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ENHANCING MATHEMATICAL RESEARCH IN HIGH SCHOOL

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We present a didactic experience in the context of enhance mathematics research in high school. The encounter of three different profiles (a skilled student, a mathematics school teacher and a researcher in mathematics) is the starting point of this activity. We explain the context in which this activity was developed and the main exploited actions by the tutors in order to create a stimulating context for the student. We present some evidences of the success of the experience as well as the results and conclusions of this experience. As a collateral finding, a new method of work between secondary school teacher and mathematic researcher has emerged.
Key words: Secondary school research, gifted students.

INTRODUCTION AND CONTEXTUALIZATION

We present our particular experience of tutoring the research of a gifted and motivated student during his last years of secondary school. The collaboration of both, the high school teacher and the researcher, created favourable conditions to produce a fruitful high school mathematical research. In this communication, we want to focus our attention in the different actions taken for creating such a favourable environment.

To contextualize the pedagogical situation, it is necessary to describe The Research Work (literally translated from the common noun Treball de Recerca in catalan), which is a structured oriented towards the research. The Research Work includes the documentation into the field, the development of the research and the report of a particular study (between 50 and 100 pages). This activity is compulsory for all students in high school in Catalonia and it is developed under the guidance and orientation of the teaching staff. The main scope of this activity is achieving the basic competences in research. It can be framed into a disciplinary, multidisciplinary or transversal discipline. The total amount of time needed for the student is approximately over 70 hours, commonly distributed amongst the last two years of the high school (17 and 18 years old).

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In the context of scientific research in high school, it is frequent for students to choose experimental sciences (such as chemistry, biology or physics) to develop their investigations, for being areas where they can apply in a more direct way the scientific method. They usually design their own experiments and then analyse the collected data. Due to the abstract-nature of mathematics, there exist fundamental difficulties to proceed with research and results analysis. As a consequence, students usually avoid this subject. If this is not the case, they either reduce the work to bibliographic search or to an extremely basic research level.

We assume this reality, but we think that gifted students could be an exception to this rule, because they have the capacities to develop real mathematical researches. We would like to introduce what we understand by gifted in the context we are dealing with. As Laurence Chisholm Young (1905-2000) defines, "mathematical talent is a combination of ingenuity, insight, creativity, the willingness to experiment, and persistence; it is not merely a skill in manipulation. By working on problems, you can develop your talent for mathematical thinking and problem solving". However, we agree with Leikin (2010) that "mathematical potential is a complex function of mathematical ability, affective characteristics, characteristics of personality of an individual and learning opportunities the person meets in his life". As we will show in this paper, these four topics are really important in our didactical experience.

DIDACTIC EXPERIENCE

A rigorous study of the aforementioned lack of real mathematical research in Research Works would imply the analysis of a complex network of causes and experiments. In our modest opinion they can be simplified to the following two main factors: firstly, the lack of appropriate skills and motivation in the vast majority of students and secondly, an almost inexistent experience in mathematical research in high school teachers making them unable to properly orient the students.

With these two important factors in mind, three years ago a favourable environment let us test the experience we present in this communication. Three different profiles, which complemented one another, met: Ferran, a skilled 14 year-old student very interested in science and more specifically in the interaction between mathematics and theoretical computer science; Laura, a mathematics secondary school teacher who also researches in mathematics education; and finally Juanjo, a mathematical researcher in the area of discrete mathematics (more concretely, in analytic combinatorics and combinatorial number theory).

The aim when introducing this project to Ferran was to enhance the capacities of his talent, providing guidance or scaffolding which allowed him to do some interesting research in abstract mathematics in the framework of his Research Work. The tutors tried also to provide him with the necessary tools to find connections between mathematics and computer science, field where the tutors were confident he could develop his potential and interests.

The proper development of the project required to focus the attention into two essential aspects to have a satisfactory experience: the mathematical knowledge and the attitude and motivation of the student.

As for the *mathematical knowledge* of the student, a deep analysis of his background and the new required concepts to learn were compulsory, being the secondary school teacher extremely useful for the former analysis and the mathematical researcher for the latter. Not only does the student need to learn new concepts, but also new research methodologies and argumentative processes. As Bransford and his colleagues (1986) present in their work, the combination of both, specific *knowledge* that is organized in ways that are appropriate for individual needs and general *strategic and metacognitive knowledge* is important for the learning of mathematical thinking and problem solving.

Notwithstanding, personal aspects, such as the *motivation and attitude* of the student, deserve a special attention. Although a large amount of work is required and no face-to-face classes are organized, students need to find time to do it out of school. The tutors were aware of this inconvenience since the beginning of the experience. In most cases, the first activities to quit by stressed students are the non-compulsory ones. This is the reason why the tutors always tried to generate an intrinsic and autonomous motivation (Vansteenkiste, Sierens, Soenens, Luyckx & Lens, 2009) to promote an enactment of the activity for its own sake.

THE PROJECT AND CONCRETE ACTIONS

The proposed project was based on a natural question arising from enumerative combinatorics. Without entering in technical details, the main question is: how many words of length *n* are there (written only with zeroes and ones) so that they do not contain a fixed pattern? The study of this question is not trivial (despite the easiest cases could be treated with elementary arguments) and requires the learning of specific techniques (as generating functions and Markov chains), which are quite intuitive. In fact, all these new concepts complement smoothly the syllabus in mathematics of the high school.

Once the student was acquiring these new concepts (working progressively in more complex problems), he could manage to take benefit of his informatics knowledge in order to conjecture and propose by itself new lines of research. Additionally, he managed to apply these techniques to different fields, as biology (in order to detect efficiently certain sequences in long protein chains) or financial mathematics.

To promote this new mathematical knowledge, different actions were carried out:

Firstly, a system of sheets of problems was organized, facilitating gradually and slowly the learning. These sheets of problems were based on the new concepts needed and were very useful also to learn the new terminology. Periodically, the researcher gave him these sheets of problems, and the student had a certain time to get them solved. With this method he could get involved with the new methodologies and concepts. During this period, the meetings with the teacher were also useful to give some advices on how to get started in that new world compared with the topics he was used to. In fact, the teacher knows more how the student was used to work.

Secondly, regular meetings were organized with the researcher to build which would be the main aims of the research. Moreover, it was important to improve his learning process and set up an environment where the student could continue investigating in an autonomous way.

Thirdly, two research meetings were organized with the tutors, other researchers and colleagues. The main purpose was to exploit oral communication skills and to feel the importance of defending a research.

Finally, during the last period of the research, there were some meetings with the researcher and the teacher to work on the research document and to improve on methodological aspects of the research.

In order to guide the student in a more personal and motivational field, we can point out different actions or attitudes:

- The student starts working on the Research Work at the last year of secondary school (15 years). This point was a challenge for him. The tutors were certain that he would feel proud of himself for starting a huge project (despite not having the experience of working on a long-term project). The student understood that the sooner he started, the better he could finish the research.
- The way the Research Project has been proposed to the student, in a wide open way, but at the same time very close to real research.
- The student receives an extra amount of motivation due to the double mentorship, mainly in the point that he has the possibility to work with a researcher in order to learn in which way real research is done.
- Another important point was the inexistence of abandonment from the part of the tutors: there was always periodical communication and absolute availability by electronic mails or videoconferences.
- The meetings, which included a research meeting, were usually held at the Mathematics Faculty of Universitat Politècnica de Catalunya. The research meeting provided a real professional environment to the experience.

After showing our methodology, we would like to comment on the operability of this experience. This project was finished in February 2011 and won several local prizes (in Catalonia), joint with the highest distinction in the school. In all cases this work was classed as an own research work. This point proves that the content of this work has been positive, both from the point of view of the tutors and the external referees.

FINAL DISCUSSION

The experience has been absolutely satisfactory, both from a scientific and a didactical point of view. We can affirm that we are proud of the development of the experience, and we would also like to include the opinion of the student, who is, at the last level, the main character in this framework:

> "Working with a tutor and a researcher is both a very interesting and inspiring experience. Being with a researcher gives you much more insight about what you are learning, working on or discovering. It gives you the sense that what you do is not a simple school exercise but real research which might sound much more exciting for the student. Even more, taking it as a research the student senses that his work must step up to a higher level and meet what is expected from him, not necessarily as a consequence of more effort but above all a higher motivation. In a work which lasts as long as 2 or 3 years, much more than a high school student is used to, and in a field that might not have noticeable results after lots

of hours of work, motivation is crucial. That's where the inspiration given by the researcher and the motivation given by the tutor prove their importance.

For instance, learning such a complicated field as generating functions can be very tough for a high school student. However, the insight that only an expert in the field has is somehow transmitted to the student; who might otherwise abandoned the research for being too complicated. That is what has probably happened to many bright students who have not had this chance. In this particular case more than two months were required to understand the basics of the theme and it was not until more than a year later that we were able to understand the core of the research.

The first part of the project was to 'discover' the field of work by reading definitions and the formal mathematical language, proving small lemmas and solving simple problems, and later more complicated ones. At this point, the possibility to get questions answered by email was very important for a novice since having direct feedback on your thoughts, doubts, hypothesis and results is crucial for a better and faster understanding.

In such a long period of time the role of the tutor is also crucial: having to concentrate on schoolwork and everyday life as well, the student usually forgets to work regularly on the paper. At this point the tutor motivates the student to keep up his work with periodical recalls and activities that can help him. In that direction, we would like to point out some of those that worked very well.

Usually, one of the most undervalued activities from the point of the student is the reunions with the tutor. However, in our case those reunions between the three of us were vital to do the paper. In a curious environment with three people instead of two, an excellent communication and tune between the 3 is mandatory. Furthermore, those reunions were excellent to plan the future work and review the previous one. All in all, having this feedback was very important for the student.

Another example of those events planned by the tutor was a talk done some weeks before presenting the research. The student had to make a speech for about 20-30 minutes to the researcher, the tutor and two other scientists without any relation to the paper. This talk was very useful to the student for different reasons. First of all it was both a good motivation and obligation to have a great deal of job done before making the presentation. Moreover, having to explain the content of the research to someone who has no idea about it obliges the speaker to better know his topic, since to transmit an idea in simple terms one has to understand it very well. Finally both the tutors and the other two scientists can propose good ideas and questions that the student hasn't come up with before, enriching even more his work."

As we can release from his comment and also from the objective awards the work has received, the experience was successful for the student. However, if we focus our attention on the tutors, we could see the experience as a creation of a concrete learning community in which the teacher and the researcher are seen as co-learners. As Wagner (1997) sees the co-learning agreement,

"...The researcher and the teacher are both participants in processes of education. Both are engaged in action and reflection. By working together, each might learn something about

> the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schoolings...” (p.16)

In this sense, the tutors have experienced co-learning. It was not the aim of the didactical experience, but it was an important situation that emerged from the experimentation.

After explaining this experience, we encourage teachers to work together with mathematics researchers in the context of enhancing research to gifted students. We know that in our situation, where the Research Work is a compulsory practice in our classrooms, there is much collaboration between universities and high schools. Usually, the universities provide mathematics researchers in order that they could tutorage the students, but after this experience, we propose a change on this direction. A good collaboration between researchers and teachers is also crucial.

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