soil Alive?*

THE CHILDCARE CENTER

The University Children’s Learning Center (UCLC) is located at the border of two states in the United States, North Dakota, and Minnesota, these states are home to diverse native communities, which inspired us to create a STEM curriculum embracing indigenous ways of knowing (Gutiérrez, 2022). UCLC provides care and education for children from 18 months to five and is situated at the University of North Dakota campus, yet its main economic support comes from families’ tuition. There are three preschool classrooms at UCLC, with children from 3 to 5 years old, in this study, we worked with one of them. UCLC has a Nature Explore certification that positions the childcare playground as an outdoor classroom. This outdoor classroom continues to provide a wide variety of learning experiences for children, teachers, and preservice teachers. When you enter the UCLC Outdoor Classroom, you find a gathering area where children and teachers can talk about what they want to do. Children can choose to go to the art area and paint on the large double-sided easel, or they can draw a picture while standing at the table, which just happens to be a very large tree stump! Or they may choose to go to the sand area to dig. Other options await children on the playground, and they include a building area, with tree stumps that are meant to act as tables for their building creations. To the right of this area is the climbing structure from before and also a new “racetrack” where children ride their wheeled toys. In the center of the racetrack is a log for climbing on, over and walking along. Young budding musicians can play the Marimba, located right next to the stage in the Music Area. Children can use their voices, the cactus shakers, and drums while they move their bodies and enjoy the music they create. The outside classroom also has logs that act as a boundary around the messy/loose parts area. This area has an abundance of tree cookies (some purchased ones along with many donated ones), large wooden blocks and smaller building materials. There is also a grassy hill for lots of movement. It includes a couple of logs for walking along and stumps for climbing over. Because of the harsh North Dakota winters, the warmer months are when indoor learning extends to the outdoors. During those warmer months, there is a lot of focus on the gardening spaces in the outdoor classroom.

THE STEM TEACHER EDUCATORS

We are a group of interdisciplinary teacher educators with different racial and disciplinary backgrounds ranging from early childhood to secondary education in STEM disciplines such as mathematics, science, and social study education. The authors started working at the University of North Dakota at the same time in Fall 2023. Elizabeth is a Chilean mathematics teacher educator preparing future elementary and secondary mathematics teachers and the mother of a child attending UCLC. In her childhood years, she spent time in the countryside with her grandparents gardening and being in close contact with nature (Kastberg et al., 2019). In her career, she has created spaces for learners (Suazo-Flores & Roetker, 2021), teachers (Chapman et al., 2020), and MTEs (Kastberg et al., 2023) to share their ways of being and knowing. She also has worked on curriculum development in mathematics education (Suazo-Flores & Alyami, 2023; Suazo-Flores & Roetker, 2021; Suazo-Flores & Roetker, 2024). Elizabeth’s life experiences and academic trajectory brought her to meet the UCLC’s director who wanted to write a grant proposal to improve the childcare center’s playground. The grant proposal needed a research component and Elizabeth knew what to propose

acknowledging what she knew about the UCLC's director's diverse ways of being, interacting, and operating (Suazo-Flores et al., 2024). The research project would take children to turn their attention to the living world (Barwell et al., 2022) and represent those experiences to others. In representing their experiences to others, children would be using STEM practices. Elizabeth invited the rest of the authors to join her in such a research project as she values working in collaboration and was aware of her limited expertise in early childhood education and natural sciences.

Over the summer of 2024, we built a collaborative inquiry (Clandinin & Connelly, 2000) with which we traversed the insider and outsider positionalities to learn about each other and the childcare educators' experiences. Our interactions with the childcare teachers and director have been guided by relationships, respect, openness to multiple voices, and consideration of how such work might shift the experiences of all engaged (Clandinin, 2006). Our main form of communication has been through conversations where language and emotions are intertwined (Maturana & Davila, 2021). We met weekly and joined the early childhood educators in their professional development meetings.

THE SOCIO-ECOLOGICAL STEM TASK

Acknowledging educators' ways of knowing and being, and UCLC's Nature Explore certification, we designed a task that would take children outside, making holes, and exploring what happens with certain materials when we bury them for a week. We called the task: Is the soil alive? Preschoolers were provided with rectangular pieces of regular white copy paper, construction paper, 100% cotton fabric, plastic, and a banana peel. Children were invited to trace or glue the objects in a handout. The curricular decision of providing rectangular pieces and asking children to trace them was informed by Elizabeth's mathematics education expertise. She was hoping children would describe the shapes and wonder about their sizes. We then invited children to bury the objects one by one in individual holes in their playground. We waited for a week to dig up the objects and observe what happened.

FINDINGS

We first introduced the task to the UCLC educators and requested their feedback and ideas for the place we could use. The educators found the task fun and were excited to teach it with us. We aimed to turn children's attention to nature by exploring the interplay between the soil and the given objects. We planned to bring objects and invite children to describe them. We also wanted children to use cameras and take pictures during the activity. A day before meeting with the children, Elizabeth helped educators prepare the soil so it was less hard for the children to make holes.

On the first day, we asked children to identify the objects and describe characteristics they perceived before and after digging them. Preschoolers enjoyed talking about the materials. They recognized objects such as paper, construction paper, and fabric. They did not know a word to describe the red piece of plastic; they did recognize they could not rip it. After describing the objects, we let them trace them or glue them to a given handout and write their predictions about whether the objects would change or not after planting them in the soil (see Figure 1 below with pictures of their work). The gluing of objects was an option as we had children close to being three years old who could not trace. Children no matter their age were free to choose tracing, gluing, or both.



Figure 1: Objects glued or traced (top) and then buried in the soil (bottom).

By documenting the objects before (top row Figure 1) we aimed to have a record of them and the children’s hypothesis about what they thought was going to happen to the objects after being buried for a week. This part of the activity also allowed for a potential exploration of the concept of area in case children wondered about the size of the given objects before and after burying. We intentionally cut the objects of the same material with the same area, so we could compare them later. We wrestle with the idea that we could not control how children or soil were going to behave as they have agency and free will. The UCLC playground is also open to the public after school hours, so one possibility was to lose some of the buried objects. Thus, for us, it was important to be open to listen and be in the moment with the educators and children instead of trying to impose predetermined learning objectives.

During day one of the activity, as the preschoolers dug in the soil to prepare holes to bury the objects, they were thrilled to discover earthworms and insect larvae. They picked them up, showed them to each other exclaiming “It’s alive!”, and had impromptu discussions with the educators about what it means to be alive. They also took pictures of us, themselves, and earthworms they found.

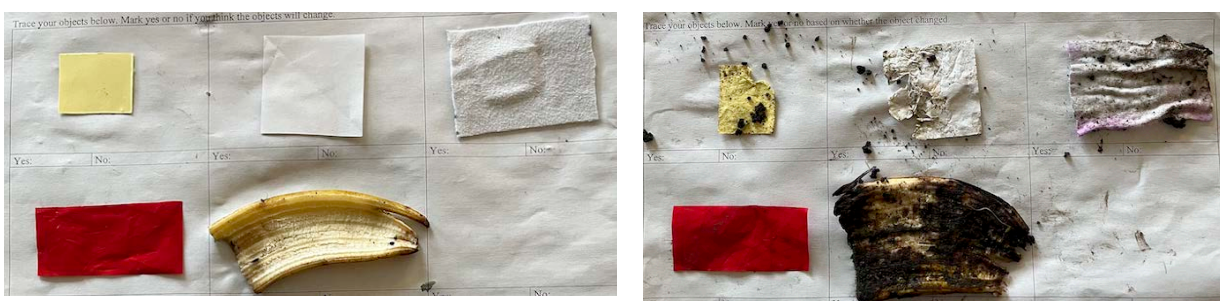


Figure 2: Materials before and after being buried for seven days. Left picture, materials from top left to right: cardboard, regular copy paper, 100% cotton, plastic, and banana peel.

On the second day of the activity, which occurred seven days later, we went outside and dug up the objects. See Figure 2 where the objects before and after being buried for a week are represented. Children were excited to play with the soil, and everybody wanted a turn to dig up objects. When finding the objects, they celebrated by shouting and clapping, they also showed interest in the earthworms, insect larvae, and ants we saw during our time outside. Figure 3 shows some pictures children took when sharing their encounter with the living world.

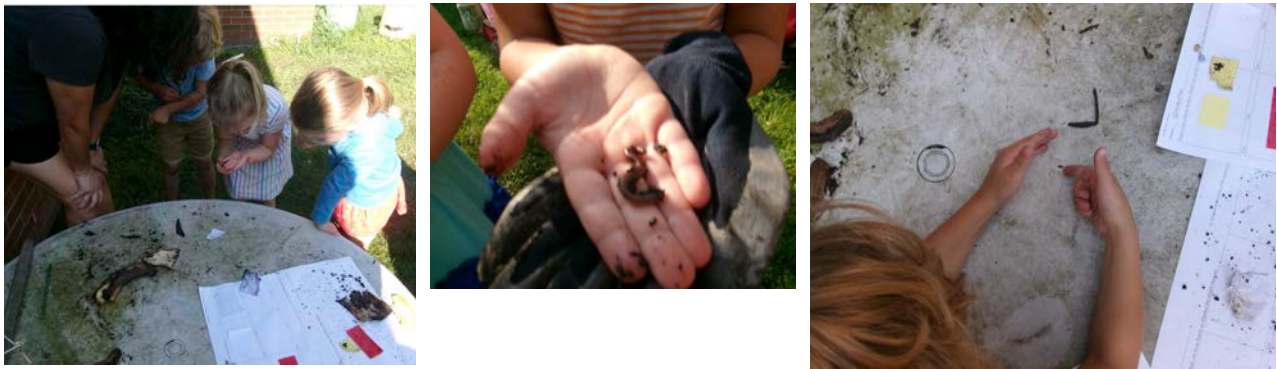


Figure 3: Children paying attention to the living world.

We then showed the kids the objects before and after being buried and asked them to share their noticing by answering the questions: How is it different? How is it the same? Some of them decided to touch the objects and others referred to the buried banana peel as a “yucky banana.” Another child started removing the soil from the banana. Figure 4 shows children touching and pointing to the buried objects. Children expressed interest in the new colors the objects presented. When comparing the pieces of papers, some of them recognized the buried piece of paper changed colors, they said: “It is dirty and it has orange.” When looking at the piece of fabric, the kids noticed the buried piece of fabric had some pink and the original was white.



Figure 4: Children touching the objects.

We worked with two kids at a time, asking the same questions. See the dialog below for an example of our conversations with the children when comparing the objects.

Elizabeth: How is this piece of paper compared to this other one? Is it the same or is it different?

Kids: It is different.

Elizabeth: How is different?

Kid 1: It has orange!

Kid 2: It is dirty!

Elizabeth: How about the fabric?

Kid 1: It has a kind of pink.

Kid 2: It is dirty.

Kid 1: It has pink or purple. Purple. My grandma would know because she likes purple.

DISCUSSION

We shared an example of an activity that merges STEM knowledge and practices with the socio-ecological in a preschool classroom. The activity resulted from teacher educators being in relationship with a childcare center and our interest in imagining a new future for STEM education. UCLC had a certification that positions the playground as another classroom and its director wanted to obtain external funding to improve it. We joined UCLC in their goals and used the opportunity to test ideas for STEM education and the socioecological. In the new future for STEM and mathematics education and the sociological, curriculum development is in coordination with the living world (Barwell et al., 2022) and the surrounding community's interests. This paper shows an example of an activity that started turning learners' attention to the soil and the living beings living on it by observing and describing the influence of the soil on given objects.

Regarding our interdisciplinary approach to build relationships with the UCLC community and among us, we were open to learning about the center and its educators before planning any lesson. We respected the childcare center's rhythms and tried to follow them instead of imposing our research agendas on them. As a diverse group of interdisciplinary STEM teacher educators, we had the common goal of creating a curriculum to promote a vision of STEM that does not consider the living world as passive or static, but as essential for itself and the existence of human beings (e.g., Barwell et al., 2022; Coles, 2023; Khan, 2020). Yet, this work is challenging as we have few examples of how it could look. We planned the activity not knowing exactly what could happen, what children would say and whether the objects were going to stay in the soil after a week. We used our mathematics education background to plan for potential discussions around describing geometric shapes and sizes. We did not anticipate children's interest in colors or in touching and looking closely at the insects and earthworms we encountered. Our traditional preparation in STEM education did not position us well to know to follow children's interest as we are used to teach towards a learning goals targeting STEM content. We redirected children's attention to describing the objects by working with a couple of them at a time. We see a need to learn about how to create spaces for exploration where our focus is only relationships instead of learning objectives.

The interdisciplinary work of creating an activity that turned learners' attention to the living world was conducted following Suazo-Flores et al.'s (2024) interdisciplinary practices. We all had a common interest in place-based STEM education. Elizabeth recognizing her limited expertise in early childhood and science education, invited her colleagues. Chris and Amber both had expertise in early childhood and science education. Elizabeth led the group in allowing everyone to feel heard and seen as well as creating spaces to discuss ideas to reach an agreement on future actions. We all met with the UCLC educators to share our interests in working alongside them; this was our approach to nurture a collaborative space where everyone felt valued. Elizabeth created other mathematics education activities before (Suazo-Flores & Alyami, 2023; Suazo-Flores & Roetker, 2021; Suazo-

Flores & Roetker, 2024), but never an activity like the one presented in this paper. She attributes it to her background and the role of being in a new physical place. UCLC is located at the border of two states in the U.S. where native communities live and many national and local parks are available. Around the same time, Elizabeth was learning about mathematics education and the socio-ecological (Boylan & Coles 2017; Vossoughi et al., 2023; Khan, 2020; Khan & Armstrong, 2019), children's lack of experiences with embodied and social activities (e.g., Haidt, 2024), and attended the 15th International Congress for Mathematical Education discussion group around mathematics education and the socioecological. It seems like everything came together to give origin to the *is the soil alive* task.

Acknowledgments

We are grateful for the University Children's Learning Center educators, families, and children who allowed us to start dreaming of a new future for STEM education.

References

- Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: An international survey before and during the pandemic. *Educational Studies in Mathematics*, 107(1), 1–24. <https://doi.org/10.1007/s10649-021-10049-w>
- Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013.
- Boylan, M., & Coles, A. T. (2017). Is another mathematics education possible?: An introduction to a special issue on mathematics education and the living world. *Philosophy of Mathematics Education Journal*, 32. <http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome32/index.html>
- Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., Cirillo, M., Kramer, S., & Hiebert, J. (2020). Improving the impact of research on practice: Capitalizing on technological advances for research. *Journal for Research in Mathematics Education*, 51(5), 518–529. <https://doi.org/10.5951/jresmetheduc-2020-0165>
- Chapman, O., Kastberg, S. E., Suazo-Flores, E., Cox, D., & Ward, J. (2020). Mathematics teacher educators' inquiry into their practice. In K. Beswick & O. Chapman (Eds.), *International handbook of mathematics teacher education, 2nd Edition. Volume 2: The mathematics teacher educator as a developing professional* (pp. 157-187). Brill-Sense Publishers. https://doi.org/10.1163/9789004424210_008
- Clandinin, D. J. (2006). Narrative inquiry: A methodology for studying lived experience. *Research Studies in Music Education*, 27(1), 44-54. <https://doi.org/10.1177/1321103X060270010301>
- Clandinin, D. J. & Connelly, M. (2000). Narrative inquiry: Experience and story in qualitative research. Jossey-Bass.
- Coles, A. (2023). Teaching in the new climate regime: Steps to a socio-ecology of mathematics education. In M. Ayalon, B., Koichu, R. Leikin, L. Ruber, & M. Taback (Eds.), *Proceedings of the 46th conference of the international group for the psychology of mathematics education (Vol. 1)* (pp.17–33). University of Haifa, Israel. <https://www.igpme.org/publications/current-proceedings/>
- Gutiérrez, R. (2022). A spiritual turn: Toward desire-based research and indigenous futurity in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 379–388. <https://doi.org/10.5951/jresmetheduc-2022-0005>
- Haidt, J. (2024). *The anxious generation: How the great rewiring of childhood is causing an epidemic of mental illness*. Penguin Press.

- Kastberg, S. E., Suazo-Flores, E., Grant, M. & Chapman, O. (2023). Seeing self-based methodology through a philosophical lens. Special Issue at the *Philosophy of Mathematics Education Journal*, 40. <https://education.exeter.ac.uk/research/centres/stem/publications/pmej/pome40/index.html>
- Kastberg, S. E., Suazo-Flores, E., & Richardson, S. E. (2019). Mathematics educator teacher stories. *Revista Brasileira de Pesquisa (Auto) Biographical*, 4(10), 48-67. <https://doi.org/10.31892/rbpab2525-426X.2019.v4.n10.p48-67>
- Khan, S. (2020). After the M in STEM: Towards multispecies' flourishing. *Canadian Journal of Science, Mathematics and Technology Education*, 20, 230–245. <http://dx.doi.org/10.1007/s42330-020-00089-4>
- Khan, S. & Armstrong, A. (2019). Math-a-POLKA: “Mathematics as a place of loving kindness and.” *Journal of the Canadian Association of Curriculum Studies*, 17(1), 1-12. <https://doi.org/10.25071/1916-4467.40441>
- Maturana, H. & Davila, X. (2021). *La revolución reflexiva: Una invitación a crear un future de colaboración*. Paidós.
- Nicol, C., Thom, J. S., Doolittle, E., Glanfield, F., & Ghostkeeper, E. (2023). Mathematics education for STEM as place. *ZDM—Mathematics Education*, 55(7), 1231–1242. <https://doi.org/10.1007/s11858-023-01498-z>
- Suazo-Flores, E., & Alyami, H. (2023). Transcending school mathematics and science while making art. In Harper, S. & Cox D. (Eds.). *Modern math tasks to provoke transformational thinking, Grades 9-12 (pp. 126-144)*. National Council of Teachers of Mathematics
- Suazo-Flores, E., & Roetker, L. (2024). Construct it! Building painted cubes: Serena's case. *Mathematics Teacher: Learning and Teaching PK-12*, 117(1), 36–40. <https://doi.org/10.5951/MTLT.2023.0069>
- Suazo-Flores, E., & Roetker, L. (2021). Pack a truck: Decontextualizing and contextualizing a task. *Mathematics Teacher: Learning and Teaching PK-12*, 6(114), 468–472. <https://doi.org/10.5951/MTLT.2020.0289>
- Suazo-Flores, E., Walker, W.S., Kastberg, S.E. et al. (2024). Mathematics education researchers' practices in interdisciplinary collaborations: Embracing different ways of knowing. *Math Ed Res J*. <https://doi.org/10.1007/s13394-024-00489-x>
- Vossoughi, S., Marin, A., & Bang, M. (2023). Toward just and sustainable futures: Human learning and relationality within socio-ecological systems. *Review of Research in Education*, 47(1), 218–273. DOI: 10.3102/0091732X231223505

APPENDIX: DISCUSSION DOCUMENT

1. Introduction and Background

This document announces a new ICMI Study ‘Mathematics Education and the Socio-Ecological,’ the 27th such study to be conducted by the International Commission on Mathematical Instruction (ICMI). The ICMI Studies are a major activity of ICMI. Their global aims are to contribute to a better understanding of the challenges faced by mathematics education in our multidisciplinary and culturally diverse world and to collaborate in advancing to their resolution. The first ICMI Study was launched in 1980. More detailed information can be found on the [ICMI web page](#).

As a named topic, the ‘socio-ecological’ is relatively new compared to many others in the field. Yet there is wide recognition of the urgent need for thought leadership to consider what is and might be the role of mathematics and mathematics education in multiple, intersecting, social, political, and ecological issues such as climate change, poverty, inequality, health crises, discrimination, and marginalization. A one-day, online, ICMI symposium in March 2023 was dedicated to the ‘socio-ecological’. It brought together 170 participants from diverse contexts to discuss work in this area of interest, in sites such as classrooms, universities and community spaces, in interdisciplinary relations with other subjects, and including issues such as teacher education, research practice and policy (see the [proceedings here](#)). The work discussed at this symposium and other forums, as set out further below, demonstrates the substantial interest in, generative substance of, and commitment to, research that locates mathematics and mathematics education within the socio-ecological.

Thus the 27th ICMI Study will bring together – in a Study Conference and culminating in a ICMI Study volume based on the Conference activities – an expert reference group comprising scholars, with diverse representation within the ICMI community, and across mathematics education sites and contexts. The task of this group is to reflect on ‘the state of the art’ by analysing the growing research and practice in the areas of mathematics education and the socio-ecological, offered from diverse traditions including, and not limited to: critical mathematics education; decoloniality; ethnomathematics; feminist thought, Indigenous ways of knowing; and mathematical modelling. Furthermore, the Study hopes to harness such contributions and anticipate new possibilities, questions, and recommendations for research, innovation and action for mathematics education located in the complexity of social and ecological interdependencies. The Study will build community (including interdisciplinary relations) and open space towards new directions in

mathematics education, as it relates to the socio-ecological across local and international levels.

2. Rationale for ICMI Study 27

The need for ICMI Study 27 has been brought into stark relief by multiple, intersecting, recent events – long experienced and documented by some, and newly by others – that highlight the rapidly changing, uncertain, precarious nature of the world; climate-change induced extreme weather events, public health crises, biodiversity loss, forced movement, growing poverty and inequality, rising totalitarianism, and reality-denying fake news (e.g. Latour, 2018; Roy, 2020; Tsing et al., 2017; van Dooren, 2014). All have made visible disproportionate impacts on racial, ethnic, class, geographical and other historically minoritized groups, for whom precarity is not new. And emerging from such precarity is growing awareness of the inseparability of human and ecological concerns. The startling events pointed to above, and many others, are connected in systems of nested, interdependent relations between humans and between humans and the Earth, that cross multiple scales of space and time. Indeed, events evidence, starkly, the destructive limits of dominant, human-centric contemporary relations, variously named ‘neoliberalism’, ‘racial capitalism’, or ‘the consumer-industrialist society’.

The current conditions of our world, as described here, are not simply the context of neutral mathematics and mathematics education practices. Mathematics education, as curriculum, pedagogy, research, policy, and so on, produces ‘knowers’, users of this mathematics. While these practices may be put to work towards sustainable, ethical relations, they may also – intentionally, or not – (re)produce related historical epistemic, ontological, linguistic, cultural, social, and ecological injustices (e.g. Skovsmose, 2021; Valero, 2023). Mathematics education cannot be assumed to function as an individual or social ‘good’, and the socio-ecological precarity of the world brings the role of mathematics education into question. Thus, there is a need for mathematics and mathematics education to expand from considering socio-political dimensions of life (e.g. Jurdak et al., 2016), to also considering: what does the socio-ecological condition of the world mean for mathematics (education), and what might mathematics (education) mean for this condition?

The past two decades have seen the emergence of and growth in mathematics education scholarship that attends to and conceptualizes – to varied extents – the ‘ecological’, and its relation to mathematics education and society. While such scholarship may draw on psychological perspectives (e.g. Louie & Zhan, 2022), most lies in critical traditions that view mathematics (education) as historical, social, and political practices. We briefly illustrate some strands of that past work, noting

that the Study itself will analyse these and other traditions in detail, we are by no means exhaustive in our choices below.

Critical mathematics education (CME) has demonstrated the potential of mathematics to ‘read’ and ‘write’ (Gutstein, 2006, following Freire, 1970) the contemporary ecological condition of the world, (e.g. Barwell, 2013; Coles et al., 2013; Skovsmose, 2023). More recently, CME is increasingly conceptualizing the social world (in particular, technology or mathematics-in-action) as having an impact on the ecological (e.g. Coles & Helliwell, 2023; Hauge & Barwell, 2017; Ödmo, Björklund Boistrup, & Chronaki, 2023; Steffensen, Herheim, & Rangnes, 2023). Socio-critical modelling, which promotes modelling as a practice for critically understanding the world, has extended from focusing on social inequities to explore human action on ecologies of the planet, in phenomena such as global warming (e.g. Basu & Panorkou, 2020), waste management (Villarreal, Esteley, & Smith, 2015), and fish stocks (Yanagimoto & Yoshimura, 2013). Critical traditions in mathematics education have focussed on peoples marginalized by coloniality and neoliberal globalisation, indeed those groups long and increasingly most affected by events located in the (inter)action of humans and the Earth, increasingly foreground these interdependencies. For example, ethnomathematics is concerned with how diverse socially and culturally situated ways of knowing and doing mathematics are used in social, cultural, political, economic and environmental practices (Rosa et al., 2012, citing D’Ambrosio, 2007). D’Ambrosio (2015) has long called attention to environmental realities in ethnomathematics, with recent examples including sustainability of Indigenous lands (e.g. de Mattos & de Mattos, 2020), and proposals for an ethno-biomathematics (Eglash, 2023). Indigenous ways of knowing foreground the ways of knowing, acting, being, and using language of variously named Indigenous communities. This work offers important direction on how to act against binaries and hierarchies of culture/nature and so-called Western/Indigenous mathematics, binaries that perpetuate the idea that mathematics can be separated from well-being or ecological considerations (e.g. Anania & Stiglitz, 2023; Gutiérrez, 2017, 2019; Kulago et al., 2021).

For thinking about mathematics education, in terms of the social *and* ecological, scholars also recruit (related) perspectives commonly considered ‘outside’ of the field, such as decoloniality, (eco)feminism, ecojustice, ethics, Indigenous futurity, linguistics, new materialism, philosophy, and posthumanism (e.g. Barwell et al., 2022; Borba, 2021; Boylan, 2017; Chronaki & Lazaridou, 2023; de Freitas & Sinclair, 2014; Gutiérrez, 2022; Khan, 2020; Kirby, 2011; Madden, 2019; Rubel & Nicol, 2020; Wolfmeyer, Lupinacci, & Chesky, 2017).

Mathematics education forums are increasingly creating opportunities for research, from all these perspectives, dedicated to mathematics education and the socio-ecological. Such forums have included four journal special issues: *Mathematics education and the living world* (edited by Boylan & Coles, 2017); *Mathematics for “citizenship” and its “other” in a “global” world: Critical issues on mathematics education, globalisation and local communities* (edited by Chronaki & Yolcu, 2021); *Mathematics education in a time of crisis—a viral pandemic* (edited by Chan, Sabina, & Wagner, 2021); and *Innovating the mathematics curriculum in precarious times* (edited by le Roux, et al. 2022). Recent relevant research also includes conferences, not only research papers, discussion groups, and panels, but notably plenary presentations, for example, the 12th International Conference of Mathematics Education and Society (e.g. Barwell, 2023), and the 47th Annual Conference of the International Group for the Psychology of Mathematics Education (e.g. Coles, 2023; Valero, 2023).

A striking characteristic to emerge from a range of the contributions above (and the March 2023 ICMI Symposium) is the relational nature of the reported mathematics education research. Many studies involve ongoing, interdisciplinary and transdisciplinary relations with other knowledges, practices, languages, and values. Such work involves ongoing collaborative relations between researchers, teachers, students, communities, activist movements, government and non-government institutions, natural scientists and social scientists. The contributions move between context and materiality, and abstraction. A coherent thread is researchers’ attention in these relations to notions of power, responsibility, answerability, activism, care, agency, and democracy. At the same time, such work is strengthened by the diversity of theoretical perspectives (and interpretations thereof), being brought to and given meaning in local experiences of mathematics education in the socio-ecological, all of which are manifestations of global sustainability concerns.

This ICMI Study 27 is an important, necessary, and timely opportunity to address particular concerns about mathematics education and the socio-ecological. As suggested by the preceding, illustrative, review of existing scholarship, there is a need for an analytic description of the growing dialogue across perspectives and voices, from across the globe, of those working on issues in mathematics education that span social (including political) and ecological (environmental) problems, and ethical concerns.

3. Aims of ICMI Study 27

Areas of mathematics education, such as those cited in Section 2, have long adopted critical approaches to relations between mathematics education and the social and/or ecological. Yet, academic thought no longer has the “luxury” (Sitas, 2023, p. 55) of critique alone; there is a need for

critique alongside, for example, creativity and speculation (da Silva, 2022). There is an urgency for thought leadership on new possibilities, questions, and recommendations for research, innovation, and action for mathematics education and the socio-ecological, and to build community (including interdisciplinary relations) to act in new directions at local and international levels. There is a sense that the pace of change in the world is vastly outstripping knowledge of, and indeed capacity to ‘know’, the socio-ecological condition (Sitas, 2023). Haraway (2016) argues that “bounded individualism,” characteristic of Western thought is “unavailable to think with, truly no longer thinkable” (p. 5).

The topic of ICMI Study 27, ‘Mathematics Education *and* the Socio-Ecological,’ locates mathematics education *in* the complexity of social and ecological interdependencies, or frames mathematics education within the socio-ecological. This includes, for example, interdependent relations between all humans (individual and community; bodies, thoughts, emotions); the living and inanimate natural world; material technologies; languages; and concepts. These relations cross multiple scales of space (local, regional, national, global) and time (pasts, presents, futures). The complexity of bringing together the social-ecological and mathematics education, (re)produces a number of tensions. Such tensions are not yet well understood (beyond simple binaries), with considerable methodological, empirical, and ethical implications. Significantly, van Dooren (2014, p. 3) argues that it is only when we come to understand the complexity of our interdependencies that we may have a sense of the obligations we (as mathematics educators) have “to hold open space in the world” for all living beings.

ICMI Study 27 aims to explore different theorizations of the socio-ecological and the role of mathematics education therein, and the implications for mathematics education research and practice. The Study is arranged around four themes:

- (A) Aims of mathematics education;
- (B) Scales of mathematics education;
- (C) Resources of and for mathematics education;
- (D) Mathematics education futures.

These themes are singular in that they provoke particular questions for mathematics education, but are inherently related. We anticipate that these related themes will further develop during the Study process. Each theme description, below, invites engagements with notions of the social and ecological *and* their interdependencies (what we refer to as the ‘socio-ecological’). These invitations also

recognise that, while we all relate to and experience the ‘socio-ecological’ in some form, our experiences will be different, on account of our place-based histories and presents. Thus, the Study will itself offer conceptualizations of mathematics education and the socio-ecological. Each theme description identifies tensions emerging from mathematics education positionings in the complexities of the social, ecological and their relations. We view these tensions as creative, as generating new questions about and for mathematics education, and prompting new imaginaries of presents and futures. Departing from these socio-ecological conceptualizations, each theme then asks of the thinking and practices of mathematics education specific questions related to: knowledge; curriculum; pedagogy; learning materials; professional development; philosophy; theory; methodology; and so on.

Taken together, the four themes recognise the presents (and their histories) in the field of mathematics education, and make space/create opportunities for scholars to find lines of openings for the field, to position themselves for thinking, acting and being in contemporary and future socio-ecological realities. The first theme focuses on the aims of and conceptualizations of mathematics education in different spaces in our pasts and presents, and for our futures. It asks how those aims and their underpinning philosophical assumptions shape possibilities for social-ecological framings of mathematics education, including the curriculum. In the second theme, scales of mathematics education, we focus our attention on how mathematics education’s interactions in the complexities of the social and ecological occur at multiple scales, of the local, regional, national and global. We are interested in how these interactions play out – within and across spaces and borders, in pasts, presents and futures – with respect to the enactment of knowledge, curriculum, and teacher professional development. The third theme, resources of and for mathematics education, considers how practices, concepts and tools are reconceptualized, or used differently, when the social and ecological are acknowledged as entangled. And, finally, the fourth theme, mathematics education futures, considers our ways of conceptualizing research and practice in mathematics education for sustainable futures, for mathematics education located in uncertain and complex socio-ecological interdependencies and ethics.

4. Themes, Sub-themes, and Invitations/Questions

Theme A. Aims of mathematics education

This theme highlights the importance that our underlying assumptions and motivations have in articulating the potentialities of a socio-ecological framing of mathematics education. We know that there is much disparity amongst educators around the aims of mathematics education, with some

being constitutionally opposed to a focus on socio-ecological issues in the classroom, while others being more amenable to them, and even centring them. Not to mention that socio-ecological issues are variously conceptualized as well. These differences in aims and conceptualizations are not particular to this moment. The aims of mathematics education have changed over time and across contexts; interrogating them has become increasingly important as mathematics education is a central area of the compulsory school curriculum and of many other forms of further education. As the socio-ecological becomes a focus of attention, inviting us to think not only of the relation between mathematics and society, but also the conditions of human life on Earth – which have now become precarious in many parts of the world – the aims of mathematics education require renewed attention. The current multiple interconnected crises we find ourselves in prompt questions about the forms of knowledge and knowing that are relevant in education, the effects of such knowing in shaping peoples’ identities and qualifications, and the possibilities for individuals and communities to mobilize themselves and their knowing into the creation of new possible futures.

As we work to create new approaches to mathematics education that could adequately engage in socio-ecological issues, it is worth examining our philosophical positionings with respect to the aims of mathematics education and engage directly in exploring multiple tensions concerning:

- The reasons for providing mathematics education and to whom.
- The functions of mathematics education in current crises.
- The types of peoples, with their identities and qualifications, we want to enable through mathematics education.
- The expected gains for individuals, communities, and societies.

We arrange these concerns in two sub-themes.

Sub-theme A1. Examining the aims of mathematics education

We are interested in how the aims of mathematics education, in different times and spaces, shift/reify/transform if we locate mathematics education in the particular socio-ecological interdependencies and complexities identified above. We ask:

A1.1 What have our aims around mathematics education traditionally been, and what philosophies and theories have informed these aims? And with what implications for our presents and futures?

A1.2 What would our aims (have to) be if we centre the socio-ecological? And what philosophies and theories might help us develop or make possible such aims? For example, how might relational theories that privilege human and non-human respect and responsibly inform educational aims? How might Indigenous theories, that centre the care-taking of land, frame the aims of education in ways that consider socio-ecological interdependencies and complexities?

A1.3 How do the aims in A1.1 shape/constrain/offer openings for the thinking demanded of us in A1.2?

Sub-theme A2. Examining mathematics as a subject of education

Mathematics education research has often been focused very specifically on mathematics; how mathematics might improve reasoning or contribute to progress or democratic participation, or, on the other side, lead to oppression and anxiety. Some mathematics educators, however, have leaned more on education, seeing that as the primary field of action, with mathematics being less centred. A more mixed approach of mathematics education, as equal partners, is also possible. No matter where along the continuum between these two positions researchers find themselves, there are controversies around what counts as both mathematics and education. We offer the following questions:

A2.1 With the addition of the socio-ecological to the mix, how are mathematics, education, or mathematics education inflected?

A2.2 Does the socio-ecological focus compel a shift towards other views of mathematics (e.g., more applied or interdisciplinary understandings of mathematics)?

A2.3 Does a socio-ecological focus dramatize the centrality of education, making mathematics more of a possible context?

A2.4 Which other kinds of meanings and possible relationships can be thought for mathematics education in the socio-ecological?

Theme B. Scales of mathematics education

Mathematics education has perhaps become accustomed to focusing on one, or a limited number, of connected sites, where interactions of relevance are considered to be enacted. We are now confronted with the realization that such interactions are connected within and across sites (e.g., the classroom and home; the school and the life out of school) and occur at multiple scales simultaneously (e.g., local, regional, national, and global). Positioning mathematics and mathematics education in the

complexities of the socio-ecological requires attention to territories of interaction, by which we mean the networks of dense relationships between communities, human and non-human entities (animals, plants, rivers, mountains, spirits, etc.), stories, myths, emotions, and the spaces by/with which they are created/associated in pasts, presents, and futures in which the multiple practices of mathematics education are enacted. How do scales of interaction play out in making mathematics education? We identify tensions in:

- How we impose/(re)constitute boundaries to organize education and the effects of such boundaries, prompting us to consider: what are the histories of these boundaries; who creates them and the power that defines them; for what purposes; with what implications, and for whom; what are alternatives?
- The different scales of interventions in mathematics education and how they interact, with implications for listening and acting at these various scales.
- The many voices within and across territories themselves (e.g., human, non-human, elder, teacher, student, doctor, policy maker, socio-ecological activist), voices that are polyphonic and even conflicting. In this sense, environments also have many voices.

Sub-theme B1. Relations between the local/global, historical, ecological, and political

We are interested in how mathematics is enacted with/in bodies/territories and stories, and how bodies/territories and stories are enacted with/in mathematics. Any given body/territory will have many (even contradictory) voices. So, attending to scales helps illuminate whose voices/stories are amplified and whose are muted. Attending to scales helps us to think about what are/might be appropriate actions, and by whom these should be undertaken.

B1.1 Whose stories/histories are included in mathematics (education), at what scales, and how does that relate to who is considered in the presents or futures of mathematics (education)? For example, what power structures exist that repeat and sustain an uneven power distribution between territories and make them (and the communities in relation) easier to govern? What role might the stories/histories of non-human entities play in mathematics?

B1.2 How and why have certain communities developed small scale/local innovations around socio-ecological issues in mathematics education? How has that reflected the voices of those engaged in mathematics as a human, cultural, political, and ecological activity?

B1.3 What is/might be the role of mathematics (education) in supporting communities' defense of their stories/histories and territories?

Sub-theme B2. Curriculum innovations and different voices located in the socio-ecological

We are interested in the methodologies for capturing small-scale/local curriculum innovations and sharing the ways that voices of those engaged in mathematics as a human, cultural, political, and ecological activity are reflected. For instance, how do national/international curricular agendas impact local and micro (classroom-level) teaching practices; how do specific communities collaborate on those practices; and how are these communities defined?

B2.1 What have we learned from trying to adopt small scale/local curriculum innovations that frame mathematics within the socio-ecological in a range of scales (local, national, regional, global) and how do local ideas travel? What are the possibilities for dialogue across scales?

B2.2 What tensions do communities face in developing, implementing, and communicating local curricular innovations to address the socio-ecological crises they live?

Sub-theme B3. Learning from site-specificities

We are also interested in sharing the ways that networks of collaboration, if any, have been helpful for communities to face their local socio-ecological problems. A crucial issue for research in mathematics education would be to make explicit what transdisciplinary knowledge has been constructed through such collaborations, and what the role of mathematical knowledge in such collaborations is.

B.3.1 How do we better prepare educators, researchers, and students to simultaneously think about local and global issues in mathematics education with respect to the planet? In what way(s), if any, does a socio-ecological perspective (e.g., learning from non-humans) serve as a proper foundation for thinking about site specificity and wider relevance?

B3.2 How do communities and educators solve local tensions between curricular innovations and official programs and how do they share solutions with other teachers and school communities, while not ignoring local specificity?

Theme C. Resources for and of Mathematics Education

This theme is concerned with the (re)conceptualization and use of different forms of resources, defined as: *practices; concepts, constructs and objects; and tools which have agency in doing/performing mathematics*. The theme and two sub-themes are located within a milieu constituted by socio-ecological concerns and their inter-relatedness with mathematics education. What issues are at stake when we conceptualize resources, and their use, in a critical manner, not assuming they are only beneficial to mathematics education and the socio-ecological? Elaborations on the theme are

underpinned by a series of interconnected questions related to tensions we identify in these interdependencies.

Practices: Practices include ways of knowing and doing mathematics, modes of reasoning, argumentation, and acceptable means of knowledge creation within a learning community (e.g., mathematical modelling). Approaches employed to foster or facilitate learning by both the teacher (pedagogies) and the learner themselves are therefore also considered practices. This also means that pedagogical approaches, as practices, are in themselves a mode of knowledge creation – both in/for the learner and the teacher.

Concepts, constructs and objects: Concepts, constructs and objects, including for instance, variables, algebraic expressions, geometrical objects, statistical processes (e.g., sampling) and techniques (e.g., integration), represent the foundations of disciplinary knowledge within mathematics. These concepts, constructs and objects, in conjunction with mathematical practices and tools, constitute a relevant part of what is considered mathematics by society in general.

Tools which have agency in doing/performing mathematics: Tools that enable the knowing/doing of mathematics include physical tools (e.g., rulers, compasses), representational tools (e.g., maps, ready reckoners, comparative charts), and digital tools (e.g., calculators, computers, virtual reality, augmented reality). Tools can take the form of technologies designed specifically to support engagement with mathematical tasks (e.g., Computer Algebra Systems, Dynamic Geometry Software, Data Analysis Applications) or generic technologies that can be used to perform mathematical activity (e.g., spreadsheets, pedometers). There is currently much interest (and concern) about tools involving artificial intelligence as they relate to education.

Tools can also be considered in a much broader sense to include the quality of spaces in which teaching and learning take place and the spaces in which students spend time outside school. At issue here are the opportunities to use tools to learn – online and in-person – in these spaces. Spaces can refer to teaching and learning conditions in educational institutions, communities, and homes, and include: physical conditions, including electricity, water; physical security; and availability and accessibility of digital tools. These opportunities vary within and across local scales, according to complex historical, socio-political, and socio-economic factors, thus connecting tools with issues related to equity, inclusiveness, and social justice.

How these different forms of resources are (re)conceptualized and utilized when located in socio-ecological interdependencies gives rise to tensions to which mathematics is called on to respond, for example tensions regarding:

- Resources enabling and constraining new relationships, and the nature of relations (both old and new) enacted by resources.
- Resources leading to marginalization and empowerment of different groups, and issues relating to values in the enactment of resources.
- The visibility/transparency of resources in relationships and interdependencies.
- Agencies of resources in predicting and creating futures.

Sub-theme C1. What and how resources are/may be used in relation to socio-ecological concerns

In this sub-theme, we are concerned with how resources can be used for reading, interpreting, and understanding socio-ecological interdependencies.

C1.1 What is the role of mathematical resources in innovation for addressing challenges associated with the socio-ecological – exacerbating current issues or generating new ones? What are the tensions around how tools are utilized to both enable (new thoughts, new practices) and constrain (including by values embedded in their design and use)?

C1.2 How are mathematical resources used in understanding, interpreting and responding to the socio-ecological?

C1.3 How are mathematical resources used in shaping or evaluating claims and reports related to the socio-ecological by expert and non-expert commentators, government agencies, and alternative-to-main-stream sources of information? How are resources used when supporting or refuting arguments about the nature, and current and future state of the socio-ecological?

C1.4 What are the agencies of mathematical resources in fostering/avoiding the social ecological crisis we live in and will be inherited?

C1.5 Which mathematical resources should be the foundation of informed participatory and critical citizenship? Which should be acquired as required within specific contexts?

Sub-theme C2. How resources are embedded within histories, values and ideologies

C2.1 How does the increasing entanglement of humans with mathematical resources impact on responses to the socio-ecological? How will the disentanglement of mathematical resources from humans (e.g., artificial intelligence) or ‘different’ humans influence the significance of the socio-ecological?

C2.2 How are ideological and aesthetic dimensions of mathematics and mathematics education present in the use of mathematical resources?

Theme D. Mathematics Education Futures

This theme focuses on possible sustainable futures, futures for mathematics education located in uncertain and complex socio-ecological interdependencies and ethics. We recognise the possibilities to define ‘futures’ in different ways, for example, futures as a mapping of the present; futures as a folding of pasts and presents; futures as plural and varied (having different characteristics); futures as dependent on the actor; futures as imagining new stories, restorying or creating counterstories.

We identify tensions emerging in these interdependencies and framings, regarding:

- Multiple senses and experiences in mathematics education, of hope, creativity, imaginations, actions, uncertainties, criticality, fears, helplessness, and violences. Prompting us to ask questions of: who hopes; who owns hope; who can dream and hope; who hopes for whom/what; futures for whom/what; who/what is included/excluded, and whose hope and aspirations? Questions of: what and how to enact ethical mathematics education?
- The enfolding of mathematics education’s pasts and presents in mathematics for thinking through science to shape, imagine and restory futures.

Sub-theme D1. Contexts and communities of education that can/have yet to be imagined

D1.1 Where might mathematics learning that cultivates hope and sits with critical realities of sustainable practices happen?

D1.2 How could future schools (re)conceptualize and (re)organise themselves to respond to the varied characteristics of plural futures?

D1.2 How is awareness of socio-ecological complexities within mathematics education preparing ourselves to respond to the changing needs of a complex, uncertain, changing world that can never be fully known?

D1.3 How does knowledge and ethics embedded in informal and cultural practices equip us to use a socio-ecological lens towards sustainability?

D1.4 How and with whom/what might mathematics educators ‘speak with’, ‘come together with’ for sustainable futures: mathematicians and practitioners of mathematics; mathematics instructors and teachers of mathematics; policy makers and curriculum developers; other disciplines, etc.?

Sub-theme D2. Knowledges, curriculum and pedagogies that can/have yet to be imagined

D2.1 What mathematical concepts lend themselves to speculation about futures?

D2.2 What pedagogies and ethics can cultivate criticality and hope?

D2.3. How do we prepare future citizens, and what are their responsibilities from a socio-ecological perspective towards achieving sustainability?

D2.4 How could a socio-ecological approach to mathematics education equip learners to face the changing futures towards sustainability?

Sub-theme D3. Practices and ethics of mathematics education research that can/have yet to be imagined

D3.1 What theories and ethics do and might guide us to understand what socio-ecological interdependencies and complexities can offer to us so as to attain sustainability?

D3.2 What methodological approaches are suitable in the face of interdependencies, complexity, uncertainty? What methodological approaches can sustain hope?

D3.3 Are concepts such as ‘theory’ and ‘methodology’ useful for mathematics education research for sustainable futures?

5. References

Anania, K., & Stiglitz, C. (2023). Touching variables: Decolonial approaches and new tools for ecological data visualization. *Leonardo*, 56(6), 579-585. <https://muse.jhu.edu/article/908016/summary>

Barwell, R. (2013). The mathematical formatting of climate change: Critical Mathematics Education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16 <https://doi.org/10.1080/14794802.2012.756633>

Barwell, R. (2023). Dialogue, responsibility and mathematics education. In R. Marcone, P. Linardi, R. Milani, J. P. A. de Paulo; A. Moura Queiroz, & M. T. da Silva (Eds.), *Proceedings of the 12th International Conference of Mathematics Education and Society* (pp. 19-50). MES.

Barwell, R., Boylan, M., & Coles, A. (2022). Mathematics education and the living world: A dialogic response to a global crisis. *The Journal of Mathematical Behavior*, 68, 101013. <https://doi.org/10.1016/j.jmathb.2022.101013>

Basu, D., & Panorkou, N. (2020). Utilizing mathematics to examine sea level rise as an environmental and a social issue. In A. Sacristán, J. Cortés-Zavala, & P. Ruiz-Arias (Eds.), *Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1064-1068). PME-NA.

- Borba, M. C. (2021). The future of mathematics education since COVID-19: Humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics*, 108(1-2), 385-400. <https://doi.org/10.1007/s10649-021-10043-2>
- Boylan, M. (2017). Towards a mathematics education for ecological selves: Pedagogies for relational knowing and being. *Philosophy of Mathematics Education Journal*, 32. <https://education.exeter.ac.uk/research/centres/stem/publications/pmej/pome32/Boylan-submitted.docx>
- Boylan, M. & Coles, A. (2017). Is another mathematics education possible? An introduction to a Special Issue on “Mathematics education and the living world: Responses to ecological crisis”. *Philosophy of Mathematics Education Journal*, 32. <https://education.exeter.ac.uk/research/centres/stem/publications/pmej/pome32/index.html>
- Chan, M. C. E., Sabena, C., & Wagner, D. (2021). Mathematics education in a time of crisis—A viral pandemic. *Educational Studies in Mathematics*, 108(1-2), 1-13. <https://doi.org/10.1007/s10649-021-10113-5>
- Chronaki, A., & Lazaridou, E. (2023). Subverting epistemicide through ‘the commons’: Mathematics as re/making space and time for learning. In E. Vandendriessche & R. Pinxten (Eds.), *Indigenous knowledge and ethnomathematics* (pp. 161-179). Springer.
- Chronaki, A., & Yolcu, A. (2021). Mathematics for “citizenship” and its “other” in a “global” world: Critical issues on mathematics education, globalisation and local communities. *Research in Mathematics Education*, 23(3), 241–247. <https://doi.org/10.1080/14794802.2021.1995780>
- Coles, A. (2023). Teaching in the new climatic regime: steps to a socio-ecology of mathematics education. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-33). PME.
- Coles, A., Barwell, R., Cotton, T., Winter, J., & Brown, L. (Eds.). (2013). *Teaching secondary mathematics as if the planet matters*. Routledge/Taylor & Francis Group.
- Coles, A., & Helliwell, T. (2023). The role of mathematics teacher educators in preparing teachers of mathematics to respond to global challenges within their classrooms. *London Review of Education*, 21(1), 1-13. <https://doi.org/10.14324/LRE.21.1.02>
- D’Ambrosio, U. (2007). The role of mathematics in educational systems. *ZDM*, 39(1-2), 173-181. <https://doi.org/10.1007/s11858-006-0012-1>
- D’Ambrosio, U. (2015). From mathematics education and society to mathematics education and a sustainable civilization: A threat, an appeal, and a proposal. In B. Greer, & S. Mukhopadhyay (Eds.), *Proceedings of the Eighth International Mathematics Education and Society Conference* (Vol. 1, pp. 19-30). MES.
- Da Silva, D. F. (2022). *Unpayable debt*. Sternberg Press.
- De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. Cambridge University Press.

- De Mattos, J. R. L., & de Mattos, S. M. N. (2020). Ethnomathematics in the Brazilian Indigenous context. In M. Rosa & C. Coppe de Oliveira (Eds.), *Ethnomathematics in action: Mathematical practices in Brazilian Indigenous, urban and Afro communities* (pp. 71-90). Springer.
- Eglash, R. (2023). Ethno-biomathematics: A decolonial approach to mathematics at the intersection of human and nonhuman design. In M. C. Borba & D. C. Orey (Eds.), *Ubiratan D'Ambrosio and mathematics education: Trajectory, legacy and future* (pp. 289-303). Springer.
- Freire, P. (1970). *Pedagogy of the oppressed*. Herder & Herder.
- Gutiérrez, R. (2017). Living mathematx: Towards a vision for the future. *Philosophy of Mathematics Education Journal*, 32. <https://education.exeter.ac.uk/research/centres/stem/publications/pmej/pome32/index.html>
- Gutiérrez, R. (2019). *Mathematx: Towards a way of being*. In Subramanian, J. (Ed.), Proceedings of the 10th International Mathematics Education and Society conference. MES
- Gutiérrez, R. (2022). A spiritual turn: Towards desire-based research and Indigenous futurity in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 379-88. <https://doi.org/10.5951/jresematheduc-2022-0005>
- Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Routledge.
- Haraway, D. J. (2016). *Staying with the trouble: Making kin in the Chthulucene*. Duke University Press.
- Hauge, K. H., & Barwell, R. (2017). Post-normal science and mathematics education in uncertain times: Educating future citizens for extended peer communities. *Futures*, 91, 25-34. <https://doi.org/10.1016/j.futures.2016.11.013>
- Jurdak, M. E., Vithal, R., de Freitas, E., Gates, P., & Kolloche, D. (2016). *Social and political dimensions of mathematics education*. Springer. <https://doi.org/10.1007/978-3-319-29655-5>
- Khan, S. (2020). After the M in STEM: Towards multispecies' flourishing. *Canadian Journal of Science, Mathematics and Technology Education*, 20, 230-245. <https://doi.org/10.1007/s42330-020-00089-4>.
- Kirby, V. (2011). *Quantum anthropologies: Life at large*. Duke University Press.
- Kulago, H. A., Wapeemukwa, W., Guernsey, P. J., & Black, M. (2021). Land, water, mathematics, and relationships: What does creating decolonizing and indigenous curricula ask of us? *Educational Studies*, 57(3), 345-363. <https://doi.org/10.1080/00131946.2021.1892690>
- Latour, B. (2018). *Down to earth: Politics in the new climatic regime* (English edition. ed.). Polity Press.
- Le Roux, K., Brown, J., Coles, A., Helliwell, T., & Ng, O.-L. (2022). Editorial for a special issue on innovating the mathematics curriculum in precarious times. *Research in Mathematics Education*, 24(2), 117-127. <https://doi.org/10.1080/14794802.2022.2090422>
- Louie, N., & Zhan, W.-Y. (2022). A socio-ecological framework for research in mathematics education. *Journal for Research in Mathematics Education*, 53(5), 365-371. <https://doi.org/10.5951/jresematheduc-2022-0003>

- Madden, P. E. (2019). *Evaluating mathematics curriculum from anti-colonial and critical mathematics perspectives*. Doctoral dissertation. Boston College
- Ödmo, M., Björklund Boistrup, L., & Chronaki, A. (2023). A teacher education statistics course encounters climate change and critical mathematics education: Thinking about controversies. In R. Marcone, P. Linardi, R. Milani, J.P. A. de Paulo; A. Moura Queiroz, & M. T. da Silva (Eds.), *Proceedings of the 12th International Conference of Mathematics Education and Society* (sixth sketch, pp. 1307-1321). MES.
- Rosa, M., D'Ambrosio, U., Orey, D. C., Shirley, L., Alanguai, W. V., Palhares, P., & Gavarrete, M. E. (2016). *Current and future perspectives of ethnomathematics as a program*. Springer Nature.
- Roy, A. (2020). *Azadi*. Penguin.
- Rubel, L. H., & Nicol, C. (2020). The power of place: Spatializing critical mathematics education. *Mathematical Thinking and Learning*, 22(3), 173-194. <https://doi.org/10.1080/10986065.2020.1709938>
- Sitas, R. (2023). Publics, politics, place and pedagogy in urban studies. In A. Pande, R. Chaturedi, & S. Daya (Eds.), *Epistemic justice and the postcolonial university* (pp. 51-63). Wits University Press.
- Skovsmose, O. (2021). Mathematics and crises. *Educational Studies in Mathematics*, 108(1-2), 369-383. <https://doi.org/10.1007/s10649-021-10037-0>
- Skovsmose, O. (2023). *Critical Mathematics Education*. Springer International Publishing.
- Steffensen, L., Herheim, R., & Rangnes, T. E. (2023). Wicked problems and critical judgment in school mathematics. In N.S. Kennedy & E. Marsal (Eds.), *Dialogical inquiry in mathematics teaching and learning: A philosophical approach* (pp. 79-102). Lit Verlag.
- Tsing, A. L., Bubandt, N., Gan, E., & Swanson, H. A. (Eds.). (2017). *Arts of living on a damaged planet: Ghosts and monsters of the Anthropocene*. University of Minnesota Press.
- Valero, P. (2023). Mathematical subjectivation: Death sentence to the planet or chances for a terrestrial life? In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 53-68). PME.
- Van Dooren, T. (2014). *Flightways: Life and loss at the edge of extinction*. Columbia University Press.
- Villarreal, M. E., Esteley, C. B., & Smith, S. (2015). Pre-service mathematics teachers' experiences in modelling projects from a socio-critical modelling perspective. In G. A. Stillman, W. Blum, & M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice: Cultural, social and cognitive influences* (pp. 567-578). Springer.
- Wolfmeyer, M., Lupinacci, J., & Chesky, N. (2017). EcoJustice mathematics education: An ecocritical (re) consideration for 21st century curricular challenges. *Journal of Curriculum Theorizing*, 32(2), 53-71.

Yanagimoto, A., & Yoshimura, N. (2013). Mathematical modelling of a real-world problem: The decreasing number of bluefin tuna. In G.A. Stillman, G. Kaiser, W. Blum, & J.P. Brown (Eds.), *Teaching mathematical modelling: Connecting to research and practice* (pp. 229-239). Springer.